Alignment vs noise:

Simple models and continuous theories for dry active matter

Hugues Chaté

CEA-Saclay, France

references upon request
hugues.chate@cea.fr

Joint work with

Igor Aronson
Denis Bartolo
Eric Bertin
Francesco Ginelli
Sriram Ramaswamy
Julien Tailleur

Jean-Baptiste Caussin
Sandrine Ngo
Anton Peshkov
Alexandre Solon

Active matter

- Definition: Energy is spent *locally* to produce directed, persistent, non-random motion
- Examples abound: in biology (animals, cells, motor proteins…) but not only (micro- and nano-swimmers, ‘smart’ colloids, robots…)
- Largely unexplored, novel collective properties
- “Swarm intelligence”, self-organized dynamical structures, new materials…

Minimal setting for collective motion / active matter:

Alignment of self-propelled/active particles in competition with noise (Vicsek et al., 1995)

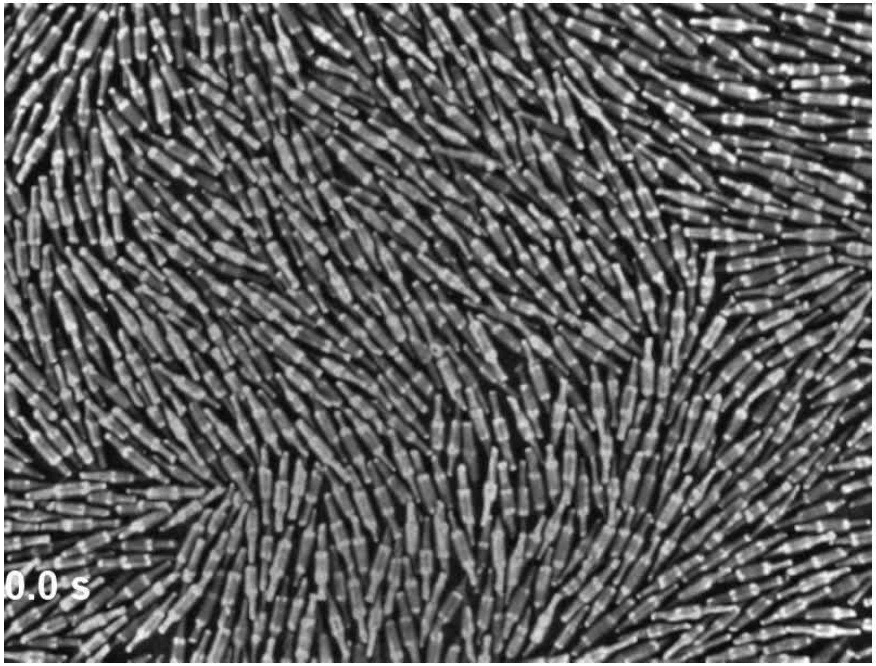
- Effective alignment
- no attraction, no repulsion, neglect surrounding fluid...
- no momentum conservation (substrate as sink)

Minimal situation of theoretical interest,
but some direct experimental relevance

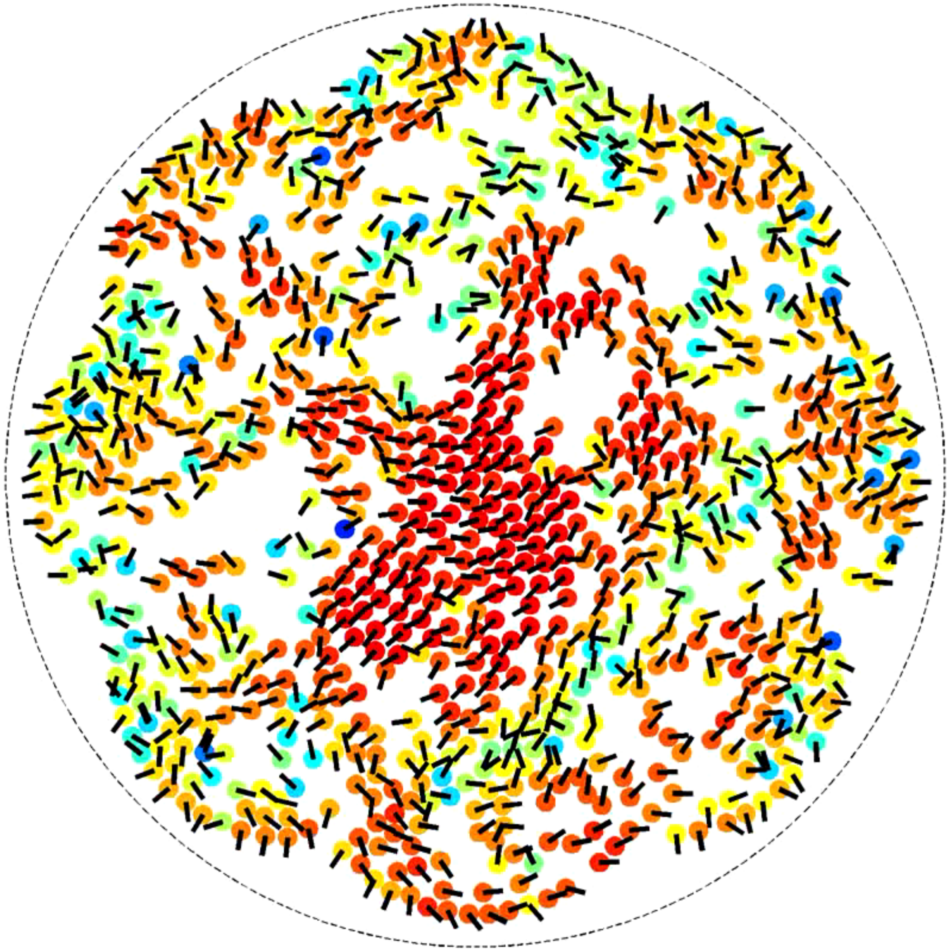
Most convincing examples so far:

- shaken granular particles,
- microtubule motility assay
- possibly Bartolo's rolling colloids

Shaken granular particles:



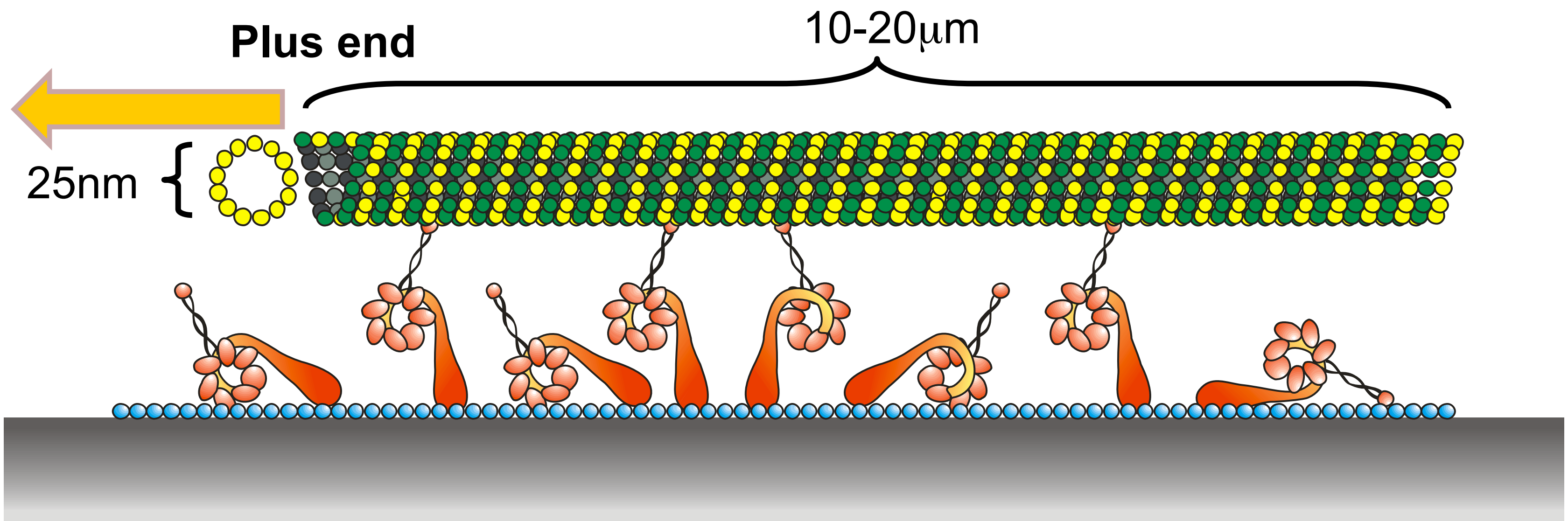
Nematic
(Narayan et al.)



Polar
(Deseigne et al.)

In vitro motility assay: dyneins + microtubules

(Sumino et al.)



- Dynein-c motor proteins, grafted on a substrate, move stabilized microtubules
- with high density of motors (1000/um^2), smooth, constant-speed motion of single MT

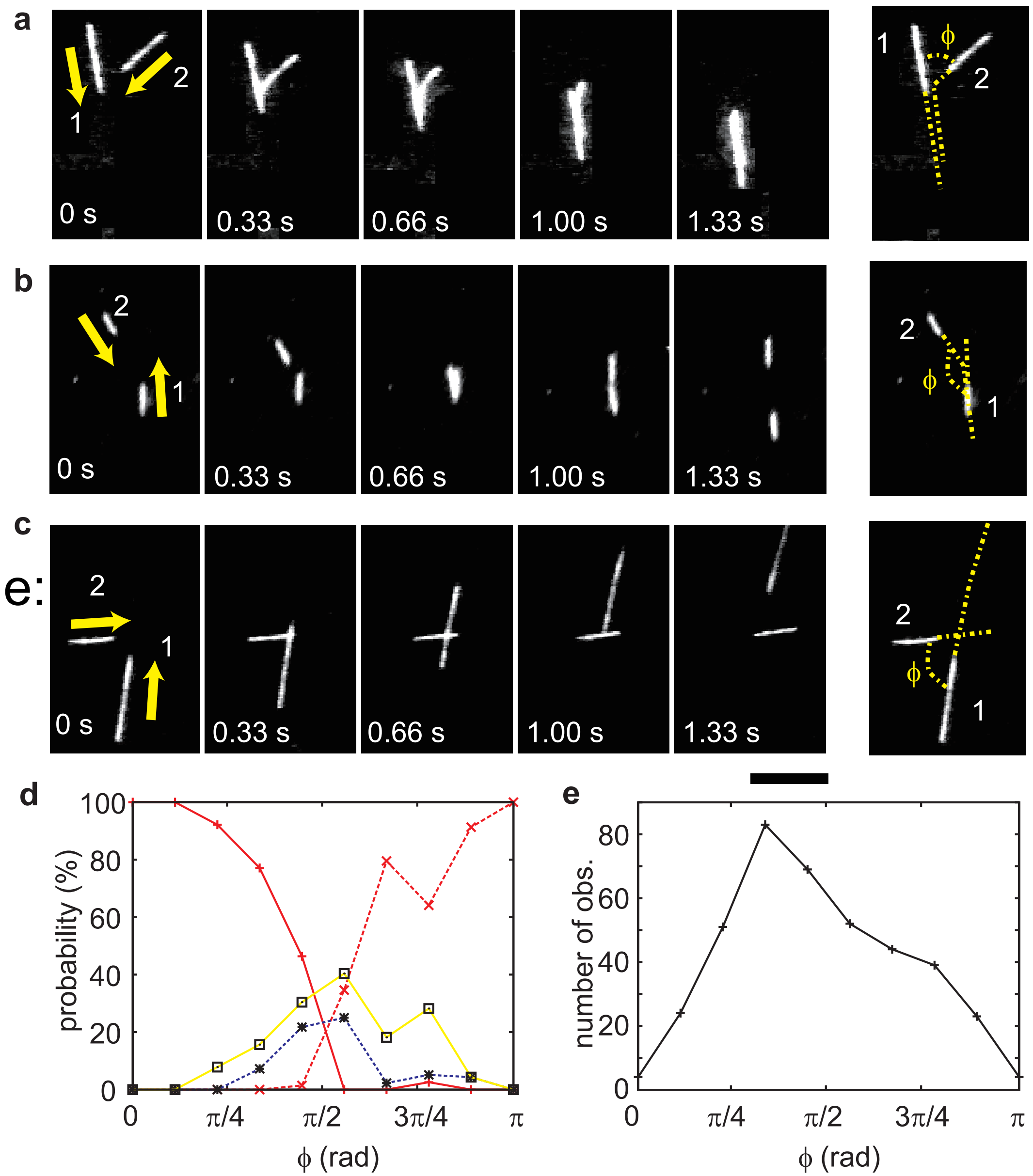
Near-perfect nematic alignment via collisions

Acute incoming angle:
Complete alignment

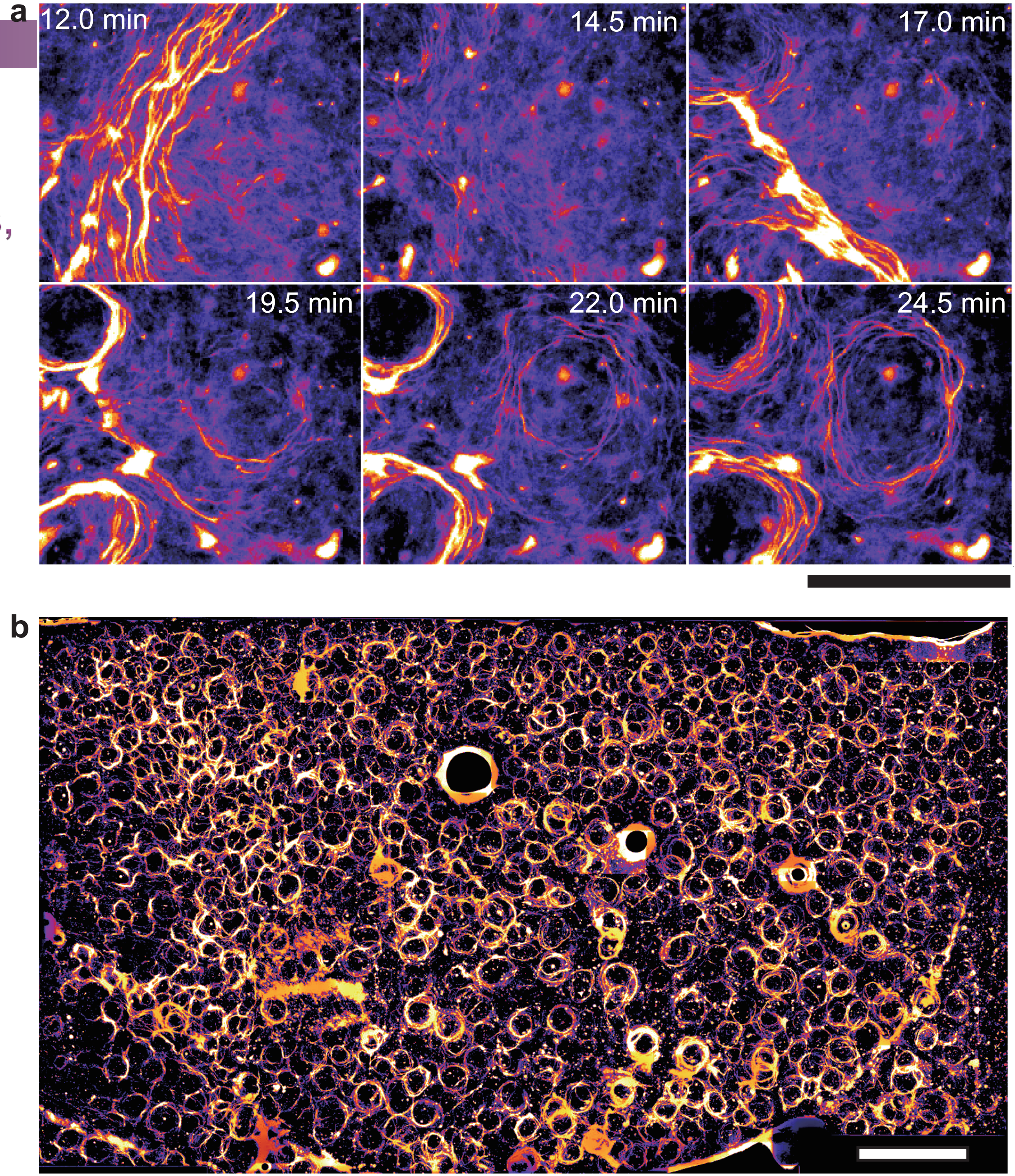
Obtuse incoming angle:
Complete anti-alignment

(Near-) right incoming angle:
Crossing (or stopping)

Statistics over some 400 binary collisions



At high density of filaments, not quite nematic order…, but lattice of large vortices



Scale bars:

500 µm (top)

2 mm (bottom)

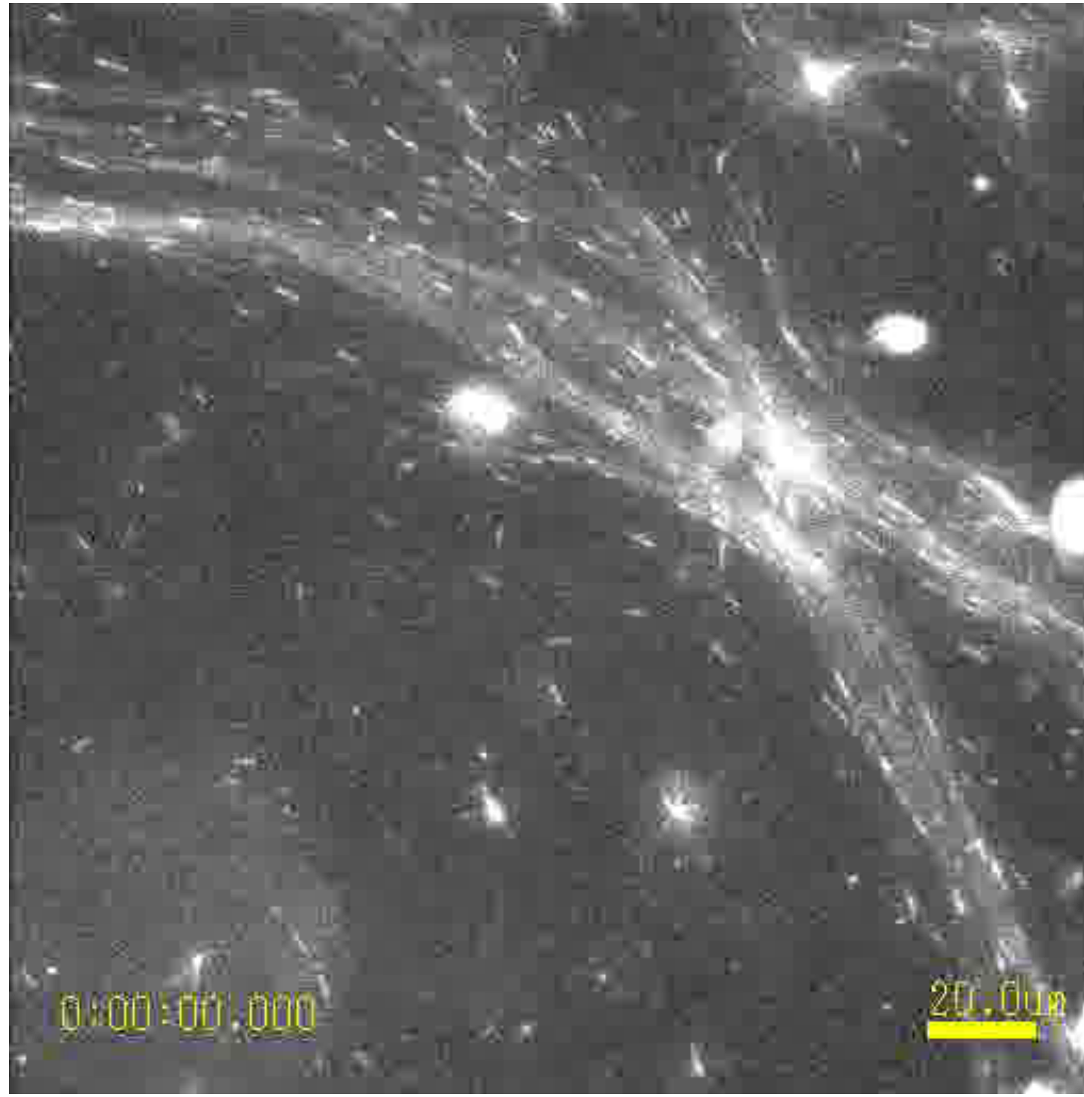
Microtubules: 10 µm

Collective motion of millions of microtubules

Local nematic order

Key ingredients (smooth random walks, nematic alignment) enough to account for emergence fo large-scale vortices

➔ Alignment vs noise



Outline of rest of talk

- 3 classes of Vicsek-style models
- Global view on phenomenology of particle models
- ‘Boltzmann-Ginzburg-Landau’ approach
- Global view on hydrodynamic descriptions

Vicsek-style models:

- Constant-speed point particles move off-lattice
- local alignment within unit distance
- In competition with noise
- 2 main parameters: global density and noise strength

3 possible classes depending on symmetry:

- Polar particles with ferromagnetic alignment (original VM)
- Apolar nematic particles with nematic alignment (“active nematics”)
- Polar particles with nematic alignment (“self-propelled rods”)

Today only original VM and active nematics

Why study such silly models?

- “Ising models” of collective motion, if not active matter (genericity, universality…)
- Good starting point to derive continuous descriptions in a controlled manner, with explicit dependence of all transport coefficients on local density and « microscopic » parameters
- Reference framework to evaluate faithfulness of continuous theories
- Continuous descriptions hopefully mostly contain crucial terms
- Most of more « realistic » models include Vicsek ingredients

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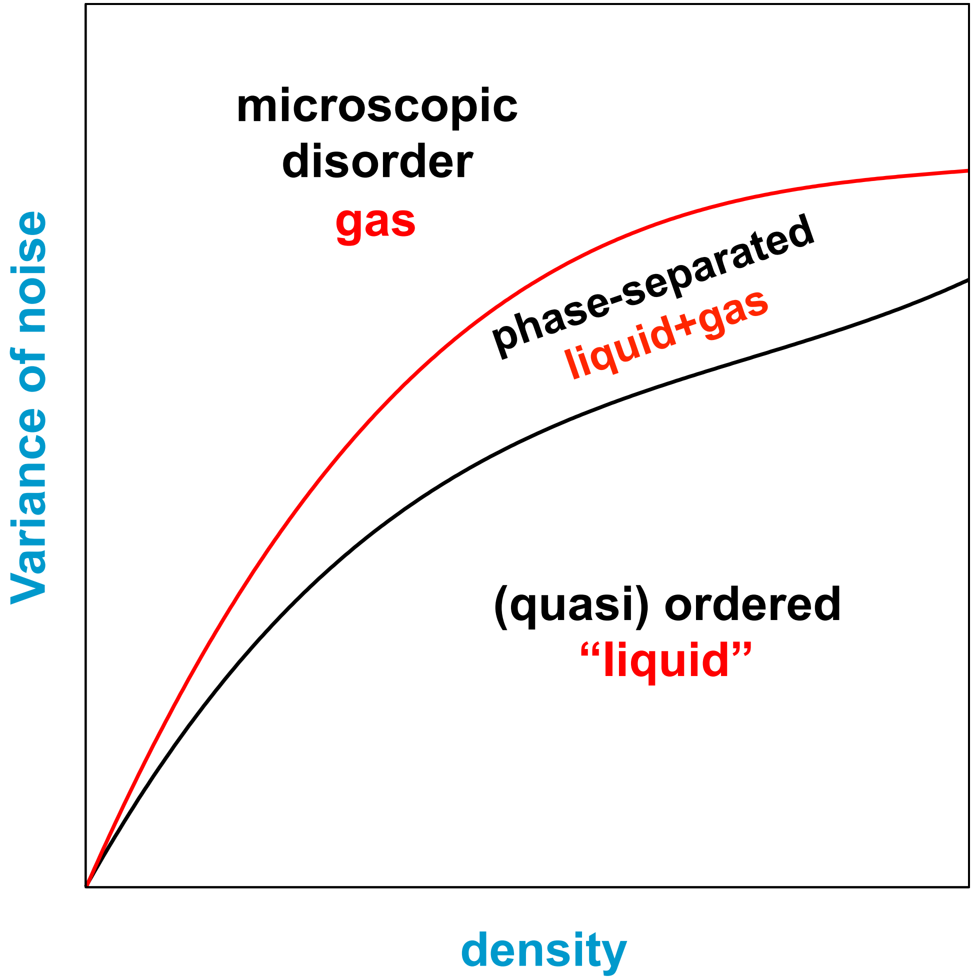
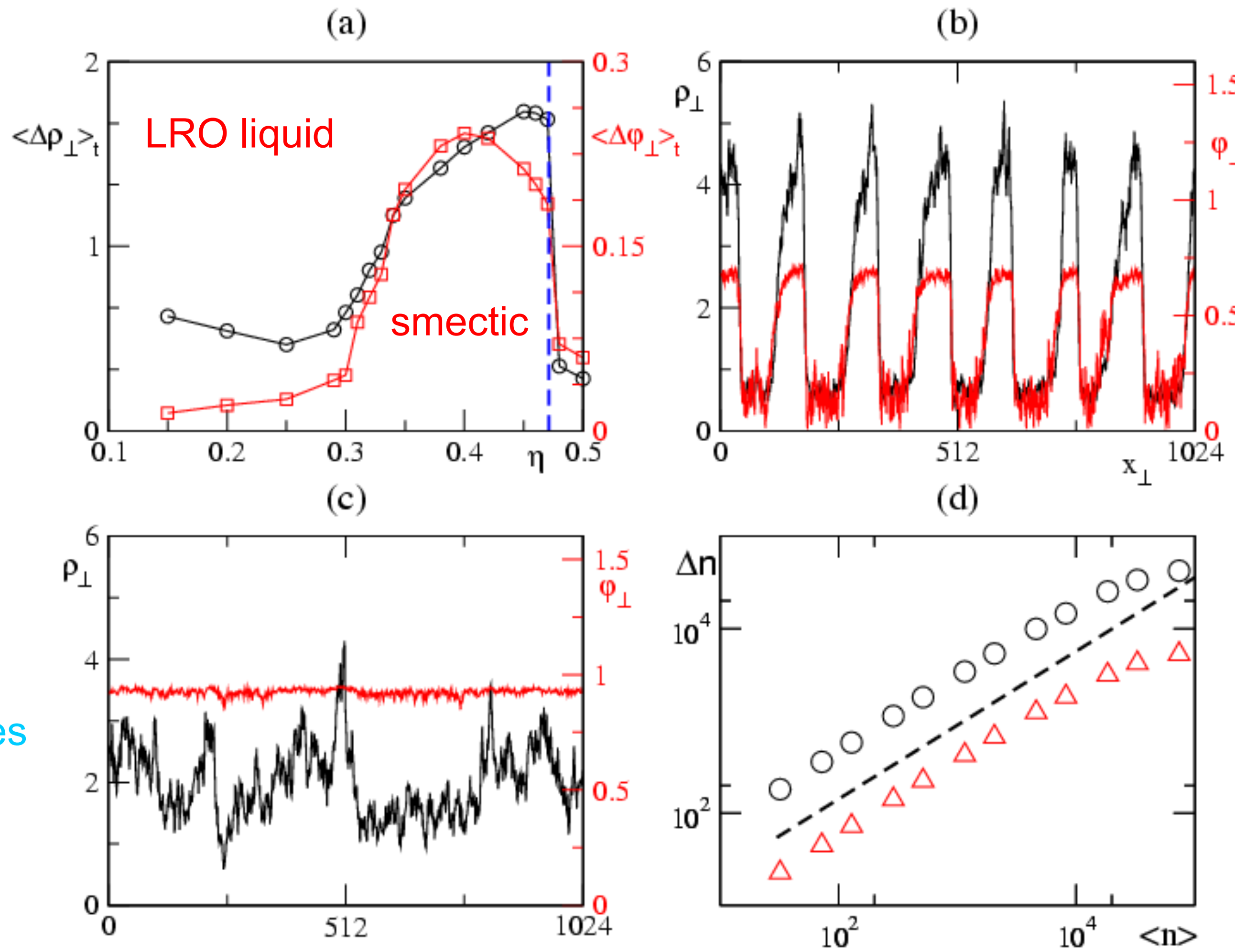
Common features at microscopic level:

- Disordered gas phase at low density/strong noise
- (Quasi-) ordered liquid phase at high density/low noise, with giant number fluctuations and superdiffusion
- In between: phase-separated inhomogeneous phase with dense and ordered regions



Vicsek model



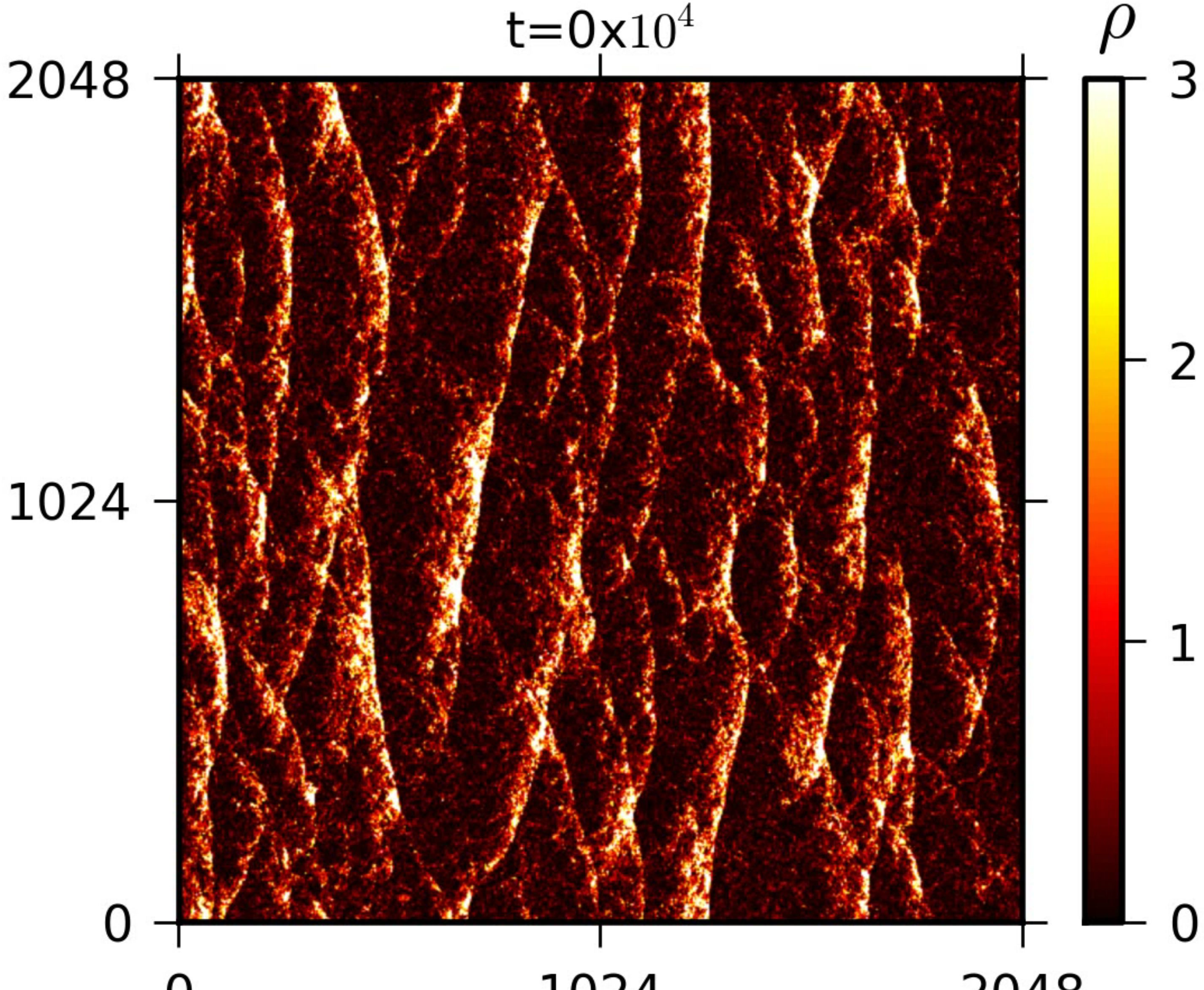
Smectic order (bands)

LRO liquid: typical profiles

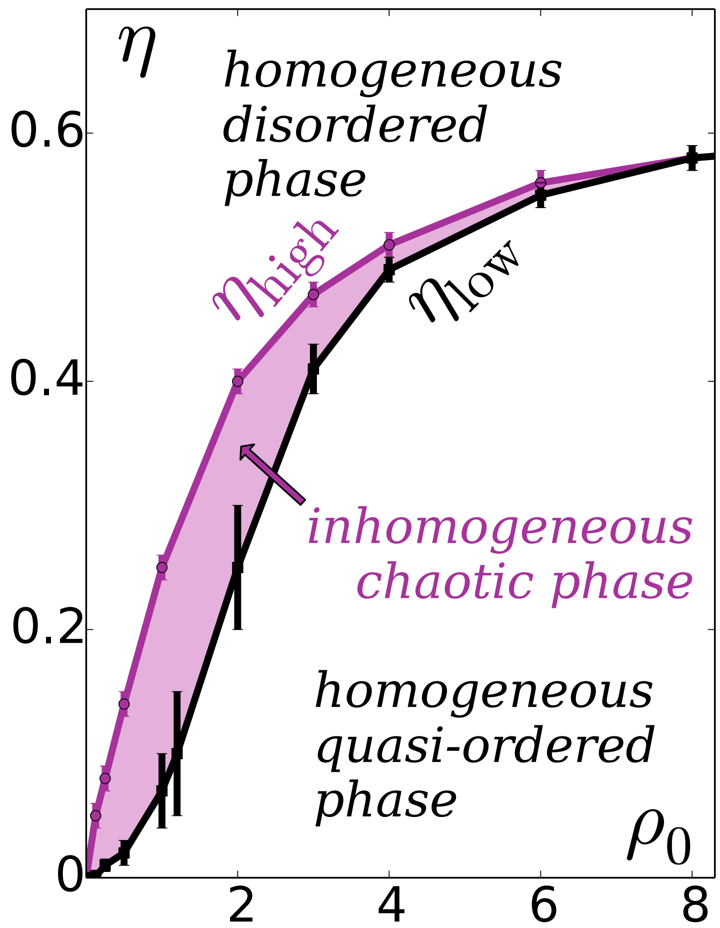
LRO liquid: « giant » density fluctuations

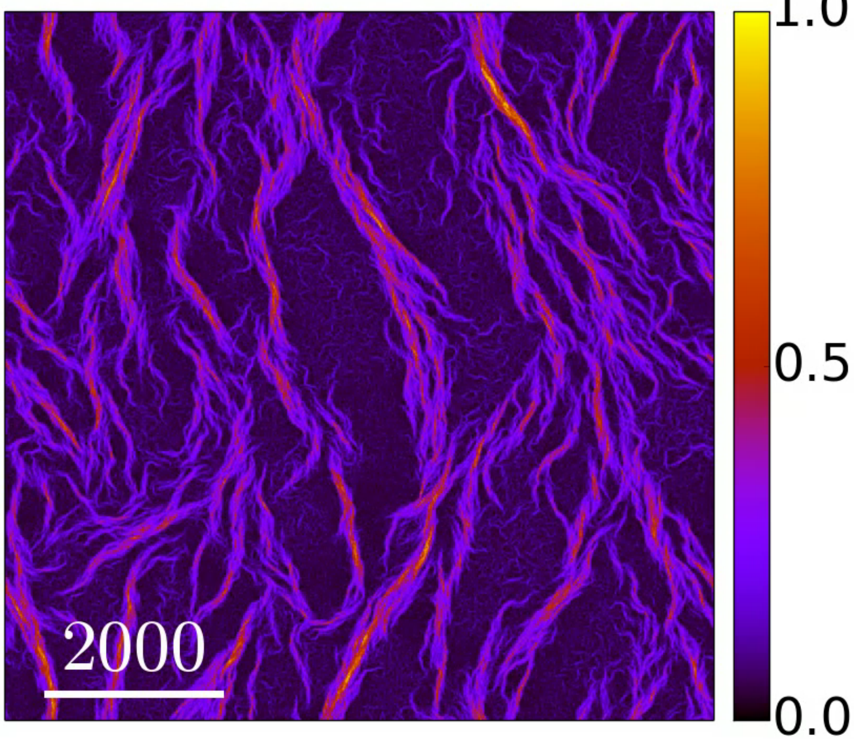
+ transversal superdiffusion

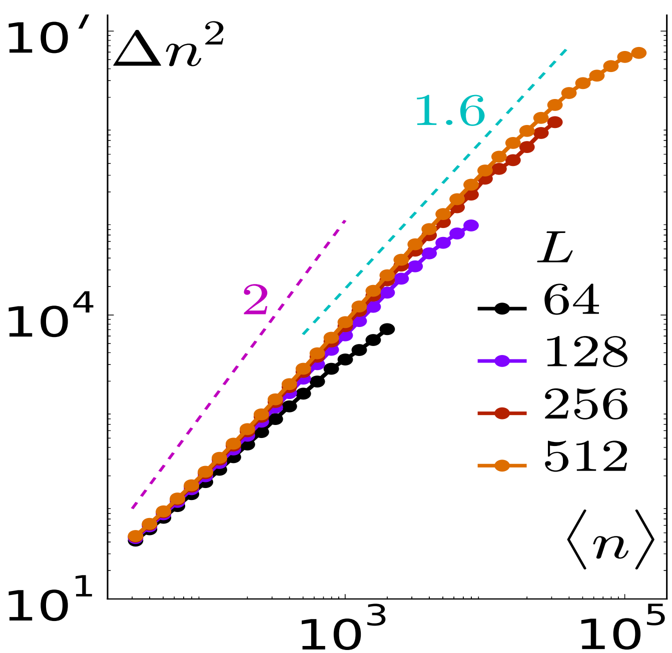
Vicsek model



Active nematics







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From particle models to (deterministic, mean-field) continuous theories: "Boltzmann Ginzburg-Landau" approach

- Start with the simple Boltzmann equation of ideal gases for the probability function *f* (r,θ,t)

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_{\rm force} + \left(\frac{\partial f}{\partial t}\right)_{\rm diff} + \left(\frac{\partial f}{\partial t}\right)_{\rm coll}$$

- No external forces but a self-propulsion given by an advection (or diffusion) term

$$\left(\frac{\partial f}{\partial t}\right)_{\rm self-propulsion} = v_0 \, \mathrm{e}\left(\theta\right) \cdot \nabla f\left(r, \theta, t\right)$$

Angular diffusion integral

- The angular diffusion is given by the integral

$$I_{\mathrm{diff}}[f] = -\lambda f(r,\theta,t) + \lambda \int_{-\pi}^{\pi} d\theta' \int_{-\pi}^{\pi} d\xi P(\xi)\, \delta(\theta' + \xi - \theta)\, f(r,\theta',t)$$

- Where λ is a diffusion probability and P(ξ) is a wrapped-Gaussian angular distribution function of variance σ^2, which plays the role of the Vicsek angular noise strength η

Collision integral

- We suppose that our system is dilute
  - ➔Binary collision integral
- We suppose a molecular chaos hypothesis
  - ➔ $f\left(A,B\right)=f\left(A\right)\times f\left(B\right)$

$$I_{coll}\left[g,h\right]=-g\left(r,\theta,t\right)\int_{-\pi}^{\pi}d\theta_2 K\left(\theta_1,\theta_2\right)h\left(r,\theta_2,t\right)$$
$$+\int_{-\pi}^{\pi}d\theta_1\int_{-\pi}^{\pi}d\theta_2\int_{-\pi}^{\pi}d\xi.P\left(\xi\right)K\left(\theta_1,\theta_2\right)$$
$$\times g\left(r,\theta_1,t\right).h\left(r,\theta_2,t\right)\delta\left(\Psi\left(\theta_1,\theta_2\right)+\xi-\theta\right)$$

- *K* depends on particle type
- Ψ depends on collision type

Fourier expansion

- Introduce the angular Fourier expansion

$$f(r,\theta,t) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \hat{f}_k(r,t)\, e^{-ik\theta}$$

$$\hat{f}_k(r,t) = \int_{-\pi}^{\pi} d\theta f(r,\theta,t)\, e^{ik\theta}$$

- The first three modes give the density, the polar, and the nematic order parameters

$$\rho = \hat{f}_0 \qquad \rho\mathbf{P} = \begin{pmatrix} \mathrm{Re}\hat{f}_1 \\ \mathrm{Im}\hat{f}_1 \end{pmatrix} \qquad \rho\mathbf{Q} = \begin{pmatrix} \mathrm{Re}\hat{f}_2 & \mathrm{Im}\hat{f}_2 \\ \mathrm{Im}\hat{f}_2 & -\mathrm{Re}\hat{f}_2 \end{pmatrix}$$

- Use complex notations for simplicity, including:

$$\nabla \equiv \partial_x + i\partial_y\,, \quad \text{and} \quad \nabla^* \equiv \partial_x - i\partial_y$$

Closure of the expansion (polar case)

- Use Ginzburg-Landau approach to close the Fourier series

$$\alpha\psi - \beta\left|\psi\right|^2\psi + \gamma\nabla^2\psi = 0$$

- Near transition $\psi \sim \epsilon \Rightarrow \begin{array}{ccc} \nabla & \sim & \epsilon \\ \alpha & \sim & \epsilon^2 \end{array}$

- We suppose $\rho\left(r,t\right) = \rho_0 + \Delta\rho\left(r,t\right) \Rightarrow \Delta\rho \sim \epsilon$

- Near transition the angular distribution is quasi-homogeneous

$$\Rightarrow \hat{f}_k \sim \epsilon^{|k|}$$

- Keeping only terms at order 3 and below, obtain well-behaved minimal nonlinear pdes.

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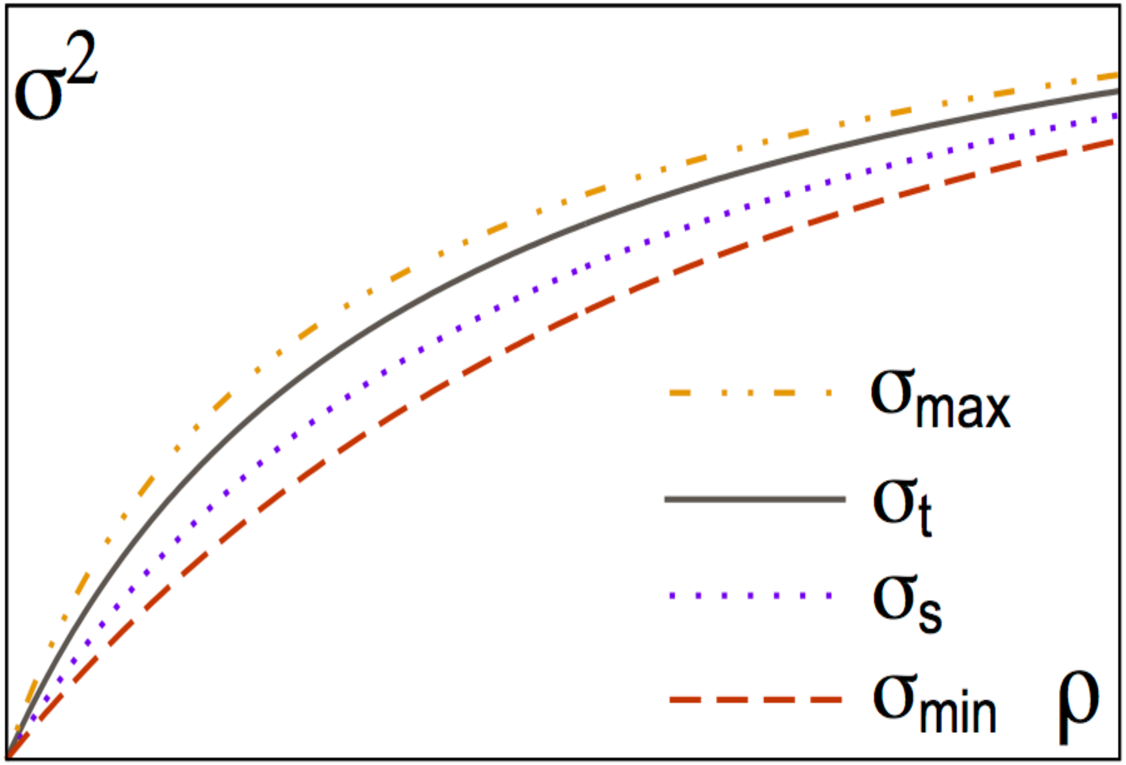
Common features (hydrodynamic level)



- Linear instability of disordered phase via continuous transition (black line)
- Bifurcated homogeneous ordered state is linearly unstable in region bordering onset (between solid black line and dotted purple line)
- Inhomogeneous solutions in region including this linear instability domain (between yellow and red lines) ➔coexistence regions
- Irrelevance of linear thresholds in fluctuating systems

Hydrodynamic equations in polar case

Continuity equation $$\partial_t \rho = -\Re\left(\nabla^* f_1\right)$$

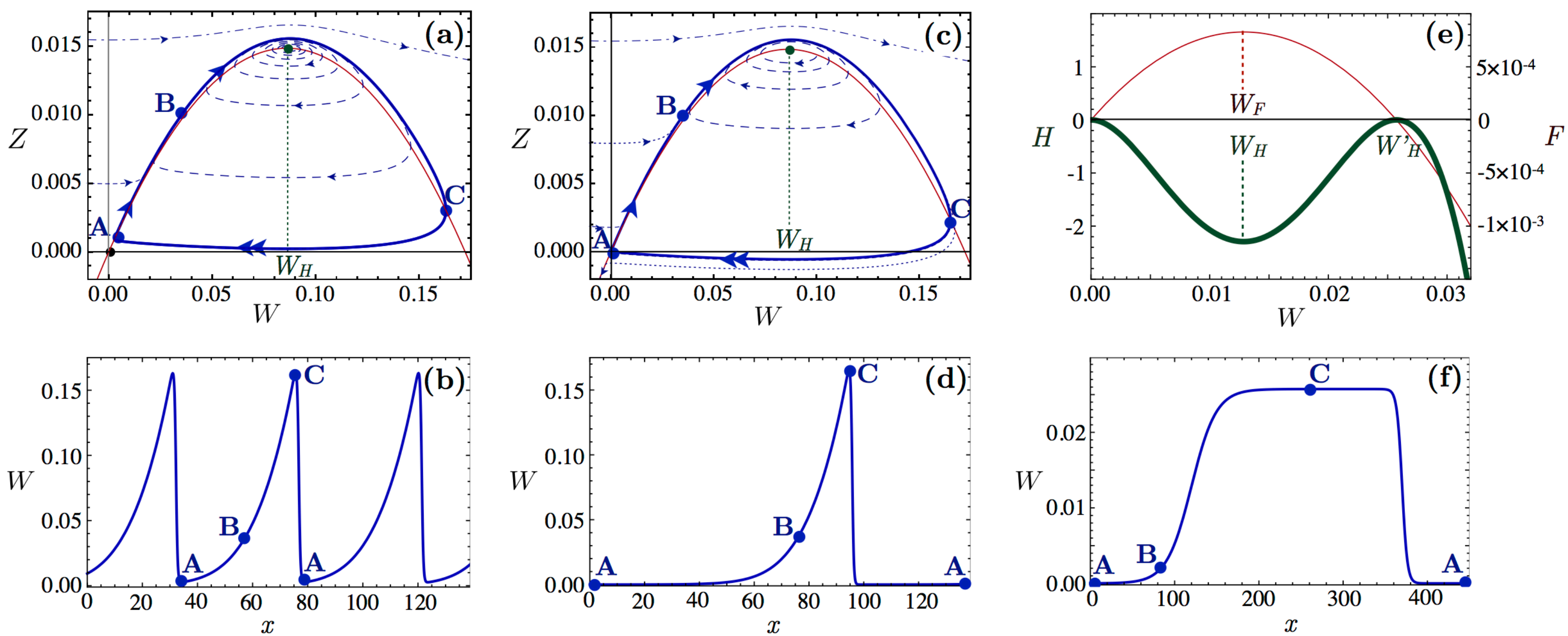
"Toner-Tu" equation

$$\begin{aligned}\partial_t f_1 + \frac{1}{2}\nabla\rho &= \left(\mu - \xi\left|f_1\right|^2\right) f_1 + \frac{\nu}{4}\Delta f_1 \\ &\quad + \iota f_1^* \nabla f_1 - \chi f_1 \nabla^* f_1\end{aligned}$$

With all transport coefficients depending on local density and noise strength
(in particular linear coefficient μ increases with ρ)

Polar case: Inhomogeneous solutions

From ODE ansatz, existence and multiplicity of inhomogeneous solutions



Periodic orbit	Homoclinic orbit	Heteroclinic orbit
Smectic phase	Solitary band	Ordered domain

Stability and selection of solutions in 2D still under investigation

Hydrodynamic equations in nematic case

Continuity equation

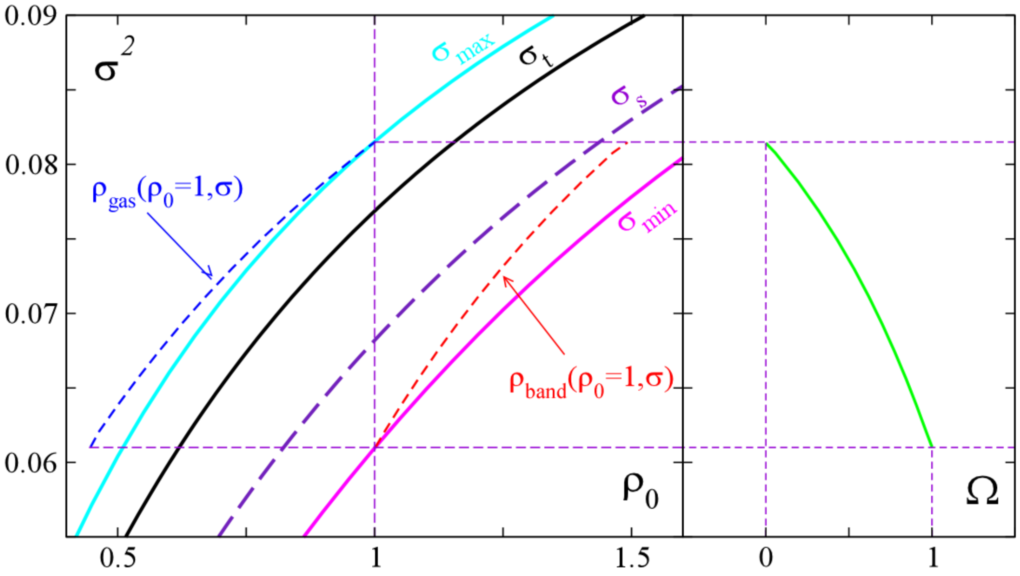
$$\partial_t \rho = \frac{1}{2}\Delta\rho + \frac{1}{2}\mathrm{Re}\left(\nabla^{*2} f_2\right)$$

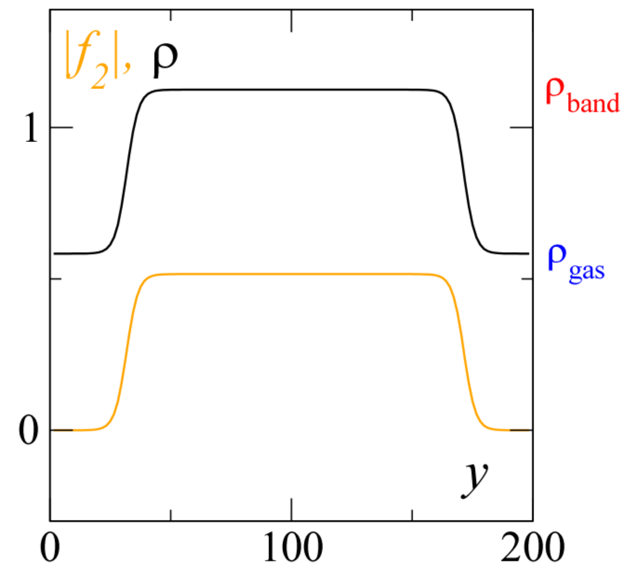
Equation for nematic field

$$\partial_t f_2 = \left(\mu - \xi\left|f_2\right|^2\right) f_2 + \frac{1}{4}\nabla^2\rho + \frac{1}{2}\Delta f_2$$

With all transport coefficients depending on local density and noise strength
(in particular linear coefficient μ increases with ρ)

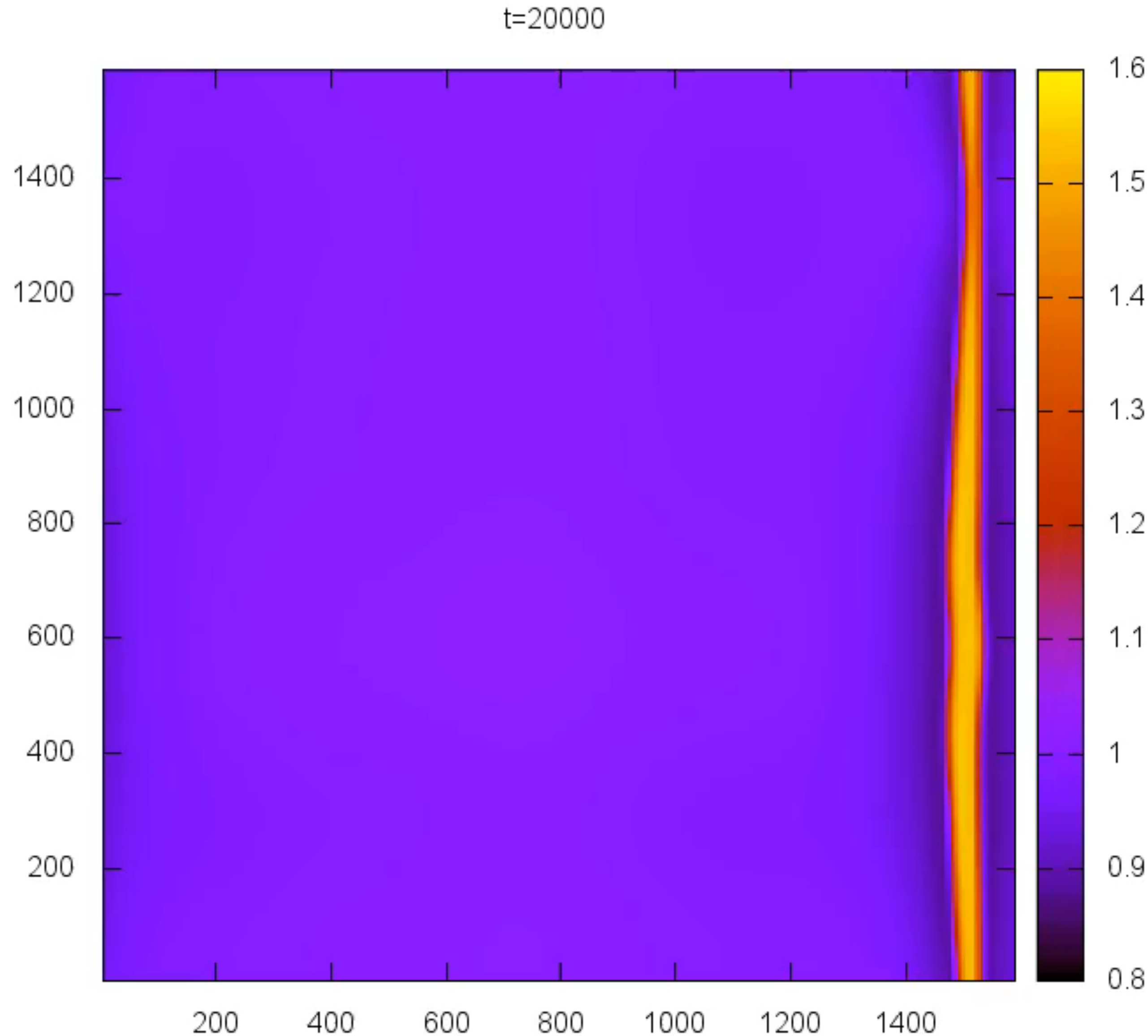
Inhomogeneous nematic band solution





- Explicit exact solution in closed form
- Observed (stable) in moderate-size domains
- But proof of linear instability in two dimensions: long-wavelength instability

In large-enough domains, spatiotemporal “band” chaos



Summary:

- Even simplest setting for active matter/collective motion reveals a wealth of unexpected collective phenomena
- Agreement between continuous field equations and particle models is very good but of course semi-quantitative at best
- General lessons:
  - density/order segregation (phase separation, liquid/gas transition) due to feedback between local density and order
  - Importance of nonlinear features/irrelevance of linear stability thresholds (inhomogeneous solutions coexist with homog.)

Outlook: a fluctuating world…

- Recap: generic long-range correlations and anomalously strong fluctuations ubiquitous in these systems.
- Evidence that fluctuations are crucial in pattern selection (polar case)
- Toner-Tu calculation predicts their existence, but the « proof » is not that solid; growing evidence of possibly more general origin.
- Next: ‘reintroduce’ fluctuations in the form of carefully calculated effective noise terms, then numerics or RG on these Langevin equations… for proper understanding of role of correlations/ fluctuations (actual thresholds, selection mechanisms)