Turbulent liquid crystals unveil universal fluctuation properties of growing interfaces

Kazumasa A. Takeuchi (Univ. of Tokyo)

Acknowledgment

Masaki Sano, Tomohiro Sasamoto,
Herbert Spohn, Michael Prähofer, Grégory Schehr

Interface Growth

Wide interest

- Ubiquitous.

  (e.g., coffee stain on a shirt, fabricating solid-state devices…)

- Obviously irreversible, thus out of equilibrium.

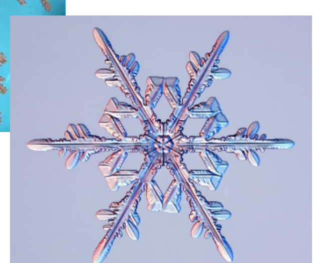
- Interesting pattern formation. (e.g., snowflakes, bacteria colony…)

  typically forming scale-invariant structures

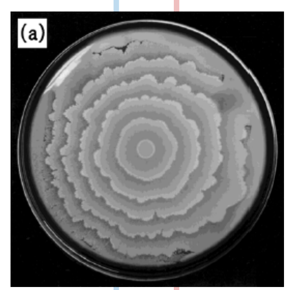
Two types of mechanism

Non-local growth

Metal dendrite



snowflake

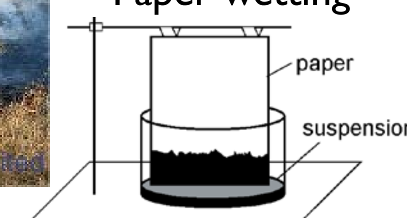


Bacterial colony

Local growth

Burning front

Paper wetting



Interface Growth

Wide interest

- Ubiquitous.
  (e.g., coffee stain on a shirt, fabricating solid-state devices…)
- Obviously irreversible, thus out of equilibrium.
- Interesting pattern formation. (e.g., snowflakes, bacteria colony…)
  typically forming scale-invariant structures

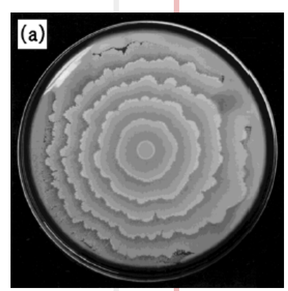
⇨ test-bed for universality out of equilibrium.

Two types of mechanism

Non-local growth

Metal dendrite

snowflake

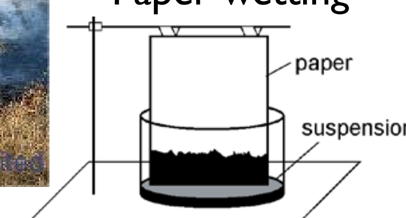


Bacterial colony

Local growth

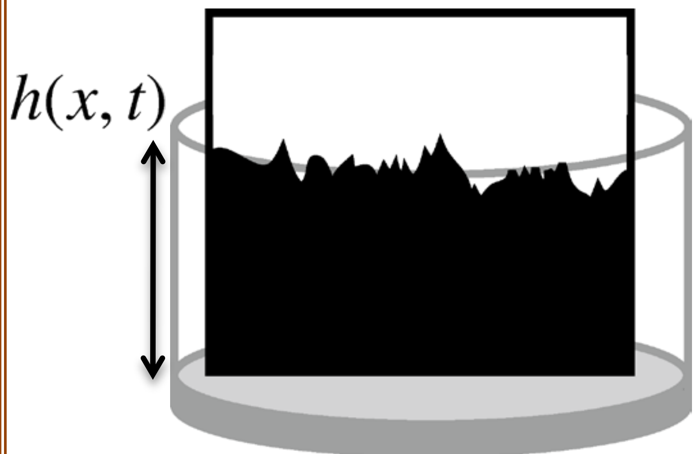
Burning front

Paper wetting



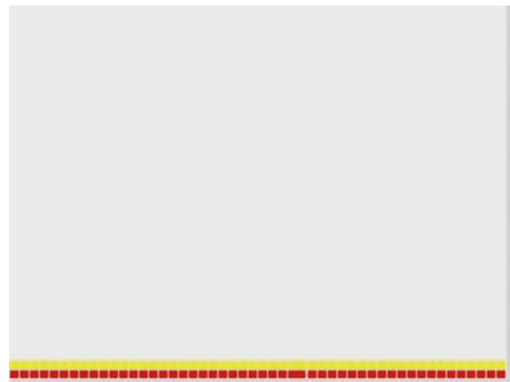
Roughening of Interfaces

Typically, local growth processes form rough, self-affine interfaces.



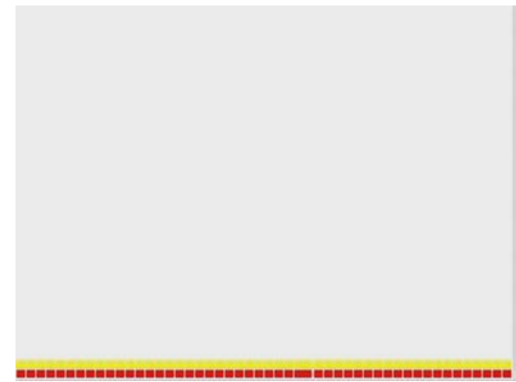
Paper wetting

(and many other experiments)

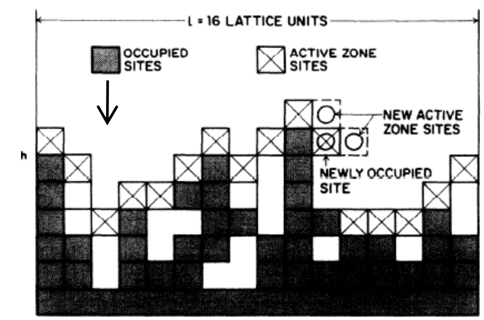


Eden model

add a particle randomly onto the interface



Ballistic deposition model

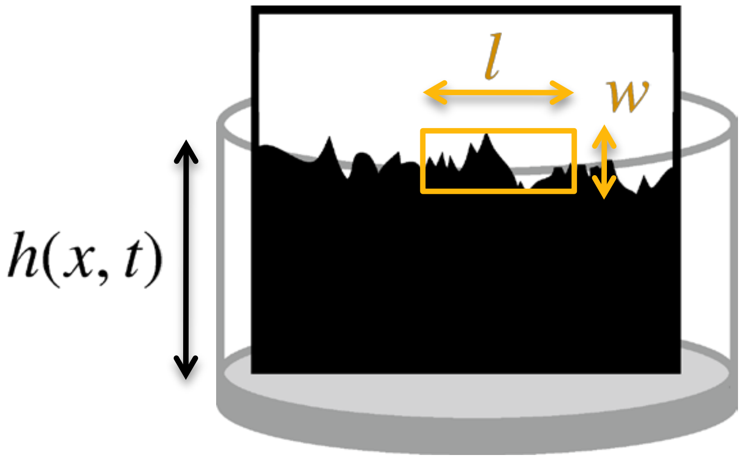


Self-affine:

fluctuation properties are (statistically) invariant under $x \to ax, \quad t \to a^z t, \quad h \to a^\alpha h$

Characterizing Self-Affinity

"Interface width" quantifies the roughness of interfaces



$w(l,t)$ = Standard deviation of $h(x,t)$ over length scale l

$$= \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_l]^2 \rangle_l} \right\rangle$$

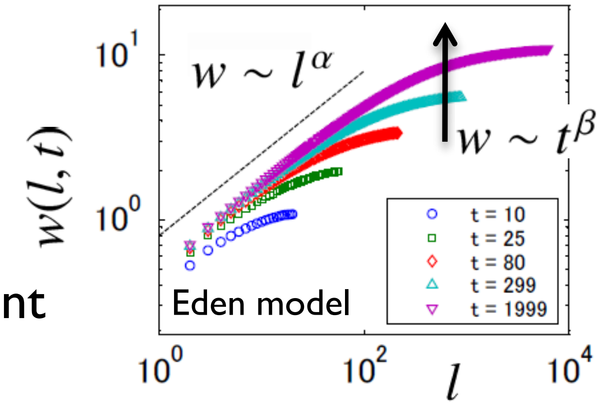
Self-affinity of the interfaces implies: (Family-Vicsek scaling)

$$\boxed{w(l,t) \sim \begin{cases} l^\alpha & (l \ll l_*) \\ t^\beta & (l \gg l_*) \end{cases} \quad (l_* \sim t^{1/z})}$$

α : roughness exponent

β : growth exponent

$z = \alpha/\beta$: dynamic exponent

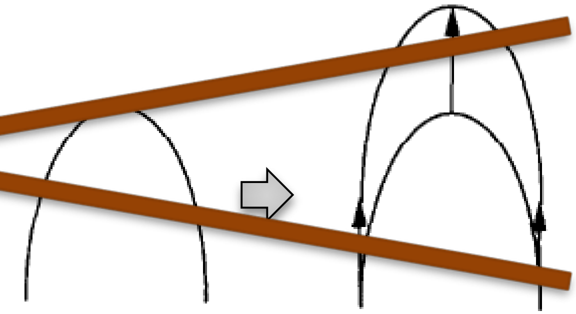


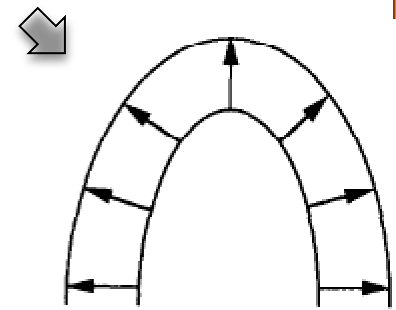
Basic Theory: KPZ Equation

- ~~Linear theory: Edwards-Wilkinson eq.~~

$$\cancel{\boxed{\frac{\partial}{\partial t}h(x,t) = v_0 + \nu\nabla^2 h + \xi(x,t)}}$$

$$\cancel{\langle \xi(x,t)\xi(x',t')\rangle = D\delta(x-x')\delta(t-t')}$$





- Kardar-Parisi-Zhang (KPZ) eq.

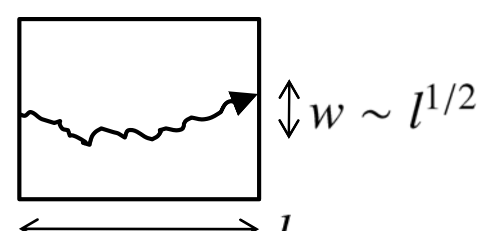
$$\boxed{\frac{\partial}{\partial t}h(x,t) = v_0 + \nu\nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \xi(x,t)}$$

※by $h \to h + v_0 t$, one can take $v_0 = 0$.

  - In (1+1) dimensions, $\boxed{\alpha = 1/2, \ \ \beta = 1/3, \ \ z = 3/2}$
  - Exponents regularly seen in numerical models ⇨ KPZ universality class
  - Why $\alpha = 1/2$?

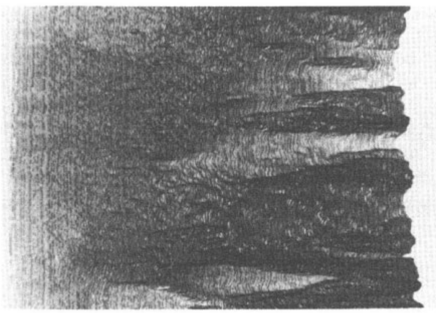
    1d EW/KPZ stationary interfaces

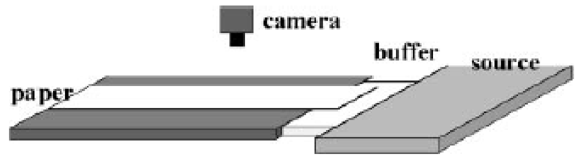
    = 1d Brownian motion $\mathcal{P}[h] \sim \exp\left[-\frac{\nu}{D}\int \mathrm{d}x(\nabla h)^2\right]$



Situation in Experiments

Rough surfaces are ubiquitous, but KPZ is seen less frequently..





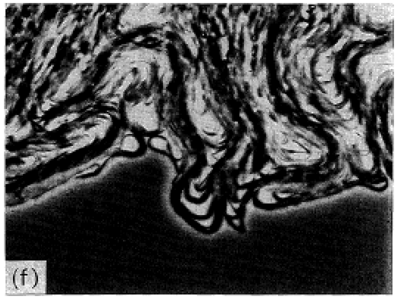
- paper wetting $\alpha = 0.73$ $\beta = 0.60$ [Kobayashi *et al.*, 2005]

- growth of plant callus [Galeano *et al.*, 2003] $\alpha = 0.86, \beta = 0.17$

- flow in porous media $\alpha = 0.81$ $\beta = 0.65$ [Horváth *et al.*, 1991]

- copper deposition [Kahanda *et al.*, 1992] $\alpha = 0.55, \ \beta > 1$

cf. $\boxed{\alpha_{\mathrm{KPZ}} = 1/2, \ \beta_{\mathrm{KPZ}} = 1/3}$



- bacteria colony $\alpha = 0.78$ [Wakita *et al.*, 1997]

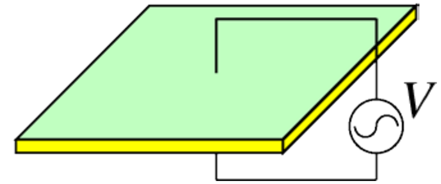
Small, but growing # of experiments showing KPZ exponents

- Colony of mutant bacteria [Wakita *et al.*, 1997]
- Slow combustion of paper [Maunuksela *et al.*, 1997-]
- Turbulent liquid crystal [Takeuchi & Sano, 2010-]
- Tumor-like & tumor cells [Huergo *et al.*, 2010-]
- Particle deposition on coffee ring [Yunker *et al.*, 2013]

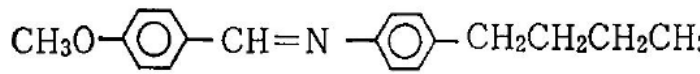
Advantages
- simple growth mechanism
- precise control
- many experimental runs

⇨ high statistical accuracy

Electroconvection



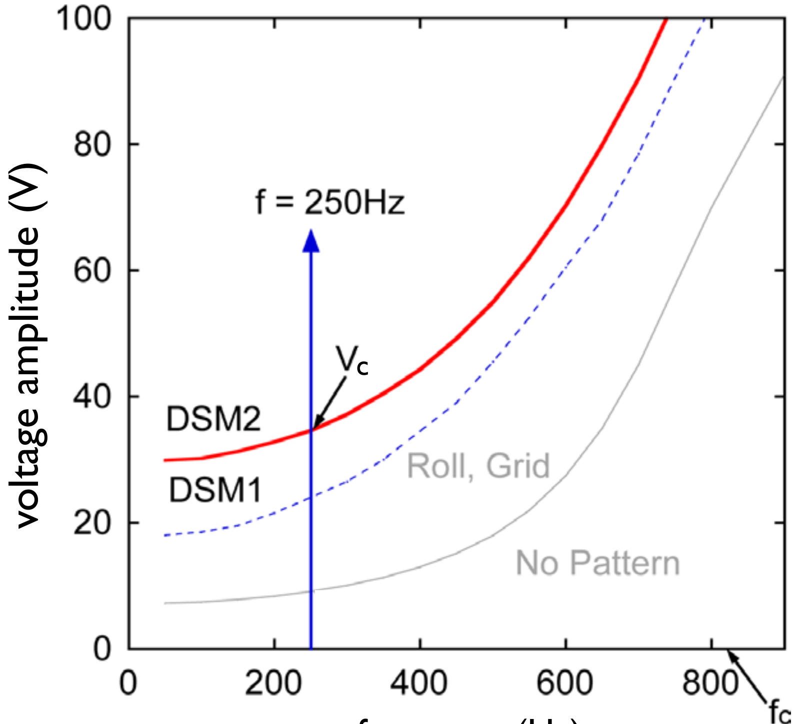
Nematic liquid crystal (e.g., MBBA)

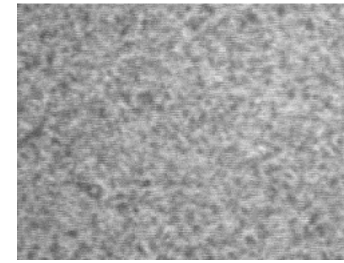
- Rod-like molecule 

- Strong anisotropy $\varepsilon_{\parallel} < \varepsilon_{\perp}, \quad \sigma_{\parallel} > \sigma_{\perp}$

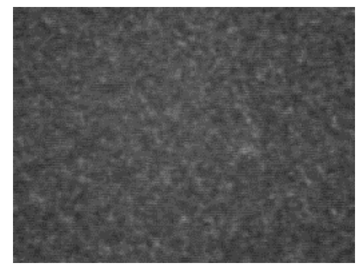
⇨ Convection driven by electric field

phase diagram (MBBA; planar alignment)

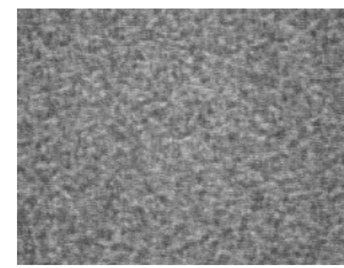




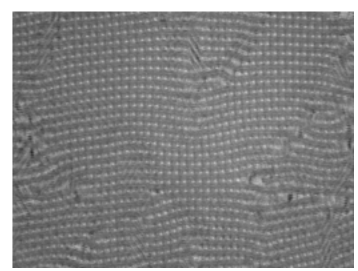
DSM2 nucleation
$(V \gg V_{\mathrm{c}})$



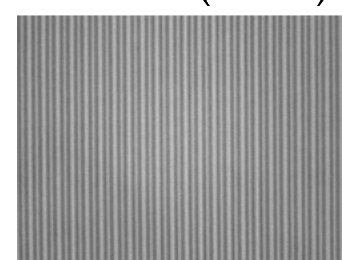
Dynamic Scattering Mode 2 (DSM2)



Dynamic Scattering Mode 1 (DSM1)



grid pattern

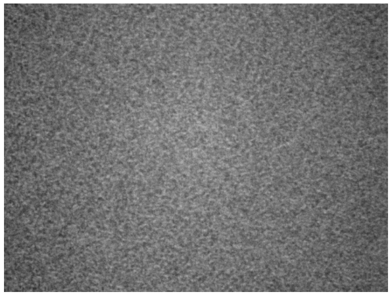


Williams domain

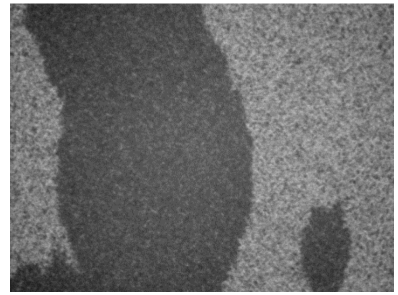
$100\,\mu\mathrm{m}$

Two Turbulent States : DSM1 & DSM2

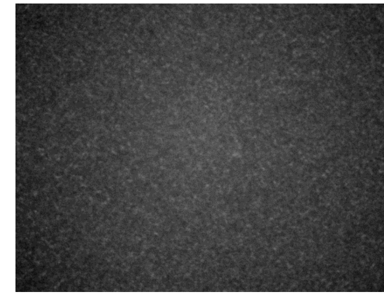
DSM1



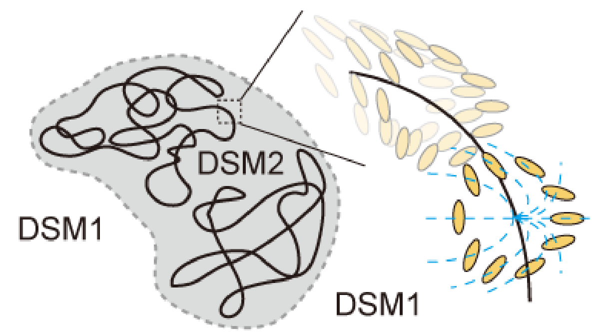
nucleation if $V \gg V_c$



DSM2



0V → 72V → 0V ($V_c \approx 30$ V, speed x3)

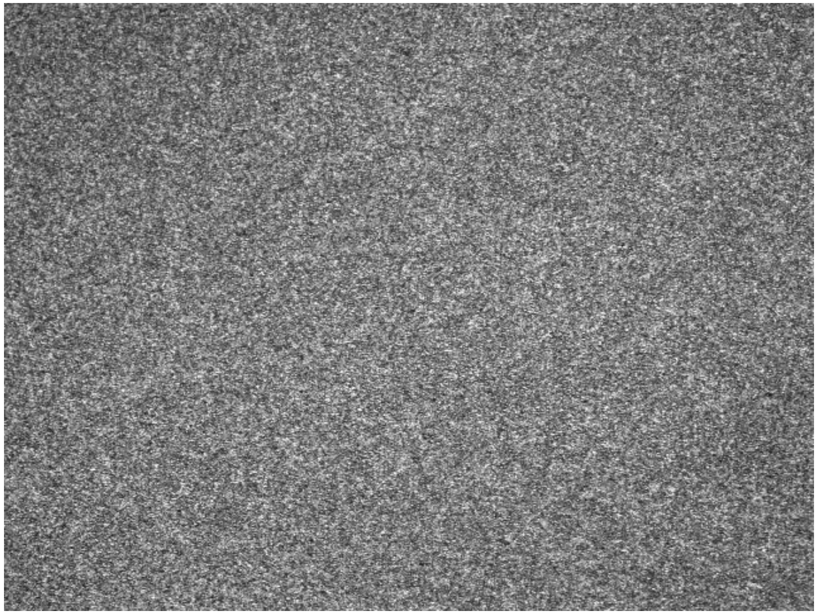


DSM2 = topological-defect turbulence
(analogy with "quantum turbulence"?)

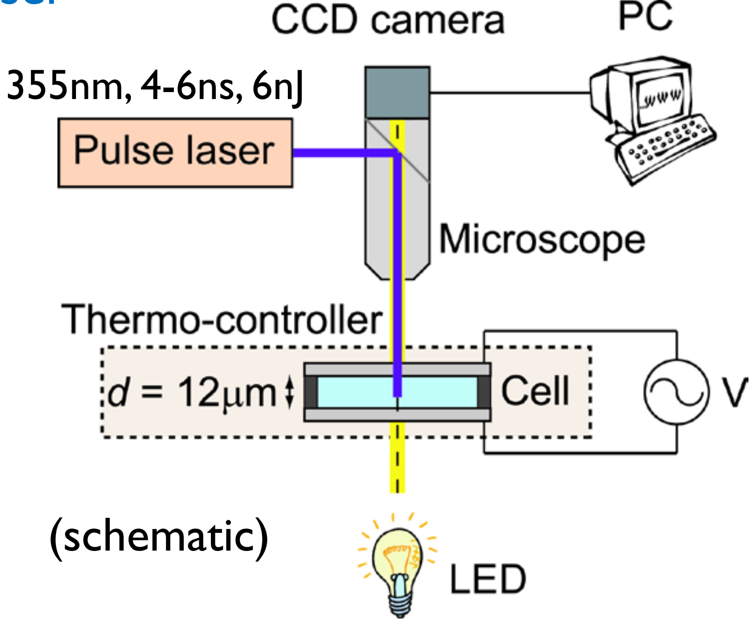
We focus on DSM1-DSM2 interfaces and study their fluctuations

Experimental Setup

- Quasi-2d cell: $16\,\mathrm{mm} \times 16\,\mathrm{mm} \times 12\,\mu\mathrm{m}$
- Nematic liquid crystal: MBBA
- Homeotropic alignment (to work with isotropic growth)
- Temperature control: $T = 25\,^\circ\mathrm{C}$
- Nucleation of DSM2 by UV pulse laser



$26\mathrm{V}, 250\mathrm{Hz}$ Speed x2, —$200\mu\mathrm{m}$

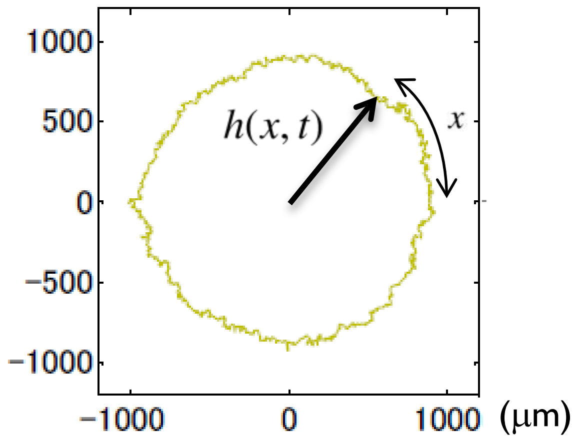


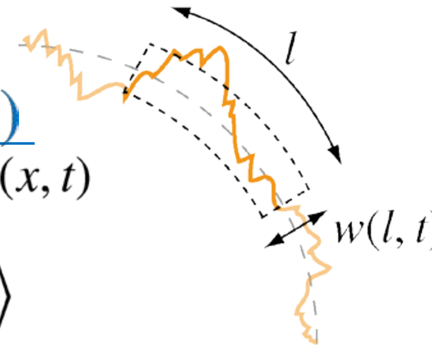
(schematic)

Rough interface appears

Scaling Exponents

interfaces at $t = 2, 7, 12, \cdots, 27\text{sec}$





interface width $w(l,t)$

= standard deviation of $h(x,t)$ over length l

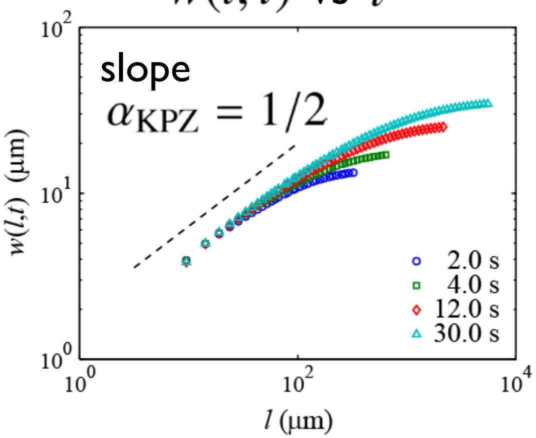
$$= \left\langle \sqrt{\langle [h(x,t) - \langle h \rangle_l]^2 \rangle_l} \right\rangle$$

Family-Vicsek scaling

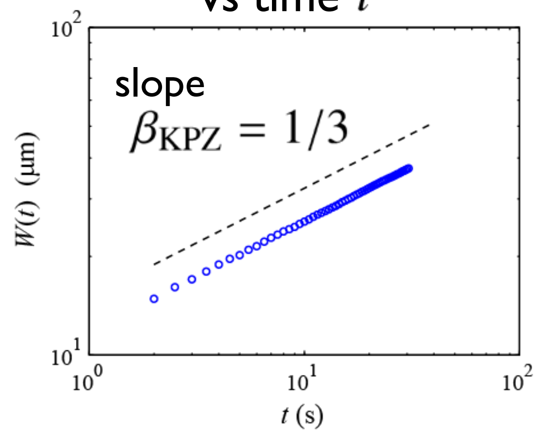
$$\boxed{w(l,t) \sim t^{\beta} F(l t^{-1/z}) \sim \begin{cases} l^{\alpha} & (l \ll l_*) \\ t^{\beta} & (l \gg l_*) \end{cases}}$$

$$l_* \sim t^{1/z}, z = \alpha/\beta$$

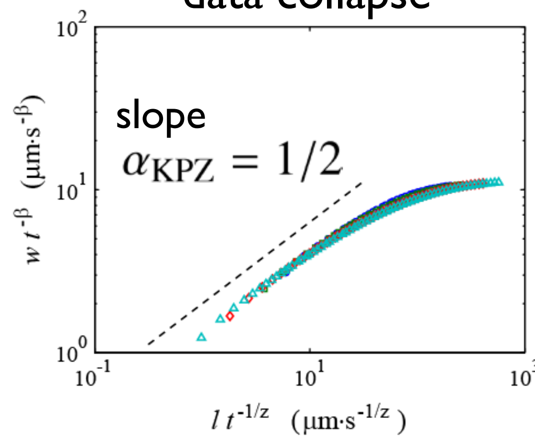
$w(l,t)$ vs l



vs time t



data collapse

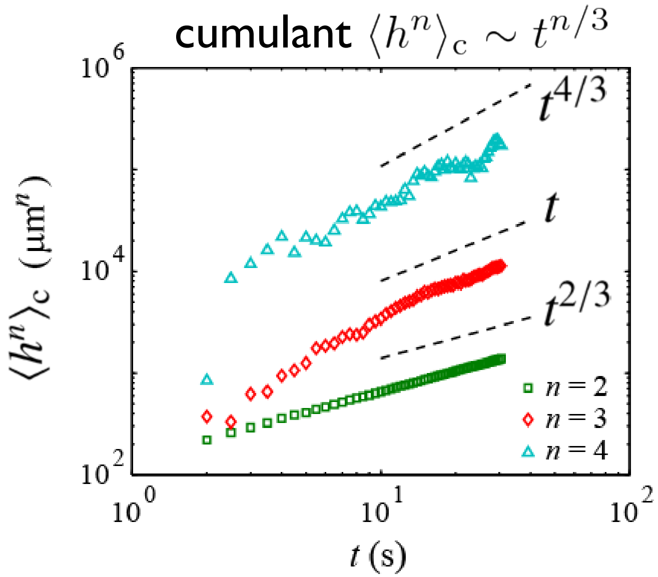


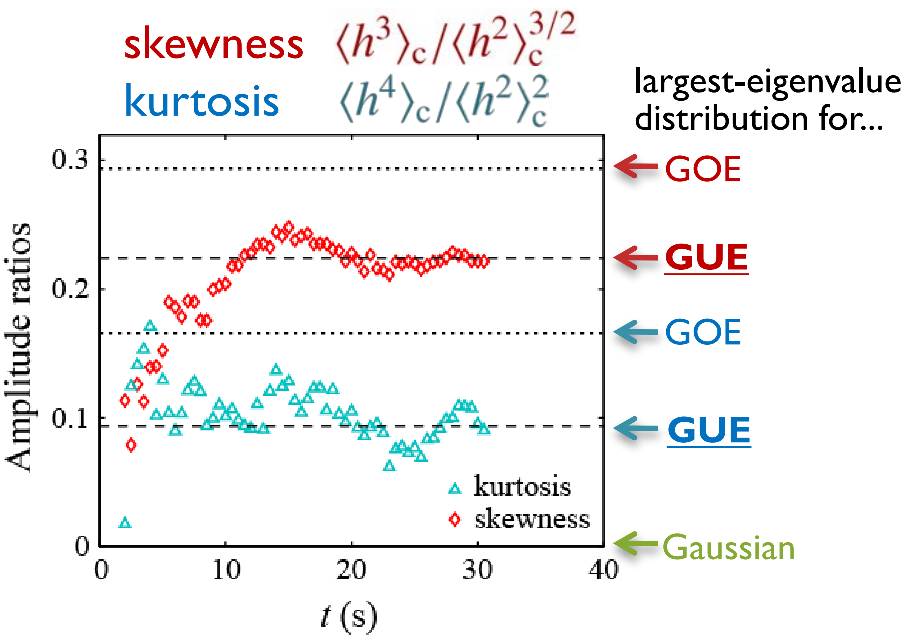
Both exponents (α, β) agree with the KPZ class

Deeper Look at Height Fluctuations

Key quantity: nth-order cumulant $\langle h^n \rangle_c$

$$\langle h^2\rangle_c \equiv \langle \delta h^2\rangle \sim t^{2/3} \qquad (\delta h \equiv h(x,t) - \langle h\rangle)$$
$$\langle h^3\rangle_c \equiv \langle \delta h^3\rangle$$
$$\langle h^4\rangle_c \equiv \langle \delta h^4\rangle - 3\langle \delta h^2\rangle^2$$





This suggests $\boxed{h(t) \simeq v_\infty t + (\Gamma t)^{1/3}\chi}$ (χ : non-Gaussian random variable)

χ obeys the largest-eigenvalue distribution [Tracy-Widom (TW) dist.] of GUE random matrices!?

Tracy-Widom Distribution

describes the largest-eigenvalue distribution of Gaussian random matrices

e.g.) Gaussian Unitary Ensemble (GUE)

complex Hermite matrix $A = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NN} \end{pmatrix}$

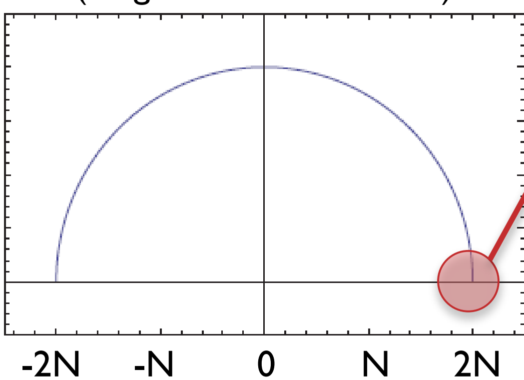
Gaussian
mean 0 variance $N/2$

$A_{ij} = \bar{A}_{ji} = a_{ij} + ib_{ij}$

$A_{ii} = a_{ii}$

mean 0 variance N

prob. density for all eigenvalues
(Wigner's semicircle law)



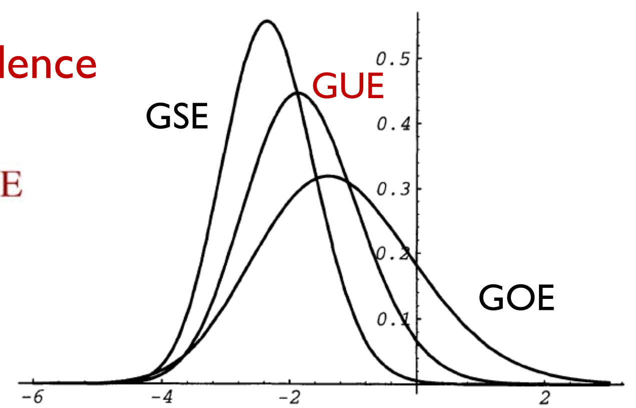
$$\boxed{\lambda_{\max} \simeq 2N + N^{1/3}\chi_{\mathrm{GUE}}}$$

$\mathrm{pdf}(\chi_{\mathrm{GUE}}) \equiv$
GUE Tracy-Widom dist.

apparent correspondence
$t \leftrightarrow N$
$\chi \leftrightarrow \chi_{\mathrm{GUE}}$

Experiment:
height fluctuations

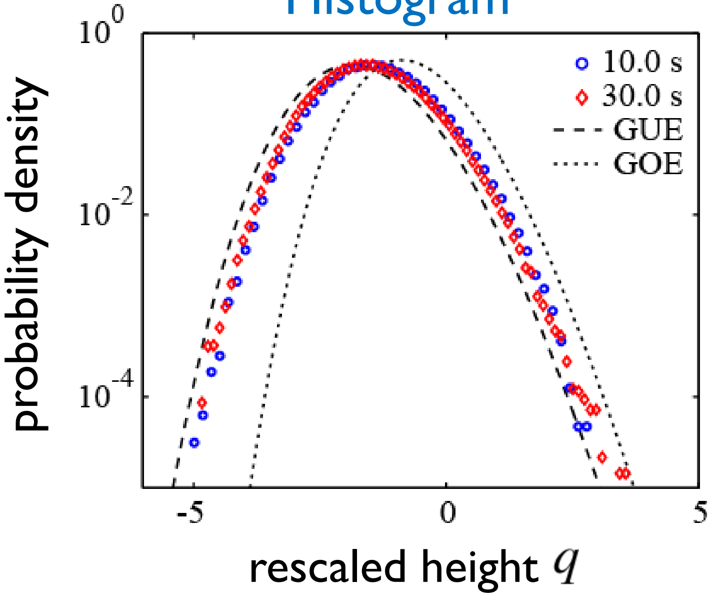
$$\boxed{h(t) \simeq v_\infty t + (\Gamma t)^{1/3}\chi}$$



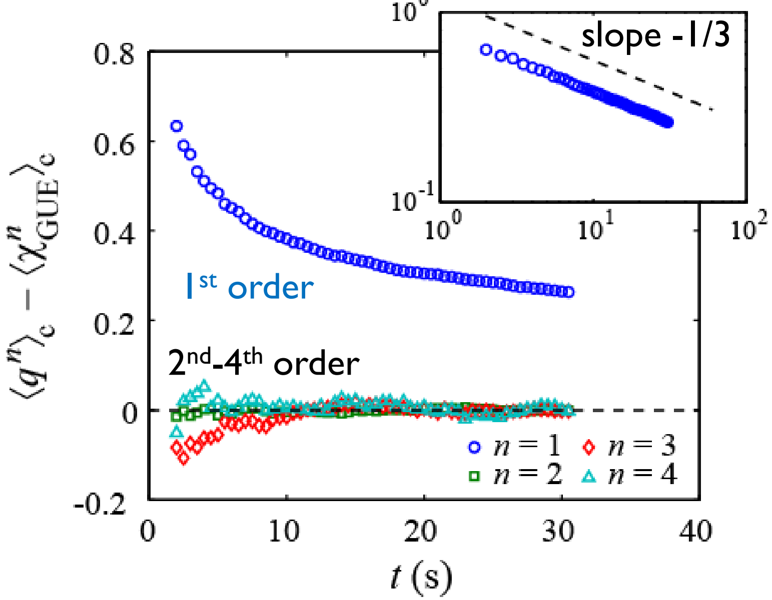
Universal Distribution!

Define the rescaled height $\boxed{q \equiv (h - v_\infty t)/(\Gamma t)^{1/3} \simeq \chi}$

Histogram



Difference from GUE-TW distribution



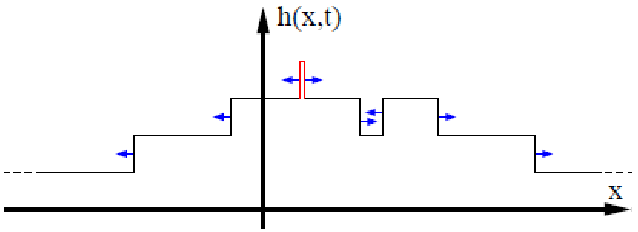
Interface fluctuations precisely agree with the GUE-TW distribution up to the 4th order cumulant! Finite-time effect $\sim t^{-1/3}$ for the mean

GUE-TW statistics was first found in solvable models [Johansson 2000; Prähofer & Spohn 2000] and recently in an exact solution of KPZ eq. [Sasamoto & Spohn, Amir et al., 2010]

Why Tracy-Widom Distribution?

In case of the PNG (= polynuclear growth) model

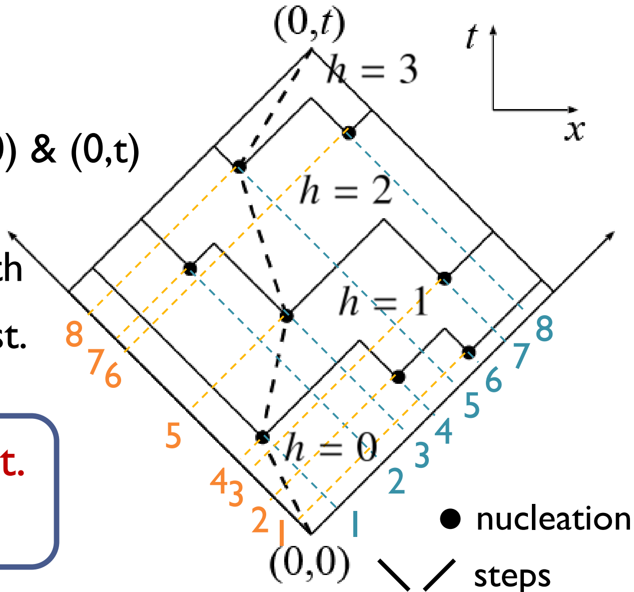
[Prähofer & Spohn, PRL 2000]



Time evolution: (1) stochastic nucleations
(2) deterministic lateral expansion

For circular interfaces, first nucleation at (x,t) = (0,0)

$h(0,t)$ = # of lines to pass when moving from (0,0) to (0,t)
= max # of dots passed by directed polymer btwn (0,0) & (0,t)
= length of longest increasing subsequences
in random permutations of Poisson-distributed length
= … (Young tableau) … = asymptotically, GUE-TW dist.



⇨ (curved) PNG fluctuations obey the GUE-TW dist.

$$h(0,t) \simeq \sqrt{2}t + (t/\sqrt{2})^{1/3}\chi_{\mathrm{GUE}}$$

related to random matrix, combinatorics, disordered systems, etc.

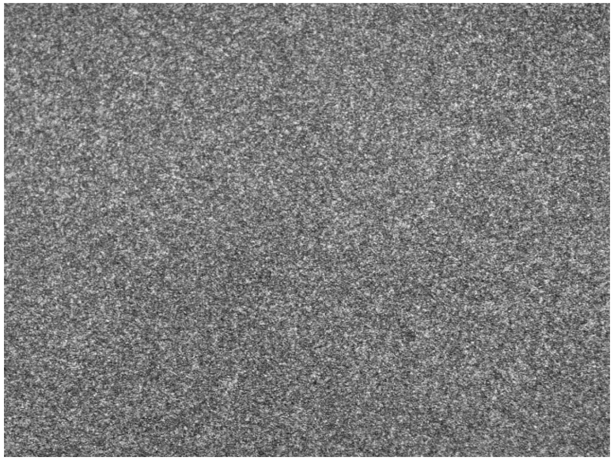
random permutation

1 2 3 4 5 6 7 8
4 7 5 2 8 1 3 6

Experiment implies universality of the GUE-TW distribution

Geometry-Dependent Universality

Flat interfaces can also be created by shooting line-shaped laser pulses

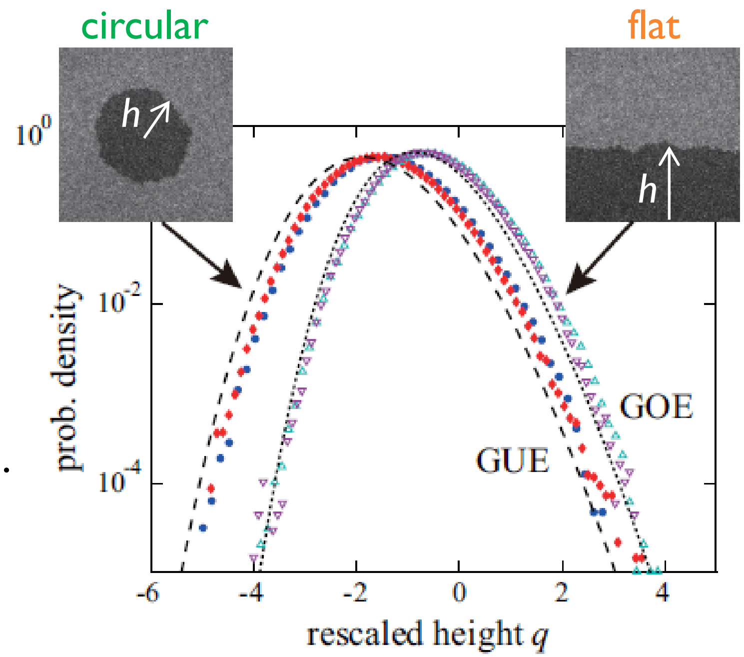


26V, 250Hz Speed x5, — 200μm

Same KPZ exponents are found.

however

measuring the distribution.



Same exponents,

but different distributions!!

circular : $h(t) \simeq v_\infty t + (\Gamma t)^{1/3} \chi_{\mathrm{GUE}}$

flat : $h(t) \simeq v_\infty t + (\Gamma t)^{1/3} \chi_{\mathrm{GOE}}$

Same results in solvable models [Prähofer & Spohn 2000]

KPZ class splits into (at least) two universality sub-classes:

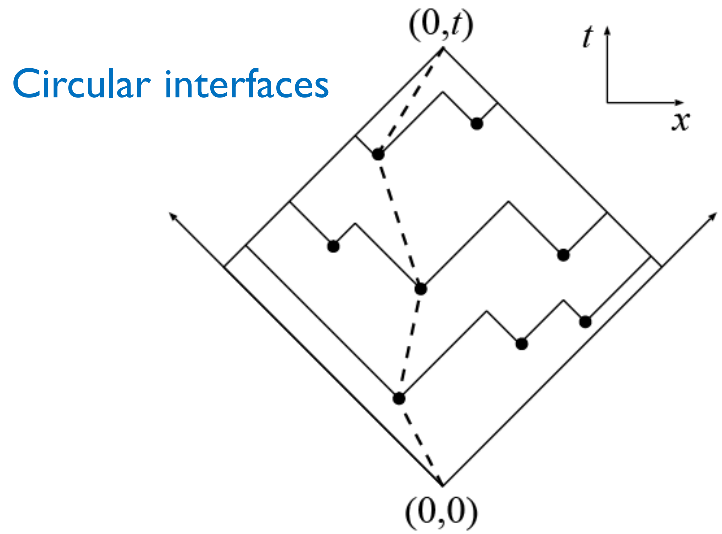
"curved KPZ sub-class" & "flat KPZ sub-class"

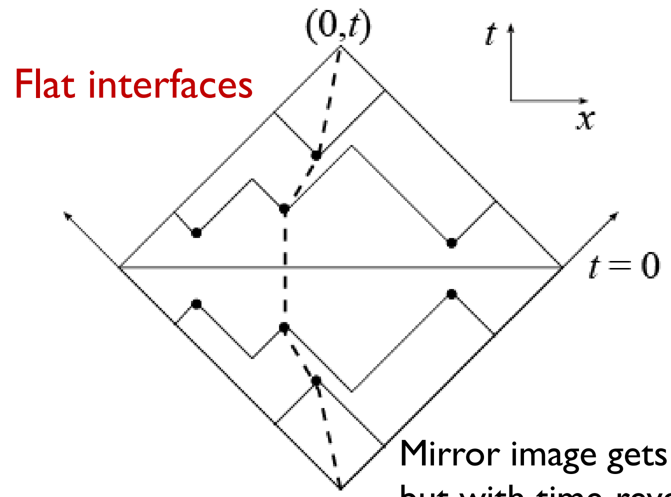
Why Different Distributions?

Quick answer: Because of different space-time symmetry

For the PNG model

- Circular ⇨ Consider a square connecting (0,0) and $(0,t)$ ⇨ GUE
- Flat ⇨ Consider a triangle connecting t = 0 and $(0,t)$ ⇨ GOE

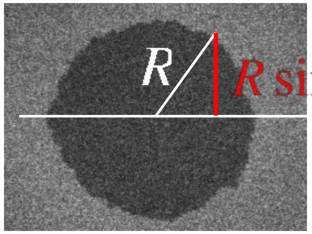




Mirror image gets back a square, but with time-reversal symmetry.

Different initial conditions (curved or not) lead to different symmetries and to different universal sub-classes! [GUE-TW (curved) & GOE-TW (flat)]

Extreme-Value Statistics (circular)

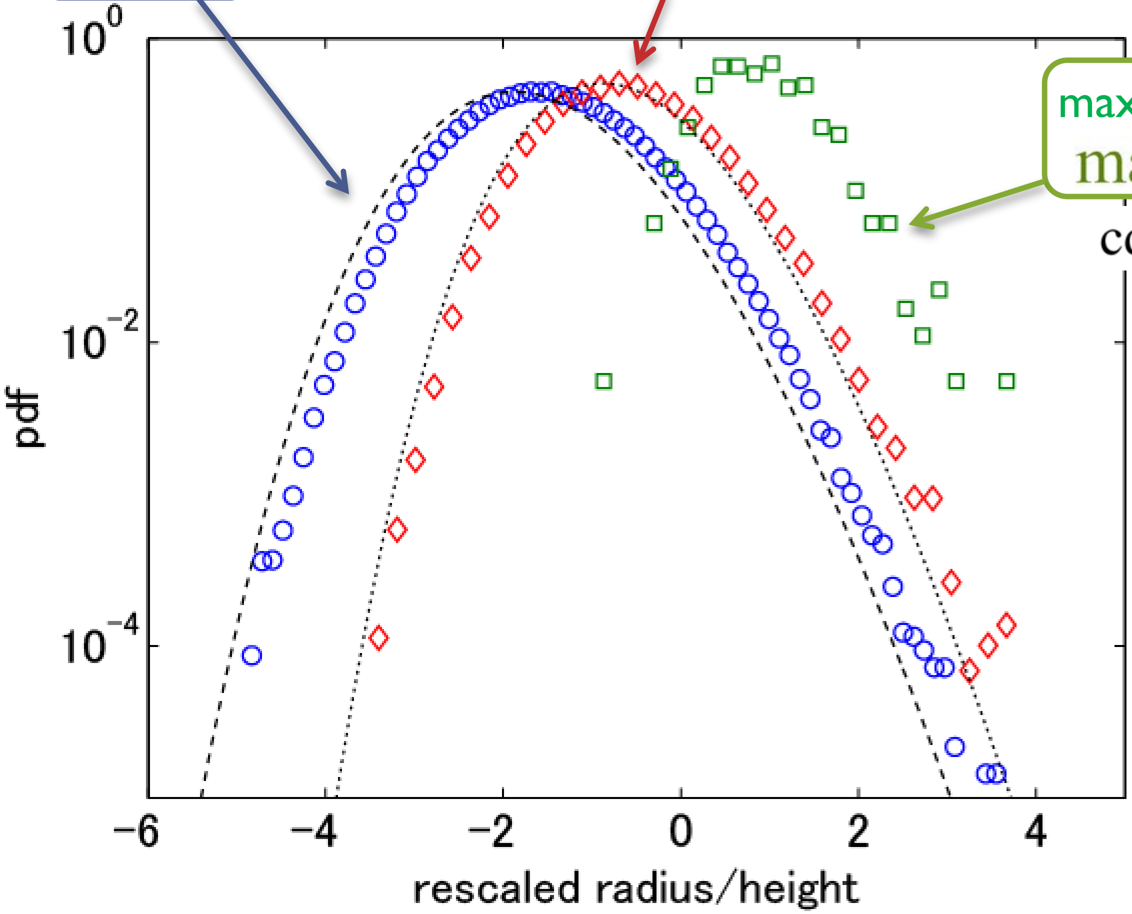


radius R : GUE-TW

max height $\max(R\sin\theta)$: GOE-TW distribution!!

max radius $\max R$: Gumbel dist.

$$\mathrm{cdf}(x) = \exp(-\mathrm{e}^{\alpha(x-u)})$$



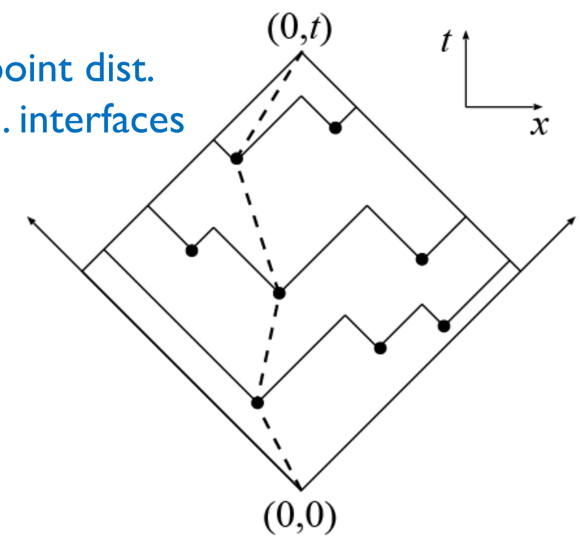
Max heights of circular interfaces obey the GOE-TW dist.!

Why GOE-TW for the Max Heights?

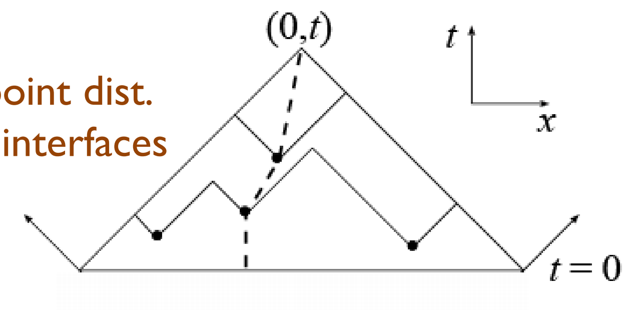
<u>For the PNG model</u>

- Circular ⇨ Consider a square connecting (0,0) and (0,t) ⇨ GUE
- Flat ⇨ Consider a triangle connecting t = 0 and (0,t) ⇨ GOE
- Max height of droplet ⇨ triangle connecting (0,0) and t = t ⇨ GOE!

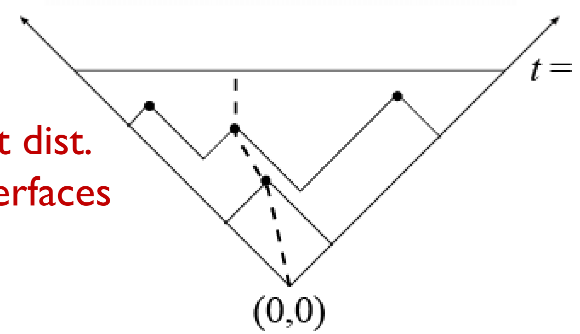
One-point dist.
of circ. interfaces



One-point dist.
of flat interfaces



Max-height dist.
of circ. interfaces



Max-height dist. for circular interfaces has the same symmetry as the one-point dist. for flat interfaces ⇨ GOE-TW dist.!

[proof: Johansson 2003]

Spatial Correlation Function

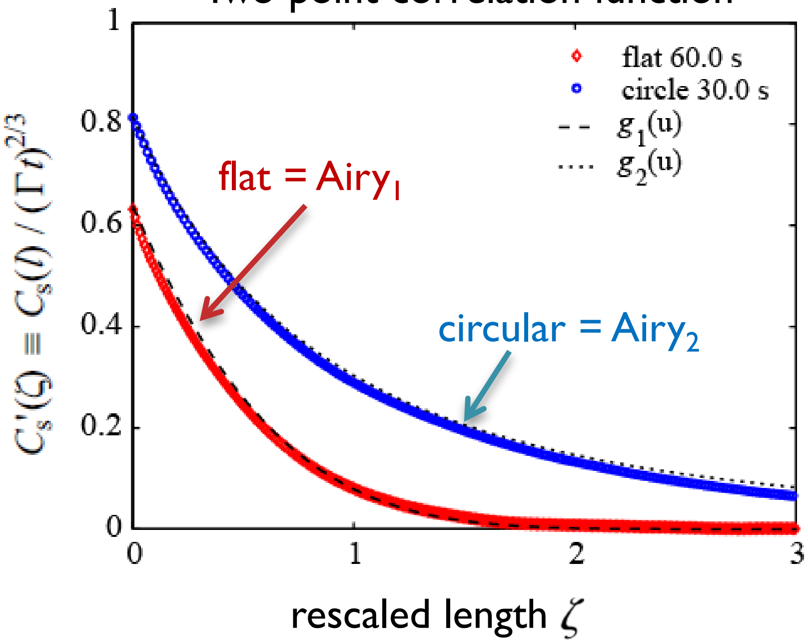
Predictions for solvable models:
$$\boxed{\begin{aligned} C_s(l,t) &\equiv \langle h(x+l,t)h(x,t)\rangle - \langle h\rangle^2 \\ &\simeq (\Gamma t)^{2/3} g_i(\zeta) \end{aligned}}$$

with $i = 1$ (flat), $i = 2$ (circular),

$$\zeta \equiv l\sqrt{\Gamma/2v_\infty}(\Gamma t)^{-2/3} \quad g_i(\zeta) \equiv \langle \mathcal{A}_i(t+\zeta)\mathcal{A}_i(t)\rangle - \langle \mathcal{A}_i(t)\rangle^2$$

$\mathcal{A}_i(t)$: Airy$_i$ process (cf. Airy$_2$ = largest-eigenvalue dynamics in Dyson's Brownian motion of GUE matrices)

Two-point correlation function



Correlation of flat / circular interfaces is governed by the Airy$_1$ / Airy$_2$ process

Qualitatively different decay

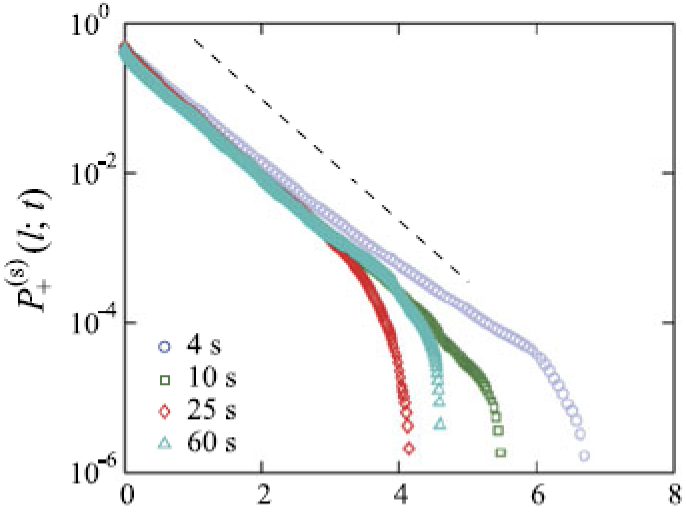
$$\begin{cases} g_2(u) \sim u^{-2} & \text{(circular)} \\ g_1(u): \text{faster than exponential} & \text{(flat)} \end{cases}$$

Spatial Persistence

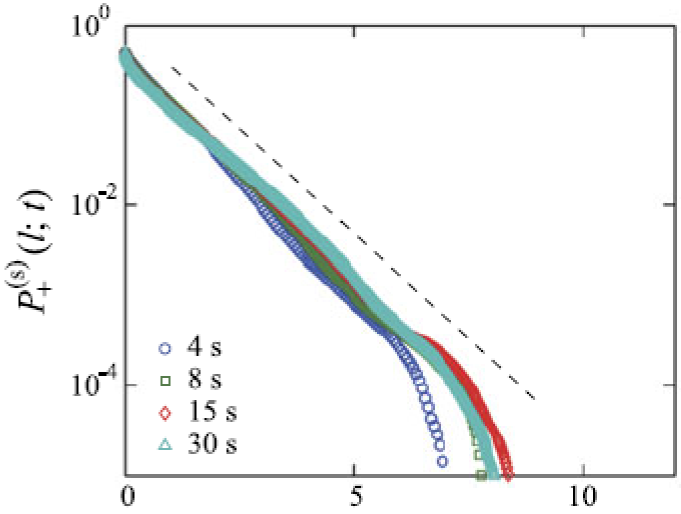
Spatial Persistence probability $P_{\pm}^{(s)}(l;t)$

= joint probability that $\delta h \equiv h(x,t) - \langle h \rangle$ keeps the same sign over length l in space at fixed time t

flat interfaces



circular interfaces



- Exponential decay $\boxed{P_{\pm}^{(s)} \sim \exp(-\kappa_{\pm}^{(s)} u)}$ with symmetric coefficients $\kappa_{+}^{(s)} = \kappa_{-}^{(s)}$
- $\kappa_{\pm}^{(s)}$ expected to be universal $\kappa_{\pm}^{(s)} \approx 2.0$ (flat) $\kappa_{\pm}^{(s)} \approx 0.9$ (circular) [cf. Ferrari&Frings 2013]
- Extension of the Newell-Rosenblatt theorem for Airy$_2$ process ?

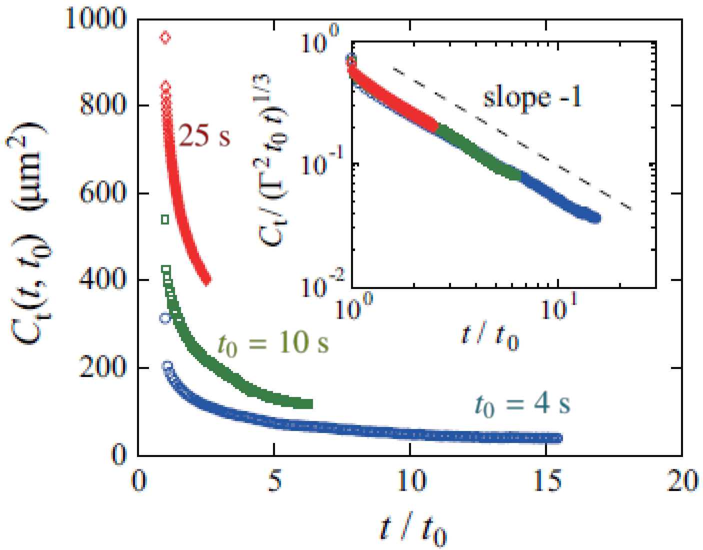
NR theorem: for stat. Gaussian processes, $P^{(s)} \sim \exp(-\kappa t)$ if $\langle \mathcal{A}(t)\mathcal{A}(0) \rangle \sim t^{-\mu}$ $(\mu > 1)$

Temporal Correlation Function

$$C_t(t, t_0) \equiv \langle h(x,t)h(x,t_0)\rangle - \langle h(x,t)\rangle\langle h(x,t_0)\rangle$$

analytically unsolved yet

flat



- Natural scaling ansatz works

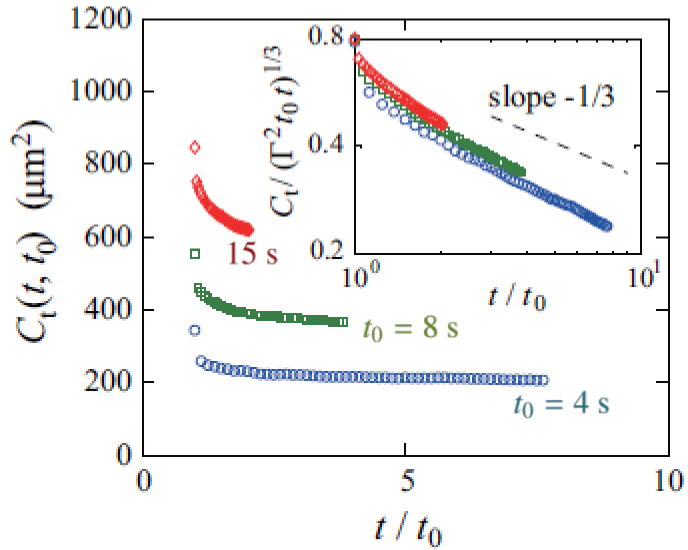
$$C_t(t, t_0) \simeq (\Gamma^2 t_0 t)^{1/3} F_t(t/t_0)$$

- In particular,

$$F_t(t/t_0) \sim (t/t_0)^{-\bar{\lambda}} \text{ with } \bar{\lambda} = 1$$

cf. Kallabis-Krug conjecture $\bar{\lambda} = \beta + d/z = 1$

circular



- The natural scaling does not seem to work as well.
- In particular,

$$C_t(t, t_0) > 0 \quad (t \to \infty)$$ (!)

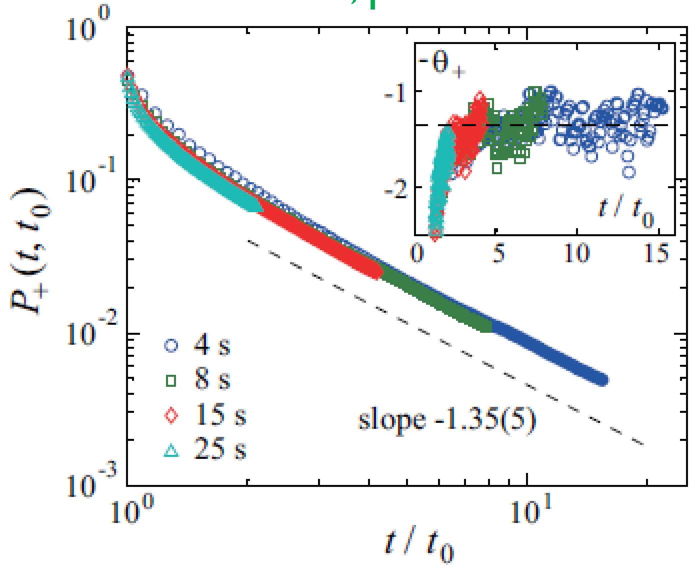
Temporal Persistence (Flat Case)

Persistence probability $P_{\pm}(t, t_0)$

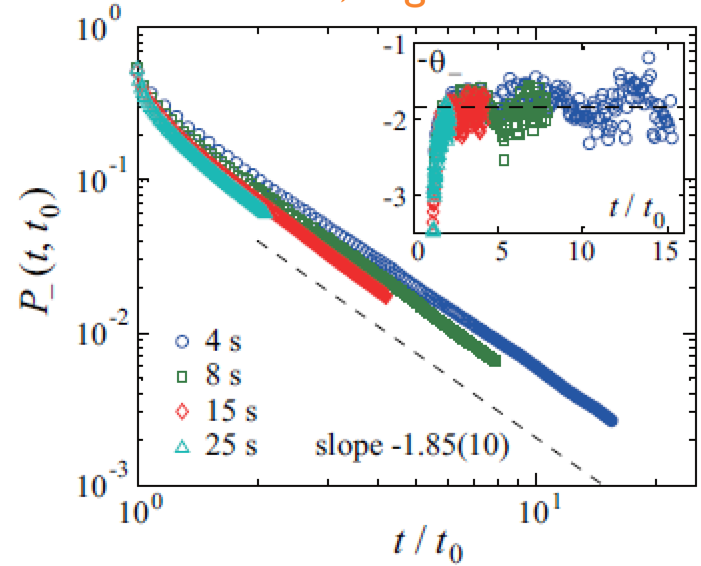
= joint probability that $\delta h \equiv h(x,t) - \langle h \rangle$ at a fixed position x is positive (negative) at time t_0 and keeps the same sign until time t

typically decay with a power law $\boxed{P_{\pm}(t, t_0) \sim t^{-\theta_{\pm}}}$

flat, positive



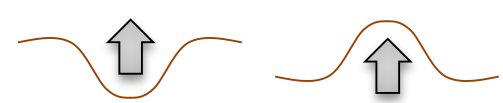
flat, negative



flat

$$\begin{cases} \theta_+ = 1.35(5) \\ \theta_- = 1.85(10) \end{cases}$$

$\theta_+ < \theta_-$ (flat)



because of the KPZ nonlinearity $\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \xi$

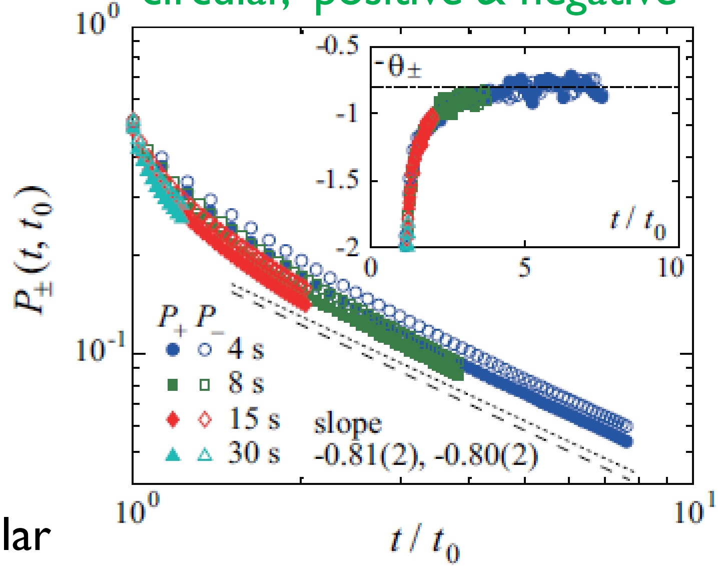
Temporal Persistence (Circular Case)

Persistence probability $P_{\pm}(t, t_0)$

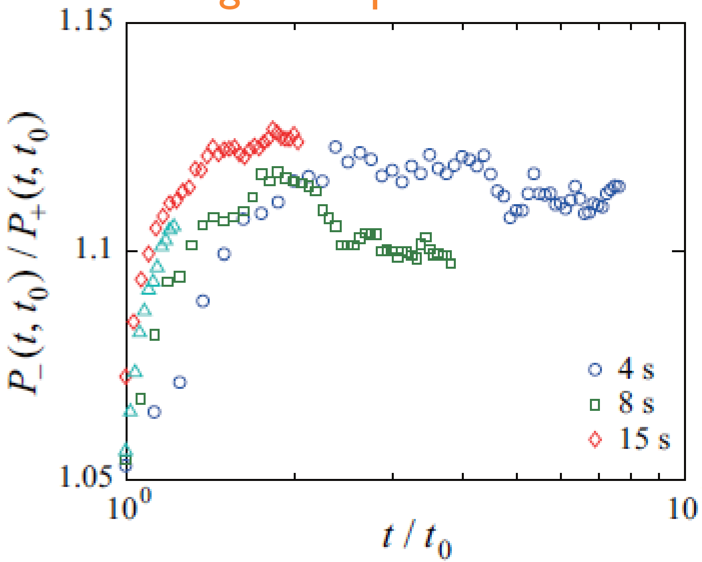
= joint probability that $\delta h \equiv h(x,t) - \langle h \rangle$ at a fixed position x is positive (negative) at time t_0 and keeps the same sign until time t

typically decay with a power law $\boxed{P_{\pm}(t, t_0) \sim t^{-\theta_{\pm}}}$

circular, positive & negative



negative / positive ratio



circular

$$\begin{cases} \theta_+ = 0.81(2) \\ \theta_- = 0.80(2) \end{cases}$$

$$\begin{cases} \theta_+ < \theta_- & \text{(flat)} \\ \theta_+ = \theta_- & \text{(circular)} \end{cases}$$

Asymmetry in persistence exponents is cancelled for the circular interfaces!

3 Important Sub-classes [I. Corwin, Random Matrices: Theor. Appl. **1**, 1130001]

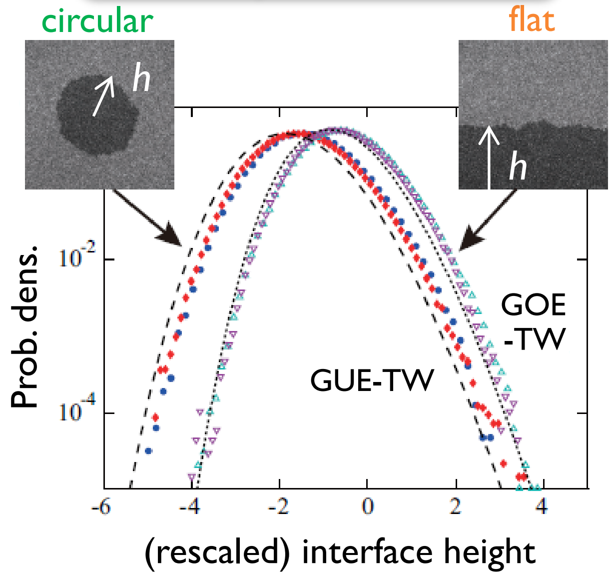
Circular (curved) interfaces

- Init. cond. : point or curved line
- Asymptotics : GUE Tracy-Widom dist., Airy_2 process
- Proved for : TASEP [Johansson CMP 2000], PNG, PASEP, KPZ eq. [Sasamoto-Spohn 2010, Amir *et al.* 2011]

Flat interfaces

- Init. cond. : straight line
- Asymptotics : GOE Tracy-Widom dist., Airy_1 process
- Proved for : PNG [Prähofer-Spohn PRL 2000], TASEP [Sasamoto JPA 2005], KPZ eq.

Liquid-crystal experiment



※ Scaling exponents are the same.

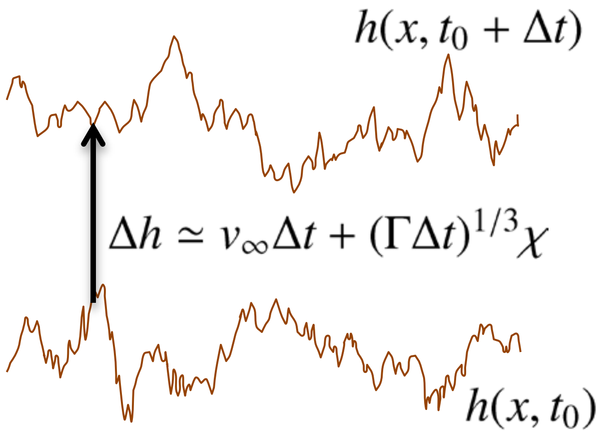
※ Other sub-classes are also argued.

Stationary interfaces

- Init. cond. : stationary interface (= trajectory of 1d-Brownian motion)
- Asymptotics : Baik-Rains F_0 distribution, $\text{Airy}_{\text{stat}}$ process
- Proved for : PNG [Baik-Rains JSP 2000], TASEP, KPZ eq. [Imamura-Sasamoto PRL 2012]

Toward the Stationary Subclass

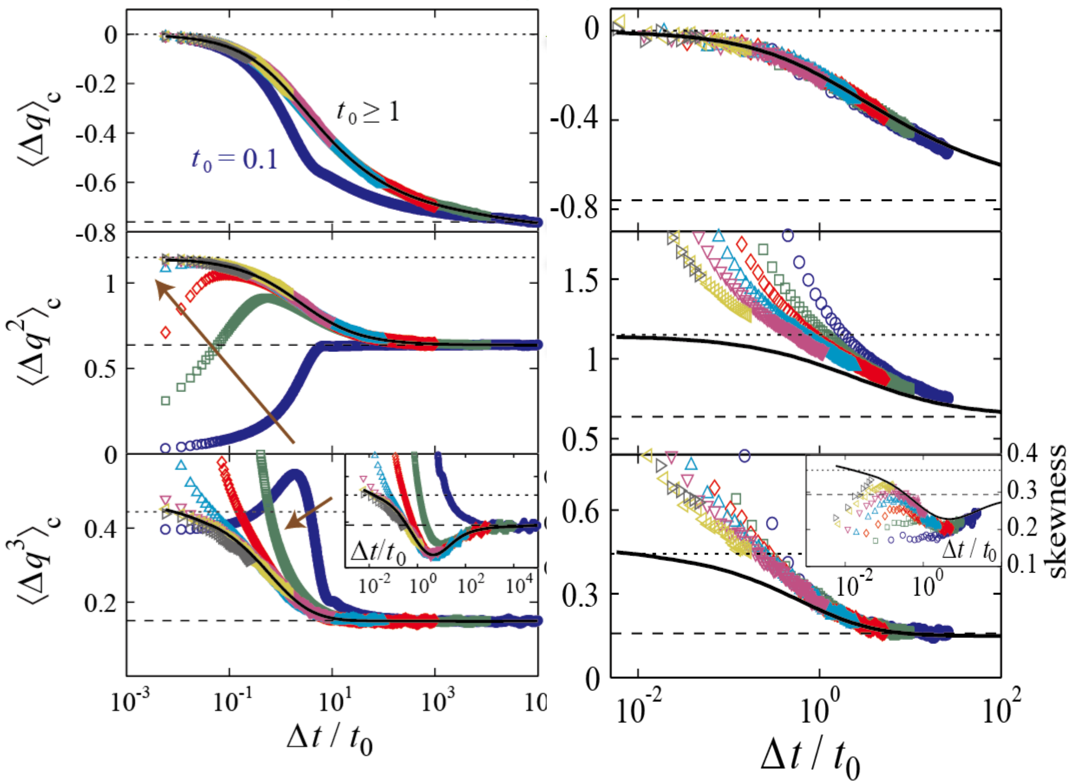
Truly stationary state is never attained unless it is taken as an initial condition, but, approach, or crossover to the stationary subclass can be studied.



rescaled height difference

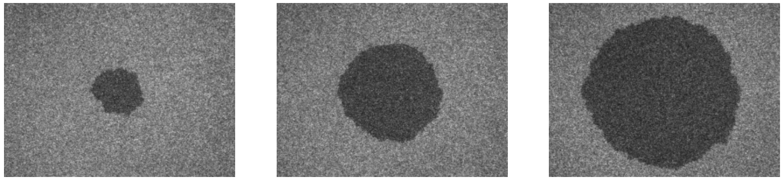
$$\boxed{\Delta q \equiv \frac{\Delta h - v_\infty \Delta t}{(\Gamma \Delta t)^{1/3}} \simeq \chi}$$

PNG model (simulation) experiment



- Scaling functions $\langle \Delta q^n \rangle_c \simeq G_n(\Delta t / t_0)$ describing flat-stationary crossover is found.
- Experiment seems to indicate the same scaling functions, so universal!

Summary



Evidence for KPZ geometry-dependent universal fluctuations in growing interfaces of liquid-crystal turbulence (DSM2)

	Flat interfaces	**Circular interfaces**
scaling exponents	$\alpha = 1/2, \quad \beta = 1/3, \quad z = 3/2$	
distribution	GOE-TW distribution (GOE largest eigenvalue dist.)	GUE-TW distribution (GUE largest eigenvalue dist.)
maximal height dist.	--	GOE-TW distribution
spatial correlation	correlation of Airy_1 process	correlation of Airy_2 process
temporal correlation	$\sim (t/t_0)^{-1}$ in rescaled units	remains strictly positive
temporal persistence	$\theta_+ = 1.35(5) < \theta_- = 1.85(10)$	$\theta_+ = 0.81(2) \approx \theta_- = 0.80(2)$

deep & direct link between quantitative experiment and exactly solvable problems

Our experiment: Takeuchi *et al.*, Sci. Rep. (Nature) 1, 34; J. Stat. Phys. 147, 853
Reviews: Kriecherbauer&Krug, J. Phys. A 43, 403001 (th); Takeuchi, arXiv:1310.0220 (exp)

YITP workshop August 20-23, 2014, Kyoto, Japan

Interface fluctuations and KPZ universality class

– unifying mathematical, theoretical, and experimental approaches

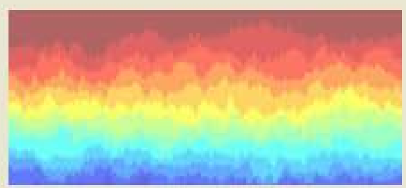
Top page | About | Program | For participants | Useful info

Updates

Apr. 21st
List of invitees updated.

Apr. 15th
Registration open!
Changed abstract instruction
(check here before submission).

Apr. 2nd
Web site open!



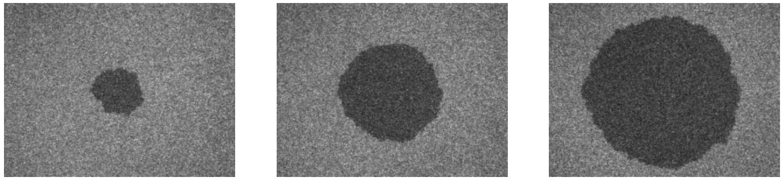
About this workshop

List of invited speakers is announced here.

Check out important dates!

Registration deadline is June 13th for those who wish to contribute a talk, apply for financial support, or need a visa document.

Summary



Evidence for KPZ geometry-dependent universal fluctuations in growing interfaces of liquid-crystal turbulence (DSM2)

	Flat interfaces	**Circular interfaces**
scaling exponents	$\alpha = 1/2, \quad \beta = 1/3, \quad z = 3/2$	
distribution	GOE-TW distribution (GOE largest eigenvalue dist.)	GUE-TW distribution (GUE largest eigenvalue dist.)
maximal height dist.	--	GOE-TW distribution
spatial correlation	correlation of Airy$_1$ process	correlation of Airy$_2$ process
temporal correlation	$\sim (t/t_0)^{-1}$ in rescaled units	remains strictly positive
temporal persistence	$\theta_+ = 1.35(5) < \theta_- = 1.85(10)$	$\theta_+ = 0.81(2) \approx \theta_- = 0.80(2)$

deep & direct link between quantitative experiment and exactly solvable problems

Our experiment: Takeuchi *et al.*, Sci. Rep. (Nature) 1, 34; J. Stat. Phys. 147, 853
Reviews: Kriecherbauer&Krug, J. Phys. A 43, 403001 (th); Takeuchi, arXiv:1310.0220 (exp)