BLACK HOLES WITH QUANTUM HAIR

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hep-th/0605295

OUTLINE

- NO (MASSIVE) HAIR THEOREMS
- LOOPHOLE: QUANTUM HAIR
  BLACK HOLES WITH QUANTUM SPIN-2 HAIR.
- STRINGY SPIN-2 AHARONOV-BOHM EFFECT
- ANALOGY WITH U(1)-AHARONOV-BOHM
- BLACK HOLES WITH QUANTUM HAIR OF MASSIVE (QCD) AXION (OR η'-MESON)
- RELATED PUZZLE AND RESOLUTION
- IMPLICATIONS (COSMOLOGY, LHC, LARGE DISTANCE MODIFIED GR
- CONCLUSIONS

COMPLEMENTARY (VERY INTERESTING)

POSSIBILITY:

BLACK HOLE HAIR UNDER
DISCRETE GUAGE SYMMETRIES

Krauss & Wilczek
Preskill & Krauss

WHEN,

$$U(1) \longrightarrow Z_N$$

BLACK HOLES CAN HAVE Z_N CHARGE.

GOOD QUANTUM NUMBERS FOR THE BLACK HOLES

MASS M

SPIN J

AND CHARGES UNDER THE MASSLESS GAUGE FIELDS

(E.G. ELECTRIC CHARGE Q)

BLACK HOLES HAVE NO MASSIVE HAIR

Bekenstein
Teitelboim

- IN PARTICULAR, NO HAIR FOR MASSIVE SPIN-2 FIELD

$h_{\mu\nu}$

PAULI-FIERZ ACTION

$$h^{\mu\nu}\mathcal{E}h_{\mu\nu} - m^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

LINEARIZED EINSTEIN

$$\mathcal{E}h_{\mu\nu} - m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = 0$$

FOR $m^2 = 0$, THERE IS A GAUGE INVARIANCE

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

BUT, FOR $m^2 \neq 0$,

$h_{\mu\nu}$ IS OBSERVABLE, AND MUST VANISH AT THE HORIZON, AND EVERYWHERE.

BUT, THERE IS A LOOPHOLE:

QUANTUM HAIR!

IN FACT, $m^2 \neq 0$ THEORY IS ALSO

HIDDENLY GAUGE INVARIANT.

$m^2 \to 0$ LIMIT IS DISCONTINUOUS:

van Dam, Veltman;
Zakharov.

For $m^2 \neq 0$,

$h_{\mu\nu} \leftarrow$ 5 DEGREES OF FREEDOM

FOR $m^2 = 0$,

$h_{\mu\nu} \leftarrow$ 2 DEGREES OF FREEDOM

WE CAN'T TALK ABOUT GAUGE SYMMETRY, UNLESS WE KNOW HOW DEGREES OF FREEDOM TRANSFORM.

$$h_{\mu\nu} \equiv \hat{h}_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu$$

NOW PAULI-FIERZ BECOMES:

$$\hat{h}_{\mu\nu} \mathcal{E} \hat{h}^{\mu\nu} - m^2 \left[\left(\hat{h}_{\mu\nu} + \underline{\partial_{\{\mu} A_{\nu\}}} \right)^2 - \left(\hat{h} + 2 \underline{\partial_\mu A^\mu} \right)^2 \right]$$

WHICH IS GAUGE INVARIANT UNDER:

$$\hat{h}_{\mu\nu} \rightarrow \hat{h}_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

$$A_\mu \rightarrow A_\mu - \xi_\mu$$

THE EQUATIONS:

$$\mathcal{E}\hat{h}_{\mu\nu} + m^2\left[\left(\hat{h}_{\mu\nu}+\partial_{\{\mu}A_{\nu\}}\right) - \eta_{\mu\nu}\left(\hat{h}+2\partial_\alpha A^\alpha\right)\right] = 0$$

$$\partial^\mu F_{\mu\nu} = -\partial^\mu\left(\hat{h}_{\mu\nu} - \eta_{\mu\nu}\hat{h}\right)$$

$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$.

THERE IS A LOCALLY-PURE-GAUGE SOLUTION:

$$h_{\mu\nu} = 0 \longleftrightarrow \hat{h}_{\mu\nu} = -\partial_{\{\mu}A_{\nu\}}$$

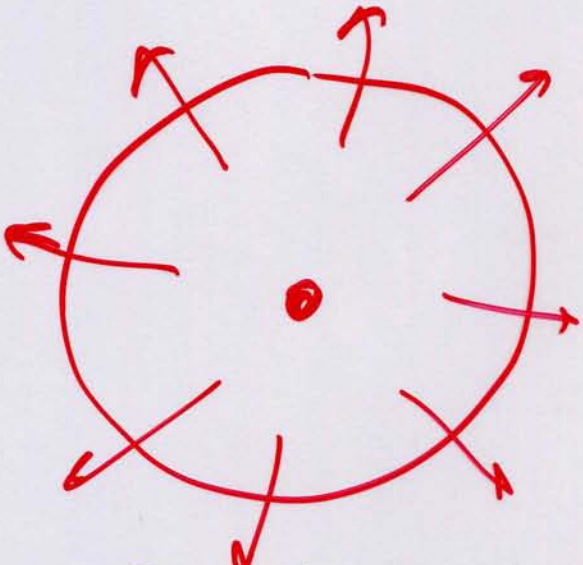
$$F_{\mu\nu} = \mu\,\frac{\epsilon_{\mu\nu}}{r^2}$$

MAGNETIC MONOPOLE TYPE CONFIGURATION FOR A_μ:

$$F_{\mu\nu} = \mu \frac{\epsilon_{\mu\nu}}{r^2}$$

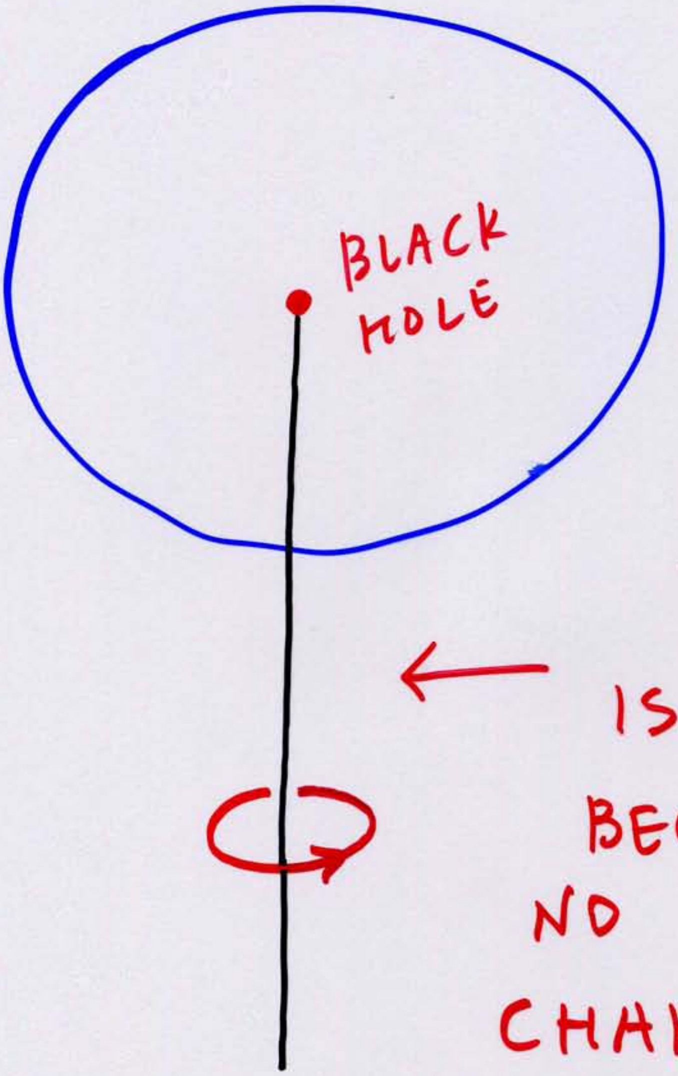
$$A_\phi = \mu \frac{1-\cos\Theta}{\sin\Theta}, \quad A_\Theta = A_r = 0$$

$$\vec{M} = \mu \frac{\vec{r}}{r^3}$$



BUT, THIS IS <u>NOT</u> A DIRAC MAGNETIC MONOPOLE, BECAUSE THERE IS NO DETECTABLE MAGNETIC FIELD.

REMEMBER $h_{\mu\nu} \equiv 0$!!!



DIRAC STRING IS UNOBSERVABLE, BECAUSE THERE ARE NO ELECTRICALLY CHARGED PARTICLES UNDER A_μ

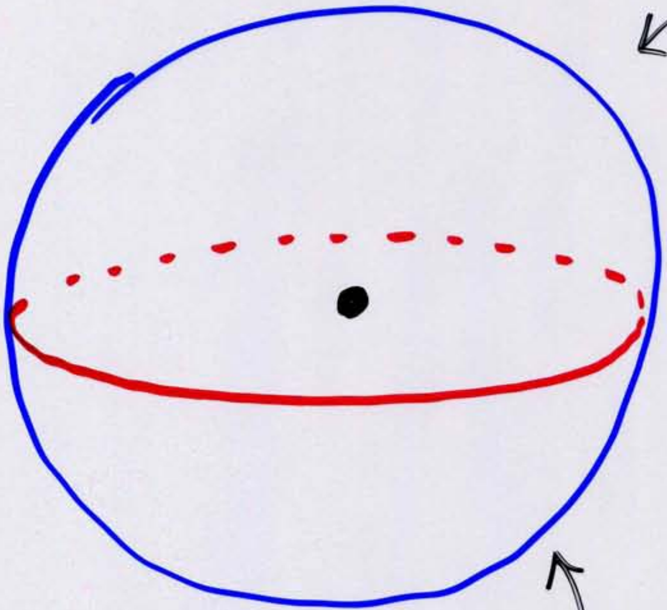
$$\int dX^\mu A_\mu$$

IS NOT GAUGE INVARIAT UNDER:

$A_\mu \rightarrow A_\mu +$ [redacted] ξ_μ

NO REFERENCE TO DIRAC STRING

Wu & Yang



$$A^U_\phi = \mu \frac{(1-\cos\Theta)}{\sin\Theta}$$

$$A^L_\phi = -\mu \frac{(1+\cos\Theta)}{\sin\Theta}$$

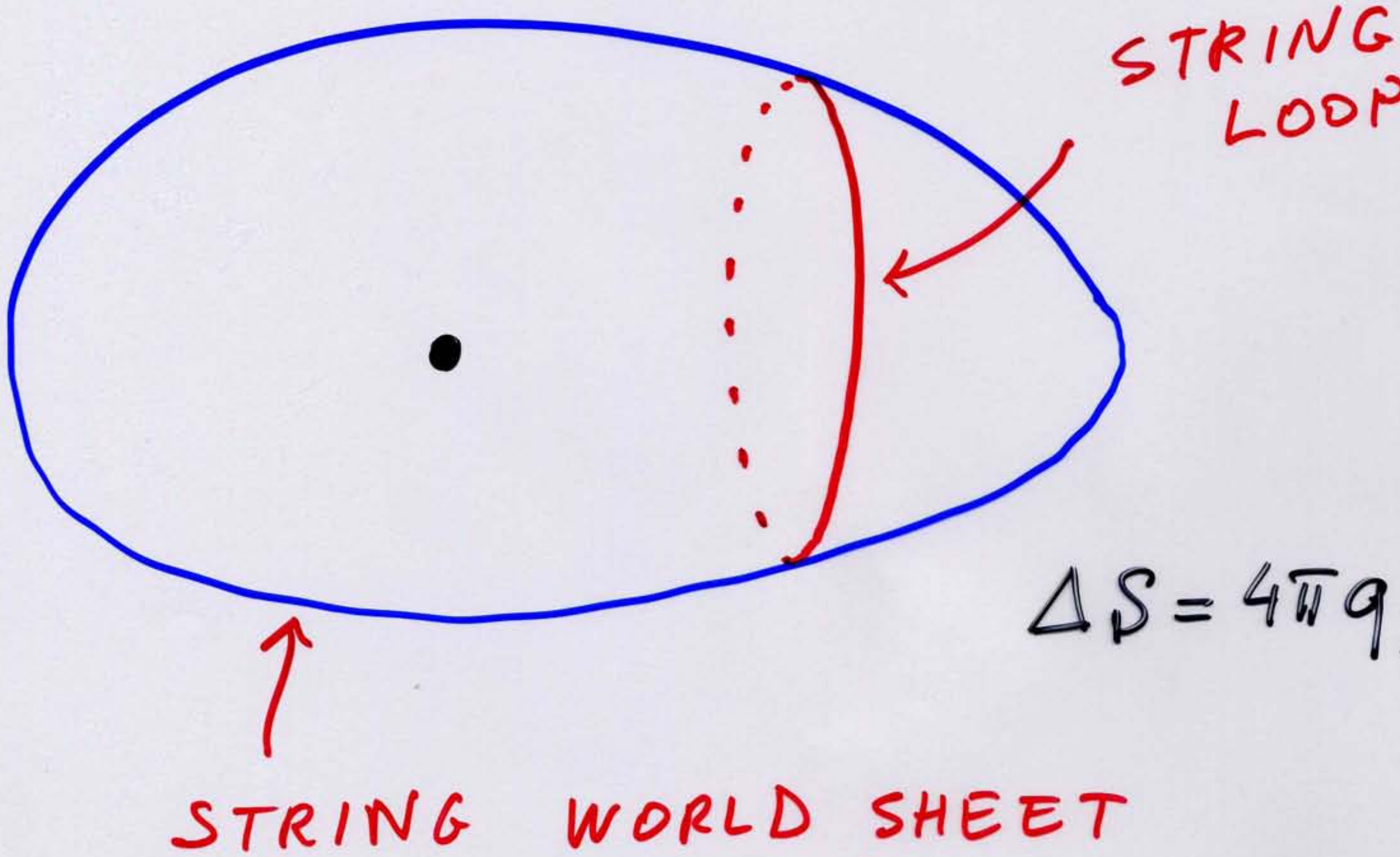
$$A^U_\mu - A^L_\mu \Big|_{\Theta=\frac{\pi}{2}} = \mu\, \partial_\mu \phi \Big|_{\Theta=\frac{\pi}{2}}$$

$$h^U_{\mu\nu} - h^L_{\mu\nu} \Big|_{\Theta=\frac{\pi}{2}} = 2\mu\, \partial_\mu \partial_\nu \phi \Big|_{\Theta=\frac{\pi}{2}}$$

$h^U_{\mu\nu}$ and $h^L_{\mu\nu}$ describe the same physics.

THE MASSIVE SPIN-2 HAIR IS CLASSICALLY UNDETECTABLE, BUT CAN BE DETECTED QUANTUM-MECHANICALLY BY STRINGY AHARONOV-BOHM EXPERIMENT

$$q \int dx^\mu \wedge dx^\nu F_{\mu\nu}$$



$$\Delta S = 4\pi q \mu$$

TO SUMMARIZE, IN THE PRESENCE OF STRINGS, PAULI-FIERZ ACTION CAN BE REPRESENTED IN THE FORM:

$$\hat{h}^{\mu\nu}\mathcal{E}\hat{h}_{\mu\nu} - m^2\left(\left(\hat{h}_{\mu\nu} + \partial_{\{\mu}A_{\nu\}}\right)^2 - \left(\hat{h} + 2\partial_{\alpha}A^{\alpha}\right)^2\right)$$

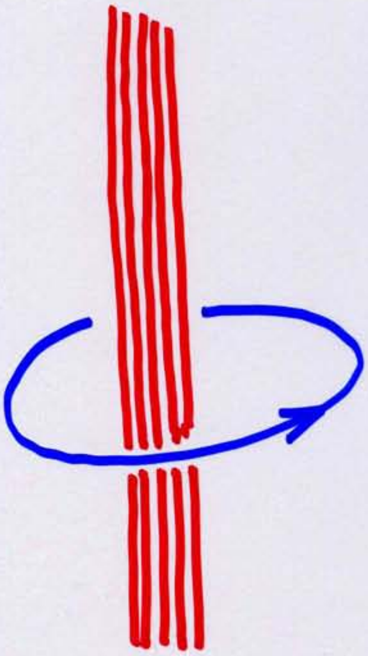
$$+ \, q\int dx^{\mu}\wedge dx^{\nu}\, F_{\mu\nu}$$

BLACK HOLES ARE LABELED BY THE QUANTUM SPIN-2 CHARGE μ.

IT IS DETECTABLE IF:

$$q\mu \neq \frac{n}{2}$$

PARALLEL WITH AHARONOV-BOHM EFFECT FOR MASSIVE $U(1)$.



MASSIVE SPIN-1 FIELD Z_μ IN THE BACKGROUN MAGNETIC FLUX

PROCA THEORY:

$$\mathcal{L} = -F_{\mu\nu}F^{\mu\nu} + m^2 Z_\mu Z^\mu$$

$$Z_\mu \to 0 \quad \text{FOR} \quad \rho \to 0$$

AND ONE MAY THINK THAT THERE IS NO EFFECT.

BUT,

FOR $m^2 \neq 0$,

$$Z_\mu \leftarrow \text{3 DEGREES OF FREEDOM}$$

FOR $m^2 = 0$,

$$Z_\mu \leftarrow \text{2 DEGREES OF FREEDOM}$$

SO AS IN THE CASE OF PAULI-FIERZ, THERE IS A HIDDEN GAUGE SYMMETRY IN PROCA THEORY.

REWRITE

$$Z_\mu \equiv \hat{Z}_\mu + \partial_\mu a$$

THEN, PROCA THEORY

$$L = - F_{\mu\nu} F^{\mu\nu} + m^2 (\hat{Z}_\mu + \partial_\mu a)^2$$

IS GAUGE INVARIANT UNDER

$$\hat{Z}_\mu \rightarrow \hat{Z}_\mu + \partial_\mu \omega$$

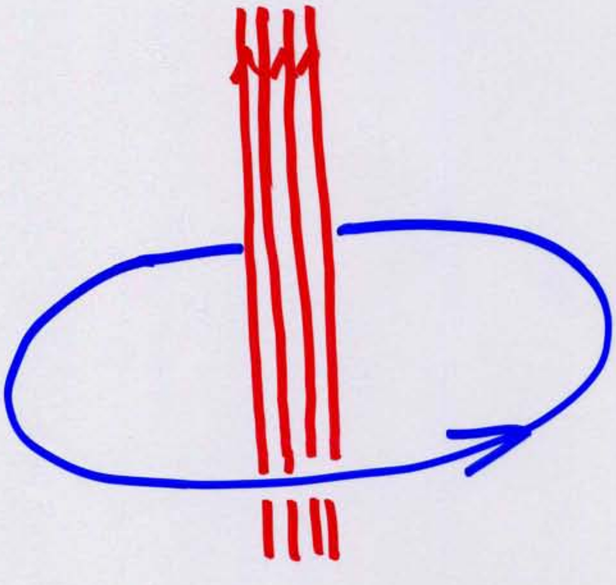
$$a \rightarrow a - \omega$$

SO OUTSIDE FLUX

$$Z_\mu = \hat{Z}_\mu + \partial_\mu a = 0$$

BUT,

$$\hat{Z}_\varphi = -\frac{\partial_\varphi a}{\rho} = \frac{1}{\rho} \neq 0$$



$$\oint \partial Z = -\oint \partial a = 2\pi$$

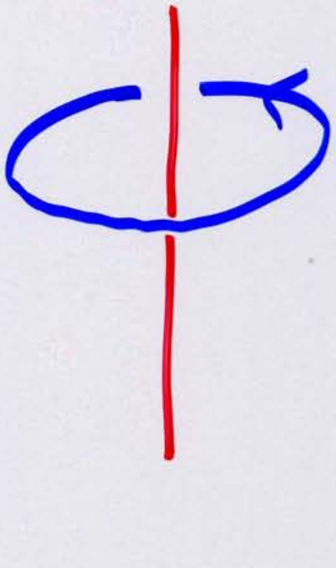
THE CONDITION FOR NON TRIVIAL AB - EFFECT IS THAT PARTICLES COUPLE TO A COMBINATION

$$\int dX^{\mu}\left[\alpha \hat{Z}_{\mu}+\beta \partial_{\mu} a\right]$$

Such that

$$\alpha-\beta \neq n$$

IN SUCH A CASE:



$$\Delta S = 2\pi(\alpha-\beta)$$

LESSON: IN THE PRESENCE OF PHOTON MASS AHARONOV-BOHM EFFECT IS NOT DETERMINED BY THE ELECTRIC CHARGE OF A PARTICLE, BUT BY THE VALUE OF

$$(\alpha - \underline{\beta})$$

FOR EXAMPLE IF $\alpha = 0$

A PARTICLE IS ELECTRICALLY NEUTRAL, BUT THE AB-EFFEC WILL PERSIST FOR

$$\underline{\beta} \neq n$$

THE COMPLETE ANALOGY WITH SPIN-2 CASE IS $\alpha = 0$:

SPIN-1 PROCA $\Longrightarrow$ $B \int dx^\mu \partial_\mu a$

SPIN-2 PAULI-FIERZ $\Longrightarrow$ $q \int dx^\mu \wedge dx^\nu F_{\mu\nu}$

ANALOGIES:

$$Z_\mu \longleftrightarrow h_{\mu\nu}$$

$$\hat{Z}_\mu \longleftrightarrow \hat{h}_{\mu\nu}$$

$$a \longleftrightarrow A_\mu$$

BLACK HOLES WITH QUANTUM (QCD) AXION HAIR

A MASSLESS KALB-RAMOND FIELD

$$\mathcal{L} = H_{\alpha\beta\gamma} H^{\alpha\beta\gamma}$$

$$H_{\alpha\beta\gamma} \equiv \partial_{[\alpha} B_{\beta\gamma]} \equiv dB$$

INVARIANT UNDER

$$B_{\alpha\beta} \rightarrow B_{\alpha\beta} + \partial_\alpha \xi_\beta - \partial_\beta \xi_\alpha$$

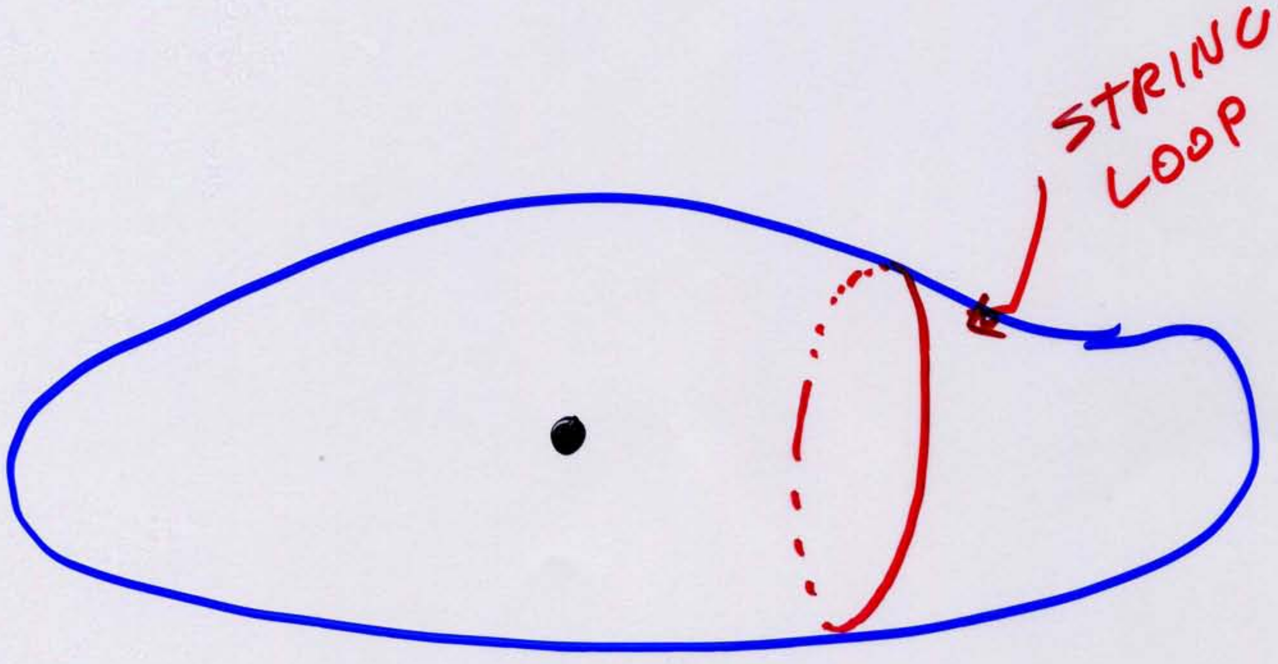
FOR MASSLESS $B_{\mu\nu}$ THERE IS A BLACK HOLE SOLUTION WITH LOCALLY-PURE-GAUGE $B_{\mu\nu}$

Bowick, Giddings, Harvey, Horowitz & Strominger.

$$B_{\mu\nu} = \mu \frac{\epsilon_{\mu\nu}}{r^2} \qquad H_{\mu\nu\alpha} = 0$$

CAN BE DETECTED BY STRINGY AHARONOV-BOHM EFFECT

$$q \int dx^\mu \wedge dx^\nu B_{\mu\nu}$$



CAN $B_{\mu\nu}$ BE A MASSIVE QCD-TYPE AXION?

THERE ARE TWO WAYS TO GIVE MASS TO $B_{\mu\nu}$

(*) ONE IS TO COUPLE IT TO A SPIN-1 STÜCKELBERG FIELD. Z_μ

$$L = H^2 + m^2 \left[B_{\mu\nu} + \partial_{[\mu} Z_{\nu]} \right]^2$$

THIS IS A DUAL DESCRIPTION OF AN USUAL HIGGS EFFECT.

$$1 + 2 = 3$$

NOT THE RIGHT DESCRIPTION FOR QCD AXION

(*) THE CORRECT MASS FOR THE QCD AXION IS:

$$m^2 \left[C_{\alpha\beta\gamma} + d_{[\alpha} B_{\beta\gamma]} \right]^2$$

WHERE

$$C_{\alpha\beta\gamma} \equiv \frac{g^2}{8\pi^2} \mathrm{Tr} \left(A_{[\alpha} A_{\beta} A_{\gamma]} - \frac{3}{2} A_{[\alpha} \partial_{\beta} A_{\gamma]} \right)$$

IS QCD CHERN-SIMONS 3-FORM.

$C_{\alpha\beta\gamma} \leftarrow \underline{\underline{0}}$ DEGREES OF FREEDOM

SO WE HAVE:

$$1 + 0 = 1$$

THE CORRECT EFFECTIVE LAGRANGIAN FOR QCD AXION IS

$$\mathcal{L} = K(dC) + m^2 (C + dB)^2$$

DUALIZATION

$\downarrow$

$$\mathcal{L}(\theta) = (\partial_\mu \theta)^2 - V(\theta)$$

WHERE

$$V(\theta) = -\int \mathrm{inv}\, K'(\theta)\, d\theta$$

$K(dc)$ is a nonsingular function.

FOR

$$\mathcal{L} = K(dC) + m^2 (C + dB)^2$$

THERE IS A BLACK HOLE SOLUTION WITH

$$B_{\mu\nu} = \mu \frac{\epsilon_{\mu\nu}}{r^2}$$

$$C_{\mu\nu\alpha} = dB = 0.$$

THIS HAIR IS QUANTUM.

IS IT DETECTABLE???

NOTICE, FOR

$$L = K(dc) + m^2 (C + dB)^2$$

THE GAUGE SYMMETRY IS

$$C \to C + d\Omega \qquad B \to B + \Omega$$

THERE IS NO INVARIANT COUPLING WITH THE STRING BECAUSE

$$\int dx^\mu \wedge dx^\nu \, B_{\mu\nu}$$

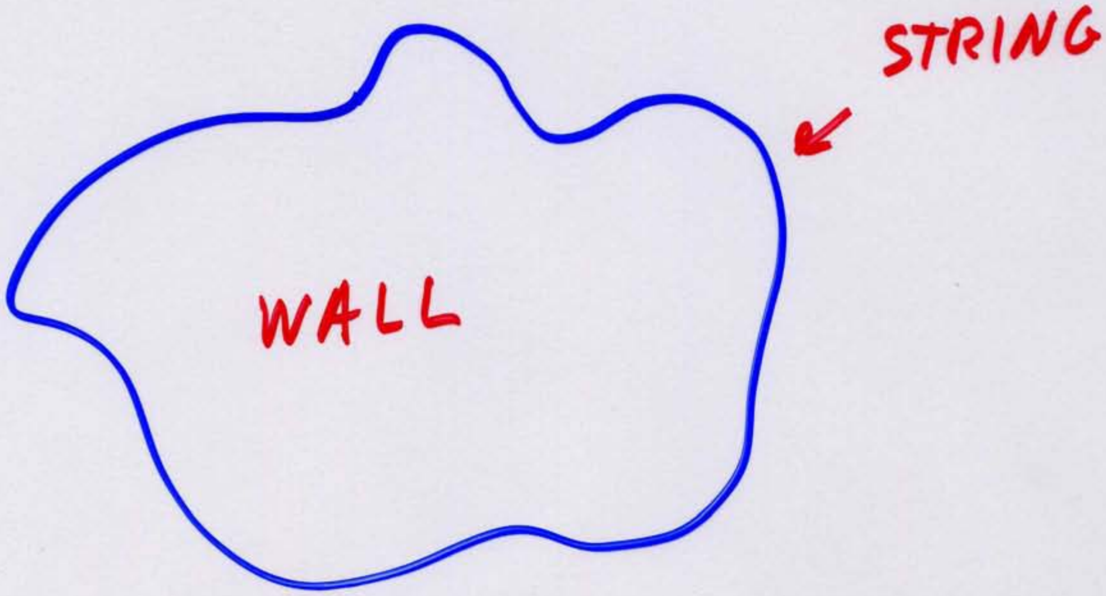
CHANGES UNDER

$$B_{\mu\nu} \to B_{\mu\nu} + \Omega_{\mu\nu}$$

BUT, ON THE OTHER HAND THERE ARE AXIONIC STRINGS IN QCD. SO WHAT IS GOING ON?

THE RESOLUTION OF THE PUZZLE IS IN DOMAIN WALLS.

AXIONIC STRING IS A BOUNDARY OF THE DOMAIN WALL, THAT SOURCES $C_{\mu\nu\alpha}$

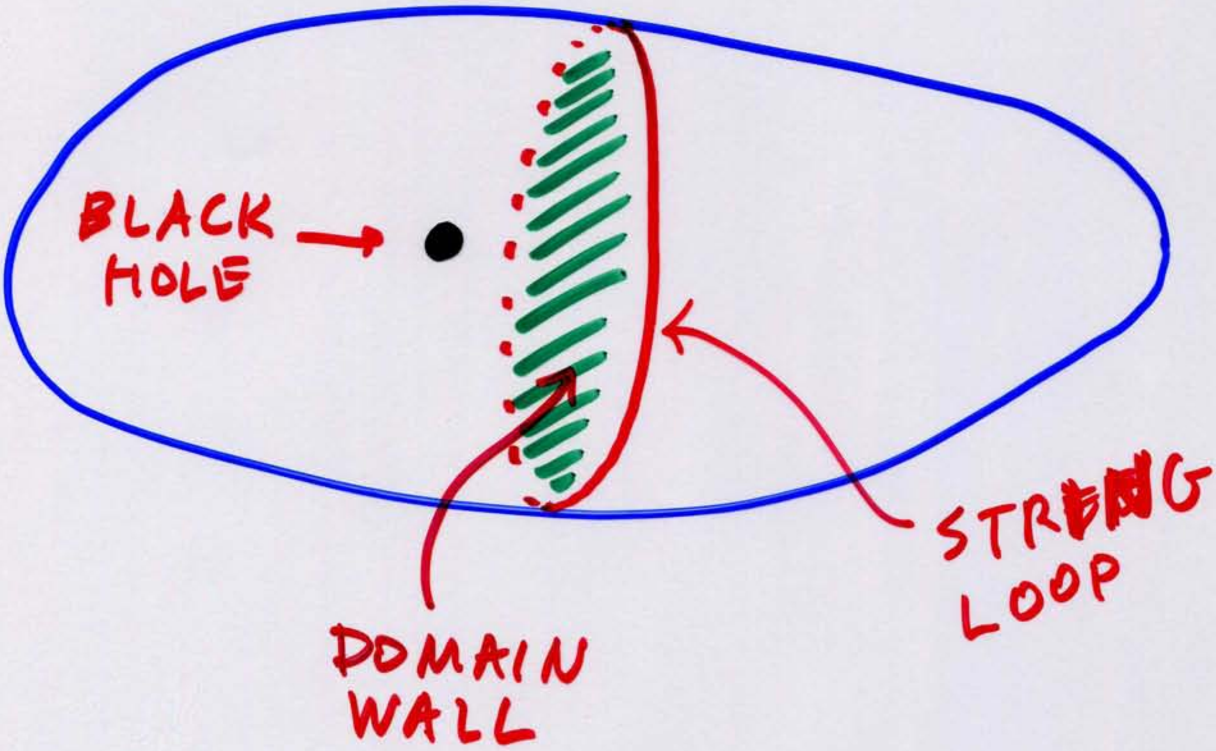


SO THE TWO COUPLINGS

$$\int dx^\mu \wedge dx^\nu \wedge dx^\gamma \, C_{\mu\nu\gamma} + \int dx^\mu \wedge dx^\nu \, B_{\mu\nu}$$

ARE INVARIANT.

BUT WHEN AXIONIC STRING LASSOES BLACK HOLE,



THE BLACK HOLE HAS TO GO TROUGH THE WALL.

CONCLUSIONS

* BLACK HOLES CAN POSSES A QUANTUM HAIR OF MASSIVE SPIN-2 FIELD

  CANDIDATES: KK, SPIN-2 GLUEBALLS, (MASSIVE GRAVITON?)

* HAIR IS DETECTABLE (AT INFINITY) BY STRINGY AHARONOV-BOHM EFFECT.

* LARGE DISTANCE EFFECT SURVIVES FOR $m \to \infty$ (M_P)

* THE SAME IS TRUE ABOUT THE MASSIVE (QCD) AXION OR η'

* BECAUSE OUR METHODS ARE TOPOLOGICAL, THEY APPLY TO ARBITRARILY SMALL BLACK HOLES, FOR WHICH THE QUASI-CLASSICAL THEATMENT BREAKS DOWN.

(*) IMPLICATION:
FOR BLACK HOLE - STRIN INTERACTIO
IN COSMOLOGY OR AT LHC

(*) INFORMATION RECOVERY AFTER
BLACK HOLE EVAPORATION.