Long Range Correlations in Driven Systems (I)

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Non-equilibrium systems

Systems with currents
(driven by electric field, T gradients etc.)

In many cases these systems reach a steady state
(but non-equilibrium steady state).

What is the nature of these steady states?

In general, the probability distribution to be in a microstate C evolves by the master equation

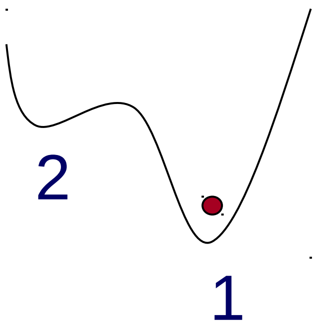
$$\frac{\partial P(C)}{\partial t} = \sum_{C'} W(C' \to C) P(C') - \sum_{C} W(C \to C') P(C)$$

in steady state $$\frac{\partial P(C)}{\partial t} = 0$$

equilibrium (non-driven) reach a steady state which satisfies the detailed balance condition for every microstates C and C':

$$W(C' \to C) P(C') = W(C \to C') P(C)$$

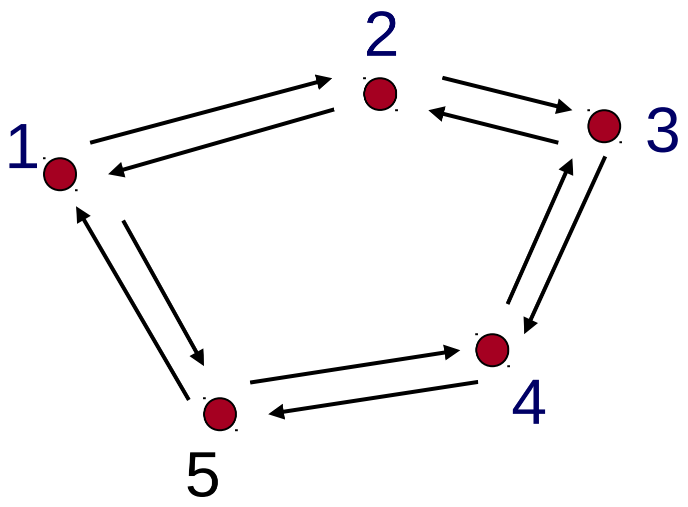
(no net probability current between two states)



Given transition rates $W(C \to C')$

a necessary and sufficient condition for detailed balance: for any set of microstates $C_1, \ldots, C_k$

$$W(1 \to 2)W(2 \to 3)\ldots W(k \to 1) = W(k \to k-1)\ldots W(2 \to 1)W(1 \to k)$$



Equilibrium states (detailed balance, no currents)

- Collective phenomena
- Phase transitions (first or second order)
- Long range order
- Spontaneous symmetry breaking
- Phase separation
- Critical behavior
- Fluctuations in the equilibrium state (spatial or temporal)
- Relaxation processes to equilibrium states
- Effect of disorder
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Rules governing equilibrium collective phenomena

- Landau's symmetry rules for the order of the transition (ferromagnets - second order ; nematic transition - first order)
- Symmetry classification into universality classes
- No long range order is low dimensional systems
- Renormalization group criteria for the order of the transition
- Gibbs phase rule (dimension of the coexistence manifold) D=2+c-n
- 180^0 rule
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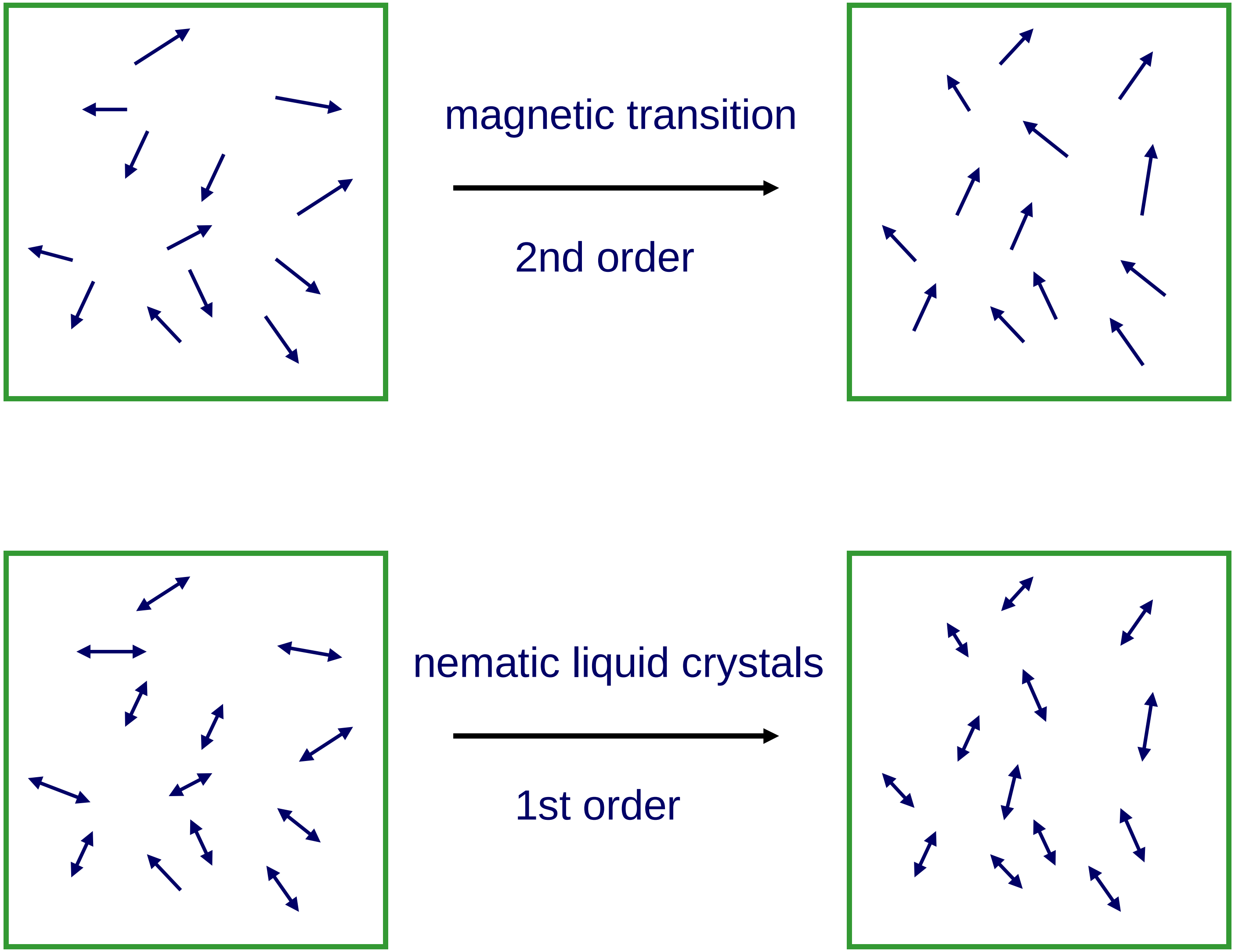
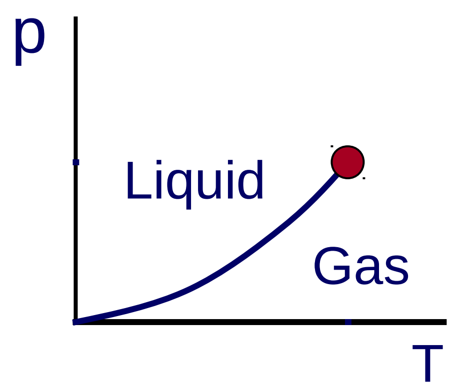

magnetic transition
2nd order
nematic liquid crystals
1st order

Gibbs phase rule

(dimension of manifold of n coexisting phases in c- components fluid mixtures)



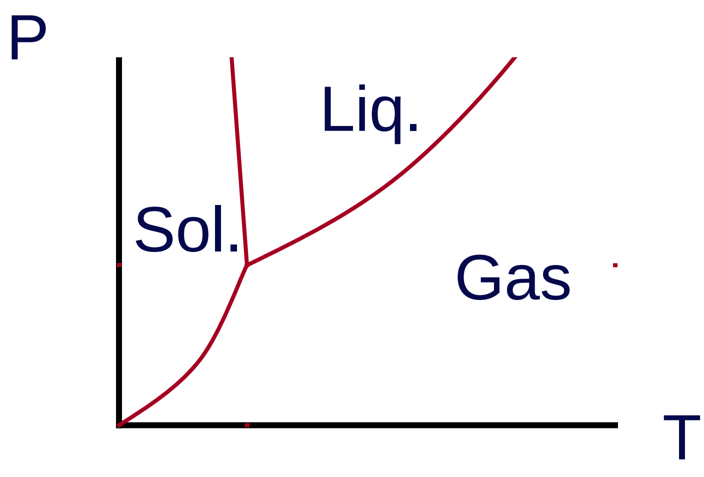
c=1
n=2

Gibbs phase rule D=2+c-n
(for the dimension of manifold of n coexisting phases
In fluid mixtures)

c- number of components in fluid mixtures
n- number of coexisting phases
D- dimension of the manifold of n coexisting phases

180^0 rule

(for coexistence lines in phase diagrams)



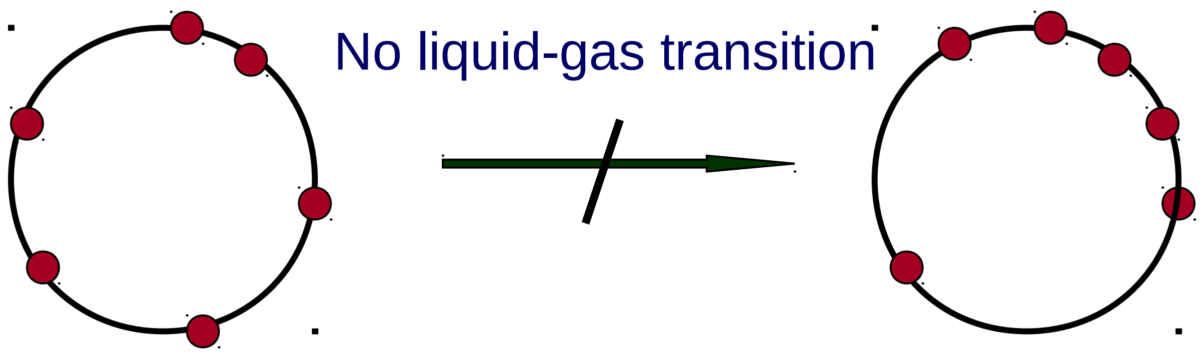
Do similar rules exist for non-equilibrium (driven) systems)?
(for which “free energy” cannot be defined)

In fact most of these rules do not apply in non-equilibrium systems.

Phase separation in 1d

In thermal equilibrium:

short range interactions
T>0 → Density is macroscopically homogeneous



Landau, Peierls 1930's: no phase separation, long range order, spontaneous symmetry breaking, phase transitions in 1d.

A simple physical argument for no long-range order in 1d

Ising model: $$H = -J\sum_n s_n s_{n+1} \qquad s = \pm 1 \;,\; J > 0$$

Ground state:

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Consider the evolution of this state: since T>0
a “wrong” droplet will be created in time

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Once created, the droplet may increase (or decrease) without energy cost.

In one-dimension “wrong” droplets are not eliminated

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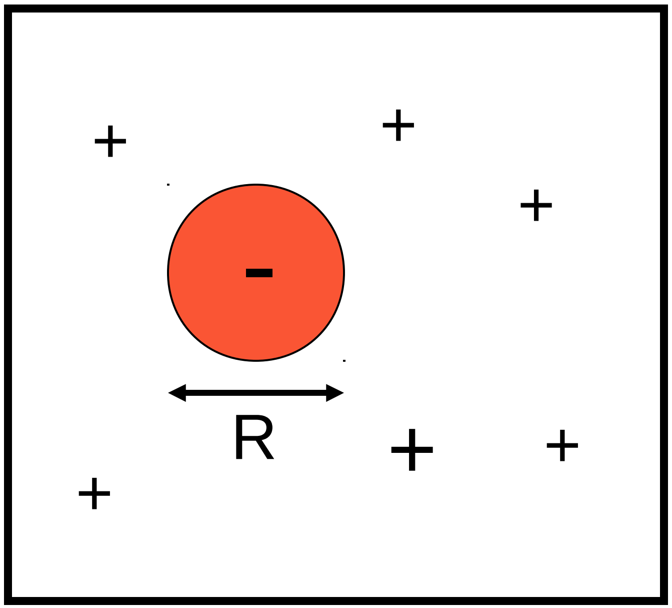
The energy of a droplet does not depend on its length (the energy cost of each droplet is 4J).

The length of droplets will fluctuate in time, droplets will merge and long range order will be destroyed in time.

Robust argument: the only ingredients are T>0 and short range interactions.

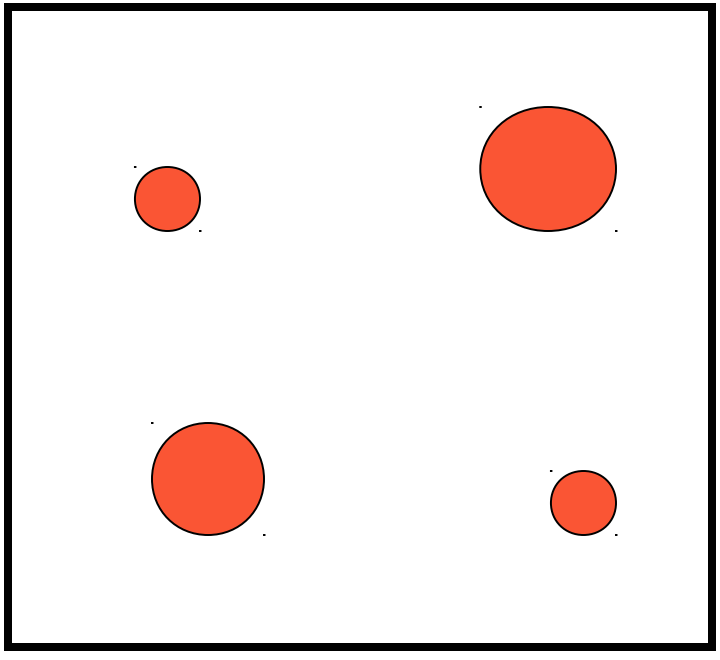
Maintaining long range order in higher dimensions:



Larger droplet cost more (surface) energy.

$$\frac{dR}{dt} \approx -\frac{\sigma}{R}$$

σ - surface tension



Wrong droplets are generated by fluctuations
but are eliminated by surface tension.

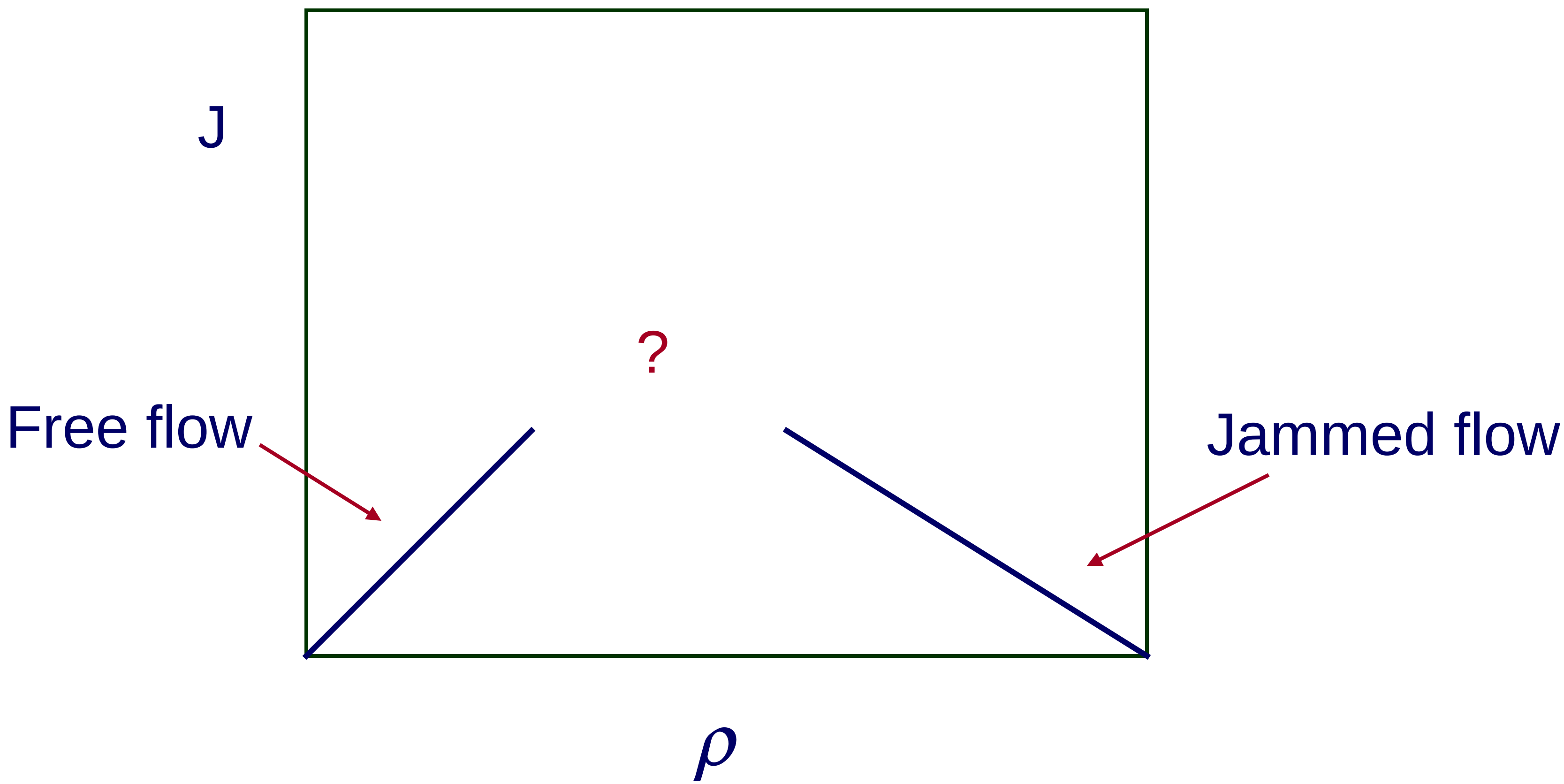
At sufficiently low T no large droplets are formed
and long range order is maintained.

Can one have phase separation in 1d driven systems (?)

local, noisy dynamics
homogeneous, ring geometry
no detailed balance

A criterion for phase separation in such systems (?)

Traffic flow - fundamental diagram



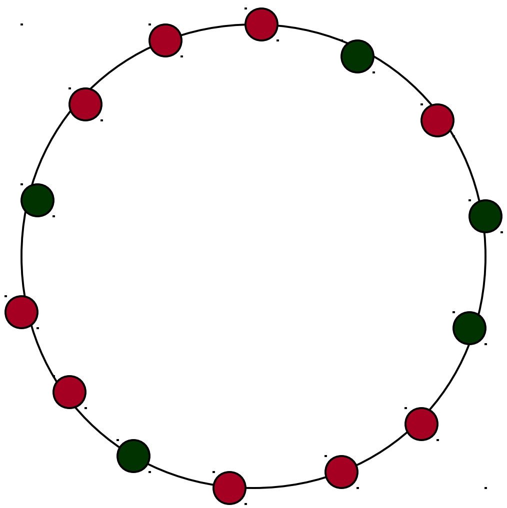
Is there a jamming phase transition?
or is it a broad crossover?

Main points

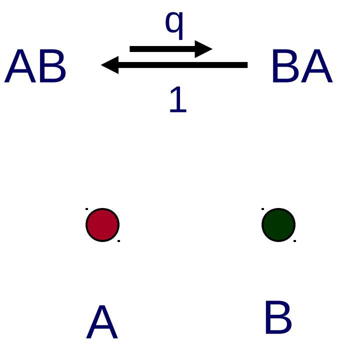
- Phase transitions do exist in one dimensional driven systems.

- In many traffic models studied in recent years Jamming is a crossover phenomenon. Usually it does not take place via a genuine phase transition.

Asymmetric Simple Exclusion Process (ASEP)



dynamics

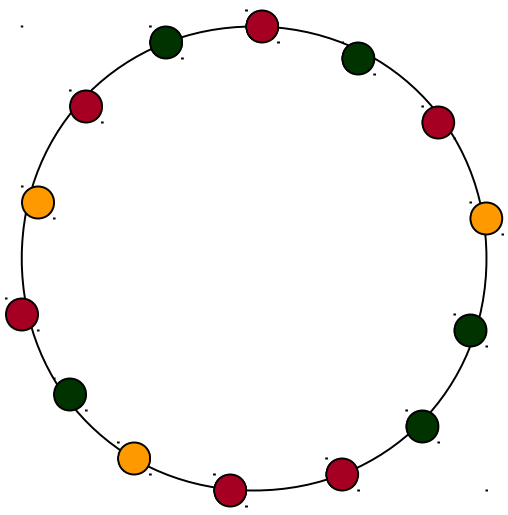


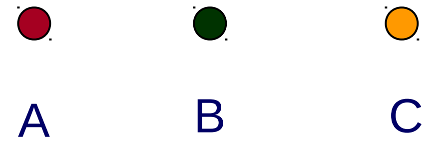
Steady State:

- ★ q=1 corresponds to an Ising model at $T=\infty$
- ★ All microscopic states are equally probable.
- ★ Density is macroscopically homogeneous.

No liquid-gas transition (for any density and q).

ABC Model





dynamics

$$AB \underset{1}{\overset{q}{\rightleftharpoons}} BA$$

$$BC \underset{1}{\overset{q}{\rightleftharpoons}} CB$$

$$CA \underset{1}{\overset{q}{\rightleftharpoons}} AC$$

Evans,Kafri, Koduvely, Mukamel PRL 80, 425 (1998)
A model with similar features was discussed by Lahiri, Ramaswamy PRL 79, 1150 (1997)

Simple argument:

ACCCC → CCCCA

CBBBB → BBBBC

BAAAA → AAAAB

$$AB \underset{1}{\overset{q}{\rightleftharpoons}} BA$$

$$BC \underset{1}{\overset{q}{\rightleftharpoons}} CB$$

$$CA \underset{1}{\overset{q}{\rightleftharpoons}} AC$$

…AACBBBCCAAACBBBCCC…

↓ fast rearrangement

…AABBBCCCAAABBBCCCC…

↓ slow coarsening

…AAAAABBBBBCCCCCCAA…

- logarithmically slow coarsening

…AAAAABBBBBCCCCCCAA…

$$t \propto q^{-l} \qquad l \propto \ln t$$

- needs n>2 species to have phase separation
- Phase separation takes place for any q (except q=1)
- Phase separation takes place for any density N_A, N_B, N_C
- strong phase separation: no fluctuation in the bulk; only at the boundaries.

…AAAAAAAAAAABBBBBBBBBBBBCCCCCCCCCCC…

Special case $N_A = N_B = N_C$

The argument presented before is general, independent of densities.

For the equal densities case the model has detailed balance for arbitrary q.

We will demonstrate that for any microscopic configuration {X}
One can define “energy” E({X}) such that the steady state
Distribution is

$$P(\{X\}) \propto q^{E(\{X\})}$$

AAAAAABBBBBBCCCCCC   E=0

……AB….. $\longrightarrow$ ……BA….. E $\longrightarrow$ E+1

……BC….. $\longrightarrow$ ……CB….. E $\longrightarrow$ E+1

……CA….. $\longrightarrow$ ……AC….. E $\longrightarrow$ E+1

With this weight one has:

$$\underbrace{W(AB \to BA)}_{=q}P(...AB...) = \underbrace{W(BA \to AB)}_{=1}P(...BA...)$$

$$P(...BA...)\,/\,P(...AB...) = q$$

This definition of “energy” is possible only for $N_A = N_B = N_C$

AAAAABBBBBCCCCC $\longrightarrow$ AAAABBBBBCCCCC**A**

E $\longrightarrow$ $E+N_B-N_C$

$$N_B = N_C$$

Thus such “energy” can be defined only for $N_A=N_B=N_C$

AABBBBCCCAAAAABBBCCCC

The rates with which an A particle makes a full circle clockwise
And counterclockwise are equal

$$q^{N_B} = q^{N_C}$$

Hence no currents for any N.

For $N_B \neq N_C$ the current of A particles satisfies $J_A \propto q^{N_B} - q^{N_C}$

The current is non-vanishing for finite N. It vanishes only in the limit $N \to \infty$. Thus no detailed balance in this case.

The model exhibits strong phase separation

…AAAAAAAABBBABBBBBBCCCCCCCCCAA…

The probability of a particle to be at a distance l on the wrong side of the boundary is q^l

The width of the boundary layer is $-1/\ln q$

$$N_A = N_B = N_C$$

The “energy” E may be written as

$$P(\{x\}) = q^{E(\{x\})}$$

$$E(\{x\}) = \sum_{i=1}^{N} \sum_{k=1}^{N-1} \left(1 - \frac{k}{N}\right) \left(C_i B_{i+k} + A_i C_{i+k} + B_i A_{i+k}\right)$$

summation over $(i+k)$ modulo N

Local dynamics

- long range

Partition sum

Excitations near a single interface: AAAAAAABBBBBB

$$Z_1(q) = \sum p(n) q^n$$

P(n)= degeneracy of the excitation with energy n

P(0)=1
P(1)=1
P(2)=2 (2, 1+1)
P(3)=3 (3, 2+1, 1+1+1)
P(4)=5 (4, 3+1, 2+2, 2+1+1, 1+1+1+1)

P(n)= no. of partitions of an integer n

$$Z_1(q) = \frac{1}{(1-q)(1-q^2)....}$$

Partition sum: $$Z(q) = N\left[\frac{1}{(1-q)(1-q^2)\ldots\ldots}\right]^3$$

Correlation function: $$\langle A_1 A_r \rangle \approx 1/3$$

with $$\langle A_1 \rangle \langle A_r \rangle = 1/9$$

for $$-1/\ln q < r < N/3$$

Summary of ABC model

- The model exhibits phase separation for any $q \neq 1$
- Needs n>2 species for phase separation.
- Strong phase separation (probability to find a particle in the bulk of the “wrong” is exponentially small.
- Phase separation is a result of effective long range Interactions generated by the local dynamics.
- Logarithmically slow coarsening process.