# Information engine with a physically realized feedback loop

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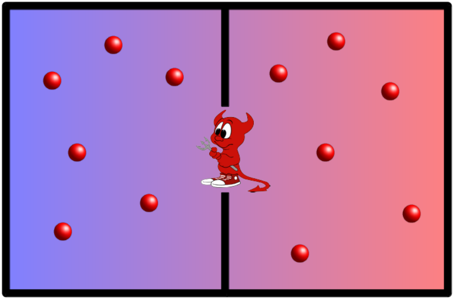
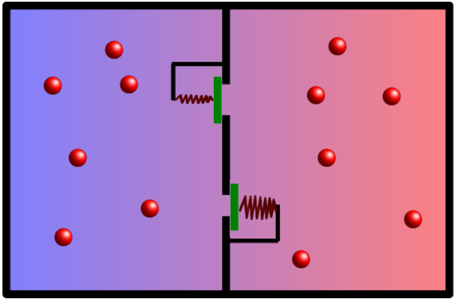
# Maxwell Demon (1867)



Does the physical implementation of a computational operation have a fundamental thermodynamic cost, purely by virtue of its logical properties?

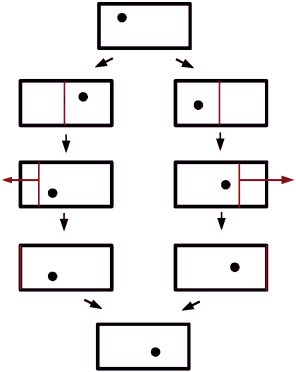
# Maxwell Demon

Smoluchowski (1914) replaces demon by a physical device.



Includes free energy of the springs.
$\Rightarrow$ Agreement with the Second Law in the long run.

# Szilard Engine (1929)



Idea: Measure where the particle is and extract work.

Second Law holds only if there was a compensating cost to perform the measurement.

# Cost of information processing

von Neumann (1949), Brillouin (1951, 1956), Gabor (1964) and Rothstein (1951)

Acquisition of information through a measurement requires a dissipation of at least $k_B T \ln 2$ energy for each bit.

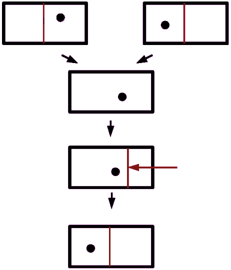
($T$ is the temperature in which the measurement takes place)

**Landauers principle** (1961):

Erasure of memory requires work.

($k_B T \ln 2$ per bit)

Reversible information processing (e.g. NOT) does not require work.



# Cost of information processing

Bennett (1973):

If the measurement device (memory) is in a well-defined state, we can carry out a **reversible measurement**:

| Sys. | Mem. | $\rightarrow$ | Sys. | Mem. |
|---|---|---|---|---|
| **0** | **0** | $\rightarrow$ | **0** | **0** |
| **1** | **0** | $\rightarrow$ | **1** | **1** |
| **0** | **1** | $\rightarrow$ | **0** | **1** |
| **1** | **1** | $\rightarrow$ | **1** | **0** |

$$\mathcal{M} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \Rightarrow \qquad \mathcal{M}^2 = 1$$

# Cost of information processing

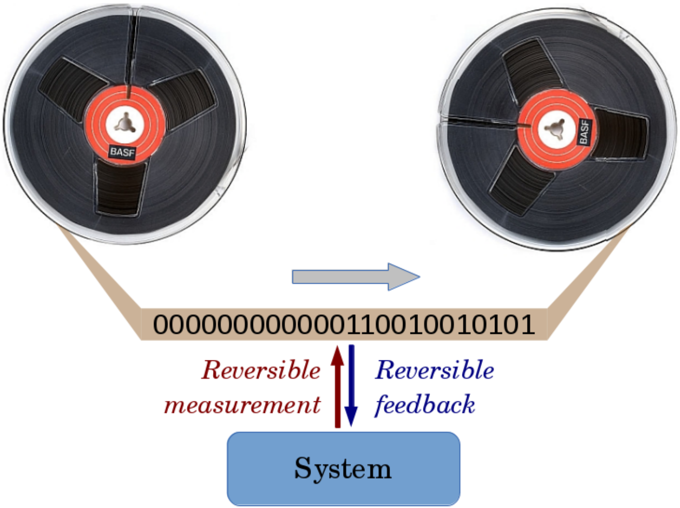
Having measured the system, bring it into a definite state.
This feedback operation is also reversible:

| Sys. | Mem. | $\rightarrow$ | Sys. | Mem. |
|---|---|---|---|---|
| **0** | **0** | $\rightarrow$ | **0** | **0** |
| **1** | **1** | $\rightarrow$ | **0** | **1** |
| **0** | **1** | $\rightarrow$ | **0** | **1** |
| **1** | **0** | $\rightarrow$ | **1** | **0** |

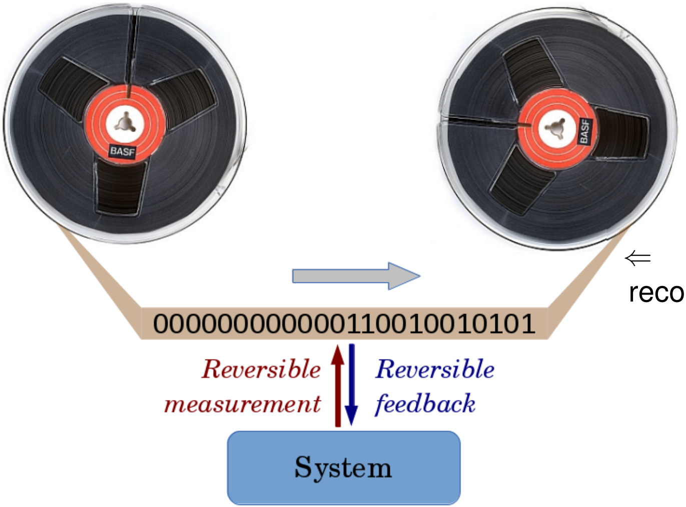
$$\mathcal{F} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \Rightarrow \qquad \mathcal{F}^2 = 1$$

**The whole feedback loop is reversible.**

# Cost of information processing

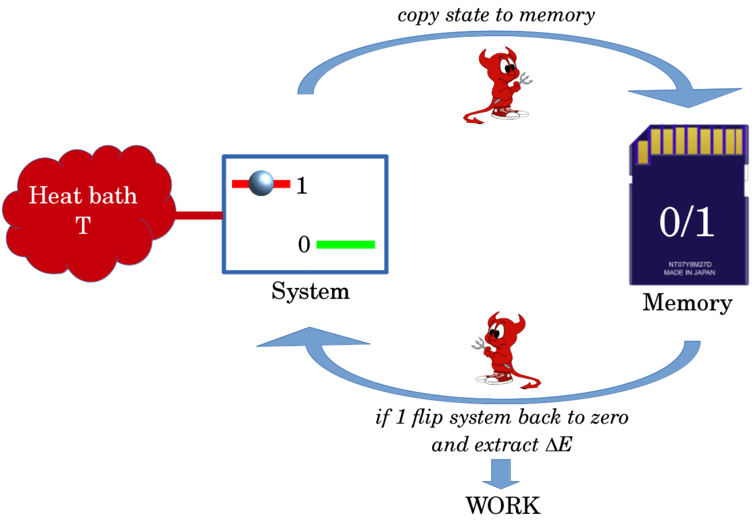


# Cost of information processing




$\Leftarrow$ Entropy recorded here!

# Information processing inequalities




Idea: Measure on which level the particle is and extract work.

# Information processing inequalities

T. Sagawa and M. Ueda, PRL 102, 250602 (2009)

$$\boxed{\text{Mean extracted work} \leq \text{Mutual information}}$$

T. Sagawa and M. Ueda, PRL 109, 180602 (2012)

Fluctuation theorem $\langle e^{-\Delta H_S^{tot} + \Delta I} \rangle = 1$

$$\boxed{\text{Mean extracted work} \leq \text{Mutual information difference}}$$

# Information processing inequalities

T. Sagawa and M. Ueda, PRL 109, 180602 (2012):

*“The entropy of a system can be decreased without any heat dissipation if we use the correlation as a resource of entropy decrease, although, in conventional thermodynamics, the entropy of the system is decreased only in the presence of heat dissipation.“*

# Information processing inequalities

$$\langle e^{-\Delta H_S^{tot}+\Delta I}\rangle = 1$$

- It is possible to obey the inequality sharply
  T. Sagawa and M. Ueda, PRE 85, 021104 (2012)
  J. M. Horowitz, T. Sagawa and J.M.R. Parrondo, PRL 111, 010602 (2013)
- Result be generalized to finite-time relaxation
  D. Abreu and U. Seifert, EPL 94, 10001 (2011)
  M. Bauer, D. Abreu, and U. Seifert, JPA 45, 162001 (2012)
- One can construct continuous feedback schemes
  H. Sandber, J-C. Delvenne, N.J. Newton, and S. K. Mitter, arXiv:1402.1010 (2014)
- Experimental confirmation (single electron box)
  J. V. Koski, V. F. Maisi, T. Sagwa, and J. P. Pekola, arXiv:1405.1272 (2014)

# Information processing inequalities

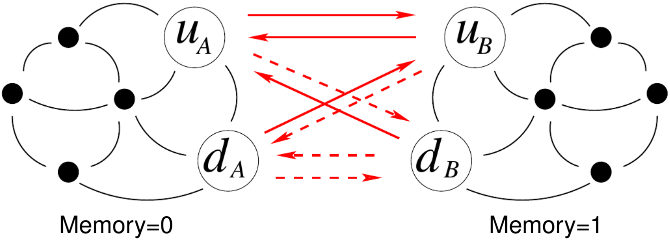
Problem: Depending on the setup there are different bounds for the extracted work.

$\Rightarrow$ **Look for unifying description.**

A. C. Barato, D. Hartich, and U. Seifert, J. Stat. Phys. 153, 460 (2013)
A. C. Barato and U. Seifert. EPL 101, 60001 (2013)
A. C. Barato, D. Hartich, and U. Seifert, PRE 87, 042104 (2013)
A. C. Barato and U. Seifert, PRL 112, 090601 (2014).

# Information processing inequalities

**Strategy:**
Implement system and memory physically.
Study the joint bipartite system.




$\Rightarrow$ **Unifying master FT for feedback and tape models.**

A. C. Barato and U. Seifert, PRL 112, 090601 (2014).

# Strategy

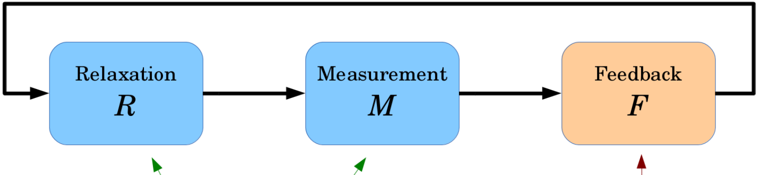
- Define a minimal model in terms of states and transition rates.
- Study entropy production and fluctuation theorem solely on this basis.
- Then introduce notion of energy and heat baths.
- Carefully distinguish flow of heat and work.

# Notations

- System/memory jointly described by configurations $c \in \Omega$
- Markov processes defined in by transitions rates $w_{c \to c'}$
- Probabilities $P_c(t)$ and currents $J_{c \to c'} = P_c(t) w_{c \to c'}$
- Master equation $\frac{\mathrm{d}}{\mathrm{d}t} P_c(t) = \sum_{c'} (J_{c' \to c}(t) - J_{c \to c'}(t))$
- For simplicity: Restrict to infinite-time relaxation
- Entropy $H = \langle H_c(t) \rangle$ with $H_c(t) = -\ln P_c(t)$
- Entropy production according to **Schnakenberg formula**: Whenever the system jumps to a different configuration, it generates a certain amount of entropy in the environment:

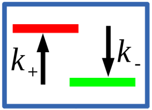
$$c \to c' \qquad \Rightarrow \qquad \Delta H^{env}_{c \to c'} = \ln \frac{w_{c \to c'}}{w_{c' \to c}}$$

# Minimal model – Closed feedback loop




Stochastic Markov processes with rates and master equation

Non-stochastic reversible mapping




# Step 1: Relaxation

$$\mathcal{L}_R = \underbrace{\begin{pmatrix} k_+ & -k_- \\ -k_+ & k_- \end{pmatrix}}_{\textit{System}} \otimes \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\textit{Memory}} = \begin{pmatrix} k_+ & 0 & -k_- & 0 \\ 0 & k_+ & 0 & -k_- \\ -k_+ & 0 & k_- & 0 \\ 0 & -k_+ & 0 & k_- \end{pmatrix}$$

Introduce rate ratio

$$\delta := \frac{k_+}{k_-}$$

Consider infinite-time relaxation into stationary state:

$$\mathcal{R} := \lim_{t\to\infty} e^{-\mathcal{L}_R t} = \begin{pmatrix} \frac{1}{\delta+1} & 0 & \frac{1}{\delta+1} & 0 \\ 0 & \frac{1}{\delta+1} & 0 & \frac{1}{\delta+1} \\ \frac{\delta}{\delta+1} & 0 & \frac{\delta}{\delta+1} & 0 \\ 0 & \frac{\delta}{\delta+1} & 0 & \frac{\delta}{\delta+1} \end{pmatrix}$$

# Step 2: Measurement

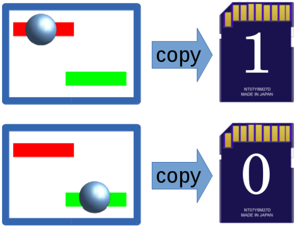
system $\otimes$ memory:

$$0 \otimes 0/1 \;\mapsto\; 0 \otimes 0$$
$$1 \otimes 0/1 \;\mapsto\; 1 \otimes 1$$

$$\mathcal{M} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$\Leftarrow$ write this as $\mathcal{M} = \lim_{t\to\infty} e^{-\mathcal{L}_M t}$



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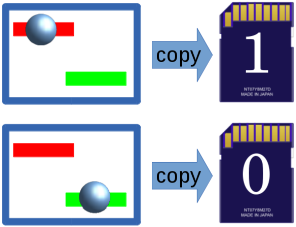
$$\mathcal{M} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$



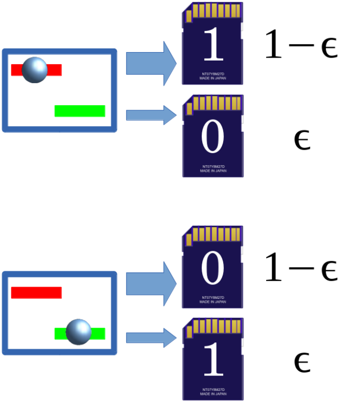

$\Rightarrow$ **Infinite entropy production**

# Step 2: Measurement

Realistic measurements:
Error parameter $\epsilon \ll 1$

$$\mathcal{M} = \begin{pmatrix} 1-\epsilon & 0 \\ 0 & \epsilon \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \epsilon & 0 \\ 0 & 1-\epsilon \end{pmatrix} \otimes \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} 1-\epsilon & 1-\epsilon & 0 & 0 \\ \epsilon & \epsilon & 0 & 0 \\ 0 & 0 & \epsilon & \epsilon \\ 0 & 0 & 1-\epsilon & 1-\epsilon \end{pmatrix}$$




$\Rightarrow$ **Finite entropy production**

# Step 2: Measurement

**The measurement can be represented as a stochastic process**

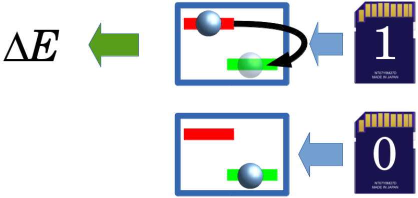
$$\mathcal{L}_M = \begin{pmatrix} \epsilon & \epsilon - 1 & 0 & 0 \\ -\epsilon & 1-\epsilon & 0 & 0 \\ 0 & 0 & 1-\epsilon & -\epsilon \\ 0 & 0 & \epsilon - 1 & \epsilon \end{pmatrix}$$

$$\mathcal{M} := \lim_{t\to\infty} e^{-\mathcal{L}_M t} = \begin{pmatrix} 1-\epsilon & 1-\epsilon & 0 & 0 \\ \epsilon & \epsilon & 0 & 0 \\ 0 & 0 & \epsilon & \epsilon \\ 0 & 0 & 1-\epsilon & 1-\epsilon \end{pmatrix}$$

# Step 3: Feedback

If memory=1 flip system state, else do nothing.

$$\mathcal{F} = \begin{pmatrix} \mathbf{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{1} \\ 0 & 0 & \mathbf{1} & 0 \\ 0 & \mathbf{1} & 0 & 0 \end{pmatrix}$$



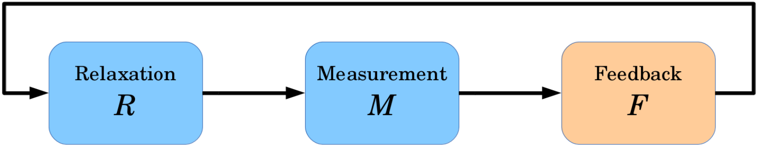

$$\mathcal{F}^2 = \mathbf{1}$$

$\Rightarrow$ $\mathcal{F}$ interchanges the second and fourth vector component

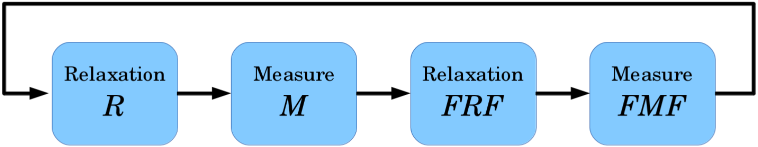
$\Rightarrow$ **Reversible, no entropy production**

$\Rightarrow$ **Not representable as a stochastic process**

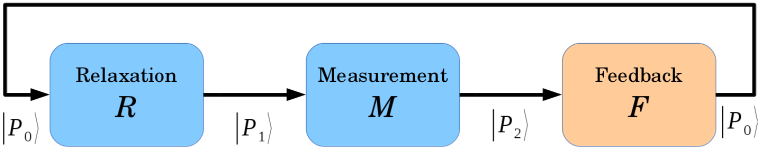
# Total cycle of the engine




is equivalent to the fully stochastic process



# Total cycle of the engine




Stationary probability vectors in the cycle:

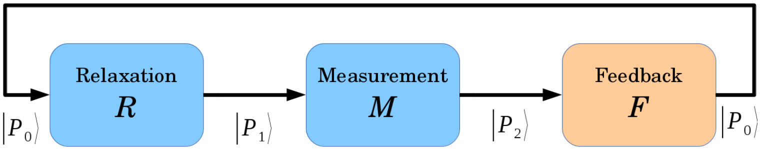
$$\mathcal{FMR}|P_0\rangle = |P_0\rangle \quad \Rightarrow \quad |P_0\rangle = \left\{ \frac{1-\epsilon}{\delta+1}, \frac{\delta-\delta\epsilon}{\delta+1}, \frac{\delta\epsilon}{\delta+1}, \frac{\epsilon}{\delta+1} \right\}$$

$$|P_1\rangle = \mathcal{R}|P_0\rangle = \left\{ \frac{(\delta-1)\epsilon+1}{(\delta+1)^2}, \frac{\delta(-\epsilon)+\delta+\epsilon}{(\delta+1)^2}, \frac{\delta((\delta-1)\epsilon+1)}{(\delta+1)^2}, \frac{\delta(\delta(-\epsilon)+\delta+\epsilon)}{(\delta+1)^2} \right\}$$

$$|P_2\rangle = \mathcal{M}|P_1\rangle = \left\{ \frac{1-\epsilon}{\delta+1}, \frac{\epsilon}{\delta+1}, \frac{\delta\epsilon}{\delta+1}, \frac{\delta-\delta\epsilon}{\delta+1} \right\}$$

...all depending only on $\delta$ and $\epsilon$.

# Questions

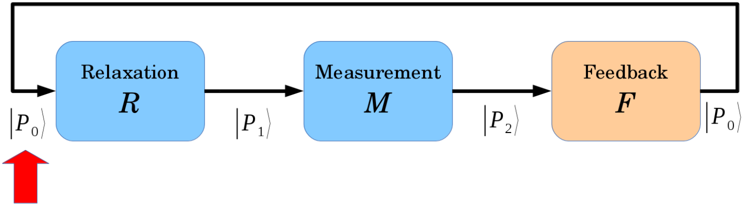



- System entropy $H^{(s)}$
- Memory entropy $H^{(m)}$
- Compound entropy $H^{(sm)}$
- Mutual information $I = H^{(s)} + H^{(m)} - H^{(sm)}$
- Entropy production $\Delta H^{(env)}$

- Heat $\Delta Q$
- Work $\Delta W$

Heat and work need thermal reservoirs.

# Component entropies and mutual information




$$H_0^{(s)} = (\epsilon - 1)\ln(1 - \epsilon) - \epsilon \ln(\epsilon)$$

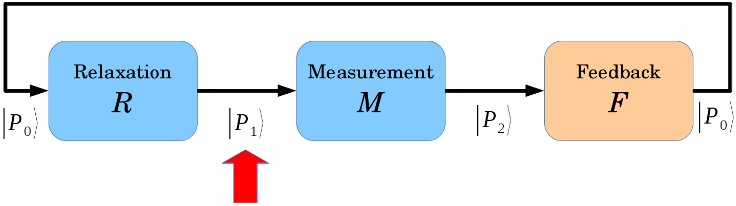
$$H_0^{(m)} = \frac{(-\delta\epsilon+\epsilon-1)\ln((\delta-1)\epsilon+1)+((\delta-1)\epsilon-\delta)\ln(\delta(-\epsilon)+\delta+\epsilon)+(\delta+1)\ln(\delta+1)}{\delta+1}$$

$$H_0^{(sm)} = \frac{(\delta+1)\epsilon\ln\left(\frac{1}{\epsilon}-1\right)+\ln\left(\frac{\delta+1}{1-\epsilon}\right)+\delta\ln\left(\frac{\delta+1}{\delta-\delta\epsilon}\right)}{\delta+1}$$

$$I_0 = \frac{(-\delta\epsilon+\epsilon-1)\ln((\delta-1)\epsilon+1)+((\delta-1)\epsilon-\delta)\ln(\delta(-\epsilon)+\delta+\epsilon)+\delta\ln(\delta)}{\delta+1}$$

Memory correlated from previous cycle.

# Component entropies and mutual information




$$H_1^{(s)} = \ln(\delta + 1) - \frac{\delta \ln(\delta)}{\delta+1}$$

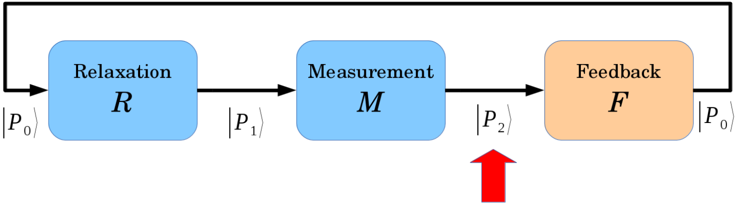
$$H_1^{(m)} = \frac{(-\delta\epsilon+\epsilon-1)\ln((\delta-1)\epsilon+1)+((\delta-1)\epsilon-\delta)\ln(\delta(-\epsilon)+\delta+\epsilon)+(\delta+1)\ln(\delta+1)}{\delta+1}$$

$$H_1^{(sm)} = \frac{(-\delta\epsilon+\epsilon-1)\ln((\delta-1)\epsilon+1)+(\delta(\epsilon-1)-\epsilon)\ln(\delta(-\epsilon)+\delta+\epsilon)-\delta\ln(\delta)+2(\delta+1)\ln(\delta+1)}{\delta+1}$$

$$I_1 = 0$$

Memory uncorrelated after infinite relaxation.

# Component entropies and mutual information




$$H_2^{(s)} = \ln(\delta + 1) - \frac{\delta \ln(\delta)}{\delta+1}$$

$$H_2^{(m)} = \frac{(-\delta\epsilon+\epsilon-1)\ln((\delta-1)\epsilon+1)+((\delta-1)\epsilon-\delta)\ln(\delta(-\epsilon)+\delta+\epsilon)+(\delta+1)\ln(\delta+1)}{\delta+1}$$

$$H_2^{(sm)} = \frac{(\delta+1)\epsilon\ln\left(\frac{1}{\epsilon}-1\right)+\ln\left(\frac{\delta+1}{1-\epsilon}\right)+\delta\ln\left(\frac{\delta+1}{\delta-\delta\epsilon}\right)}{\delta+1}$$

$$I_2 = \frac{1}{\delta+1}\left[\delta\ln(-(\delta+1)(\epsilon-1)) + \ln\left(-\frac{(\delta+1)(\epsilon-1)}{(\delta-1)\epsilon+1}\right) + \right.$$
$$\left.\epsilon\left(\delta\ln\left(-\frac{\epsilon}{(\epsilon-1)((\delta-1)\epsilon+1)}\right) + \ln\left(\frac{\epsilon(-\delta\epsilon+\epsilon-1)}{\epsilon-1}\right)\right) + ((\delta-1)\epsilon-\delta)\ln(\delta(-\epsilon)+\delta+\epsilon)\right]$$

System entropy unchanged.

# Entropy production

**Generally entropy production has to be computed by integration**

$$\frac{\mathrm{d}}{\mathrm{d}t}H^{env}(t) = \sum_{c \neq c'} J_{c \to c'} \ln \frac{w_{c \to c'}}{w_{c' \to c}}$$
$$= \sum_{c \neq c'} P_c(t) w_{c \to c'} \ln \frac{w_{c \to c'}}{w_{c' \to c}}$$

Solve master equation to get $P_c(t)$ and compute the integral

$$\Delta H^{env} = \int_0^T \mathrm{d}t \dot{H}^{env}(t)$$

But the present model all processes obey **detailed balance.** In this case the entropy production is easy to compute.

# Detailed balance

- **Standard definition:**
  Probability currents balance one another in equilibrium:

$$P_c^{stat} w_{c \to c'} = P_{c'}^{stat} w_{c' \to c} \qquad \forall c, c'$$

- **Equivalent definition without using stationary state:**
  For all *closed* stochastic paths $c_1 \to c_2 \to \ldots \to c_N \to c_1$ we have

$$\frac{w_{c_1 \to c_2} w_{c_2 \to c_3} \cdots w_{c_N \to c_1}}{w_{c_1 \to c_N} w_{c_N \to c_{N-1}} \cdots w_{c_2 \to c_1}} = \prod_i \frac{w_{c_i \to c_{i+1}}}{w_{c_{i+1} \to c_i}} = 1$$

- **Equivalent definition via entropy production:**
  The entropy production on closed stochastic paths vanishes:

$$\sum_i \ln \frac{w_{c_i \to c_{i+1}}}{w_{c_{i+1} \to c_i}} = \sum_i \Delta H^{env}_{c \to c'} = 0.$$

# Detailed balance and entropy production

Detailed balance $\Leftrightarrow$ No entropy production along closed paths.

Then there exists a potential $V_c$ such that $\Delta H^{env}_{c\to c'} = V_{c'} - V_c$

$$\Rightarrow \qquad \frac{w_{c\to c'}}{w_{c'\to c}} = \frac{\exp(v_{c'})}{\exp(v_c)}$$

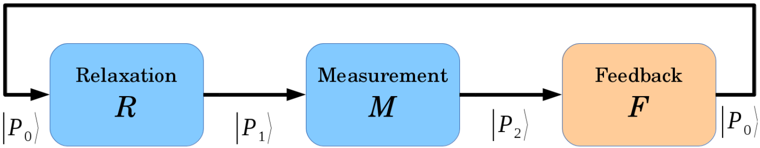
$$\Rightarrow \qquad V_c = const + \ln P^{stat}_c$$

$$\Rightarrow \qquad \Delta H^{env}_{c\to c'} = \ln \frac{P^{stat}_{c'}}{P^{stat}_c}$$

**Direct computation without integration:**

$$\Delta H^{env} = -\sum_c \Big(P_c(T) - P_c(0)\Big) \ln P^{stat}_c$$

# Entropy production



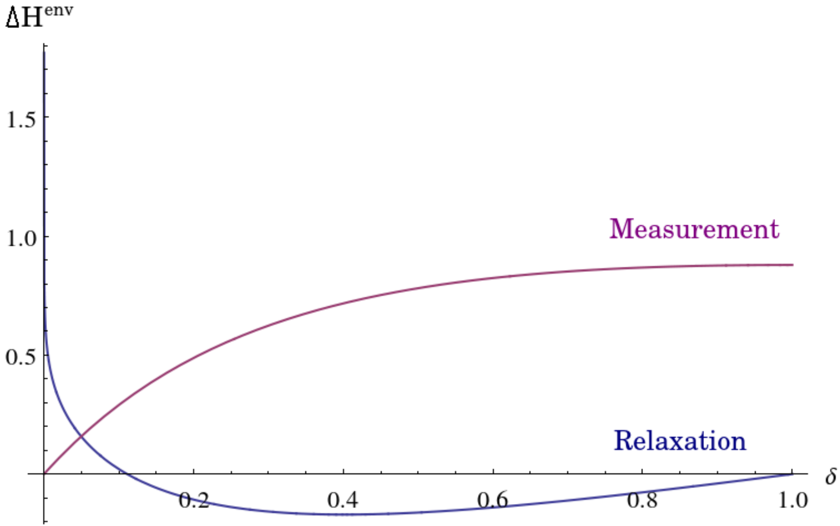

$$\Delta H_R^{env} = -\frac{(\delta\epsilon - \delta + \epsilon)\ln\delta}{\delta + 1}$$ positive or negative

$$\Delta H_M^{env} = \frac{2\delta(1-2\epsilon)\ln\left(\frac{1}{\epsilon} - 1\right)}{(\delta+1)^2}$$ always positive

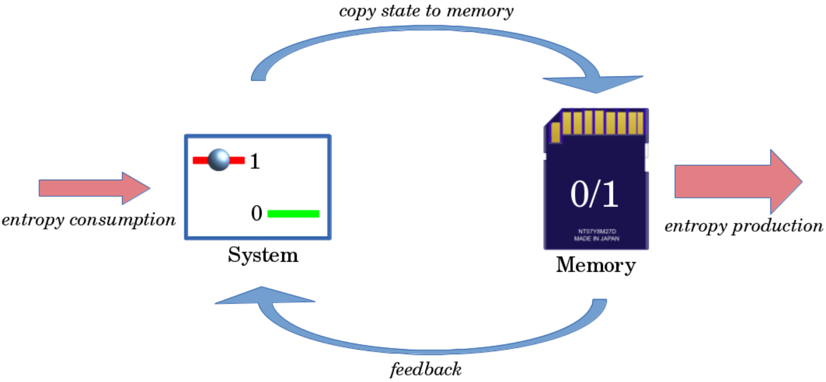
$$\Delta H_F^{env} = 0$$

# Entropy production

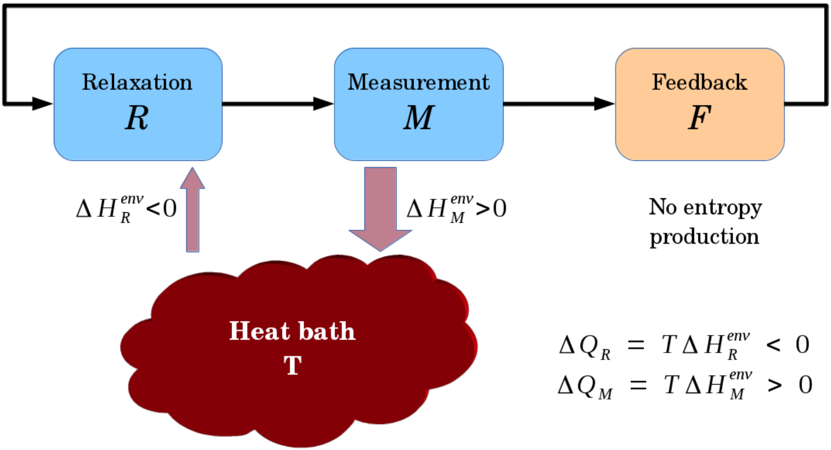
$\epsilon = 0.1$



# Negative entropy production during relaxation



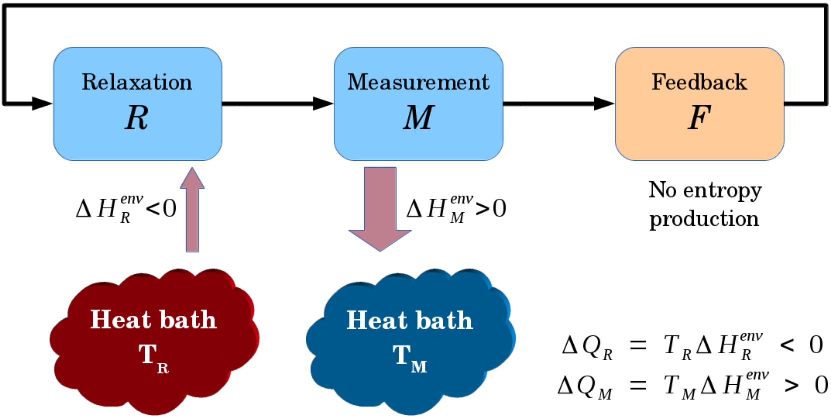
# Heat




$$\Delta Q_R = T \Delta H_R^{env} < 0$$
$$\Delta Q_M = T \Delta H_M^{env} > 0$$

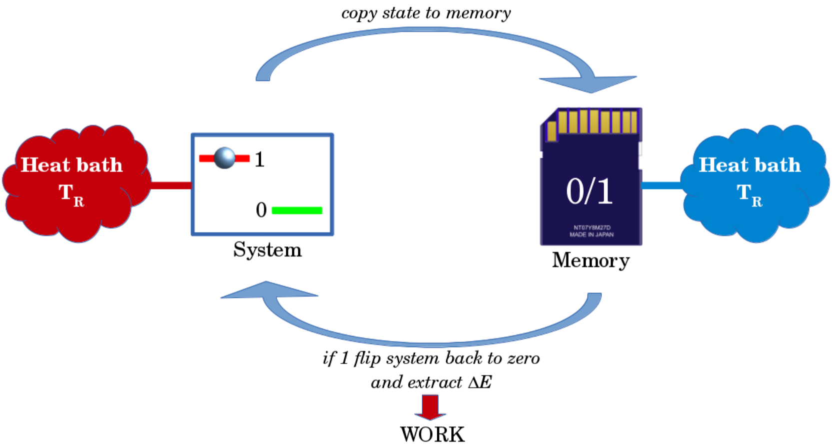
# Heat

There could be two different heat baths.



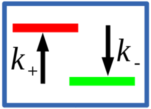

$$\Delta Q_R = T_R \Delta H_R^{env} < 0$$
$$\Delta Q_M = T_M \Delta H_M^{env} > 0$$

# Work extraction during feedback



# Work extraction during feedback

Upper level: probability $p_1 = \frac{\delta}{\delta+1} \sim e^{-\beta E_u}$
Lower level: probability $p_0 = \frac{1}{\delta+1} \sim e^{-\beta E_d}$

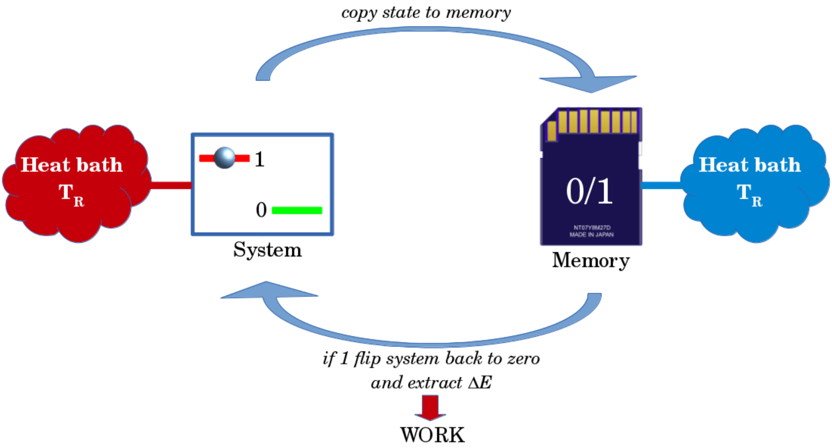



Energy difference between levels: $\Delta E = -\frac{1}{\beta} \ln \frac{p_u}{p_d} = -\frac{1}{\beta} \ln \delta$

| system | memory | probability | energy gain |
|---|---|---|---|
| 0 | 0 | $\frac{1-\epsilon}{\delta+1}$ | 0 |
| 0 | 1 | $\frac{\epsilon}{\delta+1}$ | $-\Delta E$ |
| 1 | 0 | $\frac{\epsilon\delta}{\delta+1}$ | 0 |
| 1 | 1 | $\frac{(1-\epsilon)\delta}{\delta+1}$ | $+\Delta E$ |

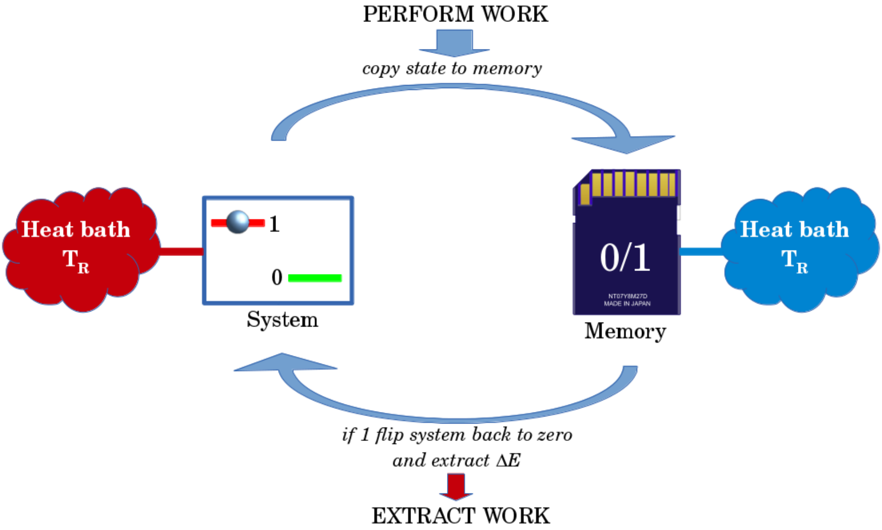
Average energy gain per cycle: $\langle \Delta E \rangle = \frac{\delta\epsilon+\epsilon-\delta}{\beta(\delta+1)} \ln \delta = \frac{1}{\beta} H_R^{env}$

# Work extraction during feedback




Energy conservation?

# Work extraction and supply



# Heat and work in general

In stoachastic dynamics, Energy is implemented as a map

$$c \mapsto E_c$$

The actual energy of a system can change by

- **the stochastic evolution:**
  Spontaneous transition $c \to c'$, causing $\Delta E = E_{c'} - E_c$, compensated by an energy export $\Delta Q = -\Delta E$ to the environment, called **heat**.
- **an explicit change of the map:**
  realiazed by an (adiabatic) modification of the Hamiltonian $E_c \to E_c + \delta E_c$, taken from or delivered to the environment, called **work** $\delta W = \delta E$.

# Heat and Work in general

- **Average flow of heat:**

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle Q(t)\rangle = -\frac{\mathrm{d}}{\mathrm{d}t}\langle E(t)\rangle = -\sum_{c\neq c'} J_{c\to c'}\,\Delta E_{c\to c'}$$
$$= \sum_{c} P_c(t)\sum_{c'} w_{c\to c'}(E_c - E_{c'})$$

- **Average work done on the system:**

$$\langle \delta Q(t)\rangle = \sum_{c} P_c(t)\delta E_c$$

Any change of the energy levels requires work.
Only in special cases the average work may vanish.

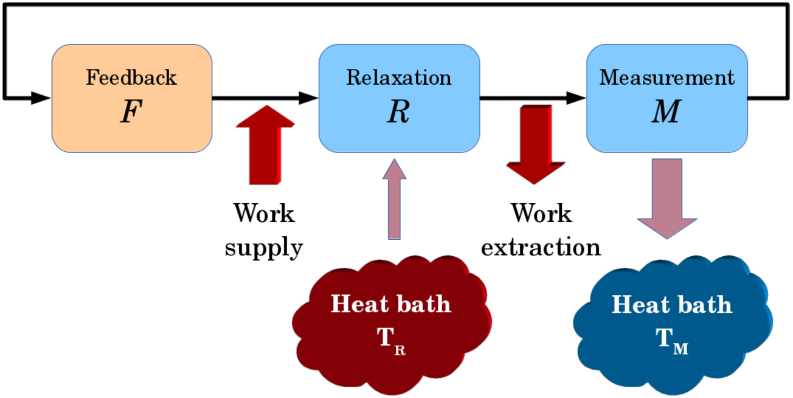
## Cost of changing transition rates

In systems in contact with a heat bath obeying detailed balance, the rates and the temperature determine the energy levels up to a constant:

$$\{w_{c\to c'}\} \quad \Rightarrow \quad \{P_c^{stat}\} \quad \Rightarrow \quad E_c = -\frac{1}{\beta}\ln P_c^{stat} + const$$

- Each change of the transitions rates requires/renders work.
- Choosing the constant the average work can be set to zero.
- In a closed cycle this cannot be done everywhere.

# Heat transfer happens in two places:



# Efficiency

Total work:

$$\Delta W = \Delta W_R + \Delta W_M$$

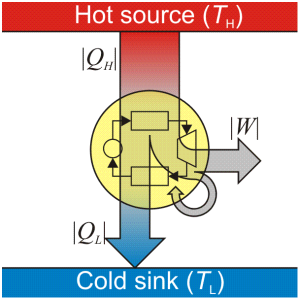
Total efficiency:

$$\eta = \frac{\Delta W}{\Delta Q_R}$$

Maximum for $\eta, \delta \to 0$:

$$\eta = 1 - \frac{2T_{cold}}{T_{warm}}$$

worse than Carnot!




Conventional heat engine

$$\eta = \frac{\gamma(1+\delta)(\delta(\epsilon-1)+\epsilon)\ln\delta + 2\beta\delta(2\epsilon-1)\ln(1/\epsilon-1)}{\gamma(1+\delta)(\delta(\epsilon-1)+\epsilon)\ln\delta}$$

# Fluctuation theorems in two steps

- **During relaxation:**
  $\Delta H_{SM} = \Delta I$

  $\Rightarrow \langle \beta e^{-\Delta W_R - \Delta I} \rangle = 1$

  $\Rightarrow \langle \beta \Delta W_R + \Delta I \rangle \geq 0$

  $\boxed{\beta \langle W_R \rangle \;\geq\; -\Delta I}$

- **During measurement:**
  $\Delta H_{SM} = -\Delta I$

  $\Rightarrow \langle \gamma e^{-\Delta W_R + \Delta I} \rangle = 1$

  $\Rightarrow \langle \gamma \Delta W_M - \Delta I \rangle \geq 0$

  $\boxed{\gamma \langle W_R \rangle \;\geq\; +\Delta I}$

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- Describe Maxell-Deamon and Szilard setups purely in terms of stochastic processes and reversible transformations.

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- Change of rates requires work from outside.
- Efficiency of the system without memory $\rightarrow$ known results.
- Efficiency of the system and memory together like ordinary heat engine.

**Thank you!**