Naturalness in Supersymmetry and Its Signatures

Yasunori Nomura

UC Berkeley; LBNL

Based on work with
Ryuichiro Kitano (SLAC)

hep-ph/0509039 [PLB]
hep-ph/0509221 [PLB]
hep-ph/0602096 [PRD]

We will be living in the Era of Hadron Collider

- Exploring highest energy regime
- Connections between signals
  and the underlying theory not so obvious

→ Input from models very important

Determination of TeV physics
through (slow) elimination processes

What contributions can theorists make?

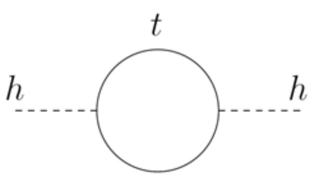
- Suggest “new” signals
- Give a list of “well-motivated” models to be tested

From what models should we start?

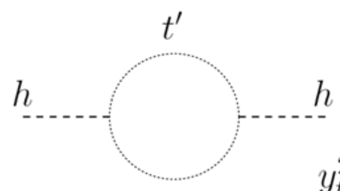
Minimality, Consistent with the existing (initial LHC) data,…

Naturalness as a Guiding Principle (still)

- Why $m_{weak} \ll M_{Pl}$?
  - Need some new particles at ~ TeV



$$\approx -\frac{y_t^2}{16\pi^2}\Lambda^2$$



$$\approx \frac{y_t^2}{16\pi^2}(\Lambda^2 - m_{t'}^2)$$

$$\approx -\frac{y_t^2}{16\pi^2}m_{t'}^2$$

→ Weak scale supersymmetry

  - Improved radiative structure (EWSB, inflation, …)
  - Gauge coupling unification
  - Theory of EWSB: radiative EWSB with large m_t
  - Relatively easy to evade constraints from EWPD

- Still leads to vast varieties of signatures
- Need to specify more

More Powerful Use of Naturalness after LEPII

- EWSB does not work well in the (simplest) minimal supersymmetric standard model (MSSM)

→ Supersymmetric fine-tuning problem

– Minimization condition (tree level)

$$\frac{M_Z^2}{2} \simeq -m_h^2 - |\mu|^2$$

In general,

$$\boxed{\frac{M_{\rm Higgs}^2}{2} \simeq -m_h^2 - |\mu|^2}$$

Natural EWSB requires

$$\frac{M_{\rm Higgs}^2}{2} \sim |m_h^2| \sim |\mu|^2$$

In the MSSM,

$$M_{\rm Higgs} \lesssim 130 \text{ GeV} \quad \longrightarrow \quad |m_h^2|,\ |\mu|^2 \lesssim (200 \text{ GeV})^2$$

$$\Delta^{-1} \geq 20\%$$

- There are several contributions to m_h^2
  - The largest contribution: top-stop loop

$$\delta m_h^2 \simeq -\frac{3y_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2)\ln\left(\frac{M_{\rm mess}}{m_{\tilde{t}}}\right)$$

M_{mess}: the scale where superparticle masses are generated

- Light top squarks and small messenger scale preferred

→ What's wrong?

  - $M_{Higgs} < M_Z$ at tree level

    need radiative corrections from top-stop loop

$$M_{\rm Higgs} \gtrsim 114 \text{ GeV} \quad \Rightarrow \quad m_{\tilde{t}} \gtrsim 1 \text{ TeV} \quad (\text{for small } A_t)$$

  - Tension between small M_{mess} and the SUSY flavor problem

    mediating SUSY breaking by SM gauge interactions

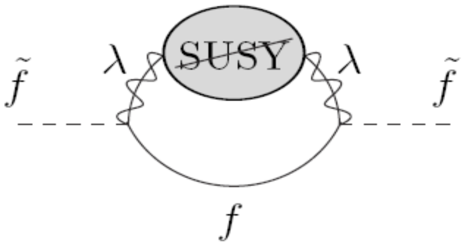
$$\frac{m_{\tilde{t}}}{m_{\tilde{e}}} \simeq \frac{(4/3)g_3^4 + \delta}{(3/5)g_1^4} \simeq (7 \sim 8) \quad \Rightarrow \quad m_{\tilde{t}} \gtrsim 700 \text{ GeV} \text{ for } m_{\tilde{e}} \gtrsim 100 \text{ GeV}$$

Suggests Several Directions to Go

- Additional contribution to M_{Higgs} and "random'' superparticle masses at low energies

Chacko, Y.N., Smith; Y.N., Tweedie, …

- Add W = S Hu Hd
- Generate soft masses at (10~100)TeV by strong dynamics



- The strong sector has an SU(5) global symmetry, but it is spontaneously broken at (10~100 TeV) as well as SUSY

$$\frac{m_{\tilde{t}}}{m_{\tilde{e}}} \neq \frac{(4/3)g_3^4 + \delta}{(3/5)g_1^4}$$ keeping gauge coupling unification

Explicit construction in warped space

- The Higgs boson may have escaped the detection at LEP II Dermisek, Gunion; Chang, Fox, Weiner
  - The Higgs boson may decay into "complicated" final states
    e.g. $h \to aa \to \tau\tau\tau\tau$ or $h \to aa \to \gamma\gamma\gamma\gamma$ (a: new scalar)
  - Complete discussion of tuning needs an underlying theory, but the tension with M_{Higgs} alleviated

- Large A_t term allows the reduction of stop masses; combined with small M_{mess} can solve the problem

Kitano, Y.N.

  - The fine-tuning problem may just be a problem of SUSY breaking mechanism, and not minimal SUSY itself
  - M_{Higgs} at tree level must be reasonably large
    - Moderately large $\tan\beta \to$ small μB
  - Complete analysis needed (including all the sensitivities of v)

Naturalness as a "model selector"

Kitano, Y.N., hep-ph/0602096

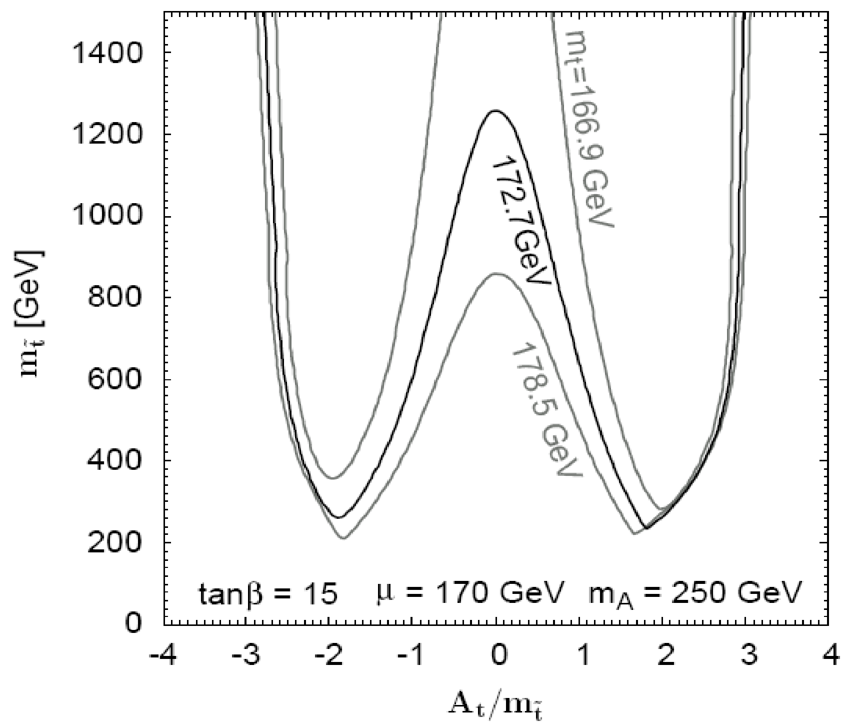
- The SUSY fine-tuning problem may just be a problem of SUSY breaking mechanism, and not minimal SUSY itself
- Large A_t term allows light top squarks, alleviating tuning

Minimal values of $m_{\tilde{t}}$ giving $M_{Higgs} \geq 114.4$GeV

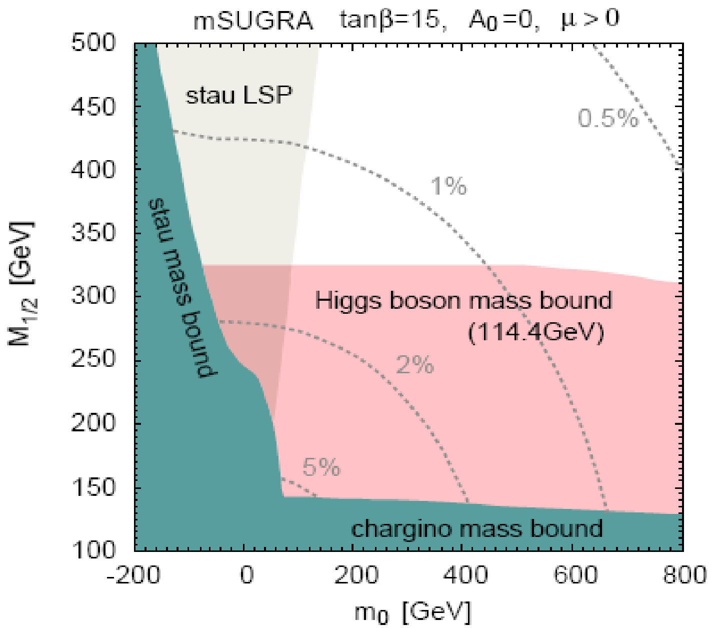
For $|A_t/m_{\tilde{t}}| \approx (1.5 \sim 2.5)$,

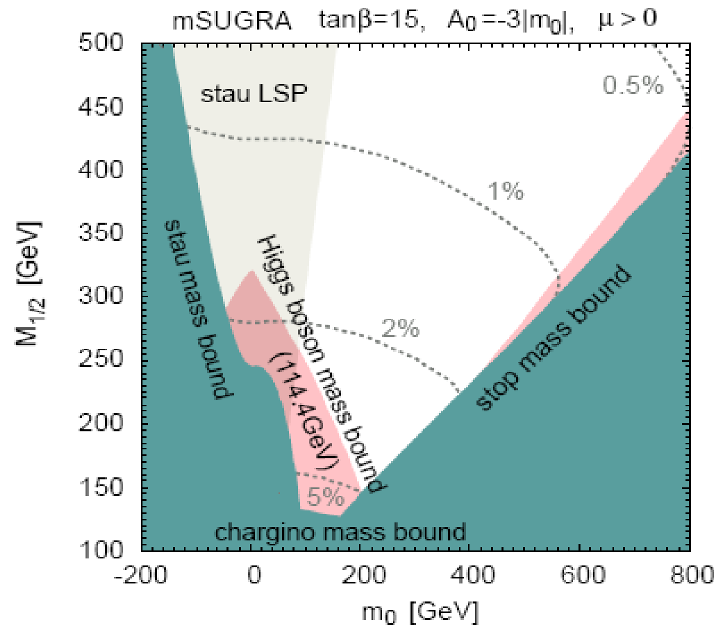
$m_{\tilde{t}} \approx (250 \sim 400)$ GeV

is allowed (for $\tan\beta \gtrsim 10$)



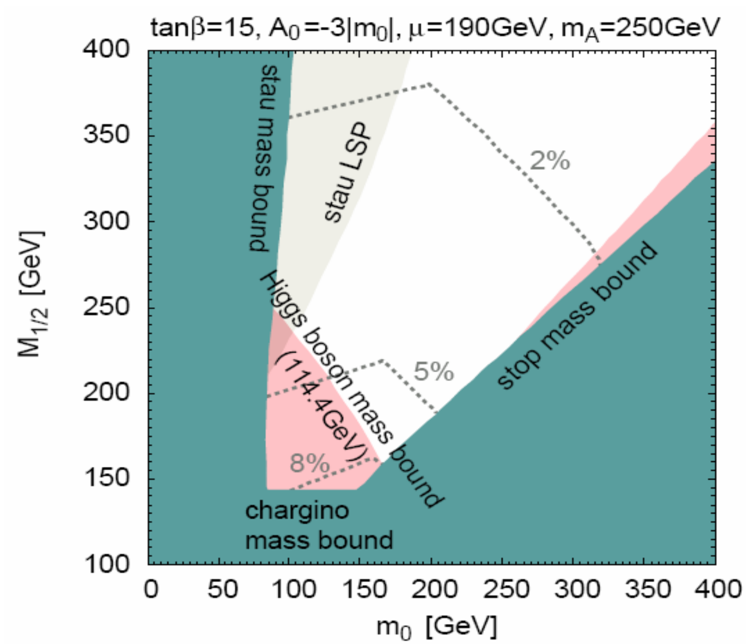
- The effect of A_t already visible at CMSSM





- Further reduction of tuning possible via non-universality

e.g. $$m_{H_u}^2, m_{H_d}^2 \neq m_0^2$$
$$M_1 \neq M_2 \neq M_3$$



- Reduction of tuning to the level of 10% possible in high scale supersymmetry breaking

Typically, $|A_t/m_{\tilde{t}}| \approx (1.5\sim2.5)$, $|\mu| \lesssim 250$ GeV , $\tan\beta \gtrsim 5$

$$m_{\tilde{t}} = (m_{Q_3}^2 m_{U_3}^2)^{1/4} \simeq 250 \text{ GeV}$$

- Further reduction of tuning requires small M_{mess}:

$$\delta m_h^2 \simeq -\frac{3y_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2)\ln\left(\frac{M_{\text{mess}}}{m_{\tilde{t}}}\right)$$

Small M_{mess} with Large A_t

→ Moduli / Boundary condition / Scherk-Schwarz SUSY breaking

e.g.

$$M_{1,2,3} = M_0, \qquad m_{Q,U,D,L,E}^2 = rM_0^2, \qquad A_{u,d,e} = -sM_0,$$

$$m_{H_u,H_d}^2 = 0, \qquad \mu B = 0,$$

"Well-ordered" spectra … reduce/eliminate tuning

Emerging Pictures

- Generic features of natural SUSY models
  - Large A_t term: $|A_t/m_{\tilde{t}}| \approx (1.5 \sim 2.5)$

    → large top squark mass splitting $m_{\tilde{t}_2} - m_{\tilde{t}_1} \approx (1.5 \sim 2.5)\, m_t$

    ($\simeq m_t$ O.K. for M_{mess} ~ TeV)
  - Light top squarks

    → How light depends on M_{mess} etc.

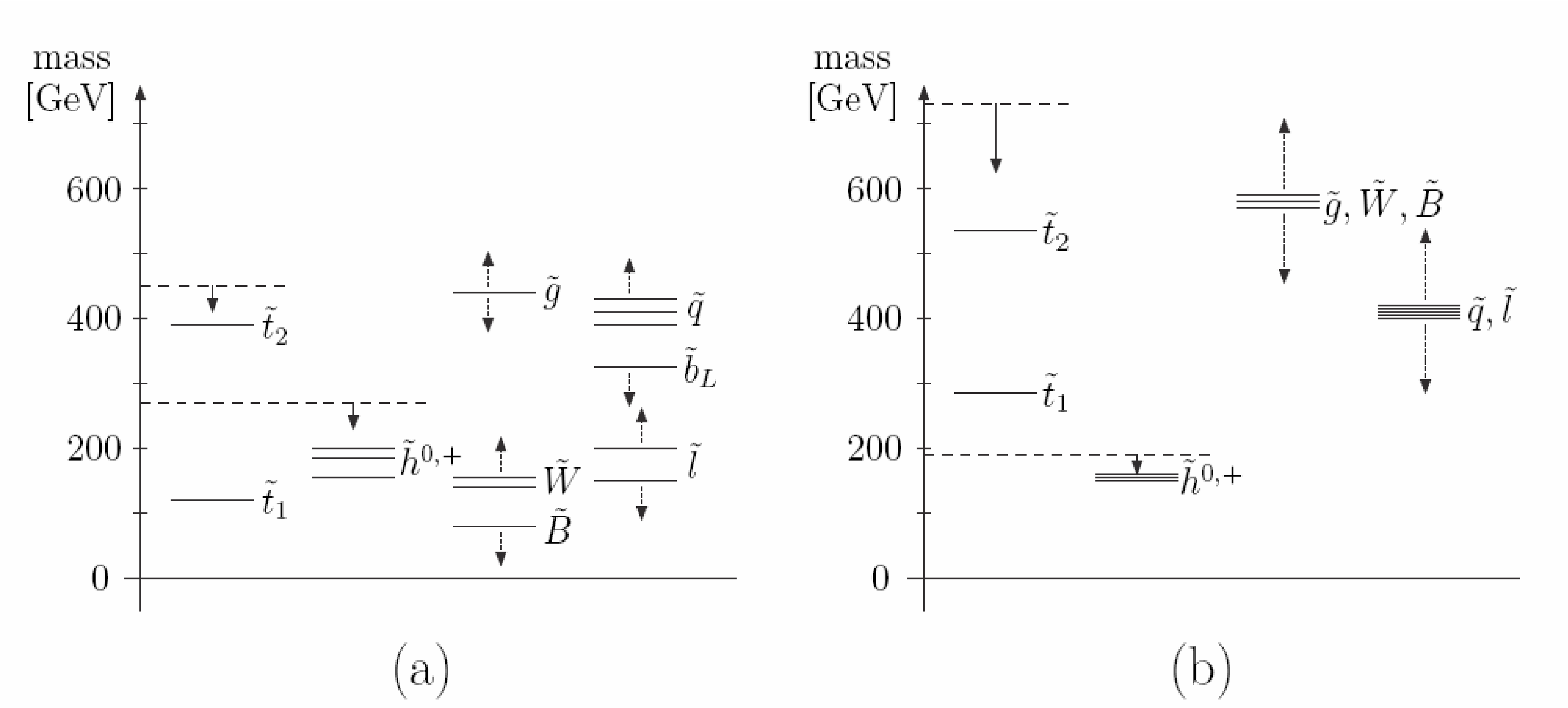
    (For the high scale case, $m_{\tilde{t}} \lesssim 300$ GeV → $m_{\tilde{t}_1} \sim 100$ GeV)
  - Light Higgs boson

    Typically, $M_{\mathrm{Higgs}} \lesssim 120$ GeV
  - "Small" μB

    Typically, $\tan\beta \gtrsim 5$
  - Small μ parameter

    $|\mu| \lesssim 190$ GeV $(270$ GeV$)$ (for $\Delta^{-1} \geq 20\%\ (10\%)$)

Characteristic Spectra



(a) "squeezed" spectra (typical in the high scale case)

(b) "well-ordered" spectra (typical in moduli-type)

→ None of these particularly well studied

A Solution to the SUSY Fine-tuning Problem within the MSSM

Kitano, Y.N., PLB631, 58 (05)

Is there any region where fine-tuning is absent?

→ Requires a careful analysis

- Consistent with various constraints?
- No "hidden" fine-tuning?
- ……

Need to specify the model

Large A_t at low energies

- $(Z+Z^+)Q^+Q$ → moduli supersymmetry breaking ($Z \rightarrow T$)

Special RG properties

Choi, Jeong,Kobayashi,Okumura;
Kitano, Y.N.

- Single moduli dominance

Effective supergravity action at ~M_{unif}

$$S = \int d^4x \sqrt{-g}\left[\int d^4\theta\, C^\dagger C \mathcal{F} - \theta^2\bar{\theta}^2 C^{\dagger 2} C^2 \mathcal{P}_{\rm lift} + \left\{\int d^2\theta \left(\frac{1}{4} f_a \mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + C^3 W\right) + {\rm h.c.}\right\}\right]$$

$\mathcal{F} = -3\exp(-K/3)$: superspace function, W: superpotential, f_a gauge kinetic function, $\mathcal{P}_{\rm lift}$: introduced to allow Λ=0 at the minimum

$$\begin{aligned}\mathcal{F} &= -3(T+T^\dagger)^{n_0/3} + (T+T^\dagger)^{r_i}\Phi_i^\dagger\Phi_i + \cdots,\\ W &= (w_0 - Ae^{-aT}) + W_{\rm Yukawa},\\ f_a &= T,\end{aligned}$$

$$\mathcal{P}_{\rm lift} = d(T+T^\dagger)^{n_P},$$

where $W_{\rm Yukawa}$ is MSSM Yukawa coupling.

(w_0~$m_{3/2}M_{unif}^2$, A~M_{unif}^3, a~$8\pi^2/N$, n_0=3 and $r_i=n_i/n$ for volume moduli)

- Moduli stabilization (supersymmetrically)

e.g. Kachru, Kallosh, Linde, Trivedi; …

$$
\begin{aligned}
aT &= \ln\left(\frac{M_{\rm Pl}}{m_{3/2}}\right), \\
\frac{m_{3/2}}{M_0} &= \frac{2}{3}\frac{\partial_T K_0}{\partial_T \ln(V_{\rm lift})}\ln\left(\frac{M_{\rm Pl}}{m_{3/2}}\right),
\end{aligned}
$$

at the leading order in $1/\ln(A/w_0) \sim 1/\ln(M_{\rm Pl}/m_{3/2})$. ($m_{3/2} = e^{K_0/2}W_0$)

M_0: moduli contribution to the soft masses

$$M_0 = \frac{F_T}{T+T^\dagger}$$

- Relation between M_0 and $m_{3/2}$

$$\frac{m_{3/2}}{M_0} \approx \ln\left(\frac{M_{\rm Pl}}{m_{3/2}}\right) = O(8\pi^2)$$

(Moduli)~(Anomaly) → Mixed moduli-anomaly mediation

Choi, Falkowski, Nilles, Olechowski, Pokorski; Choi, Jeong, Okumura; Endo, Yamaguchi, Yoshioka; ….

$$\alpha \equiv \frac{m_{3/2}}{M_0 \ln(M_{\rm Pl}/m_{3/2})} = \frac{2n_0}{2n_0 - 3n_P} + \cdots$$

"ratio": a rational number (plus corrections; see later)

- ## RG properties of soft masses

Suppose $r_i + r_j + r_k = 1$ for fields having $W = (\lambda_{ijk}/6)\Phi_i\Phi_j\Phi_k$ and $\sum_i r_i Y_i = 0$, the soft masses defined by

$$\mathcal{L}_{\rm soft} = -\frac{1}{2}M_a\lambda^a\lambda^a - m_i^2|\phi_i|^2 - \frac{1}{6}(A_{ijk}y_{ijk}\phi_i\phi_j\phi_k + \text{h.c.})$$

can be solved (at one loop) as

$$\begin{aligned}
M_a(\mu_R) &= M_0\left[1 - \frac{b_a}{8\pi^2}g_a^2(\mu_R)\ln\left(\frac{M_{\rm mess}}{\mu_R}\right)\right], \\
A_{ijk}(\mu_R) &= M_0\left[-(r_i+r_j+r_k) + 2\left\{\gamma_i(\mu_R)+\gamma_j(\mu_R)+\gamma_k(\mu_R)\right\}\ln\left(\frac{M_{\rm mess}}{\mu_R}\right)\right], \\
m_i^2(\mu_R) &= M_0^2\left[r_i - 4\left\{\gamma_i(\mu_R) - \frac{1}{2}\frac{d\gamma_i(\mu_R)}{d\ln\mu_R}\ln\left(\frac{M_{\rm mess}}{\mu_R}\right)\right\}\ln\left(\frac{M_{\rm mess}}{\mu_R}\right)\right],
\end{aligned}$$

$M_{\rm mess}$ is defined by

$$M_{\rm mess} = \frac{M_{\rm unif}}{(M_{\rm Pl}/m_{3/2})^{\alpha/2}}$$

Choi, Jeong, Okumura;
Simple proof: Kitano, Y.N.

→ $M_{\rm mess}$: effective messenger scale

($M_a = M_0,\ A_{ijk} = -(r_i+r_j+r_k)M_0,\ m_i^2 = r_i M_0^2$ at $\mu_R = M_{\rm mess}$)

Is the reduction of $M_{\rm mess}$ "real"? No hidden fine-tuning?

- $M_{\rm mess}$ ~ TeV obtained by α=2 ?

$$M_{\rm mess} = \frac{M_{\rm unif}}{(M_{\rm Pl}/m_{3/2})^{\alpha/2}} \quad \xrightarrow[\alpha = 2]{} \quad M_{\rm mess} = O({\rm TeV})$$

α is a rational number, up to corrections

$$\alpha = \frac{2n_0}{2n_0 - 3n_P} + \cdots$$

The corrections arise from terms of $\mathcal{F}$ higher order in $1/(T+T^\dagger)$.
Although $\langle T+T^\dagger\rangle \simeq 2/g_{\rm GUT}^2$ is O(1), coefficients can be O(1/8π^2).

(Technically natural)

α=2 can be obtained without fine-tuning

- Assignment for r_i (respecting RG properties)
  - SU(5) → $r_Q = r_U = r_E = \frac{1}{2}, \quad r_D = r_L, \quad r_{H_u} = r_{H_d} = 0$
  - Matter universality → $r_Q = r_U = r_D = r_L = r_E = \frac{1}{2}, \quad r_{H_u} = r_{H_d} = 0$

  arises e.g. in 6D with 5D matter and 4D Higgses

- Soft SUSY breaking masses at M_{mess} ~ TeV:

$$\boxed{\begin{gathered} M_1 = M_2 = M_3 = M_0, \\ m_Q^2 = m_U^2 = m_D^2 = m_L^2 = m_E^2 = \frac{M_0^2}{2}, \\ A_u = A_d = A_e = -M_0, \\ m_{H_u}^2 = m_{H_d}^2 = 0, \end{gathered}}$$

Corrections of $O(M_0^2/8\pi^2)$ expected for the scalar squared masses, arising from higher order terms in $\mathcal{F}$ (flavor universality assumed).

These corrections are naturally smaller than ~ v^2:

- Correction to $m_{H_u}^2$ through $m_{\tilde{t}}$ negligible even with $\ln(M_{\rm GUT}/m_{\tilde{t}})$
- $m_{H_u}^2, m_{H_d}^2 \lesssim (M_{\rm Higgs}^2/2)/20\% \approx (200\,{\rm GeV})^2$

  treated as free parameters at M_{mess} (We aim Δ^{-1} > 20%)

- μ and B parameters
  - Naturally $O(m_{3/2}) = O(100\ \mathrm{TeV})$ … too large
  - We need

$$M_{\rm Higgs}^2/2|\mu|^2 \geq 20\% \longrightarrow |\mu| \lesssim 190\ {\rm GeV}$$

$$|\mu|^2, m_{H_d}^2 \lesssim (200\ {\rm GeV})^2 \rightarrow B \approx (350/\tan\beta)\ {\rm GeV} \underset{\tan\beta \gtrsim 5}{\longrightarrow} B \approx (10\sim70)\ {\rm GeV}$$

  - Consider a field Σ having only the F-term VEV, $F_\Sigma \sim M_0$, and

$$S = \int d^4x \sqrt{-g} \int d^4\theta\, C^\dagger C \Big(-\kappa(\Sigma + \Sigma^\dagger) H_u H_d + {\rm h.c.} \Big)$$

This gives

$$\mu = \kappa F_\Sigma,$$
$$B = -(\gamma_{H_u} + \gamma_{H_d}) m_{3/2},$$

at $\mu_R \simeq M_{\rm unif}$ → $\mu \sim M_0 = O(500\text{-}1000\ \mathrm{GeV})$ naturally obtained

  - Too large B?

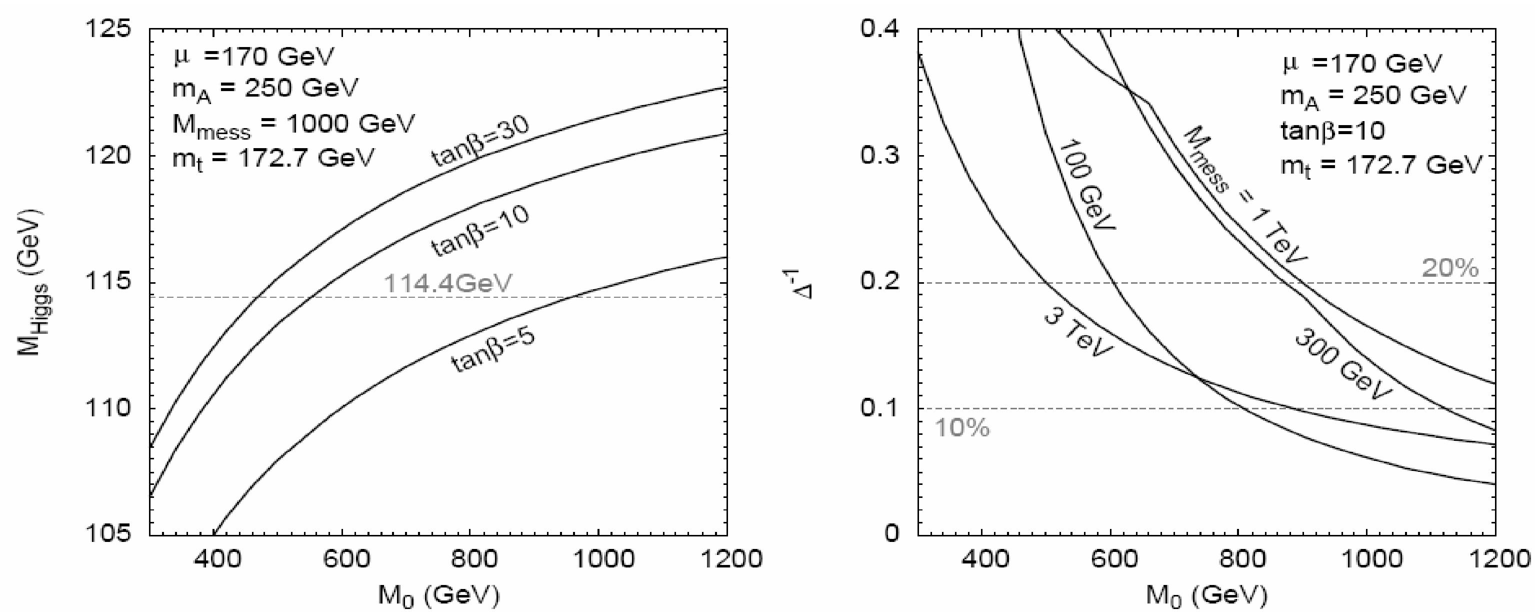
$$B(\mu_R) = 2M_0 \Big\{ \gamma_{H_u}(\mu_R) + \gamma_{H_d}(\mu_R) \Big\} \ln\left(\frac{M_{\rm mess}}{\mu_R} \right)$$

$B = 0$ at $\mu_R = M_{\rm mess}$ → Small B also obtained naturally

EWSB without Fine-Tuning

- Is there a region with $\Delta^{-1} > 20\%$?
  - M_0 bounded from below by $M_{Higgs} > 114$ GeV
    and from above by $\Delta^{-1} > 20\%$

$$m_{H_u}^2,\, m_{H_d}^2,\, \mu \text{ and } B \xrightarrow[\text{EWSB}]{} \mu,\, m_A \text{ and } \tan\beta$$



$M_0 > 550$ GeV (450 GeV) for $\tan\beta = 10$ (30) $\quad$ $M_0 < 900$ GeV

There is a parameter region with $\Delta^{-1} > 20\%$

Spectrum Summary

- Universal masses

$$m_{\tilde{b}} \simeq m_{\tilde{w}} \simeq m_{\tilde{g}} \simeq M_0$$

$$m_{\tilde{q}} \simeq m_{\tilde{u}} \simeq m_{\tilde{d}} \simeq m_{\tilde{l}} \simeq m_{\tilde{e}} \simeq \frac{M_0}{\sqrt{2}}$$

at M_{mess} ~ TeV, where

$$450 \text{ GeV} \lesssim M_0 \lesssim 900 \text{ GeV}$$

- Top squark masses light and split

$$m_{\tilde{t}_{1,2}} \simeq \frac{M_0 \mp m_t}{\sqrt{2}}$$

The lighter top squark mass as small as ~ 200 GeV

- Light Higgs boson(s)

$M_{\text{Higgs}} \lesssim 120 \text{ GeV}$ and $m_A \lesssim 300 \text{ GeV}$

- (Moderately) large tanβ

$$\tan\beta \gtrsim 5$$

- The Higgsino LSP

$$m_{\tilde{h}^0} \lesssim 190 \text{ GeV}$$

Signatures at the LHC

Kitano, Y.N., hep-ph/0602096

Characteristic Signatures
for the "well-ordered'' spectra

- Higgsino LSP at the LHC
  - $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_1^+$ close in mass

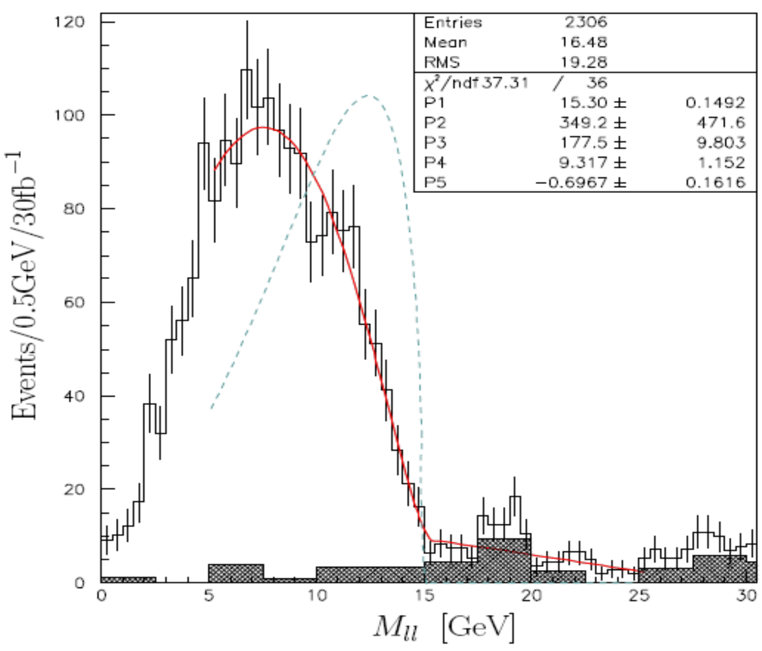
$$m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0} \simeq \frac{m_Z^2}{M_0} = O(10 \text{ GeV})$$

  - $\tilde{\chi}_2^0$ produced by $\tilde{q}/\tilde{g}$ decay:

$$\tilde{\chi}_2^0 \longrightarrow \tilde{\chi}_1^0 \, l^+ l^-$$

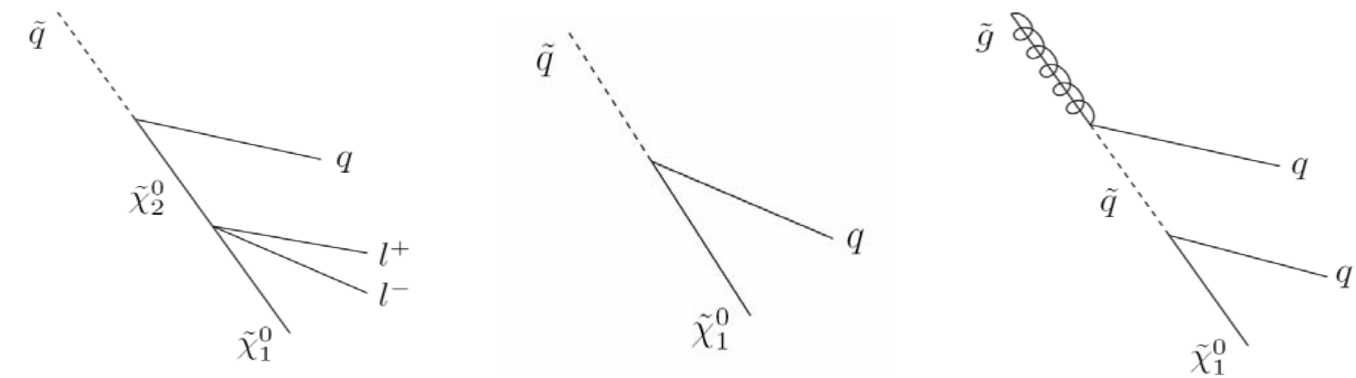
  - Small M_{ll} endpoint
  - Shape determined by the Higgsino nature of the LSP

(different from gauginos close in mass)

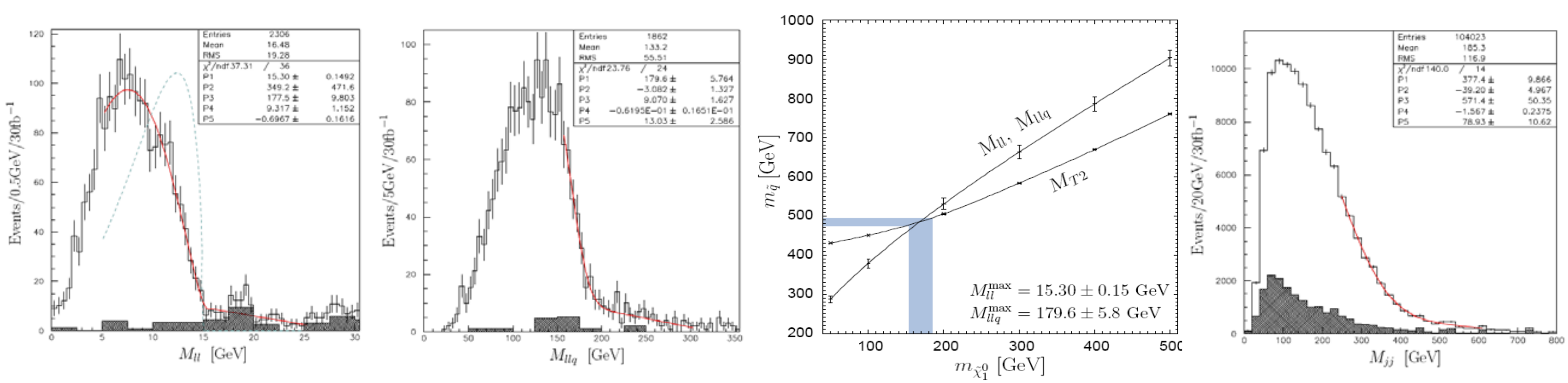


- All relevant masses determined despite short cascades
  - Use

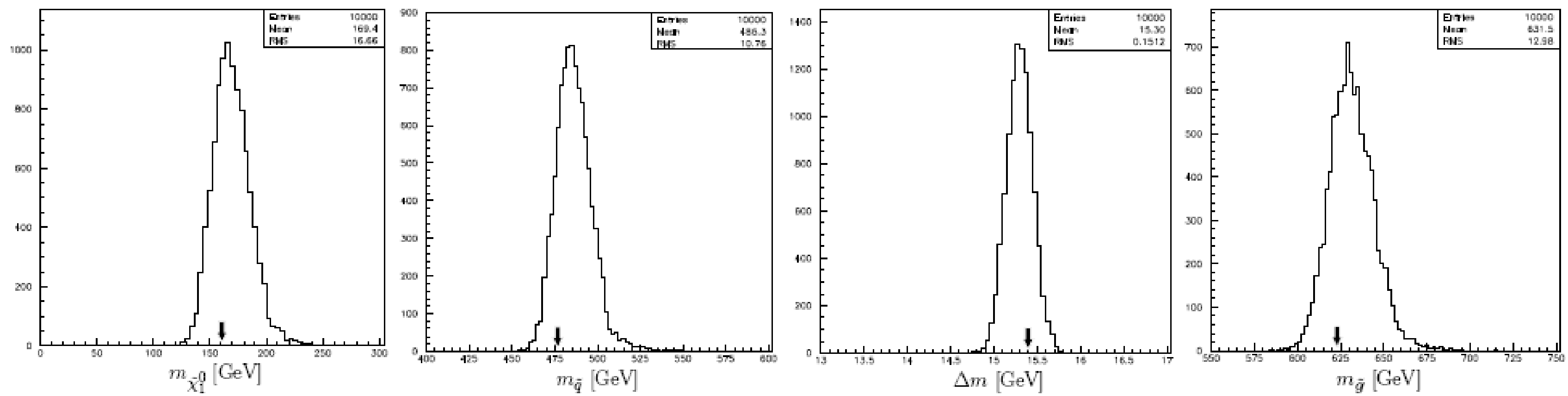
$\tilde{q} \rightarrow \tilde{\chi}_2^0 q \rightarrow \tilde{\chi}_1^0 l^+ l^- q$ $\tilde{q} \rightarrow \tilde{\chi}_1^0 q$ $\tilde{g} \rightarrow \tilde{q} q \rightarrow \tilde{\chi}_1^0 q q$



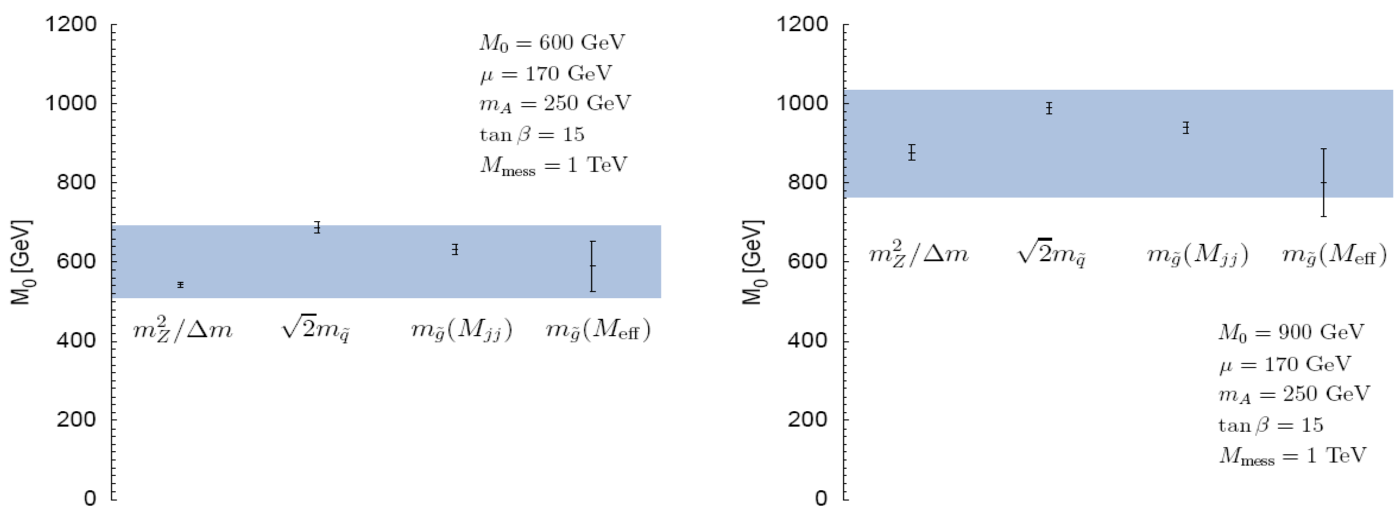
  - Fit M_{ll}, M_{llq}, M_{T2}, M_{jj} (M_{eff})



Determine $m_{\tilde{g}} \equiv M_1 \simeq M_2 \simeq M_3$, $m_{\tilde{q}} \equiv (m_Q^2)^{1/2} \simeq (m_U^2)^{1/2} \simeq (m_D^2)^{1/2}$, $m_{\tilde{\chi}_1^0}$, and $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ at a few to ten percent level.



- Model Discrimination Possible



Dark Matter (before the LHC ?)

Kitano, Y.N., PLB632, 162 (06)

- The lighter neutral Higgsino is the LSP

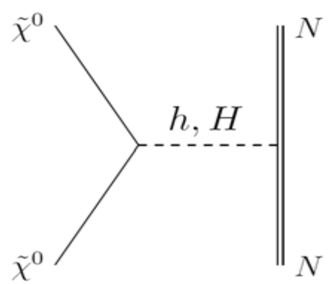
$$m_{\tilde{h}^0} \lesssim 190 \text{ GeV} \qquad \left(m_{\tilde{h}'^0} - m_{\tilde{h}^\pm} \simeq m_{\tilde{h}^\pm} - m_{\tilde{h}^0} \simeq \frac{m_Z^2}{2M_0} \right)$$

- Nonthermally produced

  e.g. Moduli → gravitino → LSP

- Direct detection

  t-channel Higgs boson exchange



Relevant parameters: bounded!

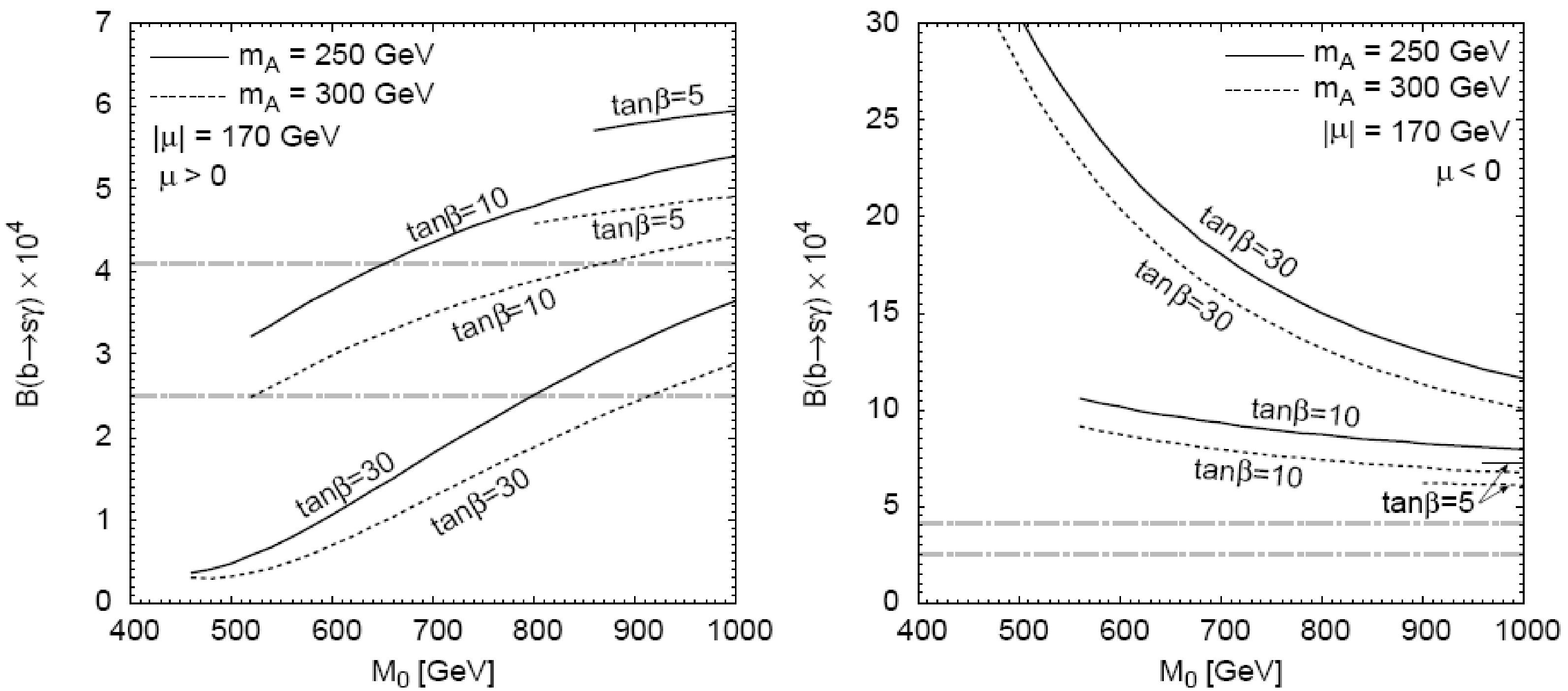
$$|\mu| \lesssim 190 \text{ GeV}, \quad M_0 \lesssim 900 \text{ GeV}, \quad \tan\beta \gtrsim 5,$$
$$m_h \lesssim 120 \text{ GeV}, \quad m_H \simeq m_A \lesssim 300 \text{ GeV}$$

Contributions from h and H^0 exchange are constructive (destructive) for sgn(μ) > 0 (< 0)

Solid lower bound on σ (SI cross section) ~ 10^{-44} obtained for $\mu > 0$!

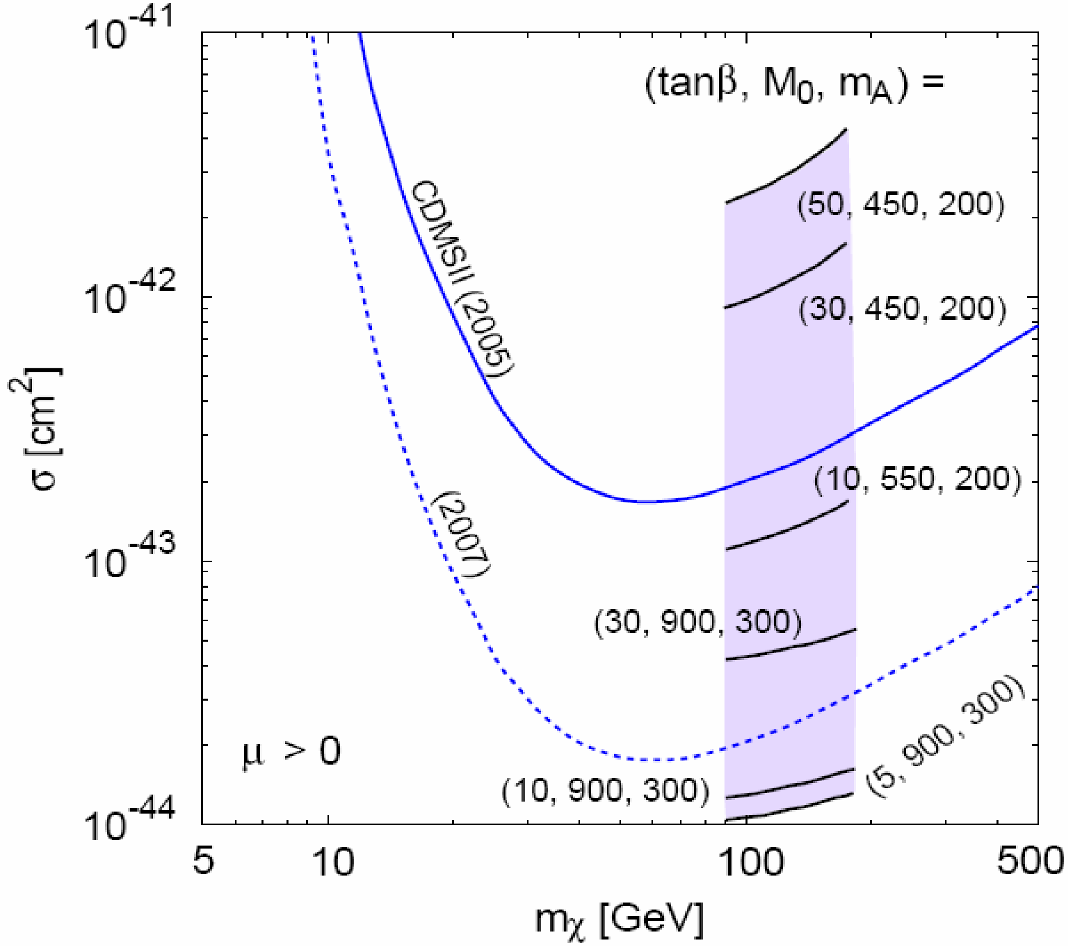
- The sign of μ determined from $b \to s\gamma$
  - The rate for $b \to s\gamma$ depends highly on sgn(μ), sgn(A_t)

Contributions from chargino and charged Higgs boson loops interfere destructively (constructively) for $\mu > 0$ (< 0)



$\mu > 0$ is chosen (also preferred from a_μ)

- Detection at CDMSII promising



  - A part of the relevant parameter space already excluded
  - A large portion will be covered by the end of 2007

Summary

- Naturalness (still) important guiding principle
- Use it as a powerful "model selector"

  (became possible after LEP II)
- What realization of SUSY at ~ TeV?
  - "squeezed" spectra
  - "well-ordered" spectra
- Mixed moduli-anomaly mediation (mirage)

  → eliminate fine-tuning
- LHC and dark matter signatures
  - Higgsino LSP
  - "degenerate" spectrum (model discrimination)