# Irreversible work, *large deviations*, critical Casimir effect & universality in quantum quenches


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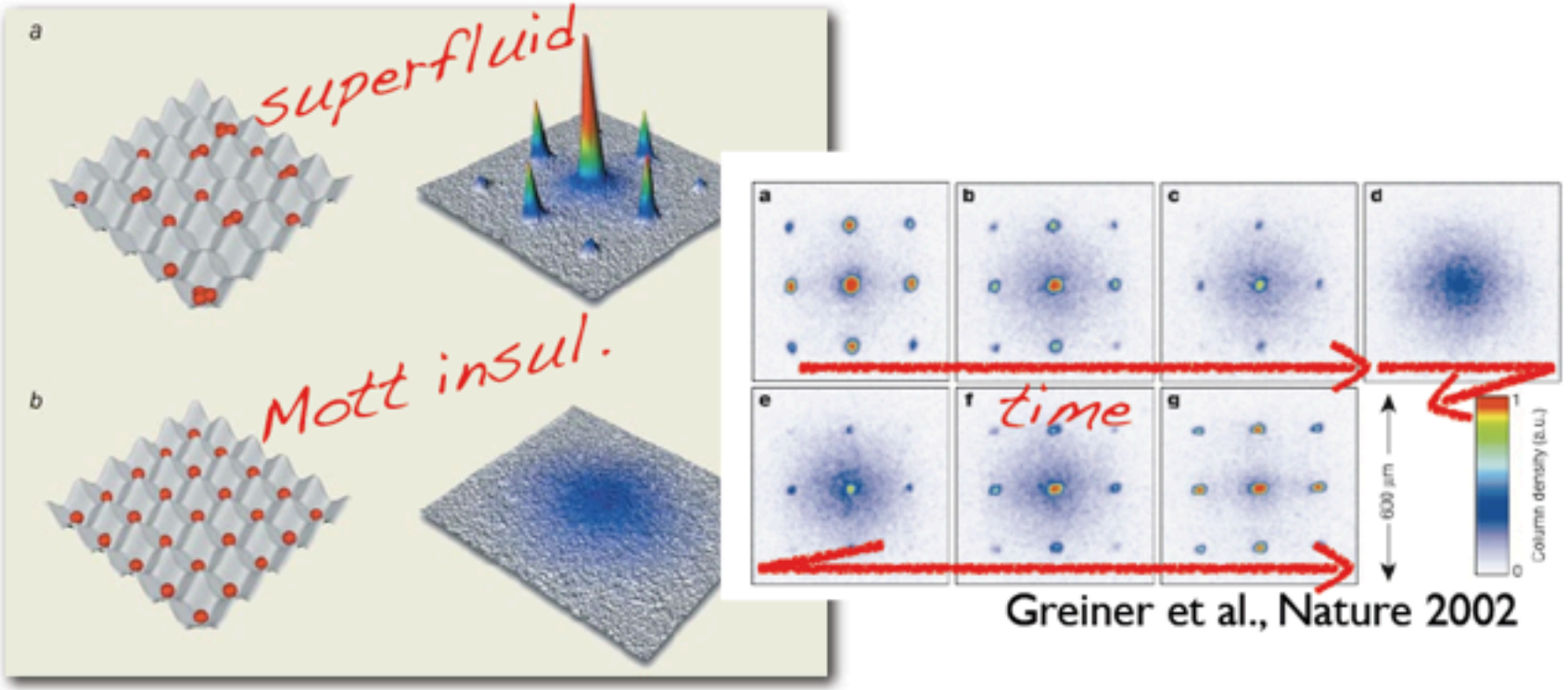
# (many-body) Quantum isolated systems

Revived interest → ultra cold atoms

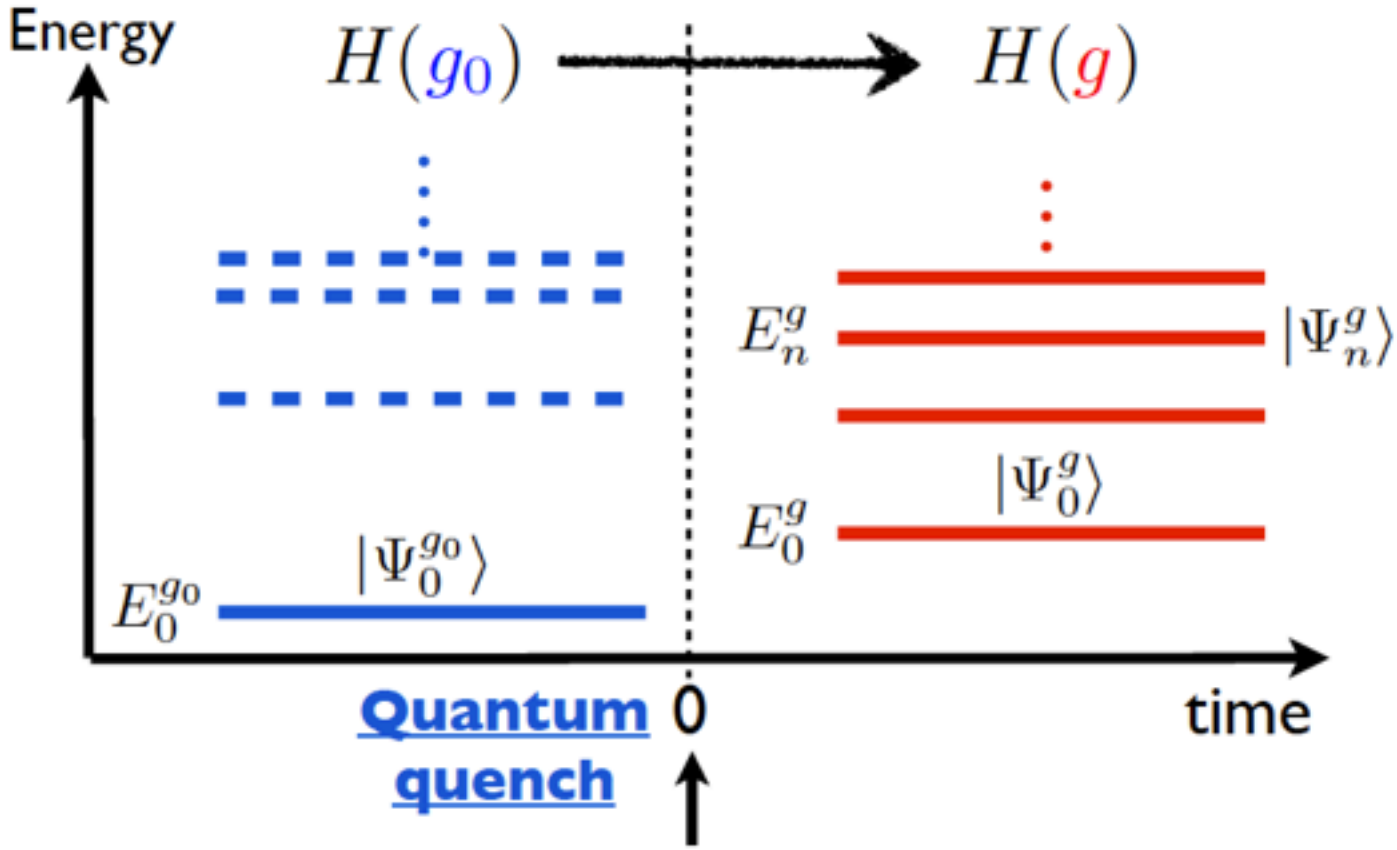
- Control of H
- Coherent (*unitary*) dynamics




Greiner et al., Nature 2002

# Quantum quench $H(g)$ (global)

$$|\psi(t)\rangle = \mathrm{e}^{-iH(g)t}|\Psi_0^{g_0}\rangle$$



# Issues...

- Dynamical transitions

  Biroli, Sciolla - Schirò, Fabrizio - Gambassi, Calabrese -...
  Heyl,Polkovnikov,Kehrein.

  stationary state?

- Long-lived non-equilibrium states

  Kinoshita, Wenger, Weiss - Kollath,Läuchli, Altman -...

  dimensionality?
  cons. laws?

- Relaxation of inhomogeneity, aging

  Shütz, Trimper - Iglói, Rieger - Carleo, Becca, Schirò, Fabrizio - ...

  integrability?

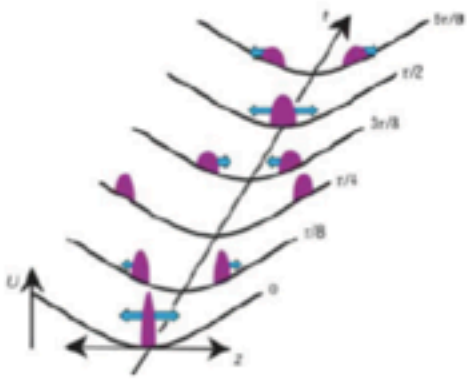
- Thermalization

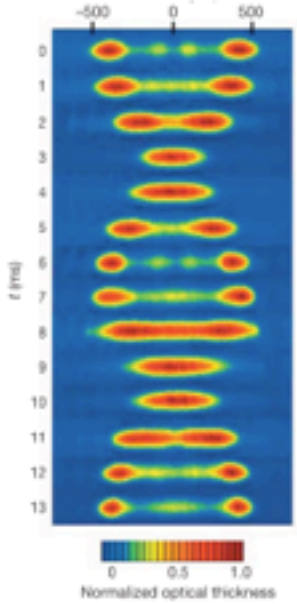
  Srednicki - Biroli,Kollath,Läuchli - Rigol,Dunjko,Olshanii - Berges -
  Calabrese,Cardy - Rossini,Silva,Mussardo,Santoro - Calabrese,Essler,Fagotti -...
  Mitra,Giamarchi -...
  Foini,Cugliandolo, Gambassi -...

  effective temp.s?

# ...look at...

- Dynamics, correlations, responses, thermaliz...






Igloi,Rieger '01 - Altman,Auebarch '02
Sengupta,Powell,Sachdev '04 - Polkovnikov '05
Zurek,Dorner,Zoller '05
Calabrese,Cardy '06 - Gritsev,Polkovnikov '07
Rossini, Silva, Mussardo, Santoro '08-'09
Fagotti, Essler, Calabrese '11-'13
Lesanovsky, Garrahan et al. '11-'13
Foini,Cugliandolo,Gambassi '11-'12

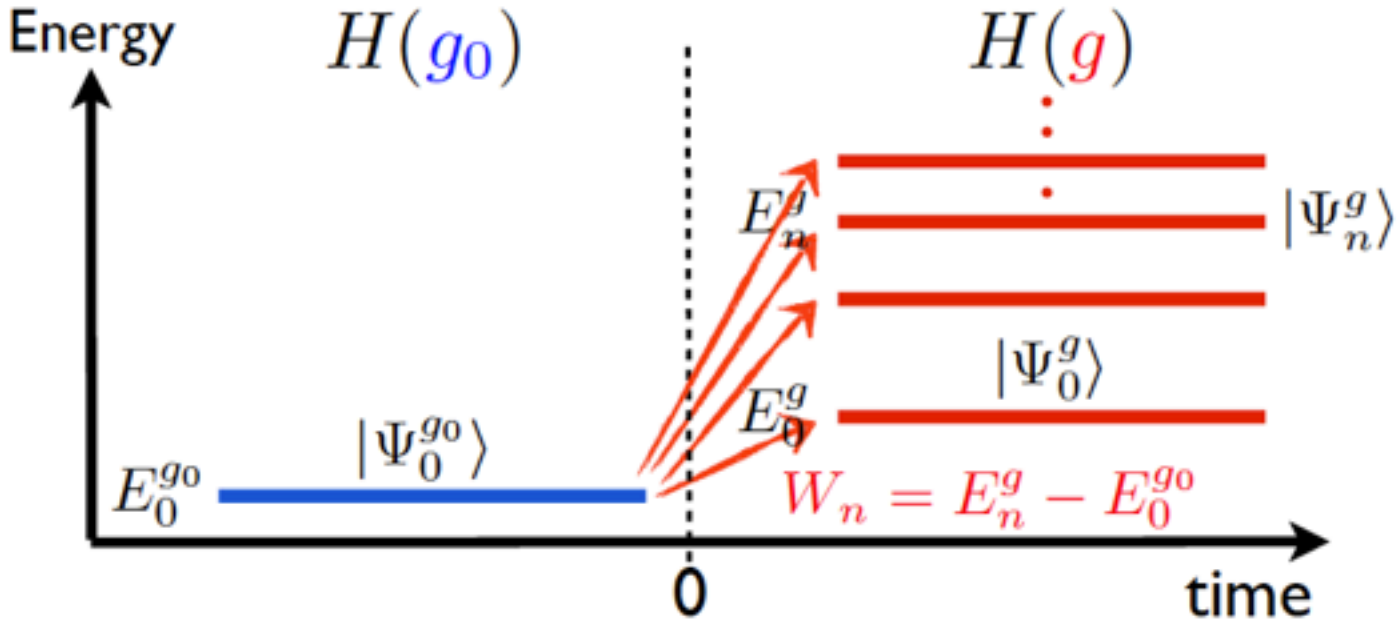
**REV: [Polkovnikov et al. RMP '11]**

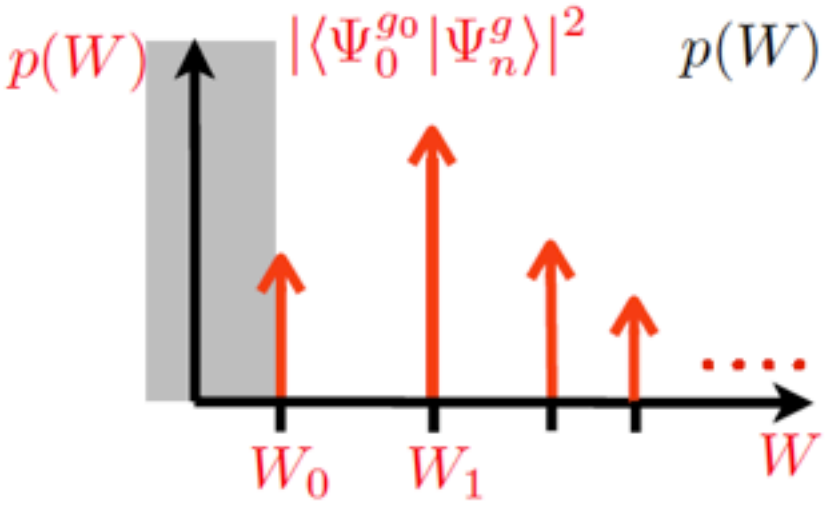
- Statistics of excitations & observ.s

Polkovnikov '08 - Silva '08 - Barankov, Polkovnikov '09
Kehrein '09-'10 - Kitagawa '11 - Gring et al. '11
Canovi et al. '11

here: **WORK** $W$ $\Delta E$

**Universality?**




$$p(W) = \sum_{n\geq 0} |\langle\Psi_0^{g_0}|\Psi_n^g\rangle|^2 \delta(W - W_n)$$

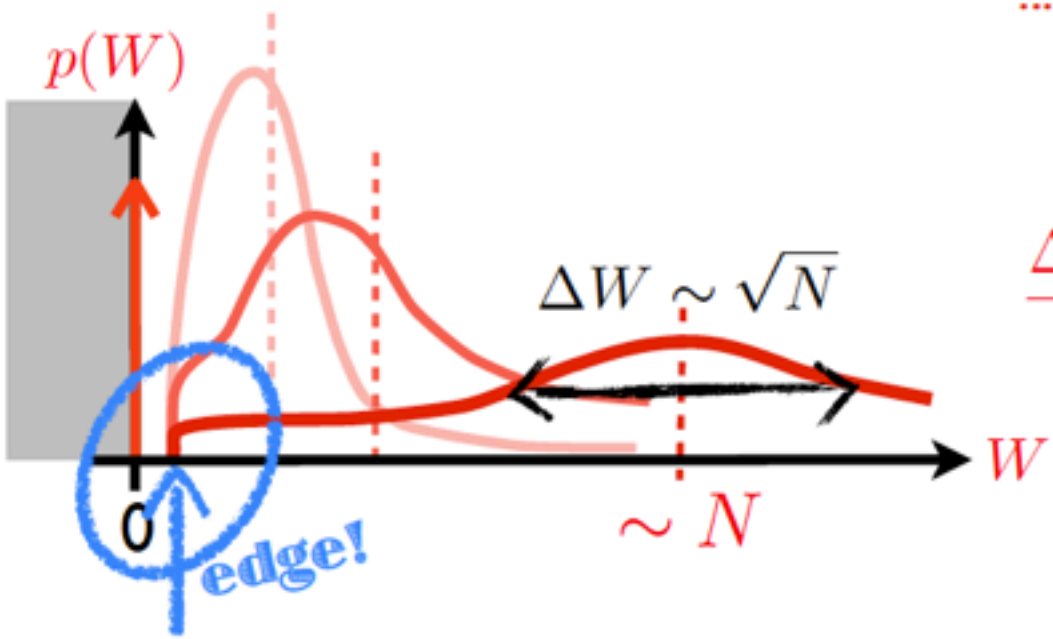
$\langle W\rangle \propto N$ extensive!

$W_0 = E_0^g - E_0^{g_0}$
reversible (min)

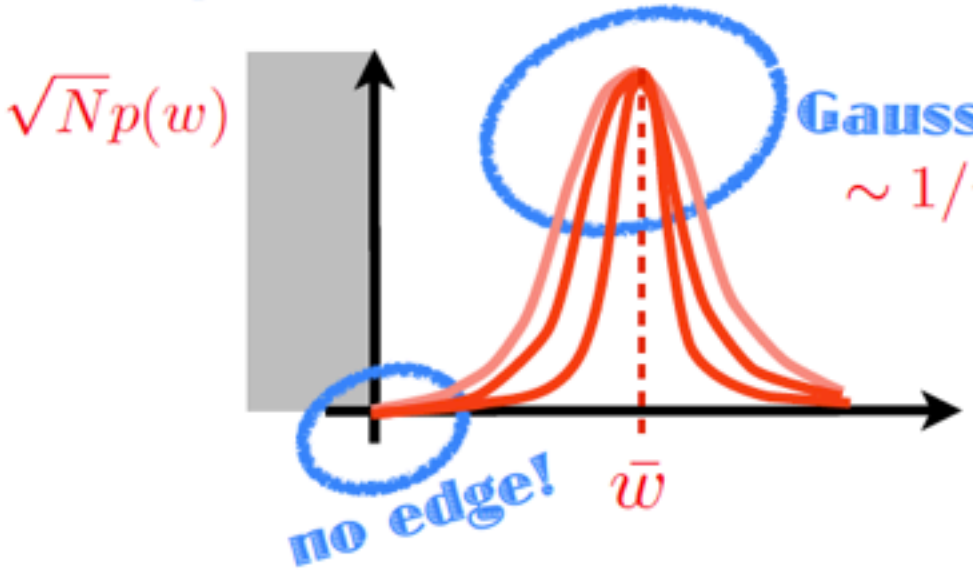
$W - W_0$ : "irreversible"

$W \to W - W_0$

.....*generic* features




$$\frac{\Delta W}{W} \sim \frac{1}{\sqrt{N}}$$




**Gaussian >>triv. univ!**

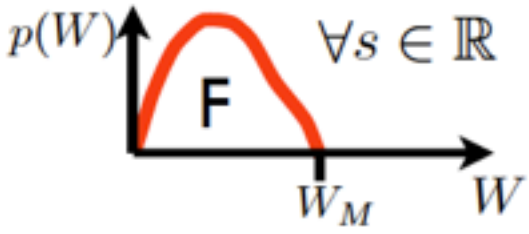
$\sim 1/\sqrt{N}$ **>>large dev.!**

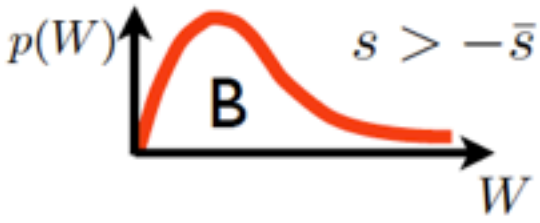
$w \equiv W/N$

intensive!

$W > 0$

$$G(s) \equiv \langle e^{-sW} \rangle \quad \text{mgf}$$




$$= \langle \Psi_0^{g_0} | e^{-s[H(g)-E_0^g]} | \Psi_0^{g_0} \rangle$$

$$\langle \Psi_0^{g_0} | e^{-sH(g)} | \Psi_0^{g_0} \rangle \quad \text{quantum } d$$

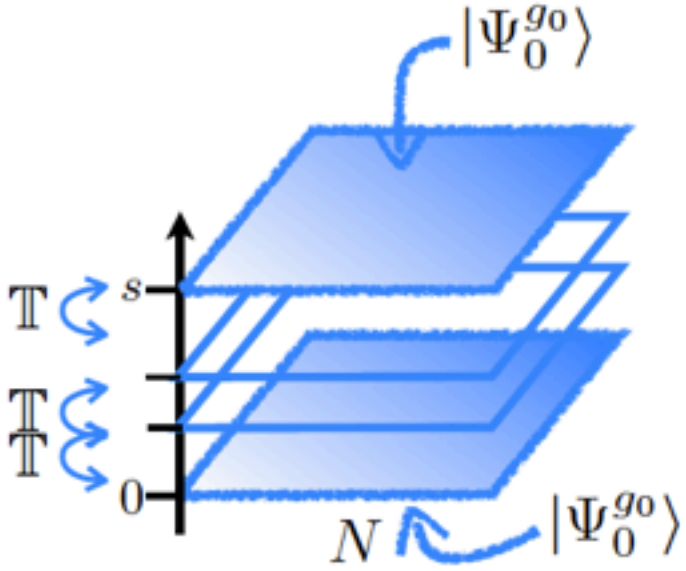
$$\mathbb{T} \equiv e^{-H(g)}$$

$$\langle \Psi_0^{g_0} | \underbrace{\mathbb{T} \times \mathbb{T} \times \ldots \times \mathbb{T}} | \Psi_0^{g_0} \rangle$$

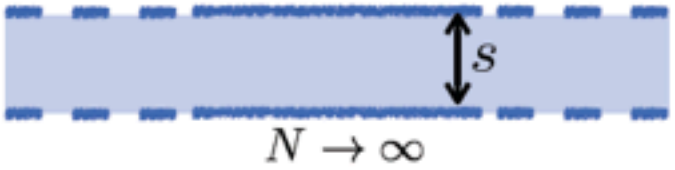
classical $d+1$
film $N \times s$

$$\langle \Psi_0^{g_0} | \mathbb{T}^s | \Psi_0^{g_0} \rangle = Z_{N \times s} \equiv e^{-\mathcal{F}}$$

confined systems: Barber, Fisher, Cardy, .. >'83




[Silva'08 - Gambassi, Silva '11]

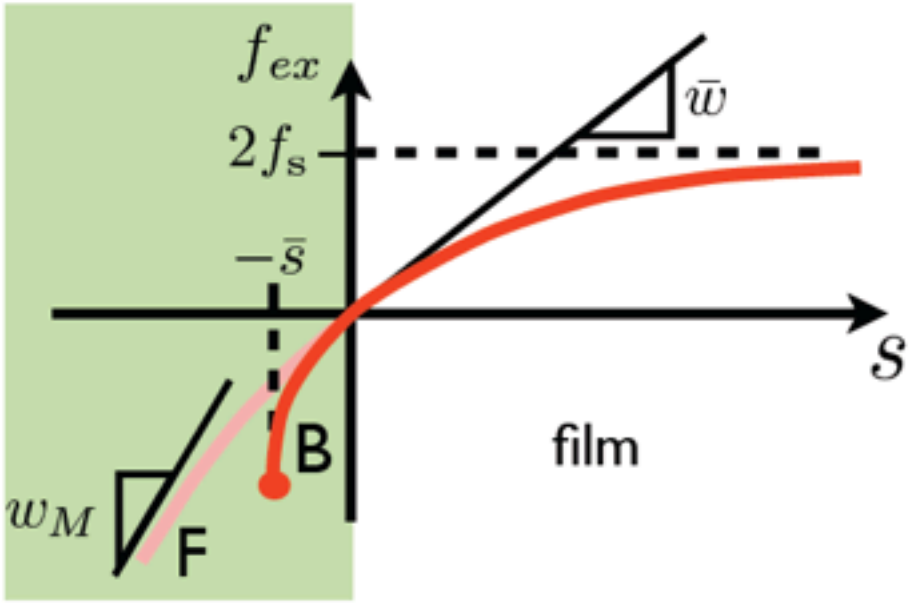



$$\mathcal{F} \overset{s\to\infty}{=} \underbrace{\text{volume} \times f_{\mathrm{b}}}_{s\,N} + \underbrace{\underbrace{\text{surface} \times f_{\mathrm{s}}}_{2\,N} + \underbrace{\text{finite-size corr.}\ (s)}_{\propto N}}_{N\,f_{ex}(s)}$$

$$f_{ex}(s) \overset{s\to\infty}{\to} 2f_{\mathrm{s}}$$

$$G(s) = \mathrm{e}^{-N f_{ex}(s)}$$

- $f_{ex}$ concave
- $f_{ex}(0) = 0$



# Universality & Casimir effect

Gambassi, Silva, arXiv:1106.2671 (2011)
Gambassi, Silva, PRL109, 250602 (2012)
Sotiriadis, Gambassi, Silva, PRE 87, 052129 (2013)

Lapl. trans.

$$p(W) \leftrightarrow G(s) \Longleftrightarrow f_{ex}(s)$$

$$\left.\begin{array}{l} H(g \to g_c) \\ \text{gap} \underset{\sim |g-g_c|^{\nu z}}{\to} 0 \end{array}\right\} \quad \xi \underset{\sim |g-g_c|^{-\nu}}{\to} \infty$$

bulk **critical** point

finite-size corr.

$$s \gg a \qquad \xi \gg a$$

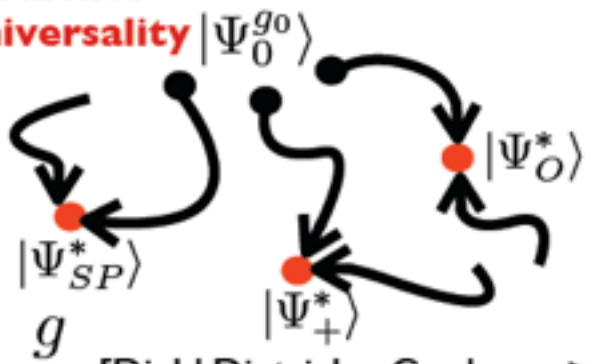
$$f_{ex}(s) = 2f_s + s^{-d}\Theta(s/\xi) + \dots$$

Universal!

$d+1$ bulk UC

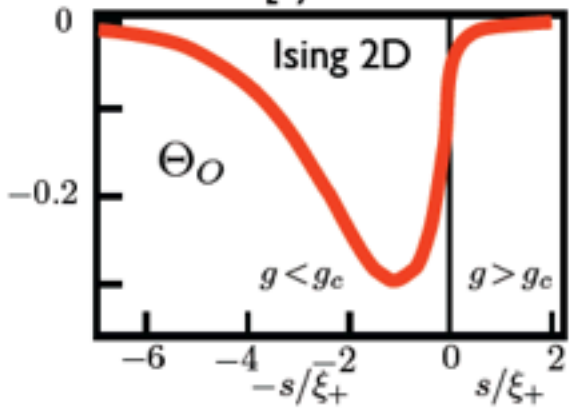
**critical Casimir eff.**

surface universality



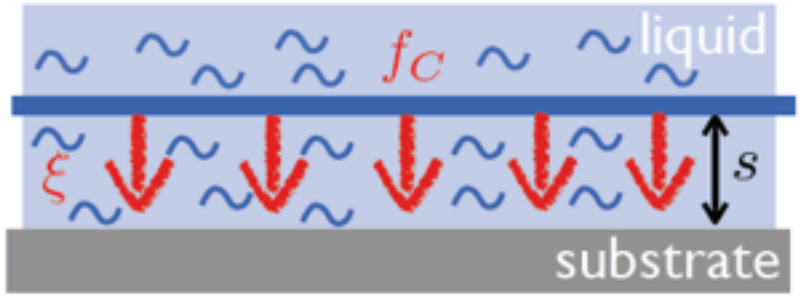

[Diehl,Dietrich - Cardy - ....>'83]

[Symanzik'83 - Fisher,deGennes'78]
[Evans,Stecki'94]




Ising 3D:
[Vasilyev,etal'08]

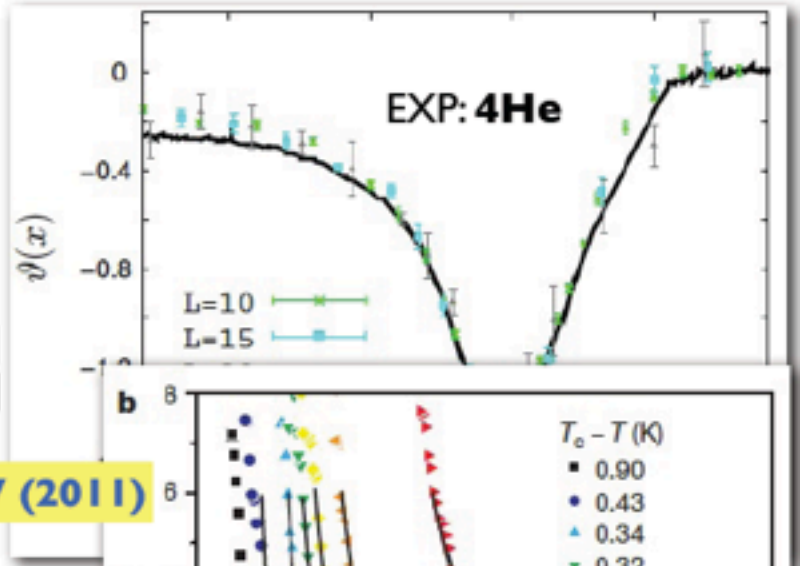
# Critical Casimir effect



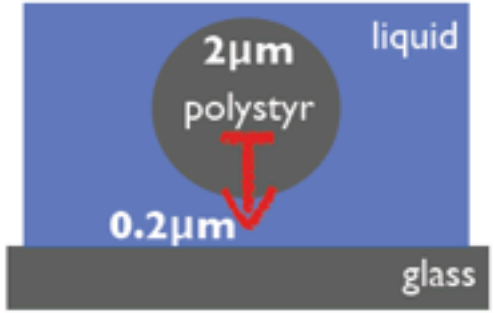

$$f_C(s) = -\frac{\partial}{\partial s} f_{ex}(s)$$


wetting films...

[exp:Garcia,Chan '99..]
[MC:Vasilyev etal. '07..]

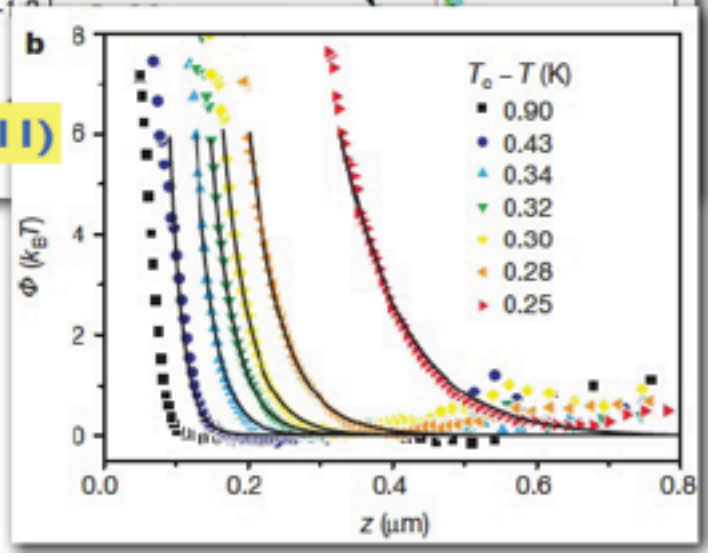



**Rev: Gambassi, Dietrich, Soft Matter 7,1247 (2011)**



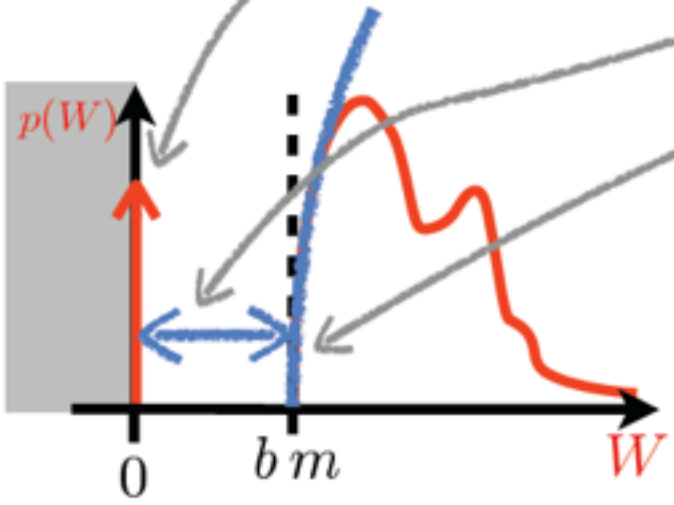

colloids...

[exp: Hertlein et al. '08..]



$$\Theta(x \gg 1) = \mathcal{C}\, x^a\, \mathrm{e}^{-b\,x} \qquad m = m(g) = \xi^{-1}$$

$$p(W) = \mathrm{e}^{-N 2 f_s}\left[\delta(W) + C\,\theta(W - b\,m)(W - b\,m)^{d-a-1} + \ldots\right]$$



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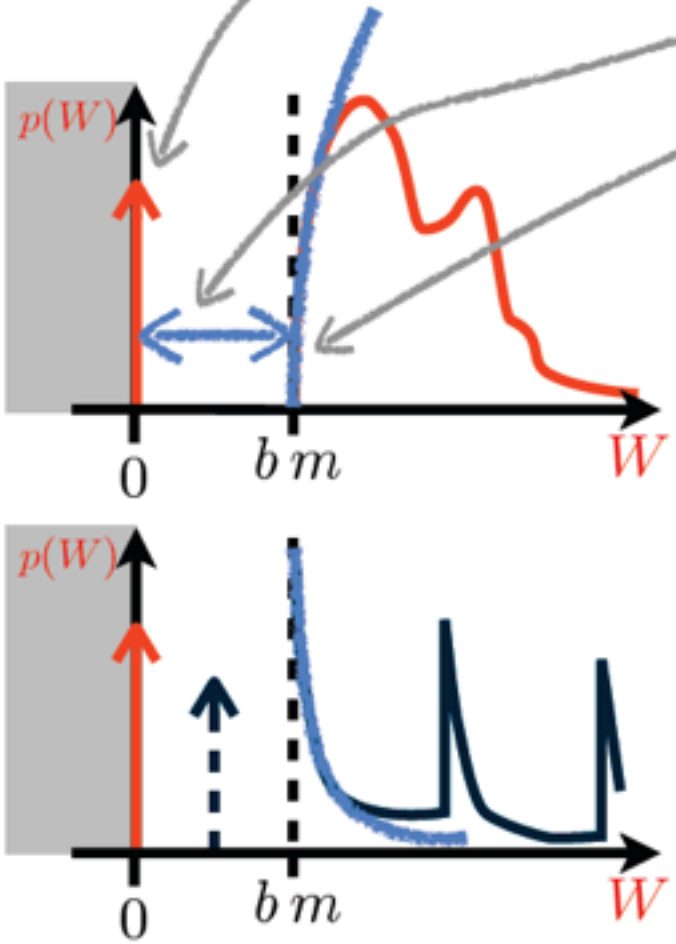
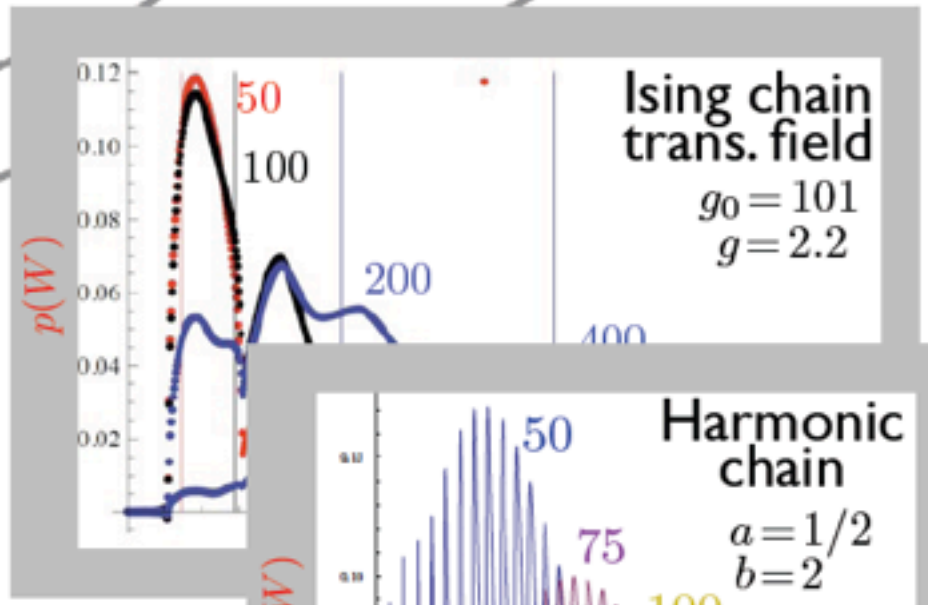
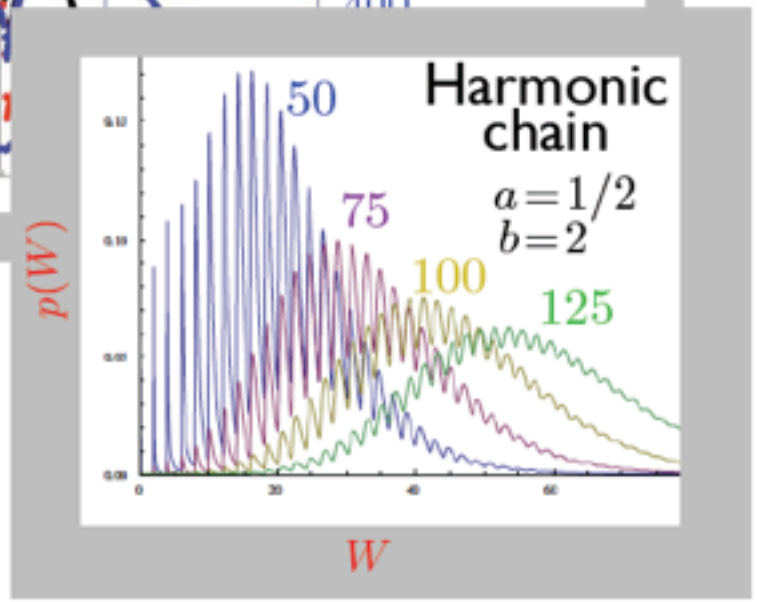
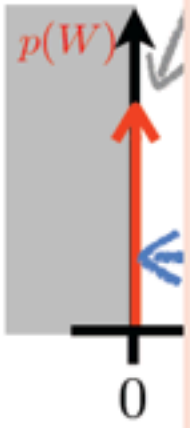


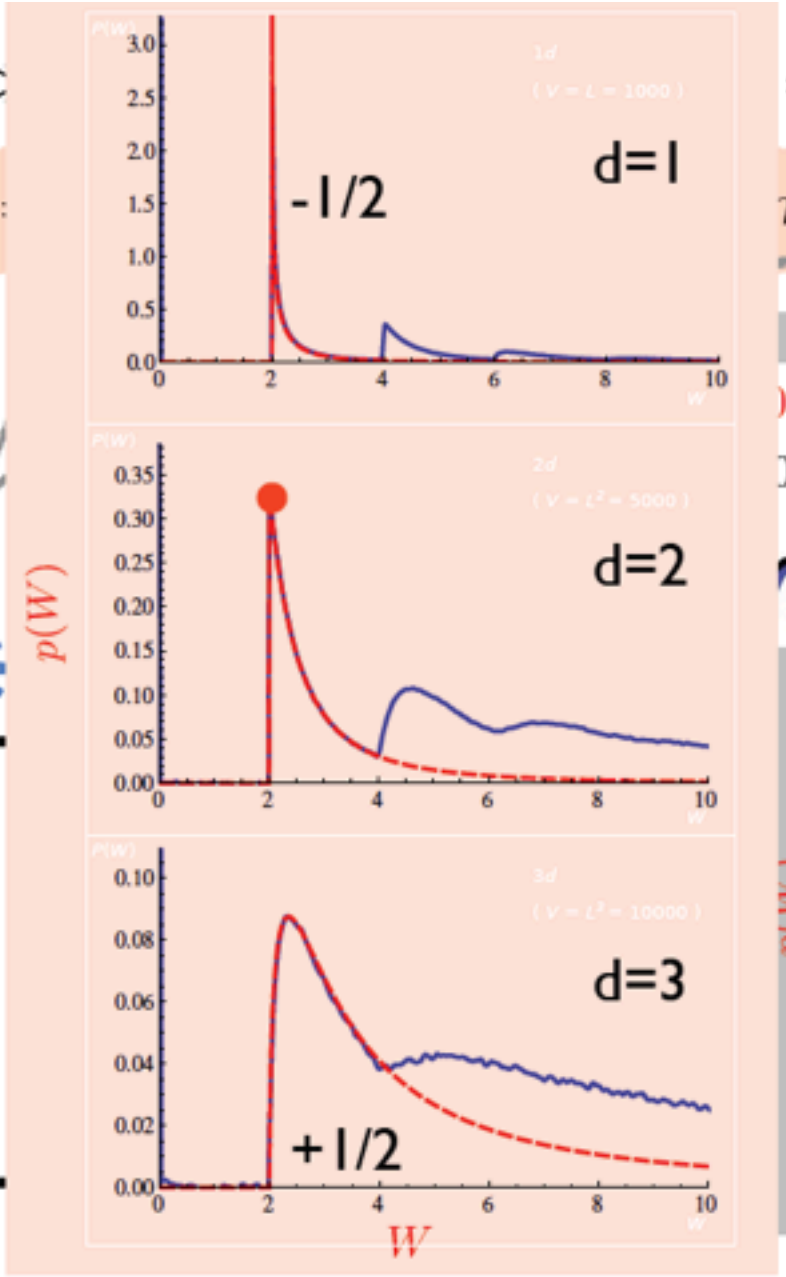



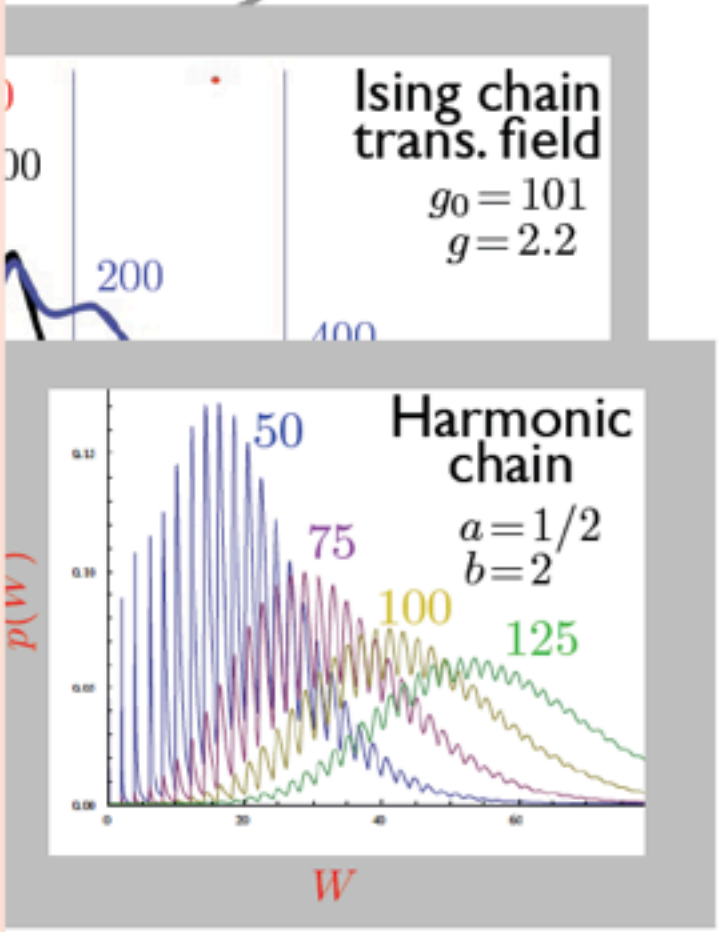
$\Theta(x>$ $= m(g) = \xi^{-1}$

$p(W) :$ $n)(W - b\,m)^{d-a-1} + \ldots]$







# Large deviations [Touchette,Phys.Rep.'09]

**Gambassi, Silva, PRL 109, 250602 (2012)**

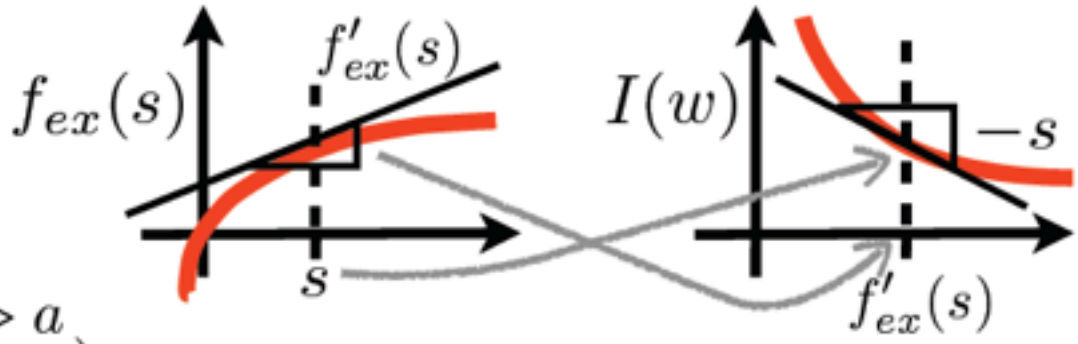
$$G(s) \equiv \langle \mathrm{e}^{-sW} \rangle = \mathrm{e}^{-N f_{ex}(s)} \qquad (W \to Nw)$$

$$p(w) \propto \int \mathrm{d}s\, \mathrm{e}^{sNw} G(s) \propto \int \mathrm{d}s\, \mathrm{e}^{N[sw - f_{ex}(s)]}$$

$$\Rightarrow \quad p(w) \propto \mathrm{e}^{-N I(w)}$$

Legendre, Fenchel:

$$I(w) = -\inf_{s \in \mathbb{R}} \{ s\, w - f_{ex}(s) \}$$

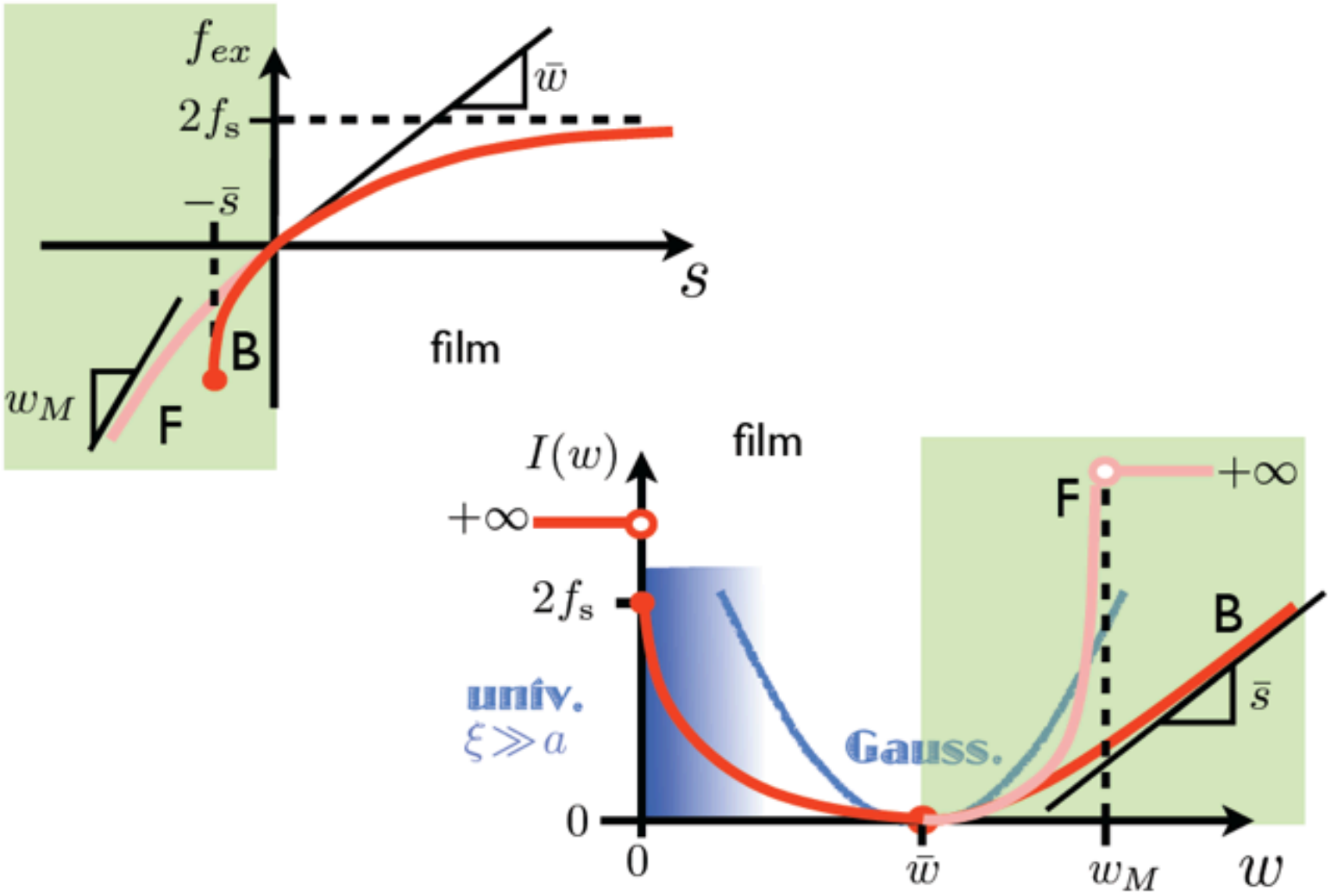



$$I(w<0) = +\infty$$

$$I(w \to 0^+) \Leftarrow f_{ex}(s \to \infty) \qquad (s \gg a)$$

**universal@CP !**

$$\xi \gg a$$

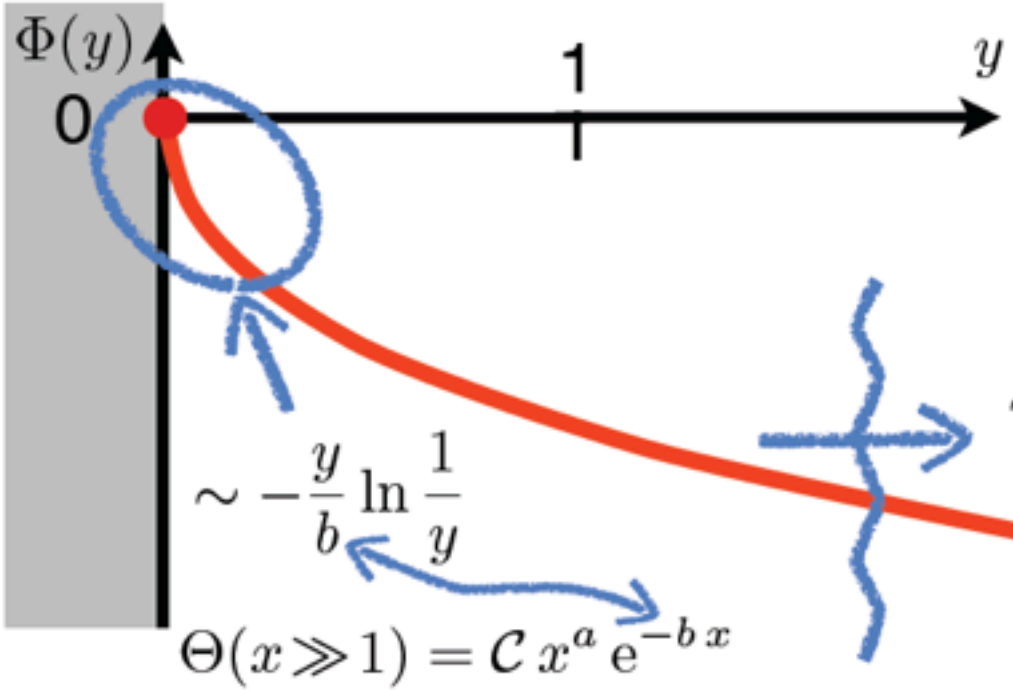

$f_{ex}$
$2f_s$
$-\bar{s}$
$\bar{w}$
$s$
B
F
$w_M$
film
$I(w)$
$+\infty$
$2f_s$
0
univ.
$\xi \gg a$
film
Gauss.
F
$+\infty$
B
$\bar{s}$
0
$\bar{w}$
$w_M$
$w$

$s \gg a$
$\xi \gg a$

$$f_{ex}(s) = 2f_s + s^{-d}\Theta(s/\xi) + \dots$$

univ. ⇓ L-F $\quad \Phi(y) = -\inf_{x \geq 0} \{ xy - x^{-d}\Theta(x) \}$

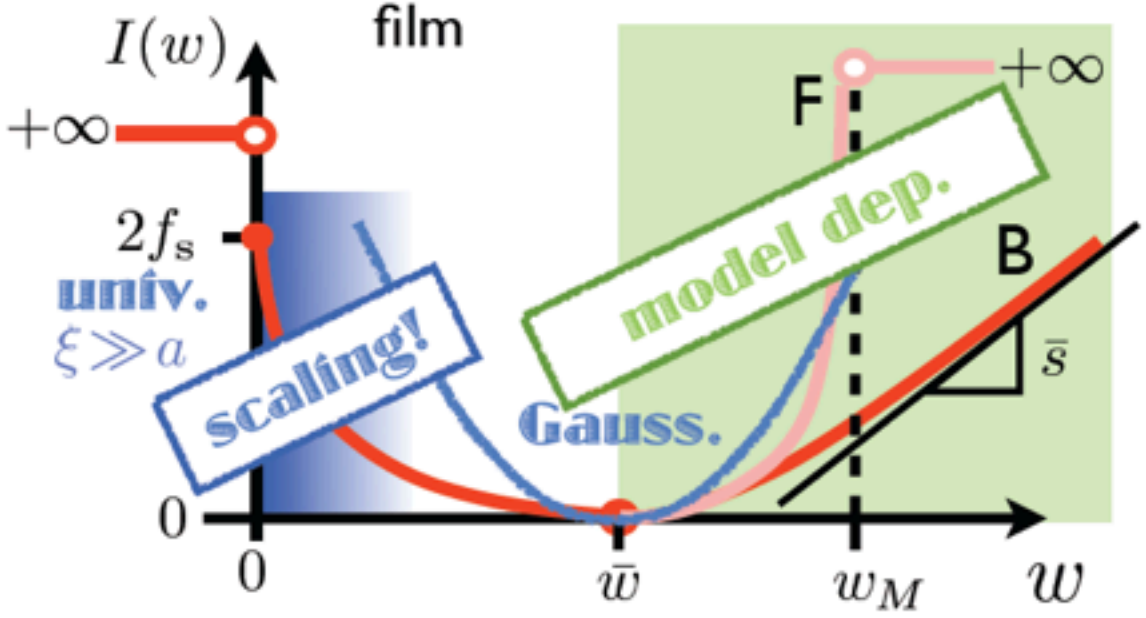
$$I(w) = 2f_s + \xi^{-d}\Phi(w\xi^{d+1}) + \dots$$




$\Delta = d|\Theta(0)|$

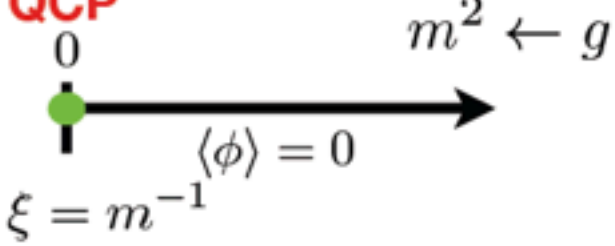
**critical Casimir amplitude**

@CP: $I(w \to 0) = 2f_s - \frac{d+1}{d}\Delta\left(\frac{w}{\Delta}\right)^{d/(d+1)} + \dots$


film
$I(w)$
$+\infty$
$2f_s$
univ.
$\xi \gg a$
scaling!
0
0
Gauss.
F
$+\infty$
model dep.
B
$\bar{s}$
$\bar{w}$
$w_M$
$w$

Free bosonic field: (phonons...) $d$-dim. latt. $a$ **QCP**

$$H(m) = \frac{1}{2}\int \mathrm{d}^d x \left[\pi_\phi^2 + (\partial_x \phi)^2 + m^2 \phi^2\right]$$




QP: $\omega_k(m)$ + quench: $\lambda_k(m, m_0)$ $\quad |\cdot| \leq 1$ [Gambassi,Sotiriadis,Silva'13]

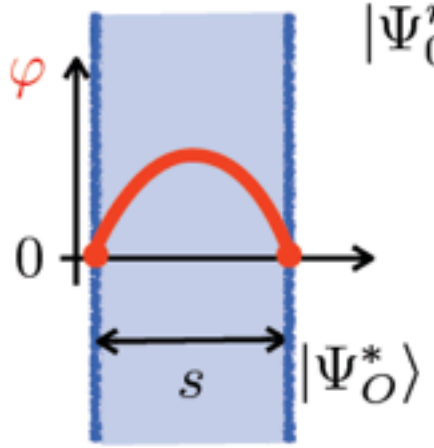
$$f_{ex}(s) = \frac{1}{2}\int \frac{\mathrm{d}^d k}{(2\pi)^d} \ln\left[\frac{1 - \lambda_k^2\, \mathrm{e}^{-2\omega_k s}}{1 - \lambda_k^2}\right]$$

→ class B
d+1-dim Gauss field $\varphi$

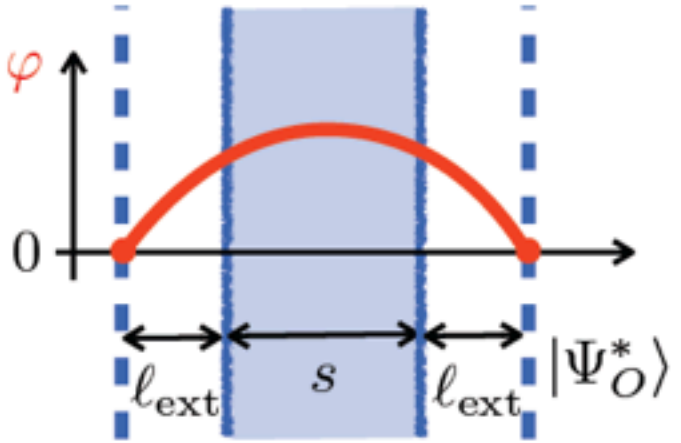
$m_0 a, \quad m \to 0 \quad (\xi \gg a)$

$|\Psi_0^{m_0}\rangle \rightarrow |\Psi_O^*\rangle$

$\varphi|_{\mathcal{B}} = 0$




$\ell_{\mathrm{ext}} = m_0^{-1}$ extrapol. length [Binder'83]

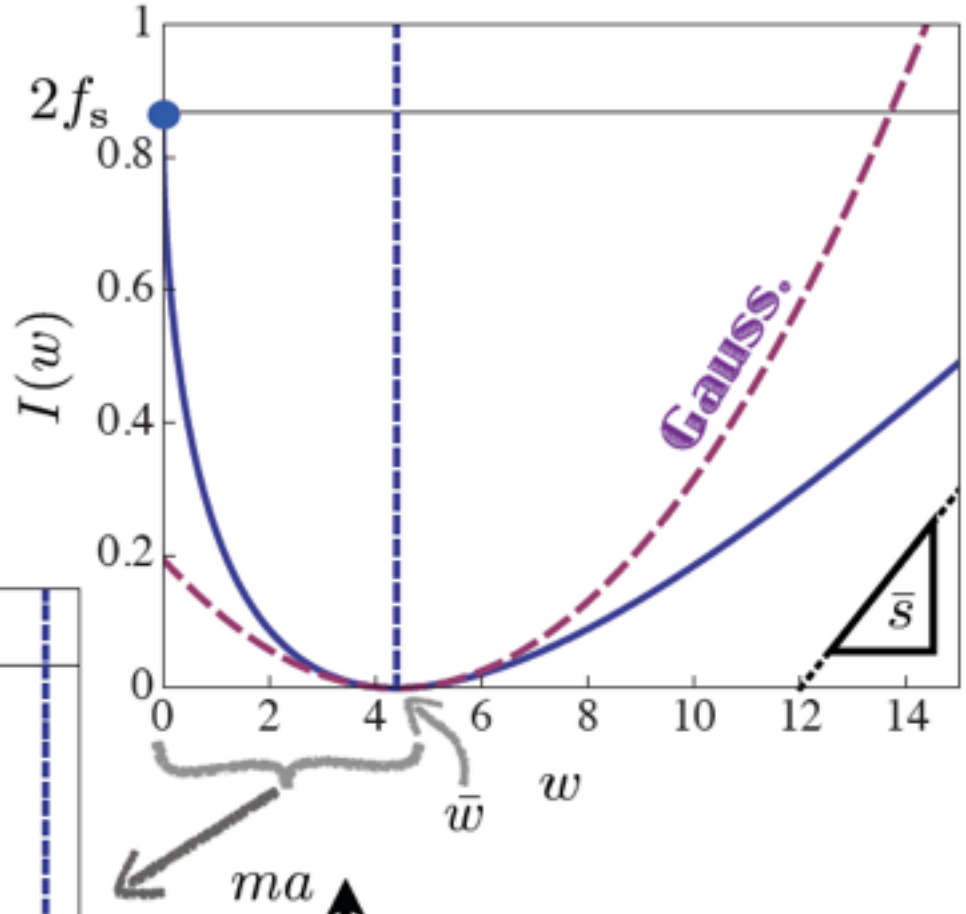


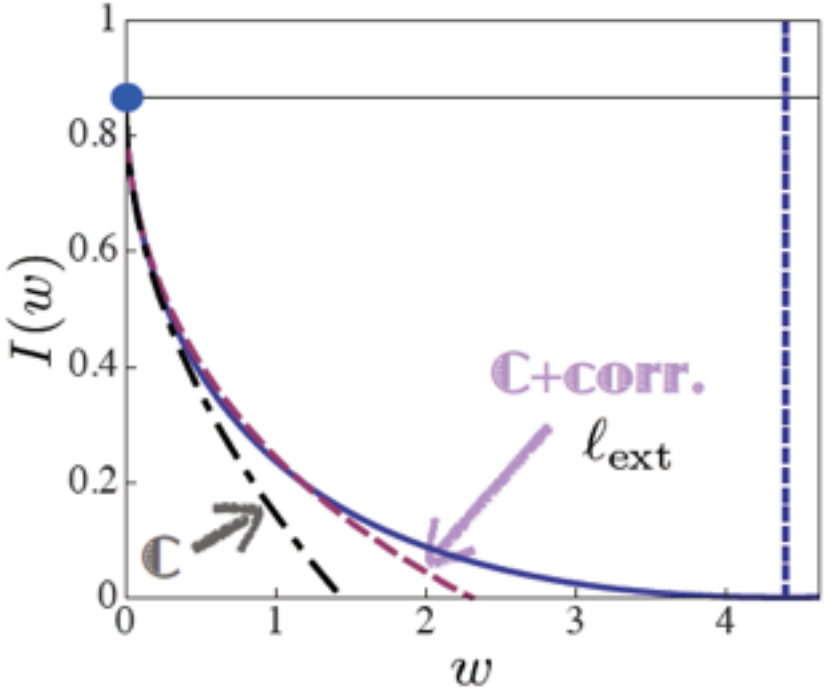
[Gambassi & Silva'12]

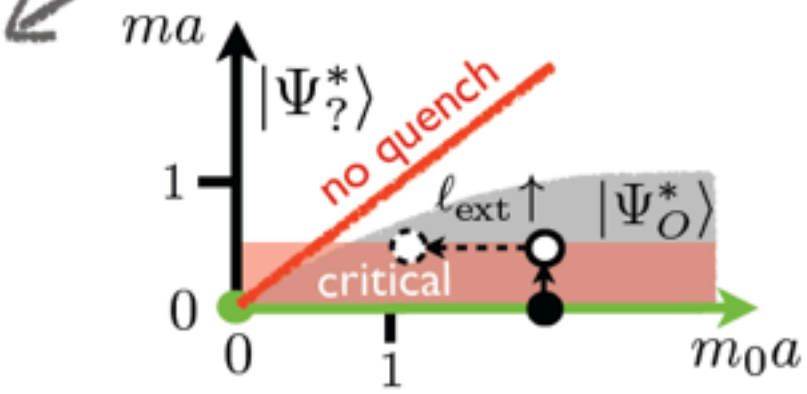
$d=1$

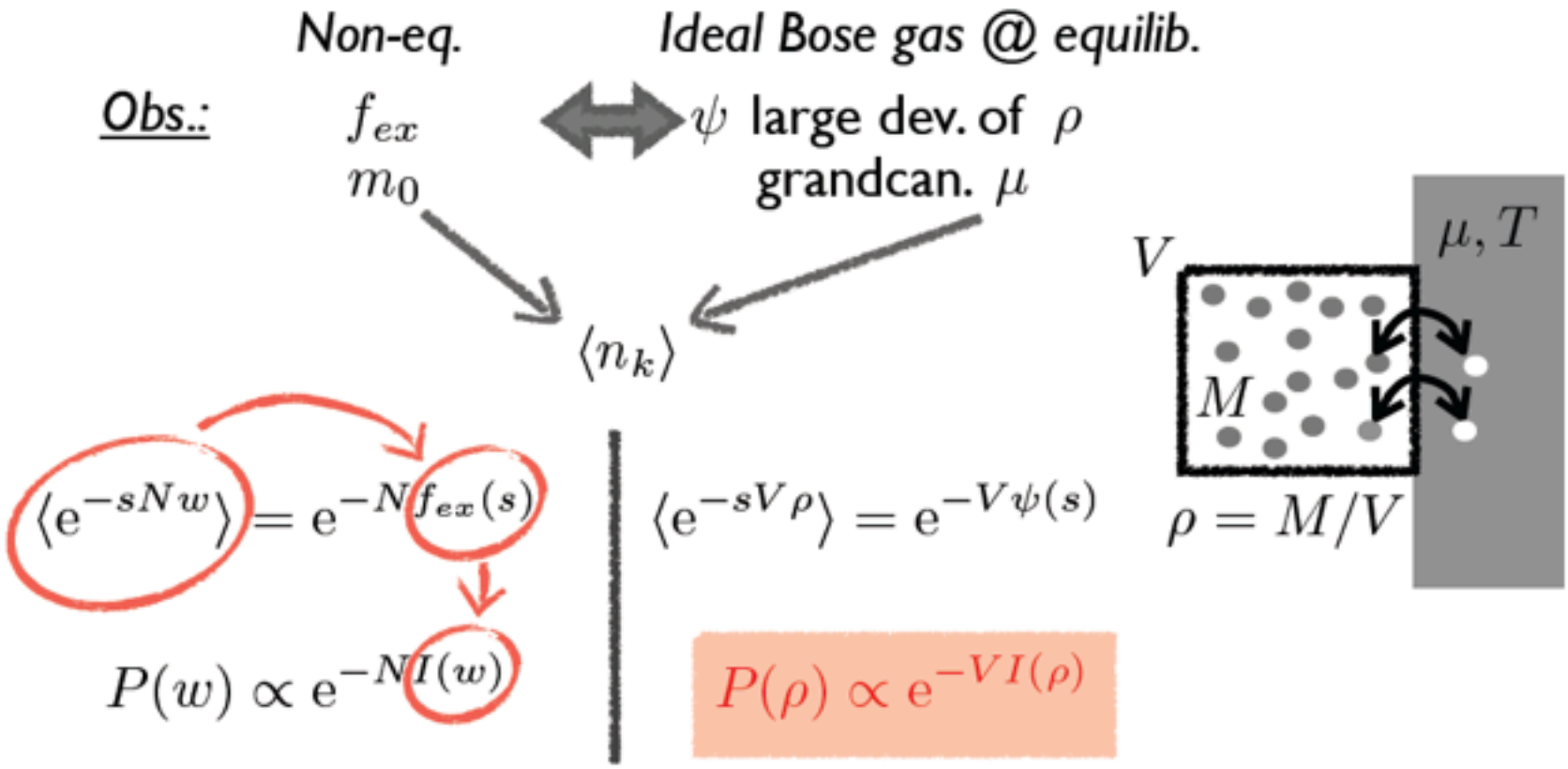
$m_0 a=20$

$m=0,\ \xi=\infty$




Non-eq.
Ideal Bose gas @ equilib.
Obs.:
$f_{ex}$
$m_0$
$\psi$ large dev. of $\rho$
grandcan. $\mu$
$\langle n_k \rangle$
$\mu, T$
$V$
$M$
$\rho = M/V$
$\langle \mathrm{e}^{-sNw} \rangle = \mathrm{e}^{-N f_{ex}(s)}$
$P(w) \propto \mathrm{e}^{-N I(w)}$
$\langle \mathrm{e}^{-sV\rho} \rangle = \mathrm{e}^{-V\psi(s)}$
$P(\rho) \propto \mathrm{e}^{-V I(\rho)}$

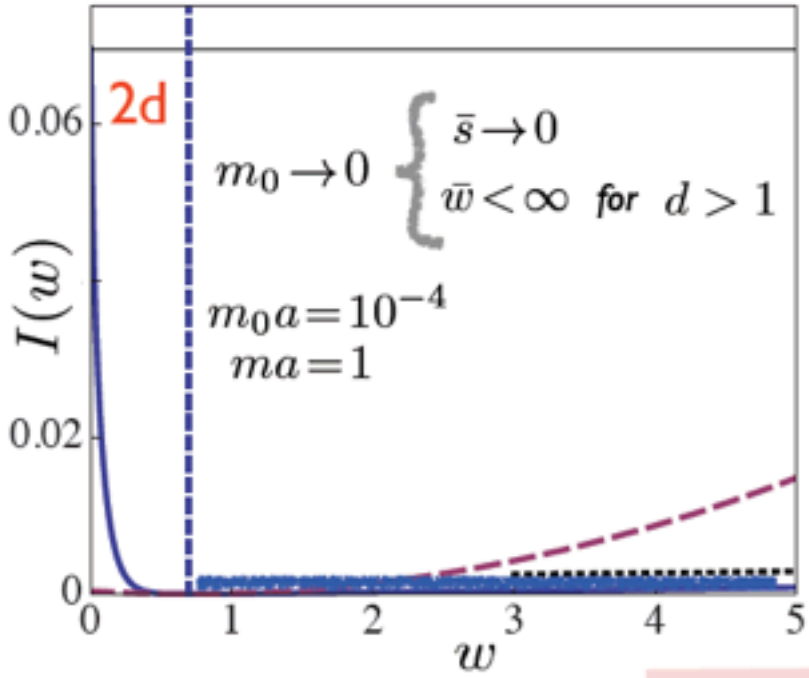
Non-eq. Ideal Bose gas @ equilib.

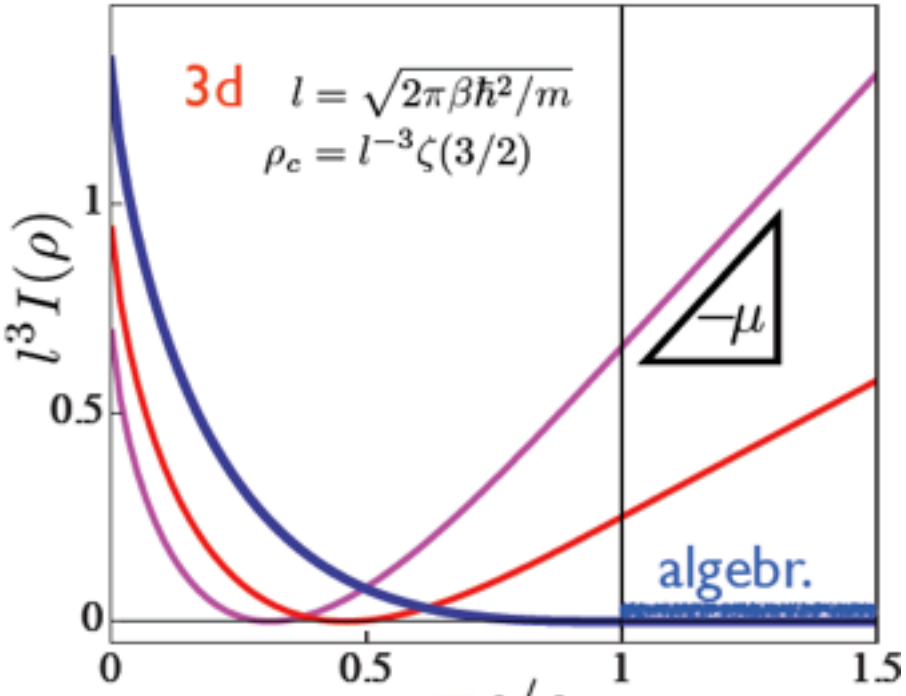
Obs.: $f_{ex}$ $m_0$ ??? $\leftrightarrow$ $\psi$ large dev. of $\rho$ grandcan. $\mu$ } ideal Bose $\mu \leq 0$

BEC @ $\mu_c = 0$

dim. $> 2$

$\langle n_k \rangle$



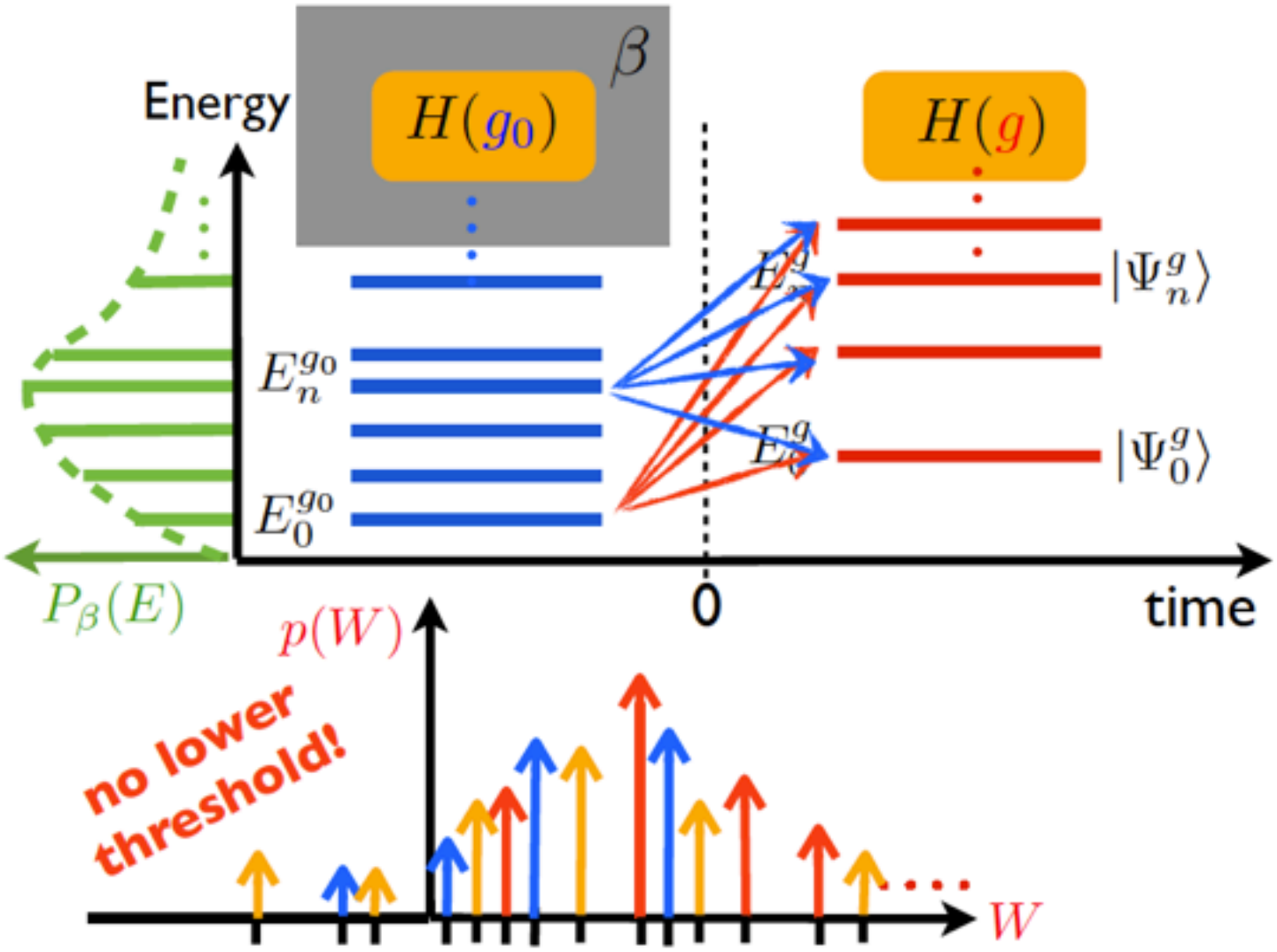

algebr. $p(W^{(k\to 0)})$

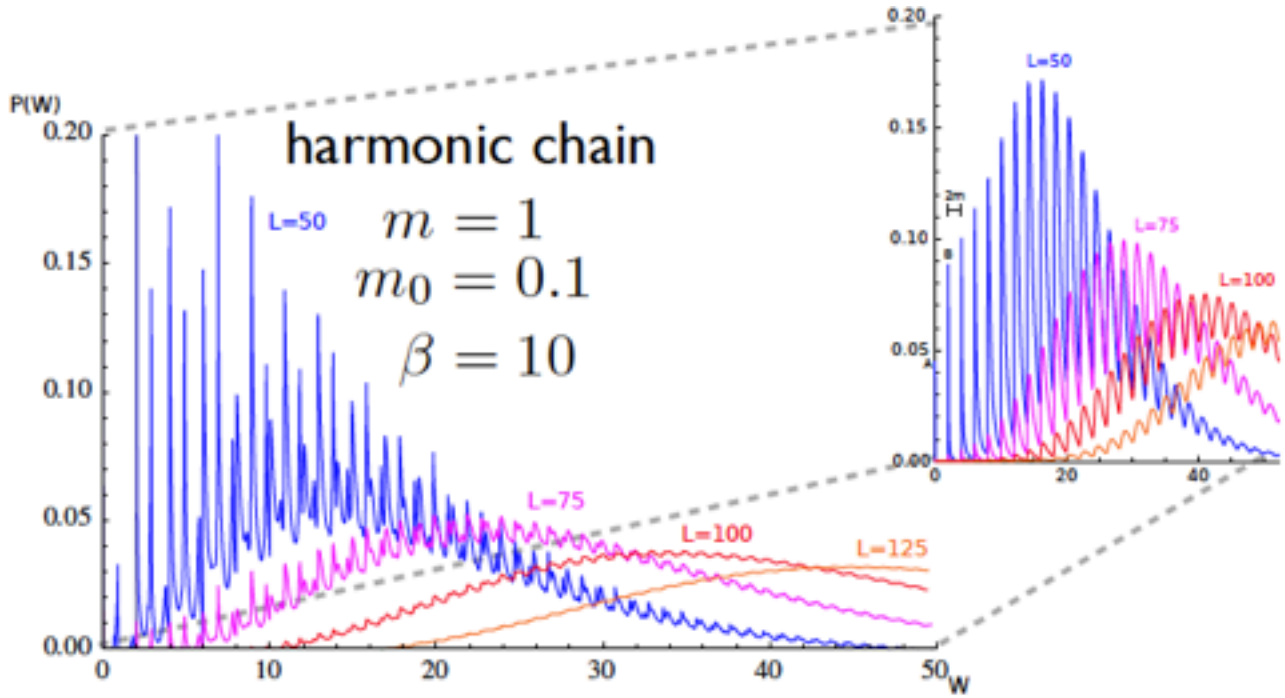
**Condensation**

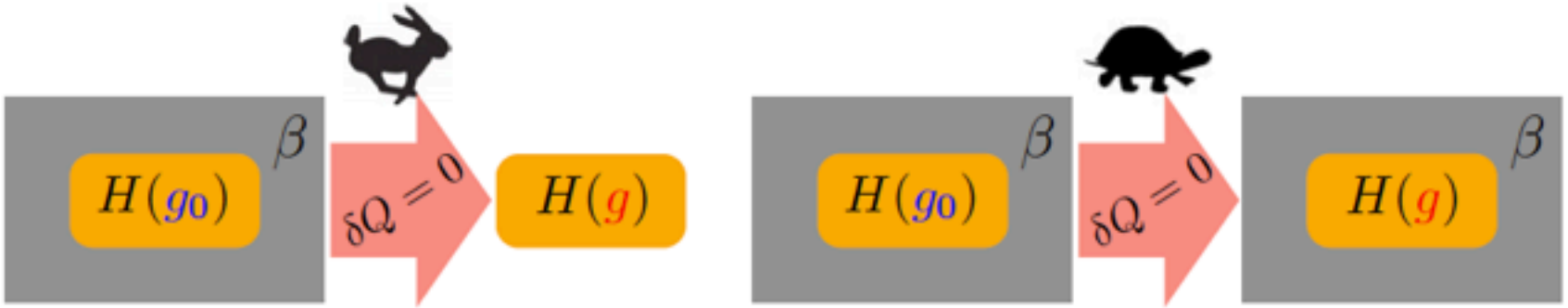
$\langle (\Delta\rho)^2 \rangle \nearrow \infty$

condens.

# Thermal


Energy
$\beta$
$H(g_0)$
$H(g)$
$E_n^g$
$|\Psi_n^g\rangle$
$E_n^{g_0}$
$E_0^g$
$|\Psi_0^g\rangle$
$E_0^{g_0}$
$P_\beta(E)$
0
time
$p(W)$
no lower threshold!
$W$




$\langle W \rangle$

$\Delta S \neq 0$

$W_{\rm rev} = \Delta F_\beta \equiv F_\beta(g) - F_\beta(g_0)$

$\Delta S = 0$

$$W_{\rm irr} \equiv \langle W \rangle - \Delta F_\beta \quad \Longleftrightarrow \quad \Delta S_{\rm irr} \equiv \beta W_{\rm irr}$$

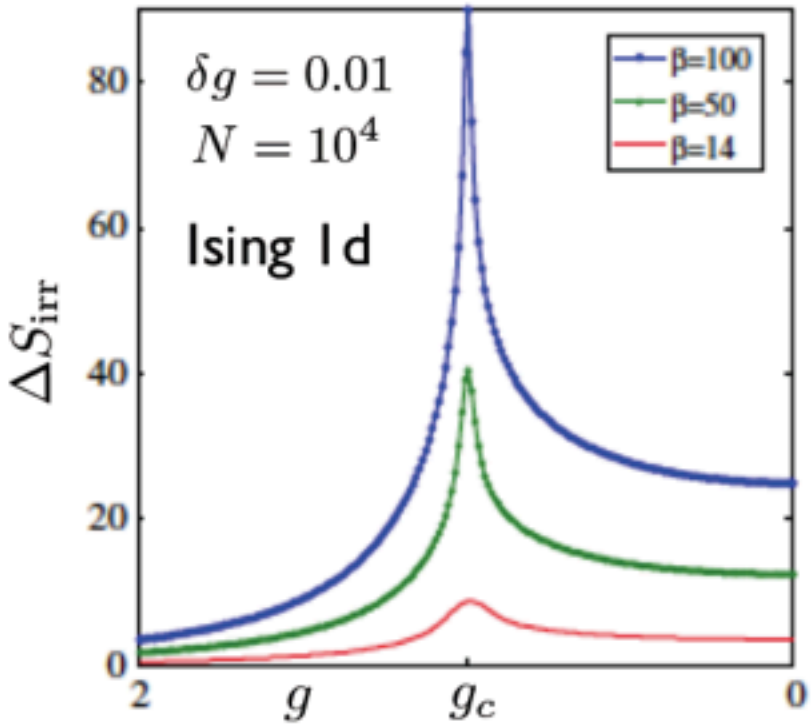
[Jarzynski '97]

$$\Delta S_{\rm irr}(g+\delta g, g)$$

[Dorner et al PRL '12]

$$F''_\beta(g) \sim |g - g_c|^{-\alpha}$$

[Sotiriadis, Gambassi, Silva '13]



$$W_{\rm irr} = \langle W \rangle - \Delta F_\beta \; = F_\beta(g_0) + (g - g_0) F'_\beta(g_0) - F_\beta(g)$$

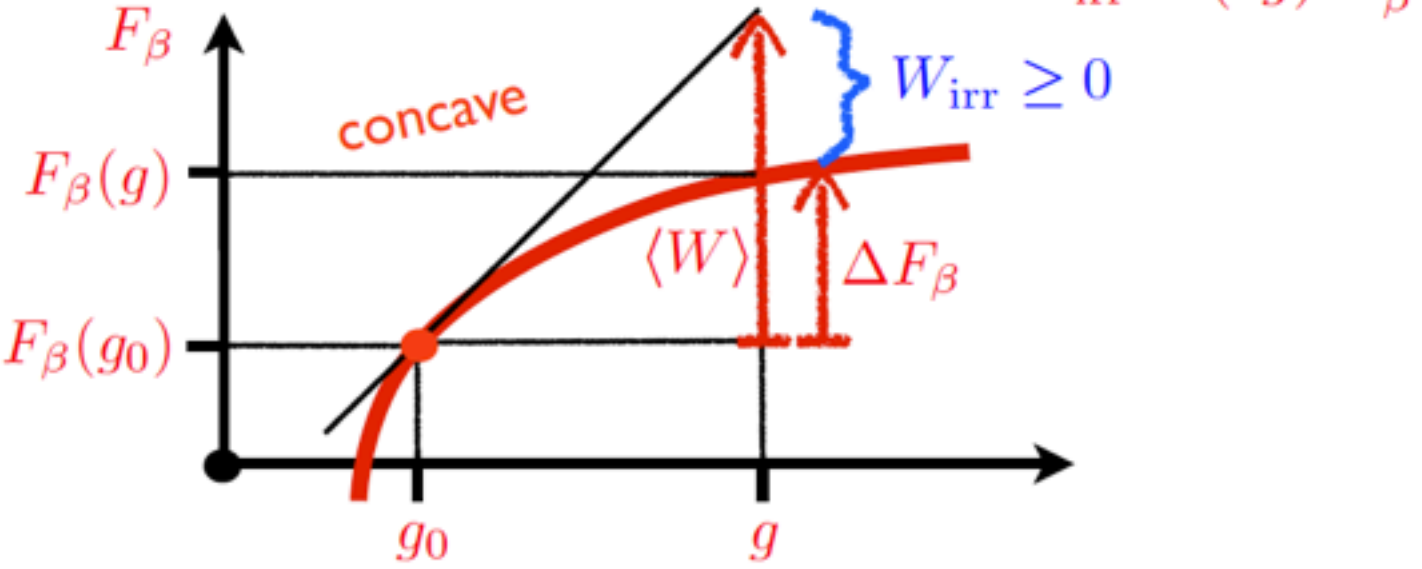
$$\langle H(g) - H(g_0) \rangle = (g - g_0) \left\langle \frac{\partial H(g_0)}{\partial g_0} \right\rangle$$

linear

$$= (g - g_0) F'_\beta(g_0)$$

$\beta$

$$\Delta S_{\rm irr} \propto (\delta g)^2 F''_\beta$$



# Conclusions

Take-home message:

- Quantum ⇔ Classical & universality dyn. transitions?
- Large deviations, edge singularity & **critical Casimir effect**
- different quantities?
- dimensionality + interaction?
- boundary states? crossover?
- *how to measure it ?* [Dorner et al PRL '13] [Mazzola et al PRL '13]

Bose:

- condensation?

# Conclusions

Take-home message:

- Quantum ⇔ Classical & universality

dyn. transitions?

Gambassi, Calabrese, EPL 95, 66007 ('11)

- Large deviations, edge singularity &

**critical Casimir effect**

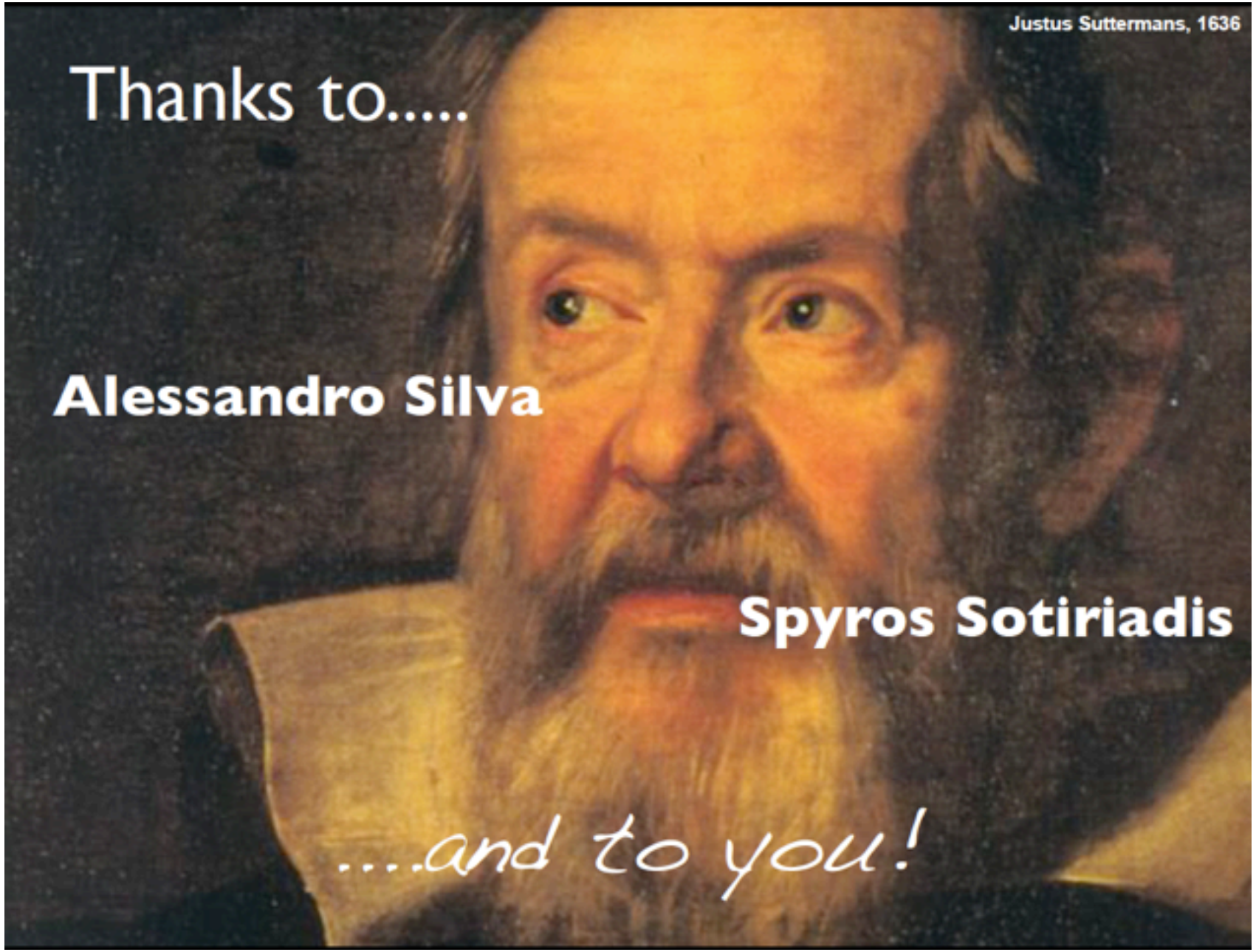
Gambassi, Silva, arXiv:1106.2671 ('11)
Gambassi, Silva, PRL 109, 250602 ('12)
Sotiriadis, Gambassi, Silva, PRE 87, 052129 ('13)

- different quant
- dimensionality + interaction?
- boundary states? crossover?
- *how to measure it ?*

[Dorner et al PRL '13]
[Mazzola et al PRL '13]

Bose:

- condensation?


Justus Suttermans, 1636
Thanks to.....
Alessandro Silva
Spyros Sotiriadis
....and to you!