Magnets: even a bit of disorder can make a great difference.

With M.Zannetti, E.Lippiello,
A.Decandia, S.Puri, A.Mukherjee …

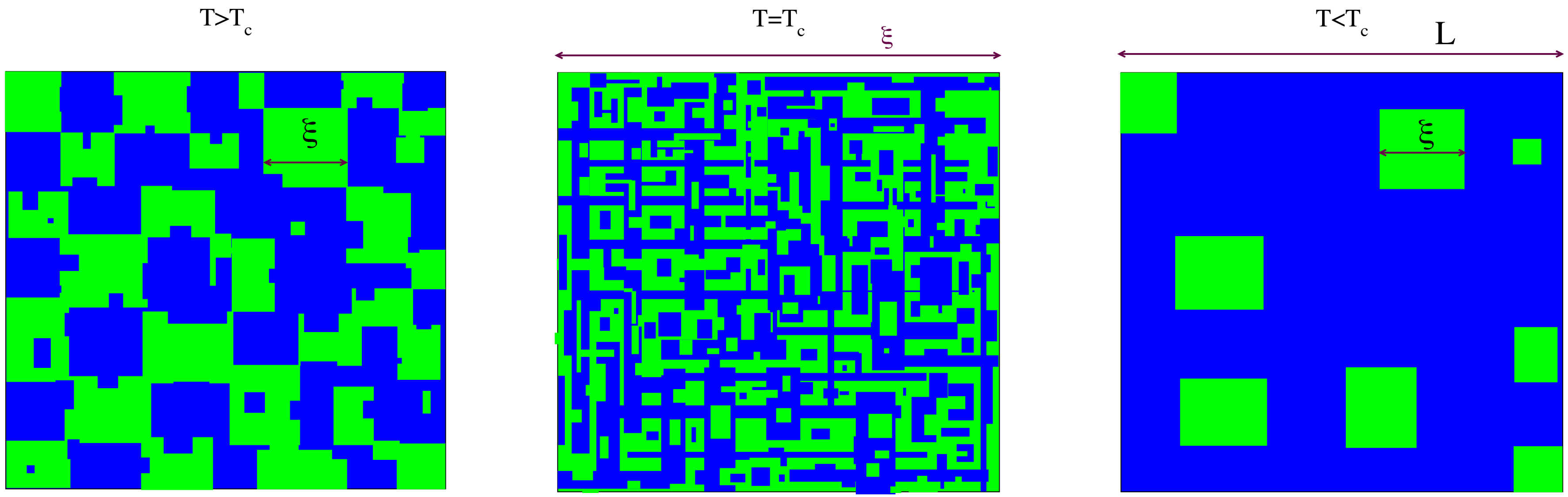
Outlook

- Coarsening in clean magnets
- Coarsening in dirty magnets (d=1)
- Coarsening in dirty magnets (d>1)

Warm up

Clean magnets

Equilibrium



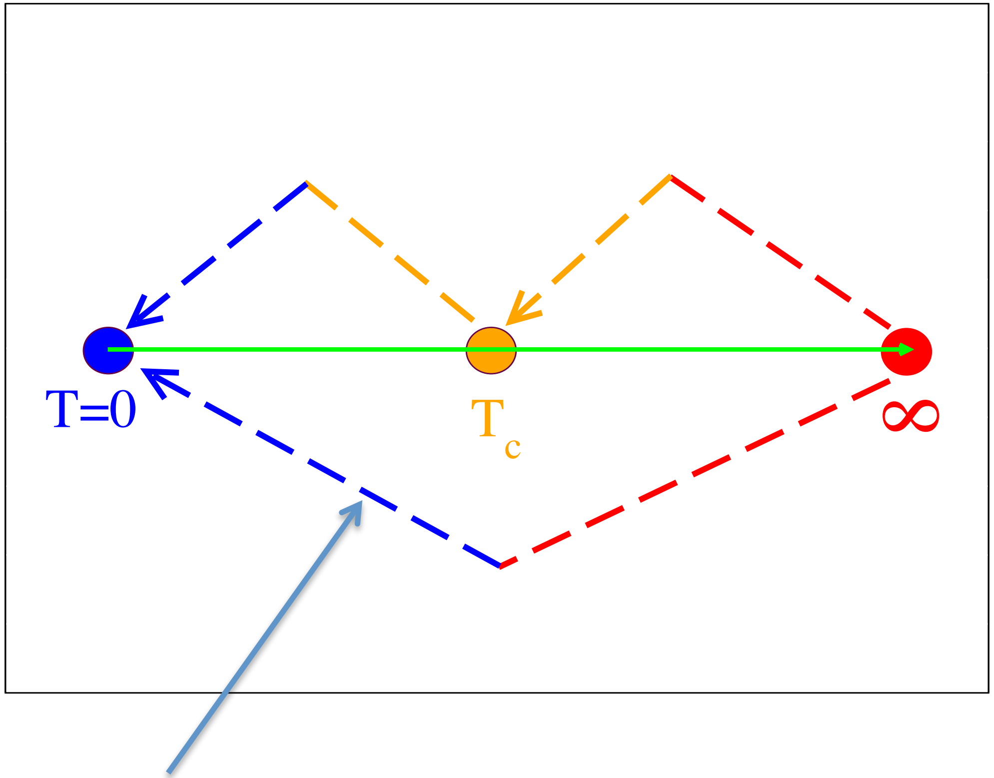
If Disorder Present Must not Alter This Structure

Model

$$H = -J \sum_{\langle ij \rangle} S_i S_j$$ (Or other)

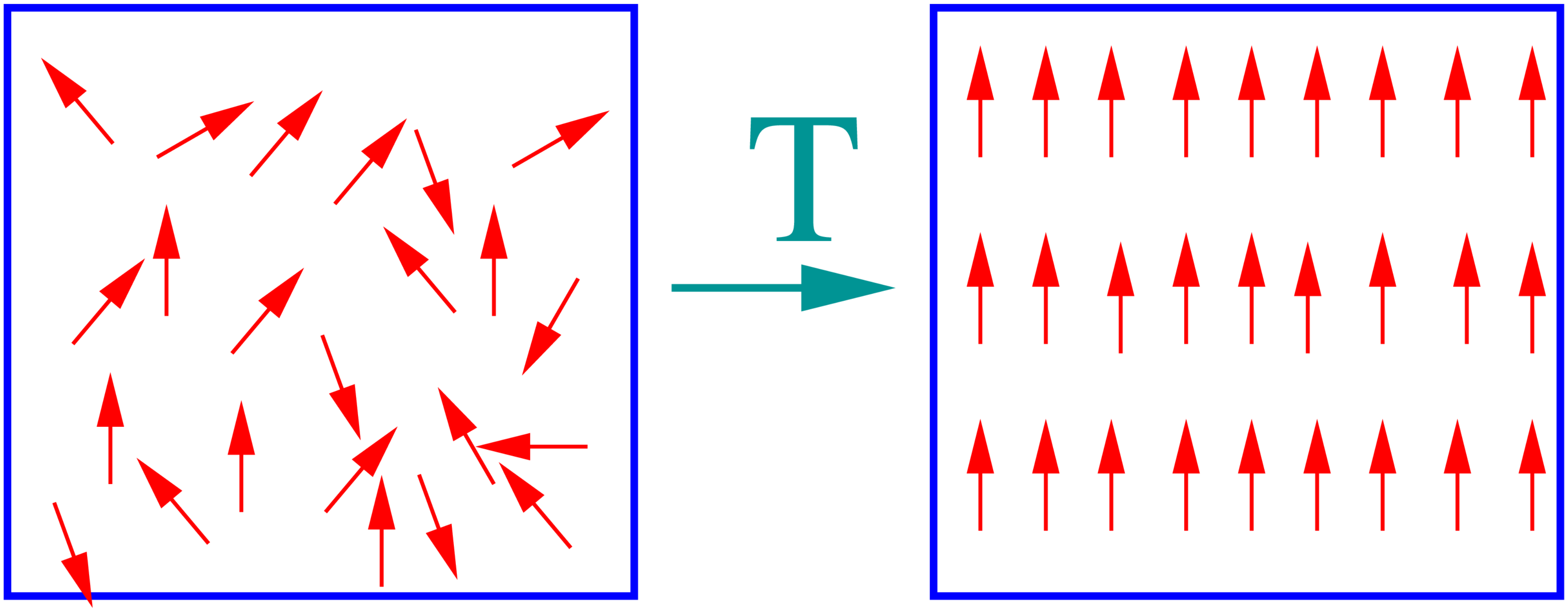
+ Dynamics

Never Equilibrating Processes

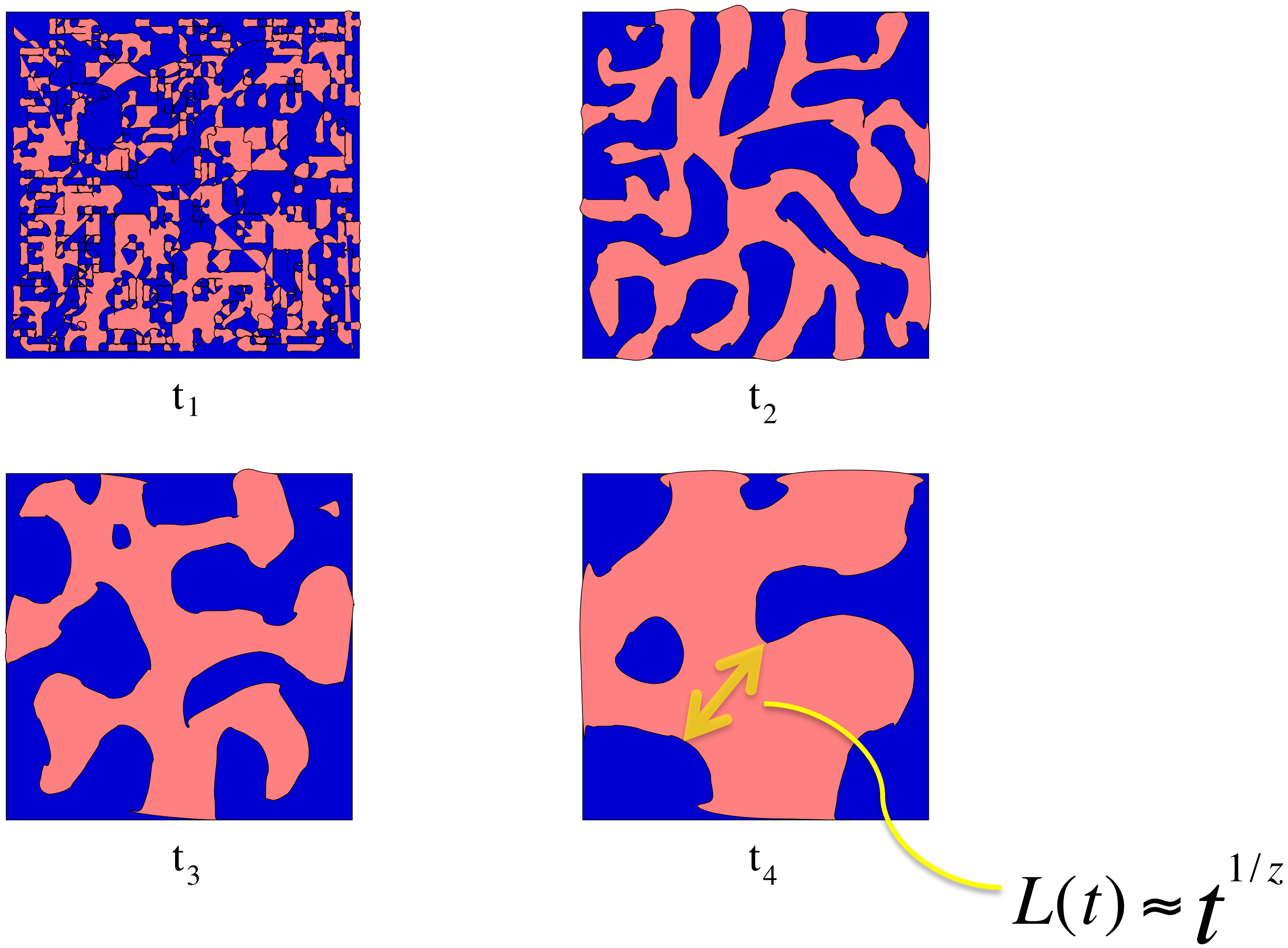


Focus on this

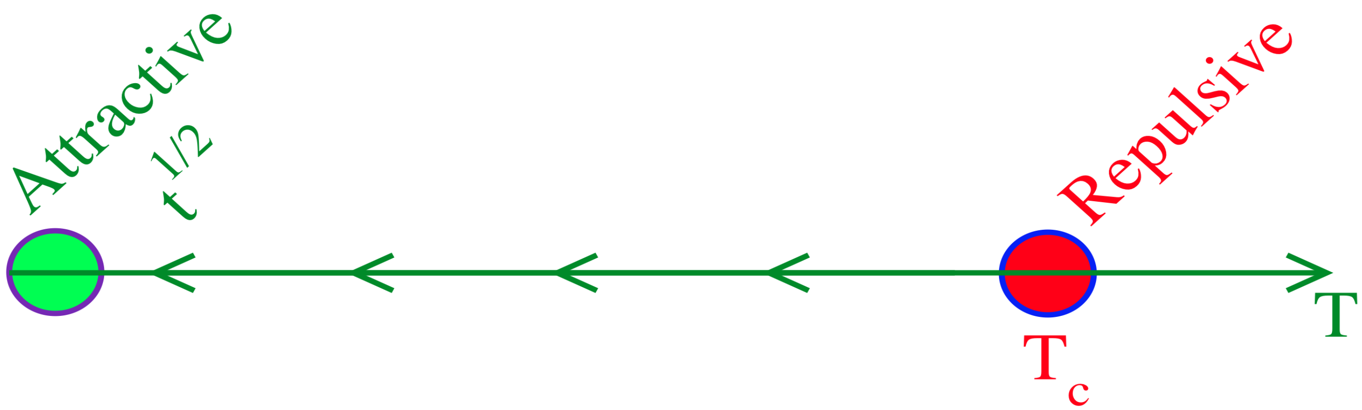
Ordering



Building up an infinite length



(Dynamical) RG interpretation



Dynamical Scaling

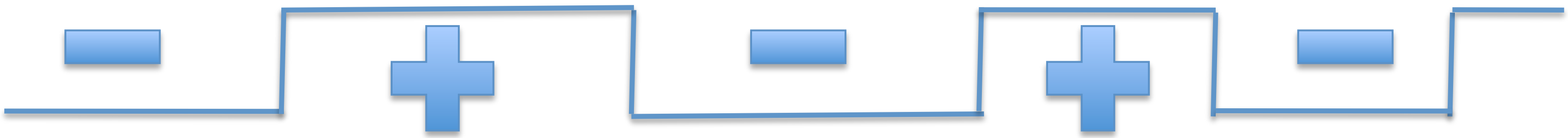
For large times $\left(t \to \infty\right)$

$$G(r,t) = \left\langle S_i(t) S_j(t) \right\rangle = g\left(\frac{r}{L(t)}\right) \quad , \quad r = dist(i,j)$$

$$C(t,t_w) = \left\langle S_i(t) S_i(t_w) \right\rangle = c\left(\frac{L(t)}{L(t_w)}\right)$$

$$L(t) \approx t^{1/z}$$

Simplest case: d=1



$$kT_c / J = 0$$

$$\xi \approx e^{2J/kT}$$

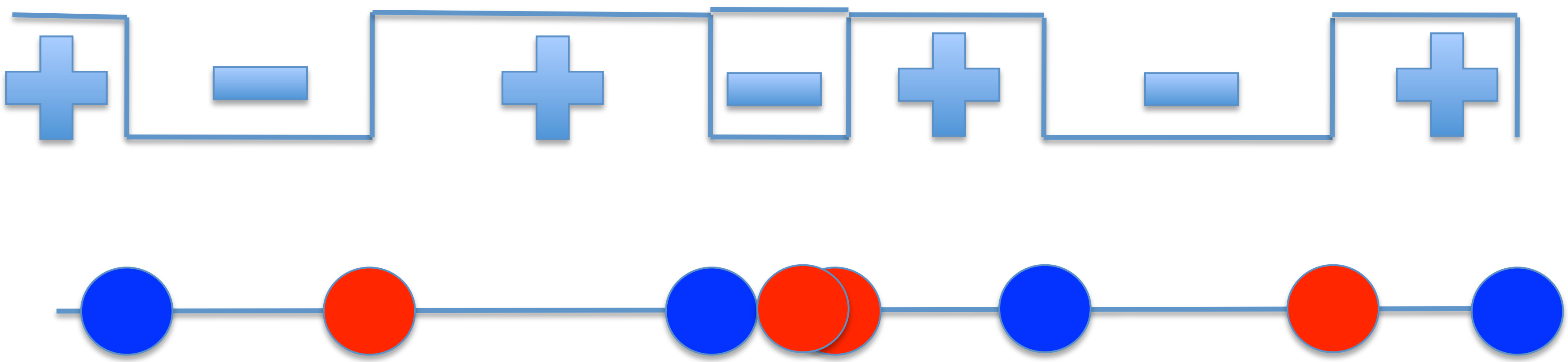
$$\Rightarrow$$

$$J = \infty$$

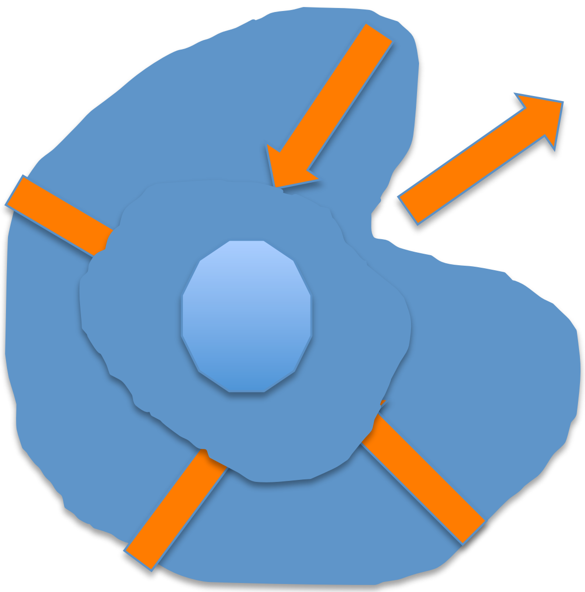
$$(or\ t \ll t_{eq})$$

Only interface diffusion
+ annihilation

$J = \infty$ Reaction-diffusion



D>1

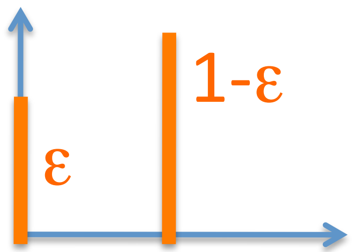


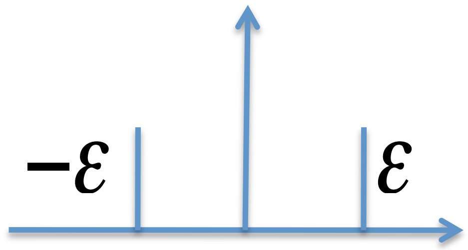
Curvature Driven

Quenched disorder

Dilution

ξ_i

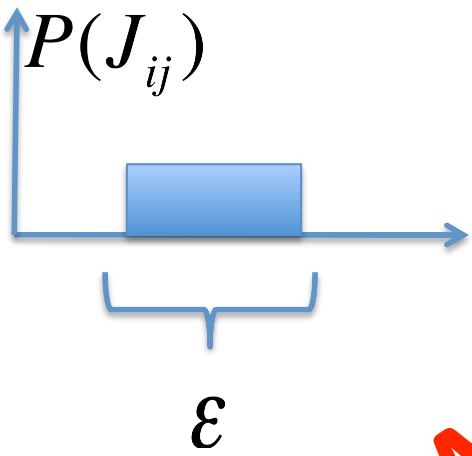




Random field

$$H = -\sum_{\langle ij \rangle} J_{ij} S_i S_j + \sum_i h_i S_i$$

Random Bond



No Frustration !

Some open issues

- Growth of L(t) ?
- Superuniversality?

Average over Quenched Disorder

$$C(t,t_w) = \overline{\langle S_i(t) S_i(t_w) \rangle} = c\left(\frac{L(t)}{L(t_w)}\right)$$

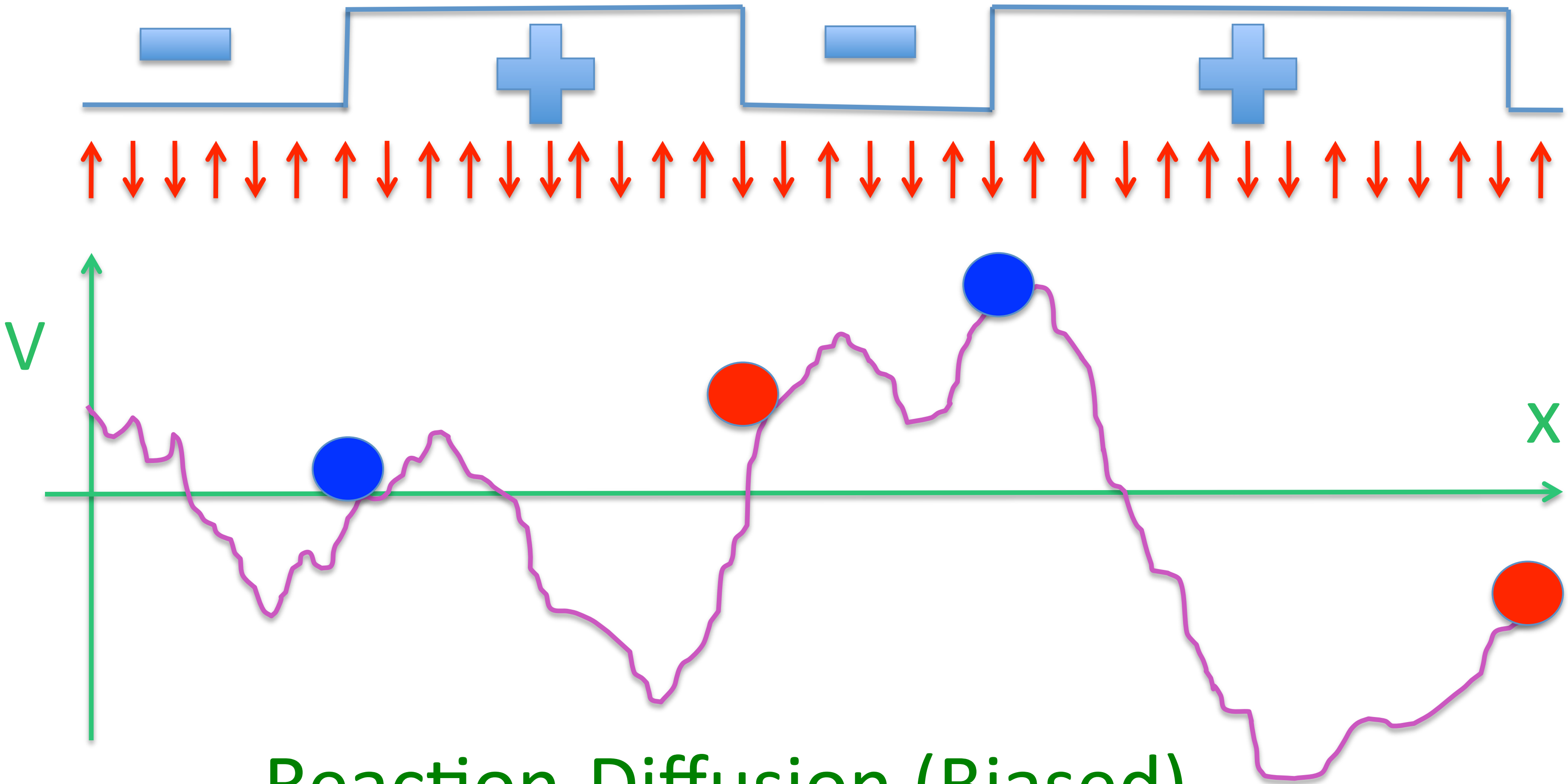
Disorder dependent

Disorder independent

Simplest Case: RF d=1

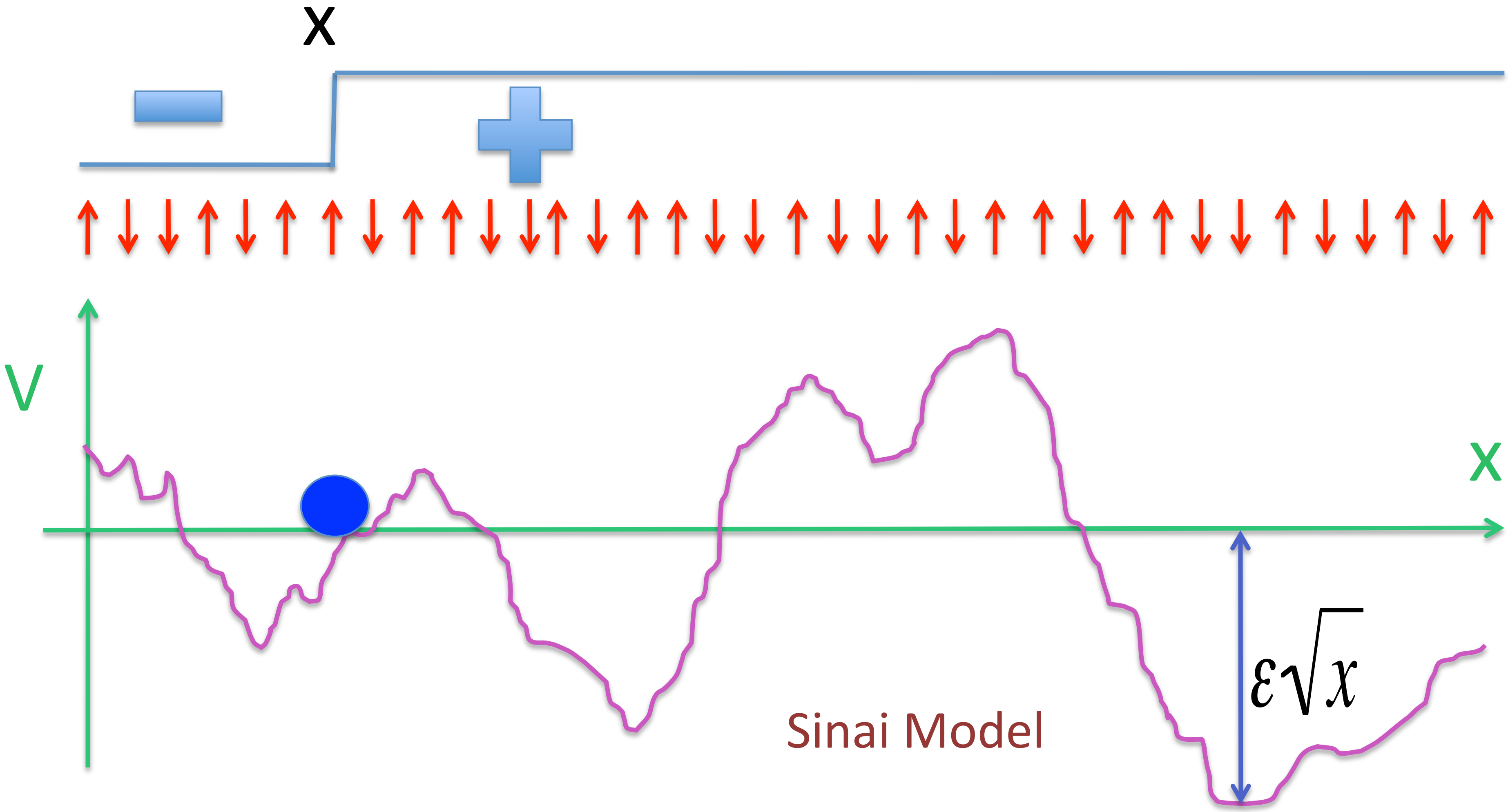
$$J = \infty$$

Only interface diffusion
+ annihilation



Reaction-Diffusion (Biased)

Single interface

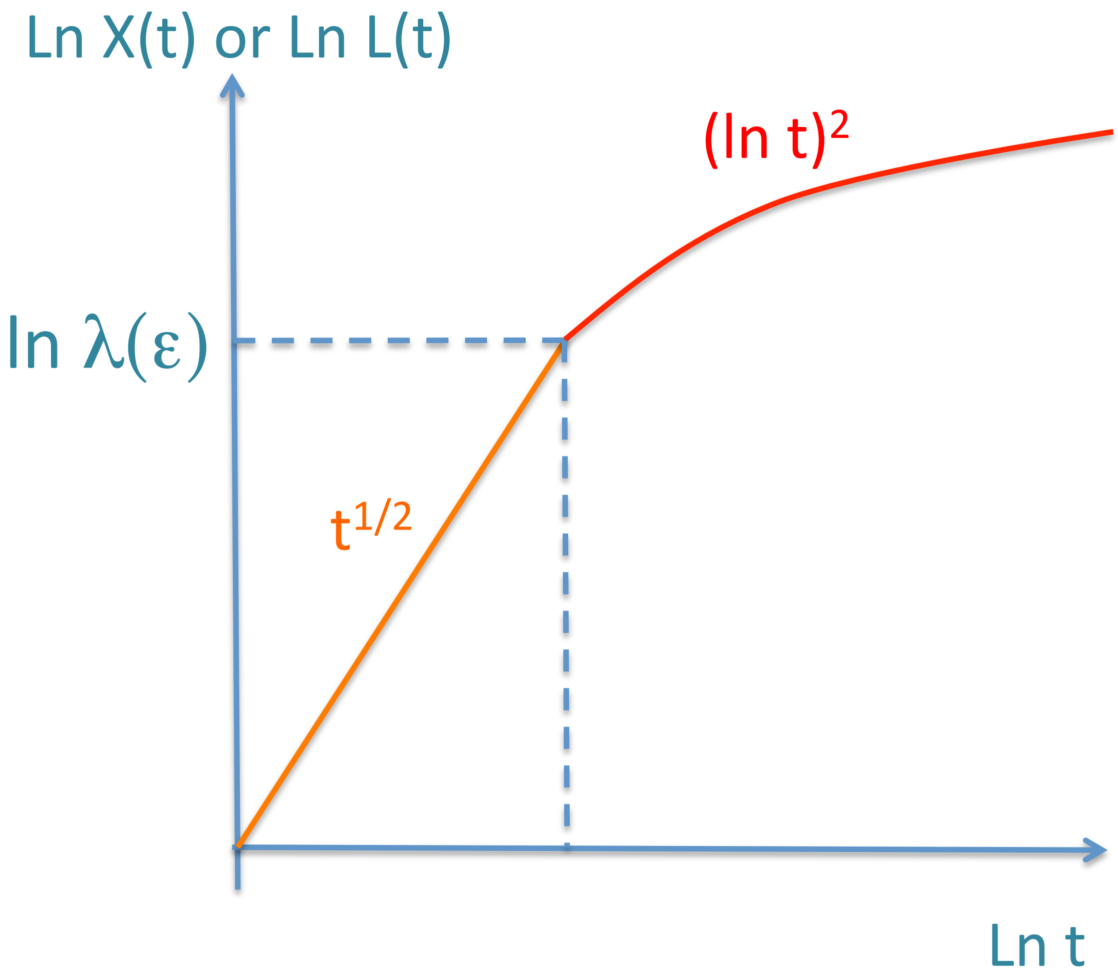


Characteristic Length associated to Disorder

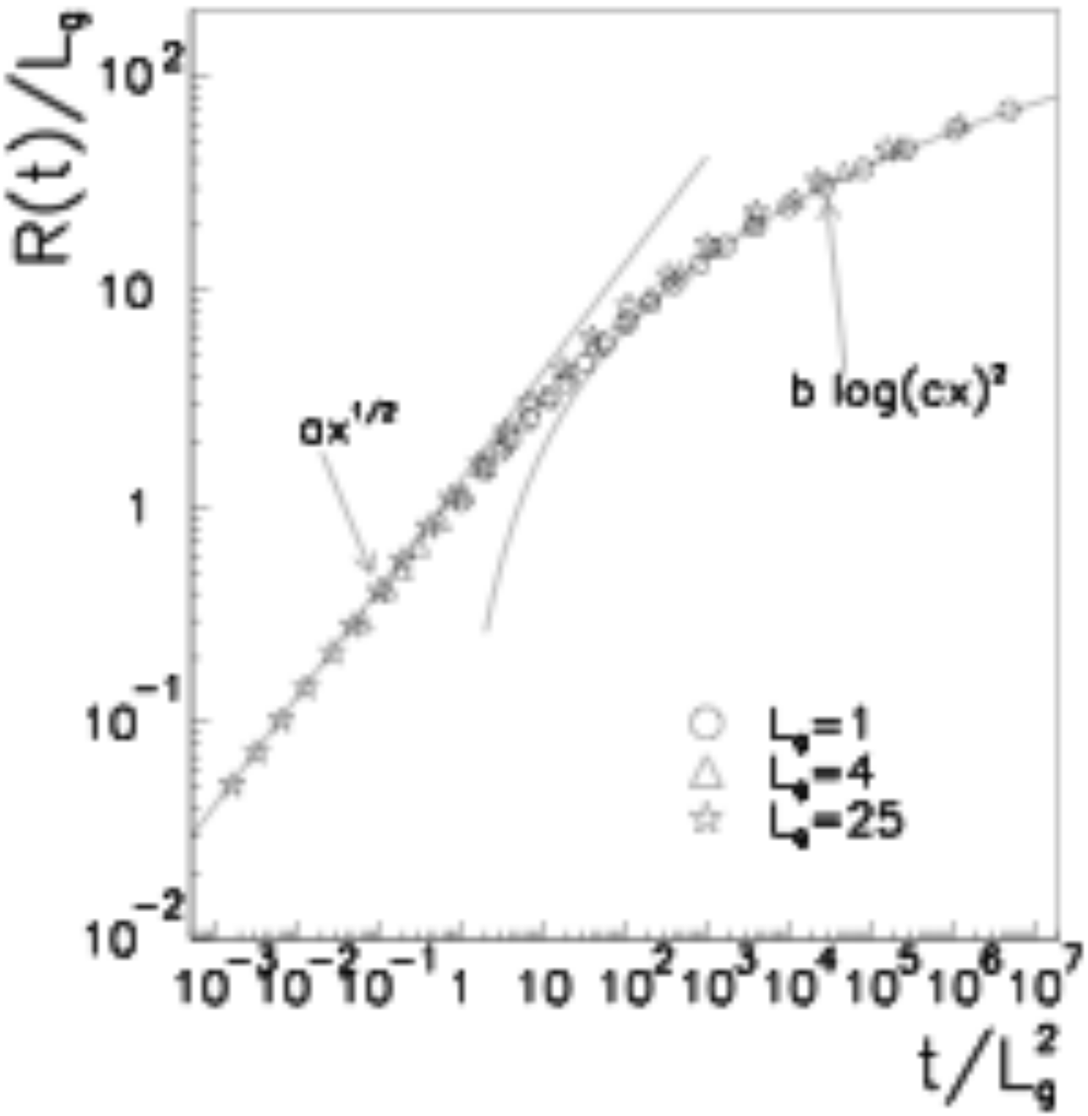
$$kT = \varepsilon\sqrt{x}$$

$$\Downarrow$$

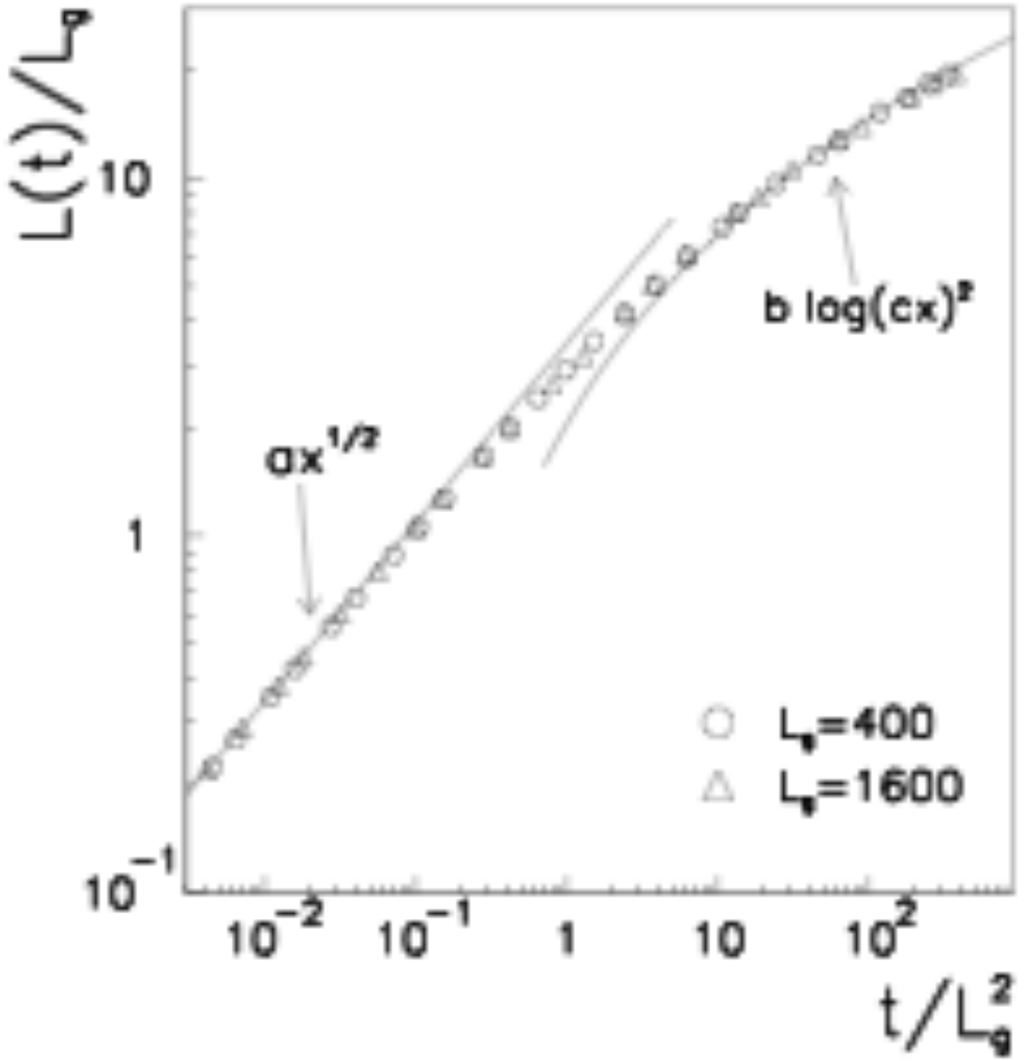
$$\lambda(\varepsilon) = \left(\frac{kT}{\varepsilon}\right)^2$$



Numerical check



Analogously for L(t)



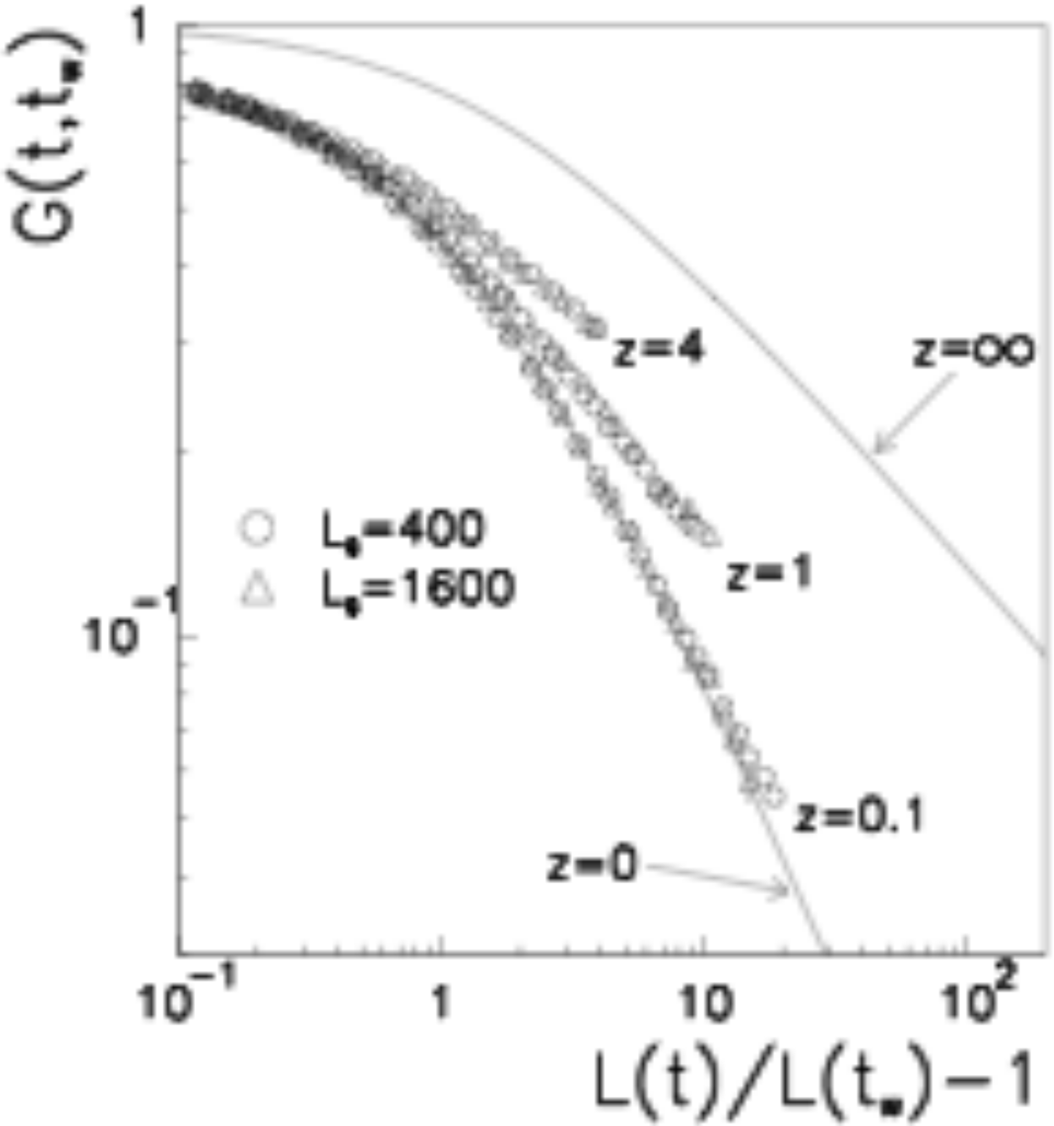
Scaling

$$L(t) = L(t,\varepsilon) = \lambda(\varepsilon)\, l\left(\frac{t^{1/z}}{\lambda\,(\varepsilon)}\right)$$

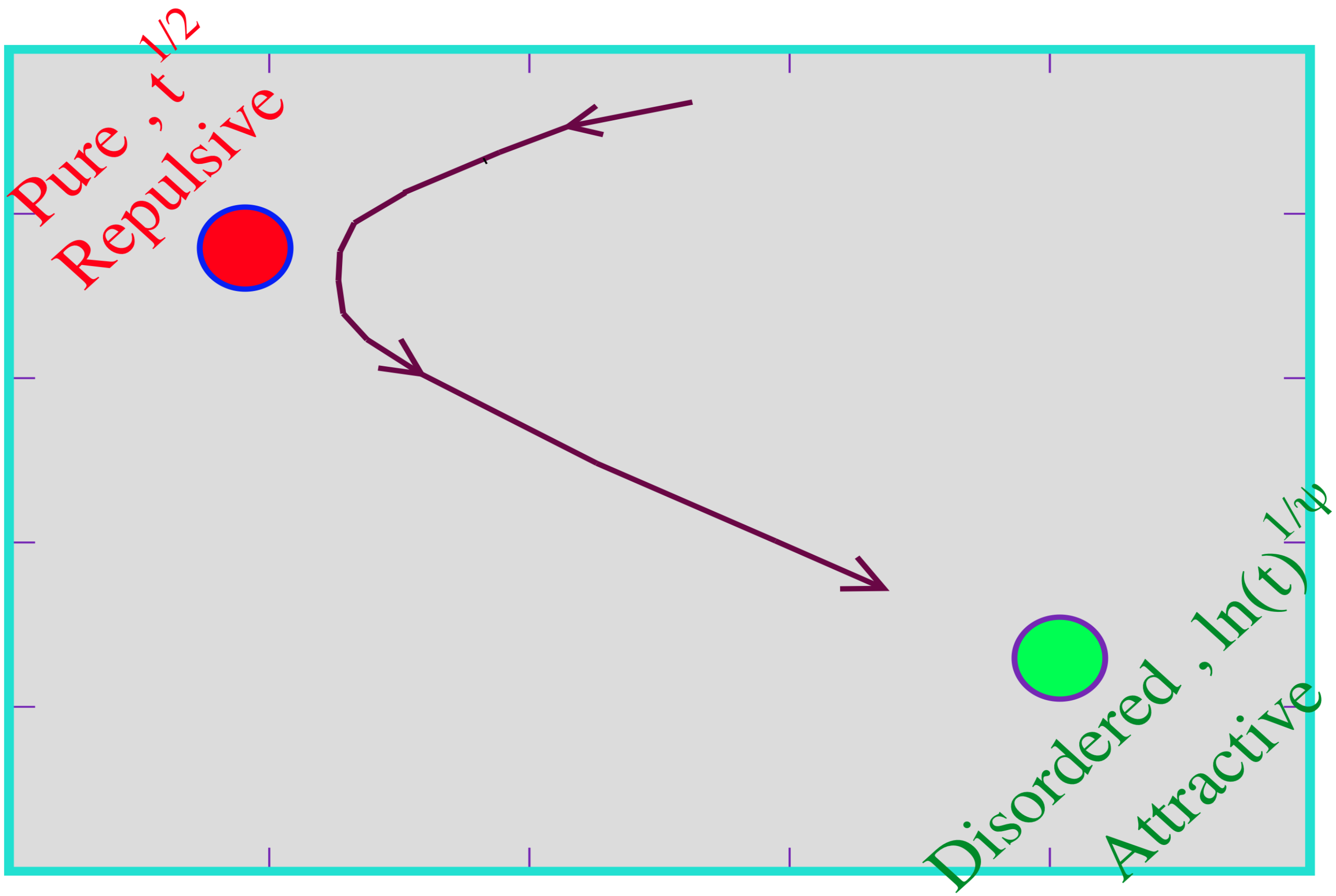
$$C(t,t_w) = C(t,t_w,\varepsilon) = \overline{\left\langle S_i(t) S_i(t_w) \right\rangle} = c\left(\frac{L(t)}{L(t_w)}, \frac{\lambda(\varepsilon)}{L(t_w)}\right) \neq c\left(\frac{L(t)}{L(t_w)}\right)$$

In general

Superuniversality? No!

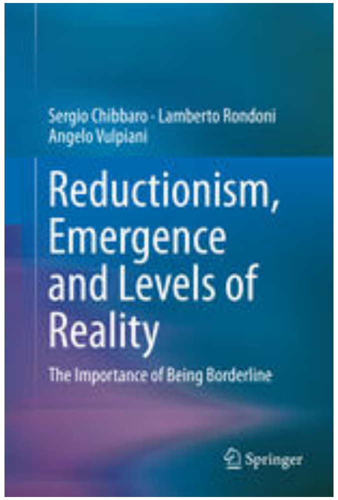


RG interpretation



$$\varepsilon \to 0$$

$$\lim_{t \to \infty} \lim_{\varepsilon \to 0} \cdots \neq \lim_{\varepsilon \to 0} \lim_{t \to \infty} \cdots$$



Linear Response Function

$$TM(t,\varepsilon) = \frac{T}{N\varepsilon^2} \sum_{i=1}^{N} \overline{\langle S_i(t) \rangle h_i}$$

Staggered Magnetization

$$T\chi(t,t_w) = \lim_{\varepsilon \to 0} \frac{T}{N\varepsilon^2} \sum_{i=1}^{N} \overline{\langle S_i(t) \rangle h_i}$$

Linear Response Function
(Susceptibility)

Switched on at t_w

Scaling

$$\chi(t,t_w) = \mathrm{X}\left(\frac{L(t)}{L(t_w)},\frac{\lambda(\varepsilon)}{L(t_w)}\right)$$

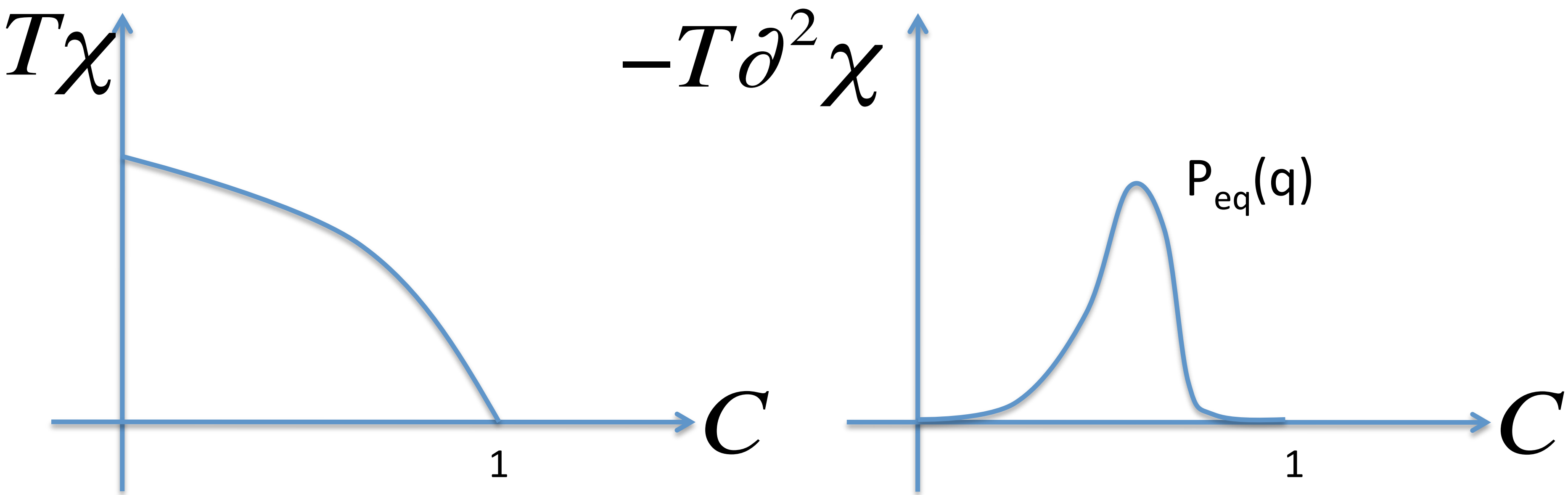
$$\chi(t,t_w) = \hat{\chi}\,(C)$$

Fluctuation-Dissipation Relation

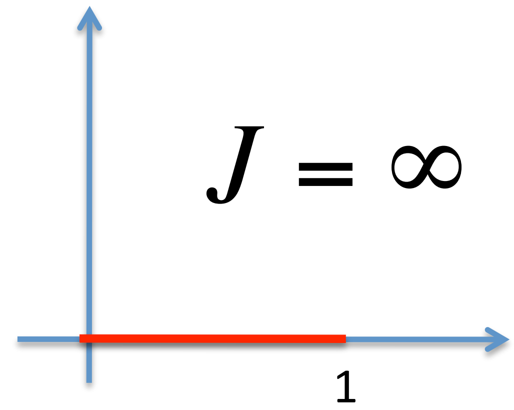
Statics-Dynamics

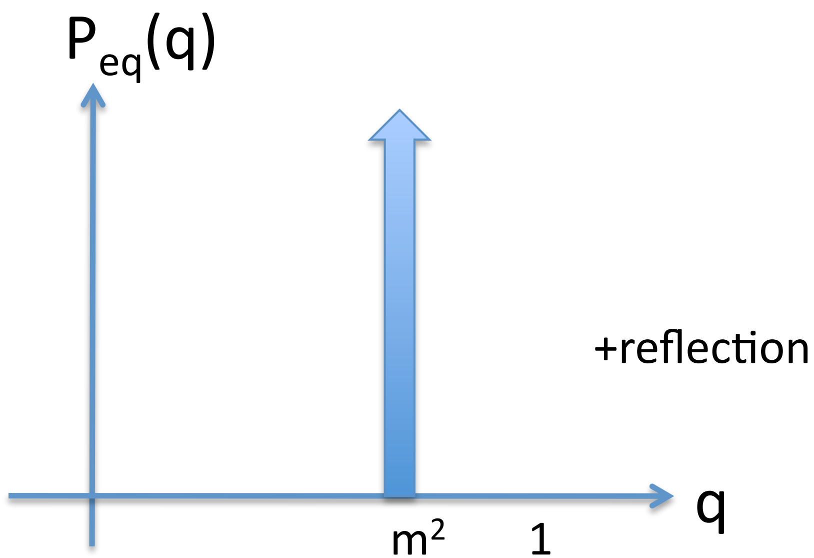
$$\lim_{t_w \to \infty} \left[-T \frac{\partial^2 \hat{\chi}(C)}{\partial C^2} \right] = P_{eq}(q)$$

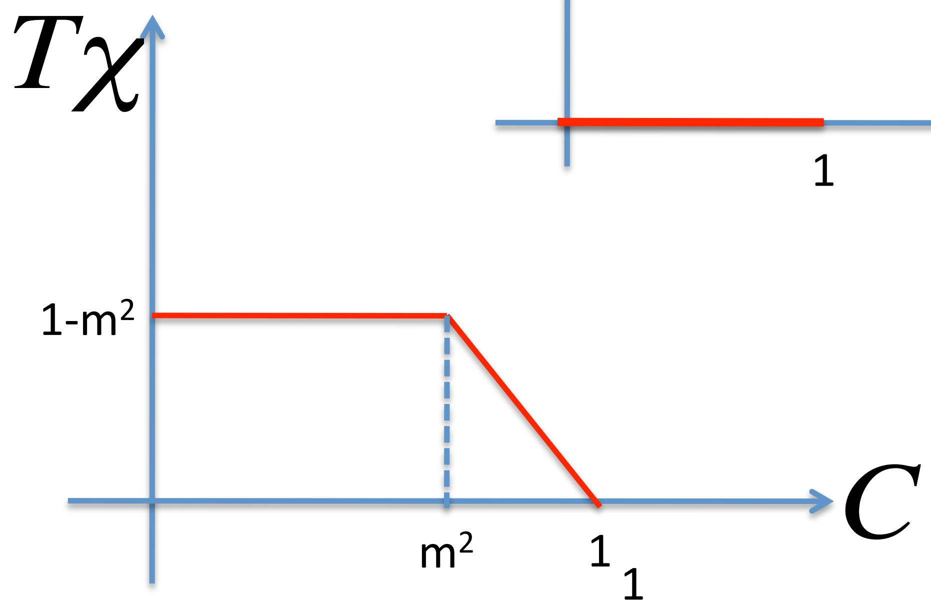
Franz-Mezard
Parisi-Peliti

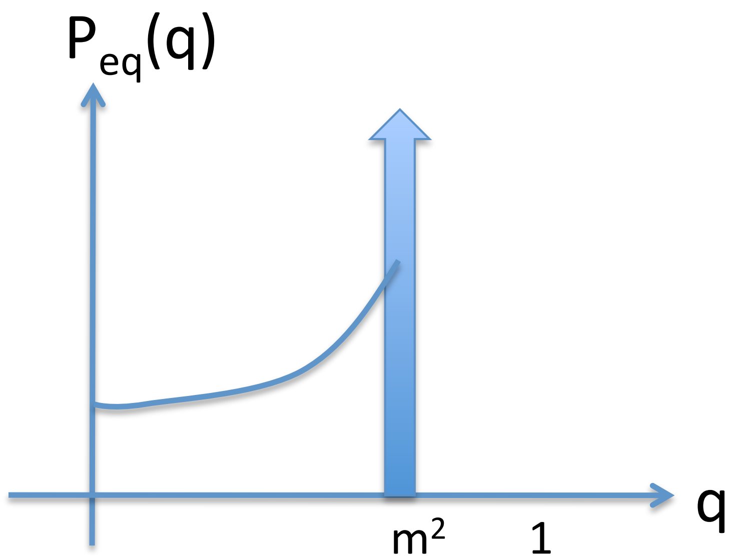


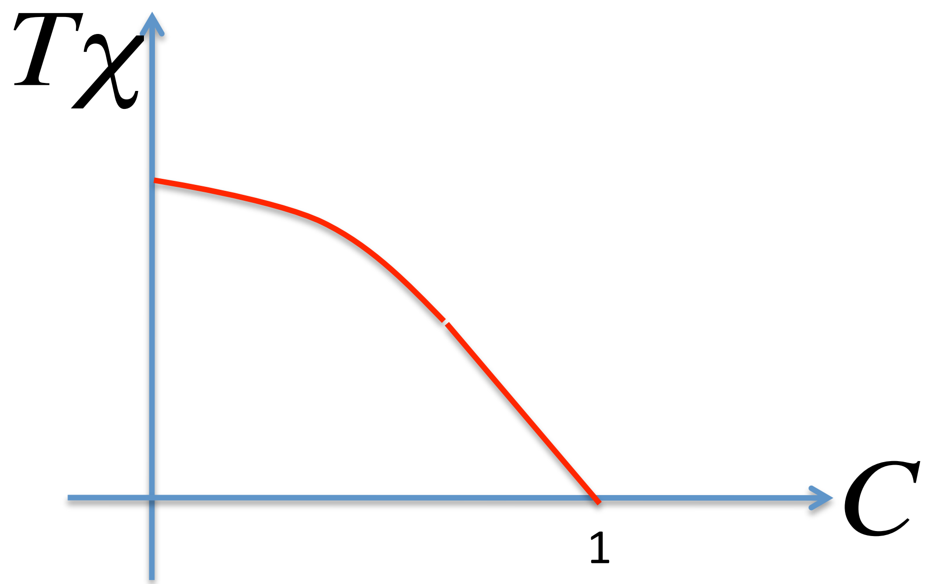
P(q) - Classification





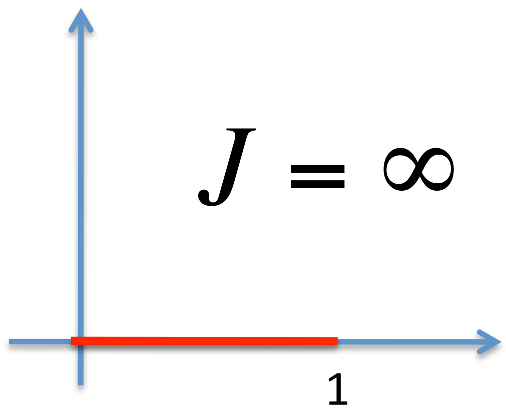




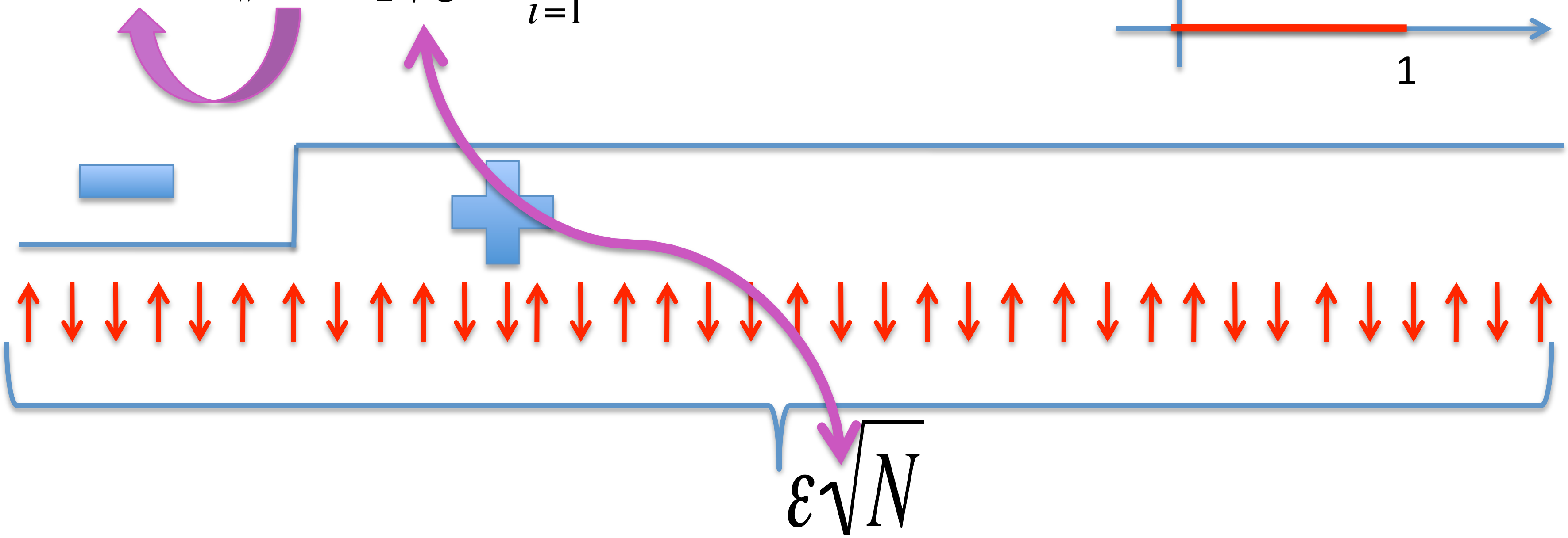


$$\lim_{t\to\infty}\lim_{\varepsilon\to 0}\cdots\neq\lim_{\varepsilon\to 0}\lim_{t\to\infty}\cdots$$

$$\lim_{t_w\to\infty} T\chi(t,t_w) = \lim_{t_w\to\infty}\lim_{\varepsilon\to 0}\frac{T}{N\varepsilon^2}\sum_{i=1}^{N}\overline{\langle S_i(t)\rangle h_i}$$



$$\lim_{\varepsilon\to 0}\lim_{t_w\to\infty}\frac{T}{N\varepsilon^2}\sum_{i=1}^{N}\overline{\langle S_i(t)\rangle h_i}=0$$



$$\lim_{t\to\infty}\lim_{\varepsilon\to 0}\cdots \neq \lim_{\varepsilon\to 0}\lim_{t\to\infty}\cdots$$

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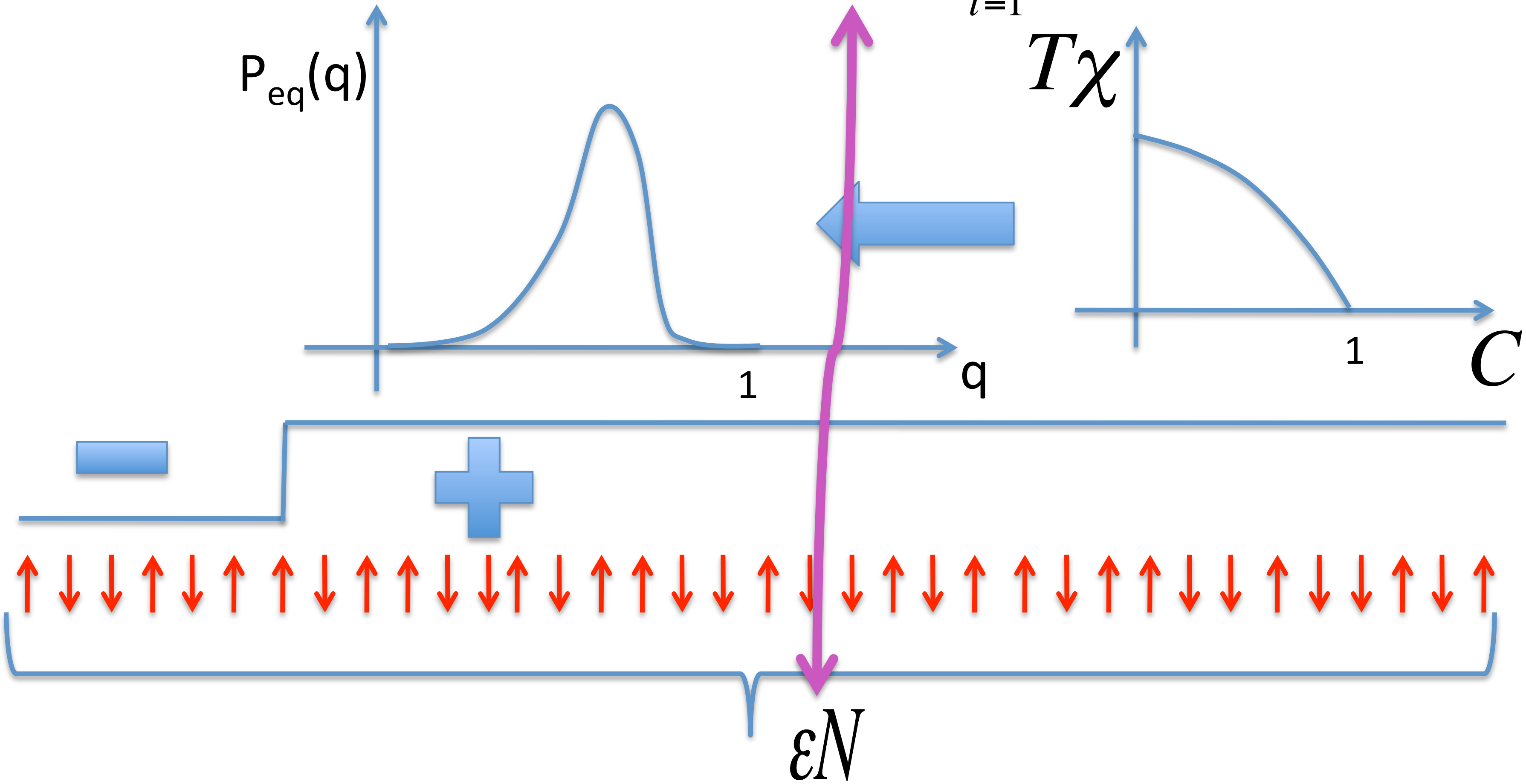
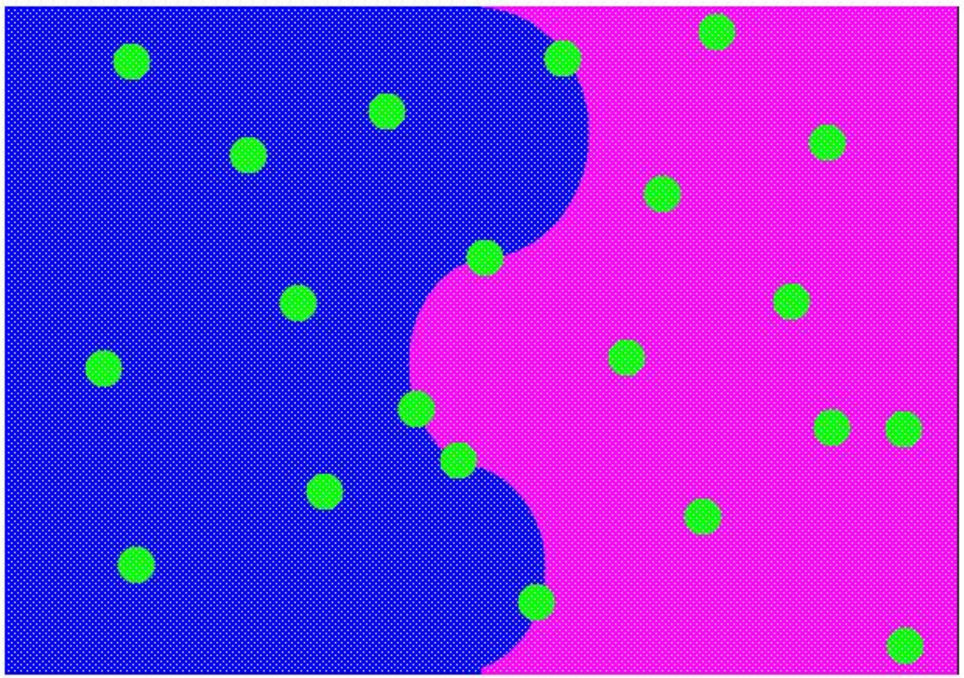


D>1

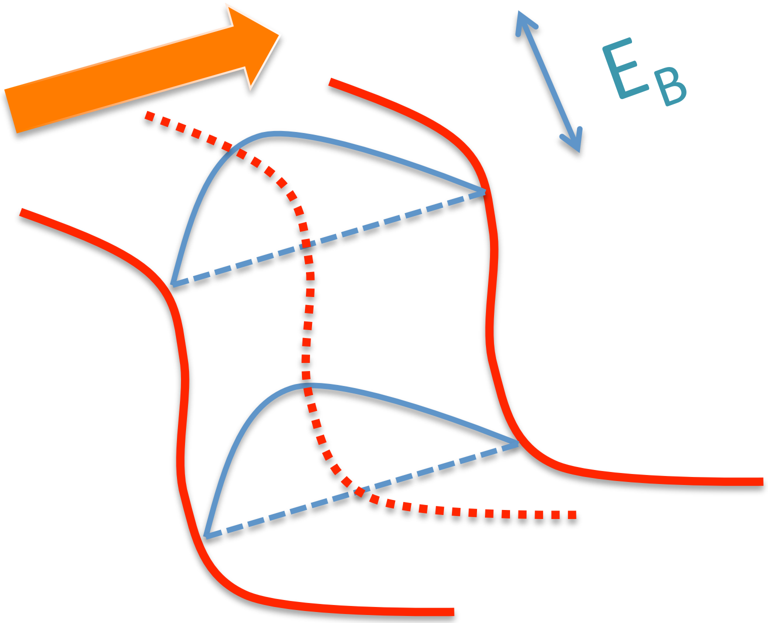


$$L(t) \ll t^{1/z}$$

Pinning

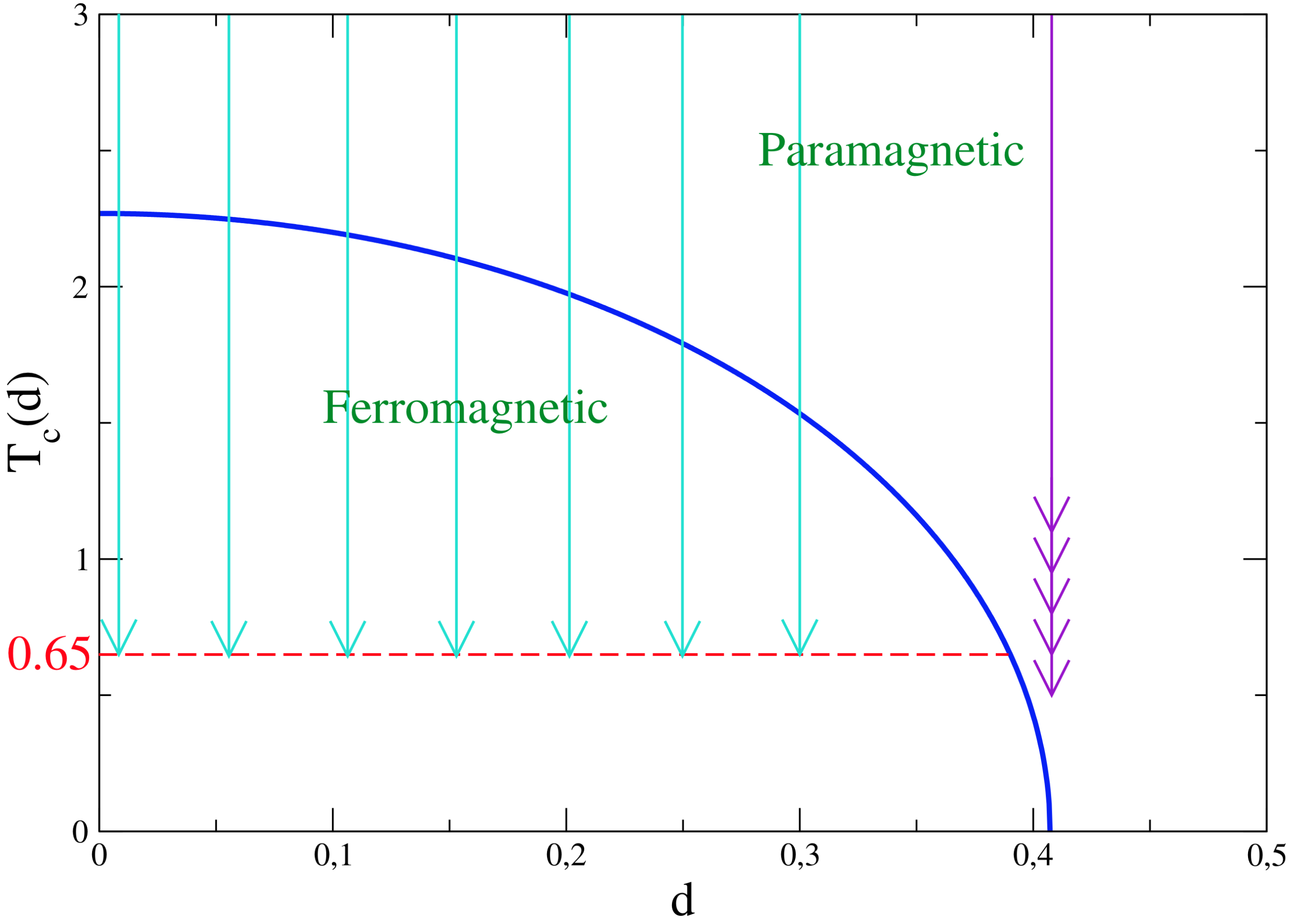
Barriers

$$t \approx e^{E_B(L)/kT}$$



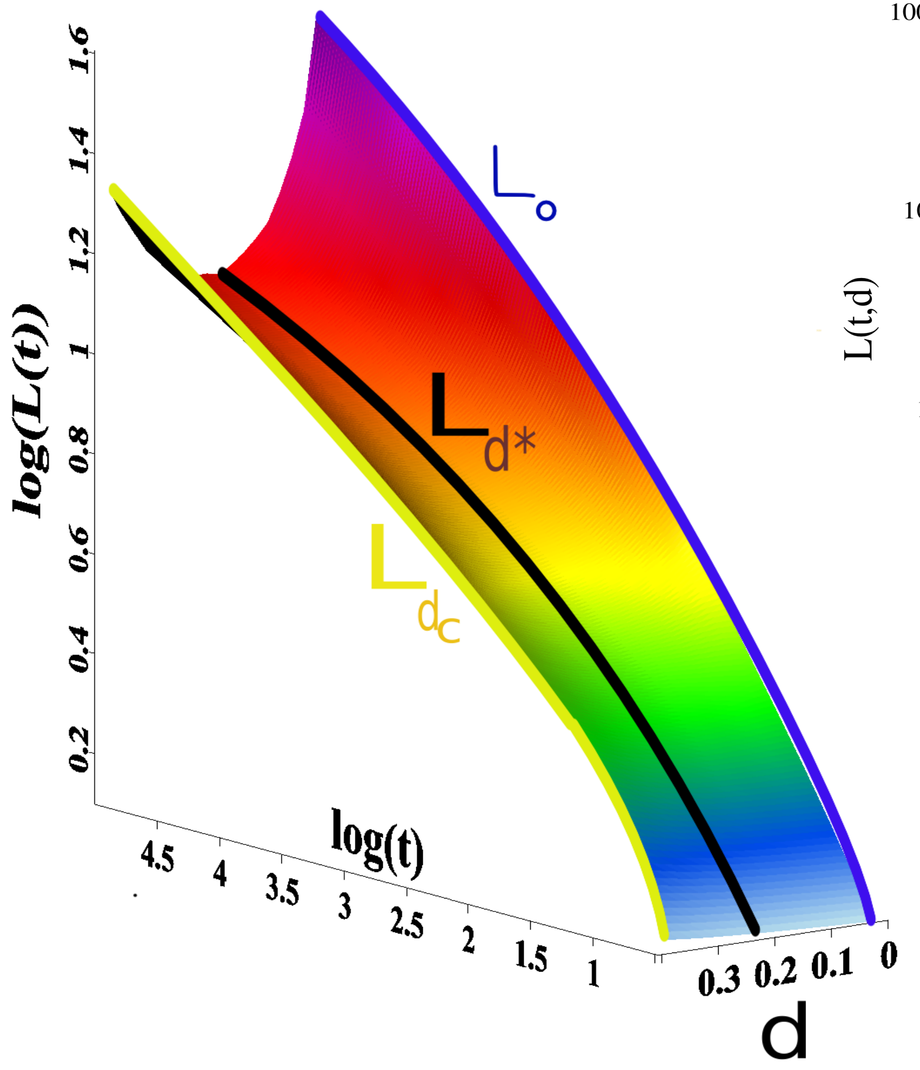
$$E_B \approx \begin{cases} L(t)^{\psi} \\ \ln L(t) \end{cases} \quad \Longrightarrow \quad L(t) \approx \begin{cases} (\ln t)^{1/\psi} \\ t^{1/\zeta} \end{cases}$$

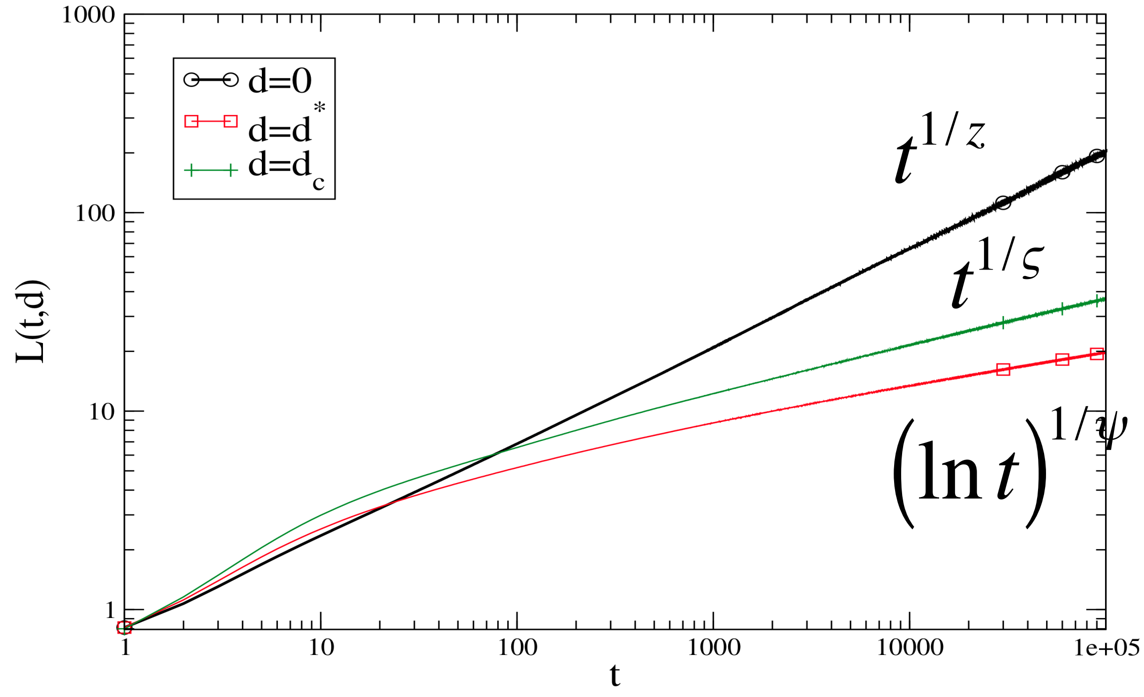
Some results (SD d=2)



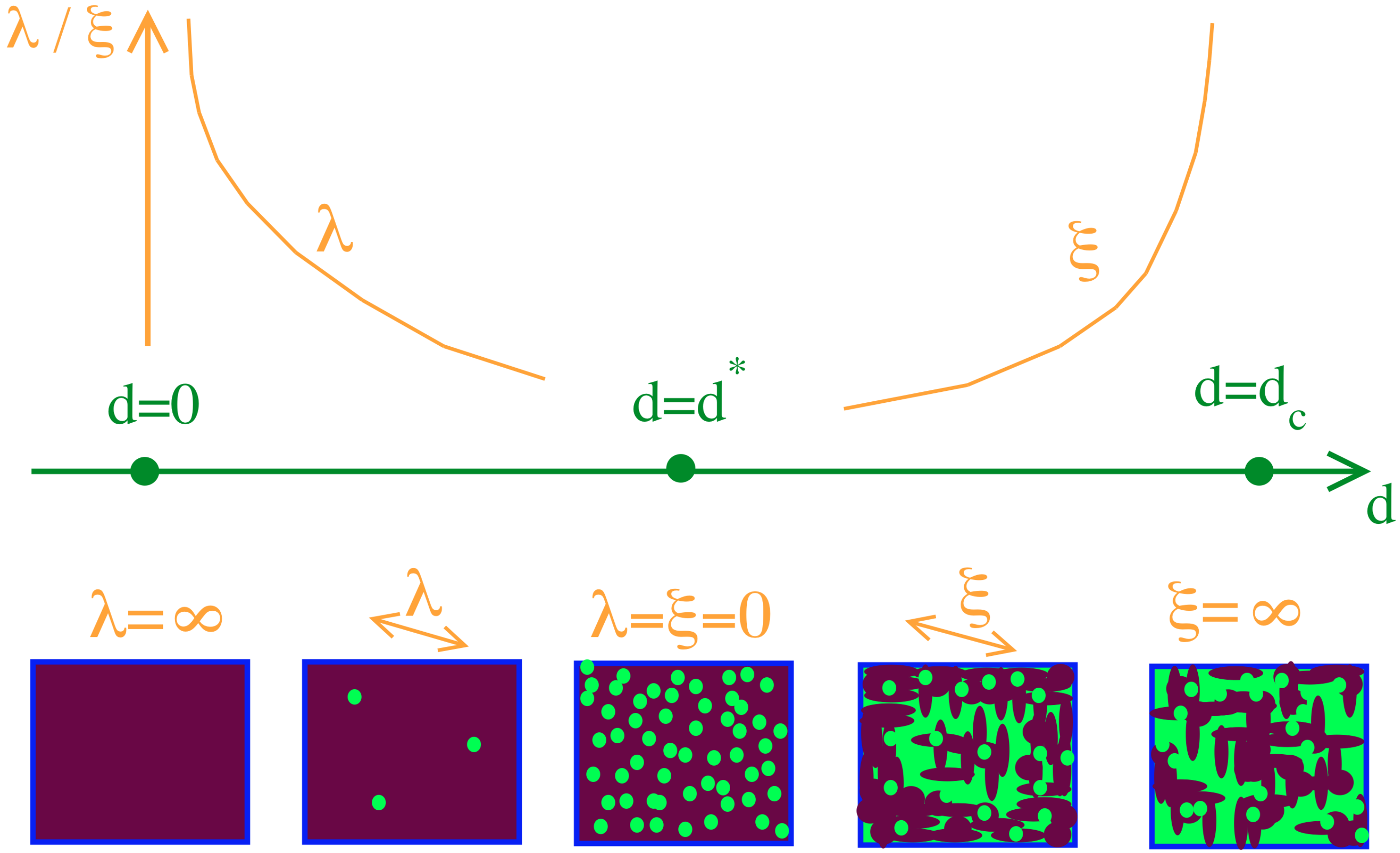
d is dilution (ε)

Some results (SD d=2)





Substrate



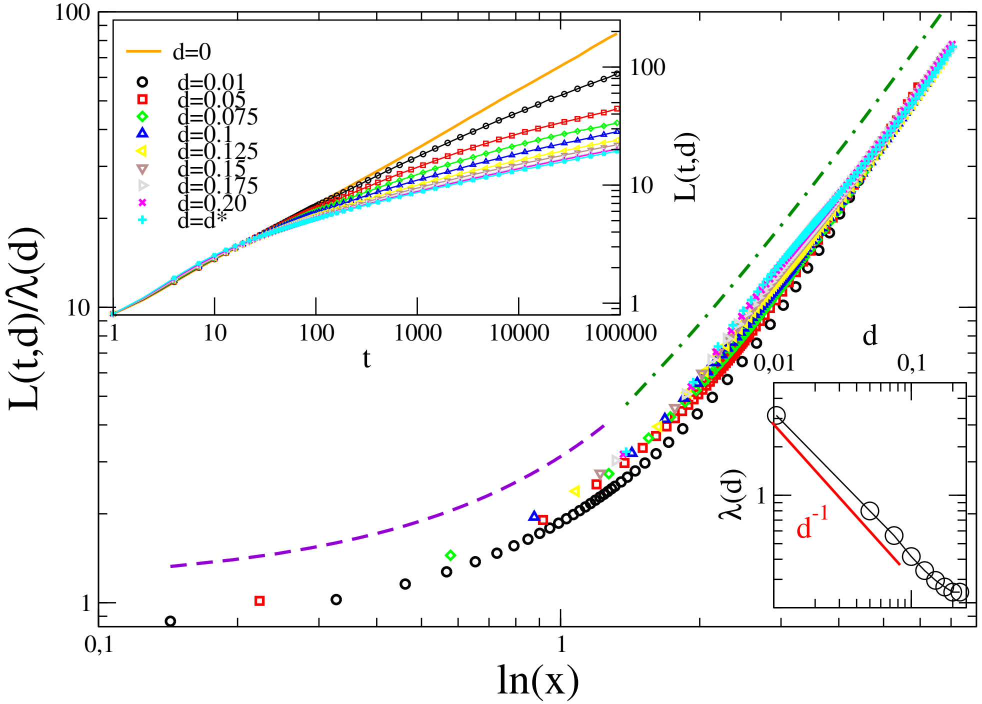
Scaling

$$L(t) = \lambda(\varepsilon)\, l\left(\frac{t^{1/z}}{\lambda(\varepsilon)}, \frac{t^{1/\xi}}{\xi(\varepsilon)}\right)$$

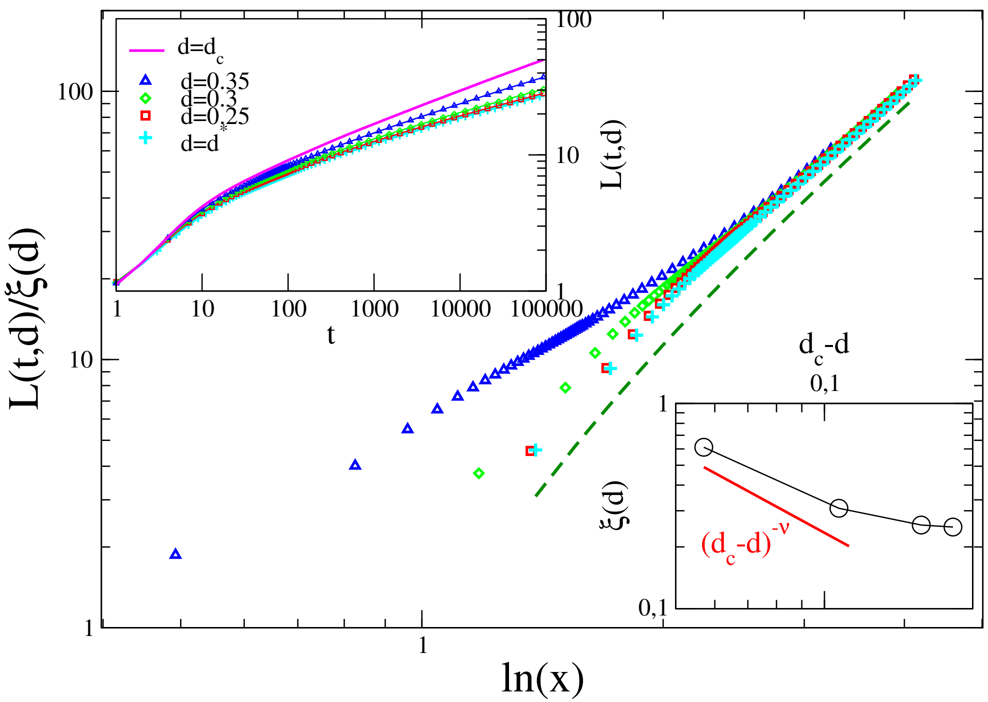
$$\lambda(\varepsilon) \approx \varepsilon^{-1}$$

$$\xi(\varepsilon) \approx \left(\varepsilon_c - \varepsilon\right)^{-\upsilon}$$

Numerics

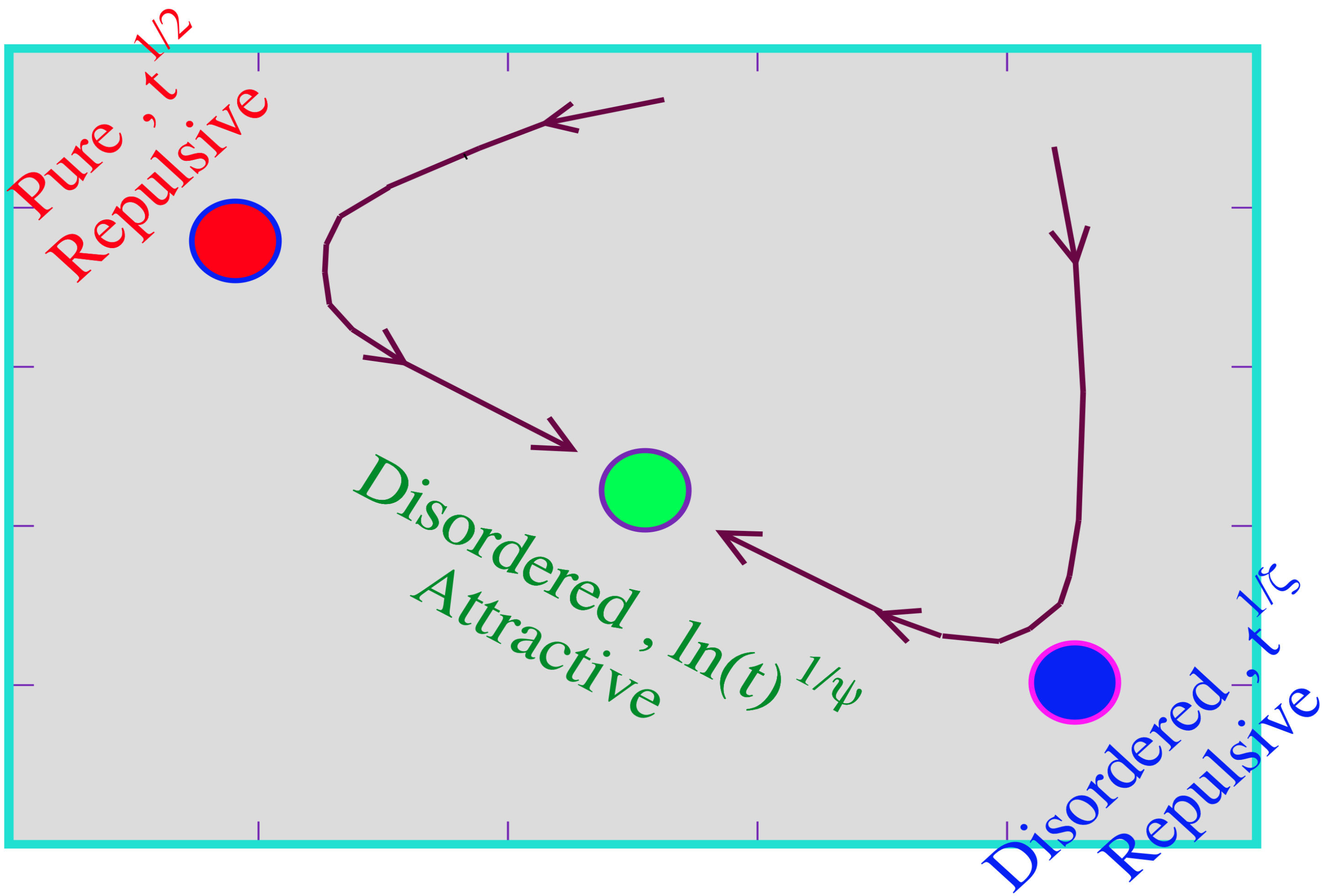


L(t,d)/λ(d)
ln(x)
d=0
d=0.01
d=0.05
d=0.075
d=0.1
d=0.125
d=0.15
d=0.175
d=0.20
d=d*
L(t,d)
t
d
λ(d)
d^{-1}



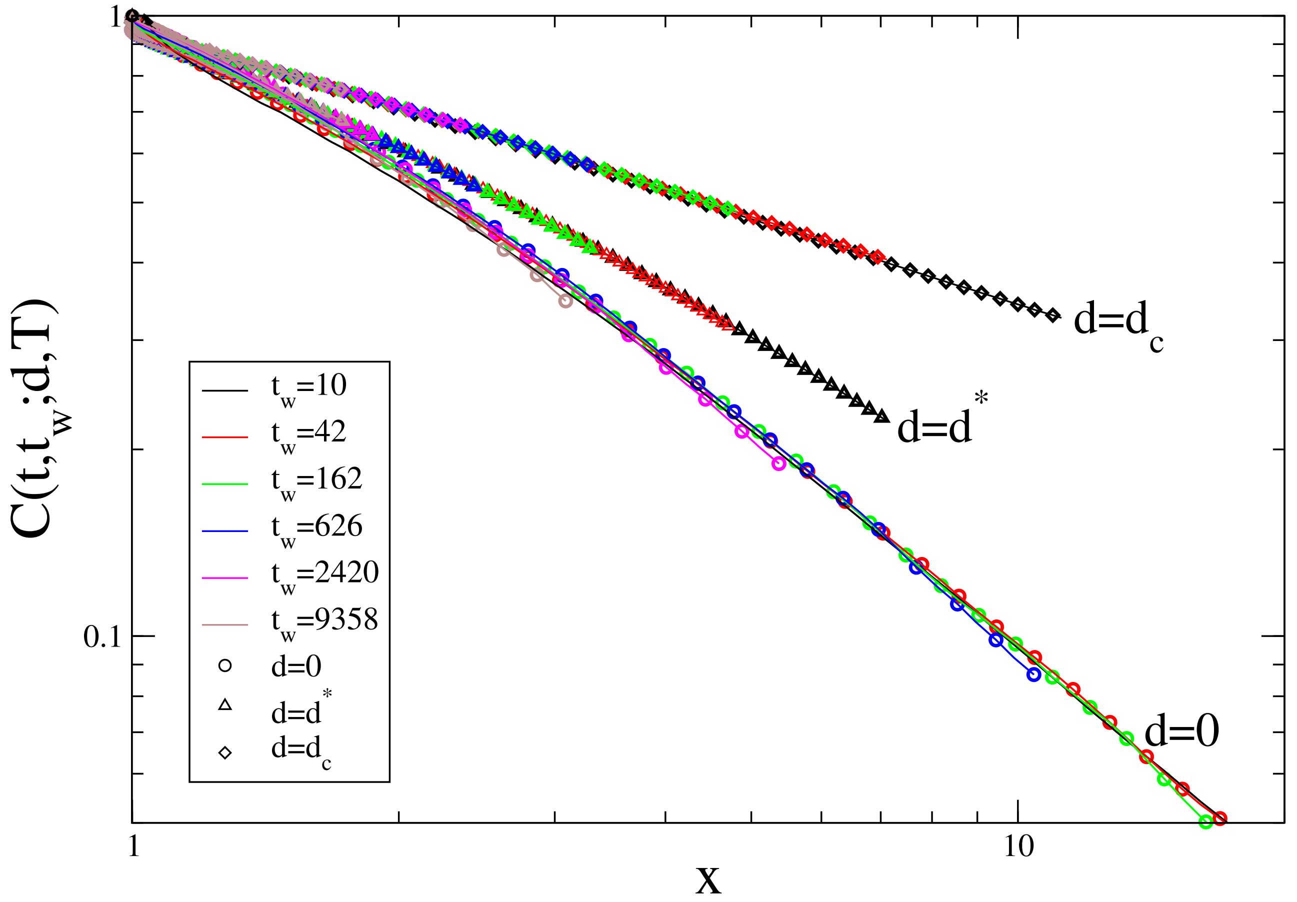
L(t,d)/ξ(d)
ln(x)
$d=d_c$
d=0.35
d=0.3
d=0.25
d=d*
L(t,d)
t
d_c-d
ξ(d)
$(d_c-d)^{-\nu}$

RG interpretation

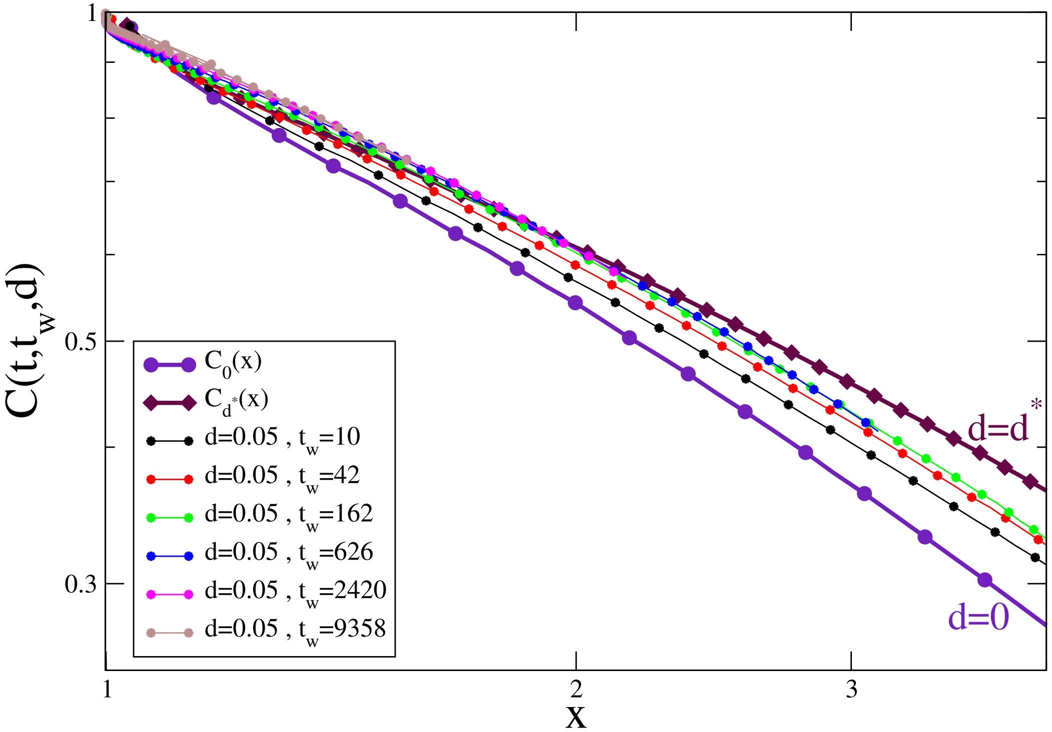


Scaling of C

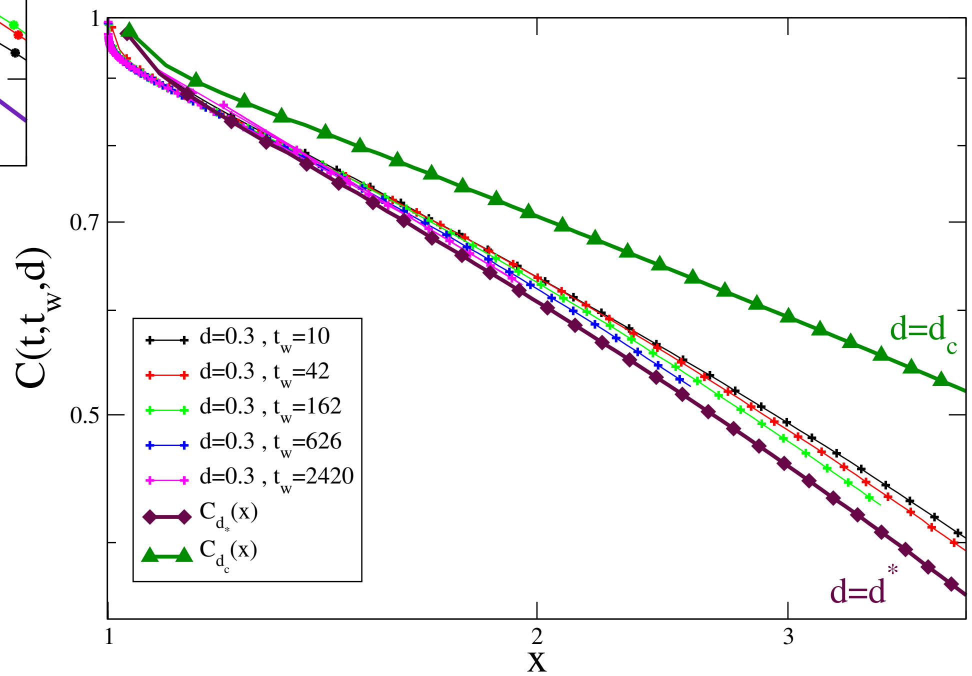
$$C(t,t_w) = \overline{\langle S_i(t) S_i(t_w) \rangle} = c\left(\frac{L(t)}{L(t_w)}, \frac{\lambda(\varepsilon)}{L(t_w)}, \frac{\xi(\varepsilon)}{L(t_w)} \right)$$



Scaling of C

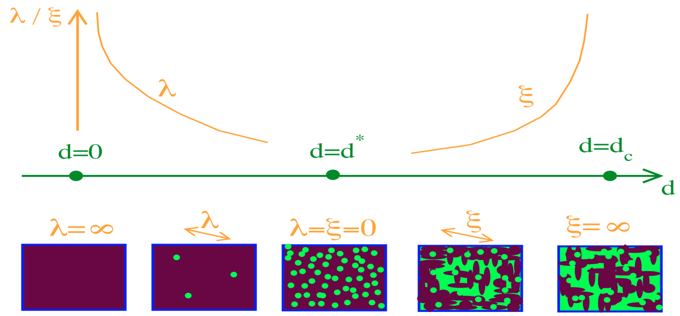


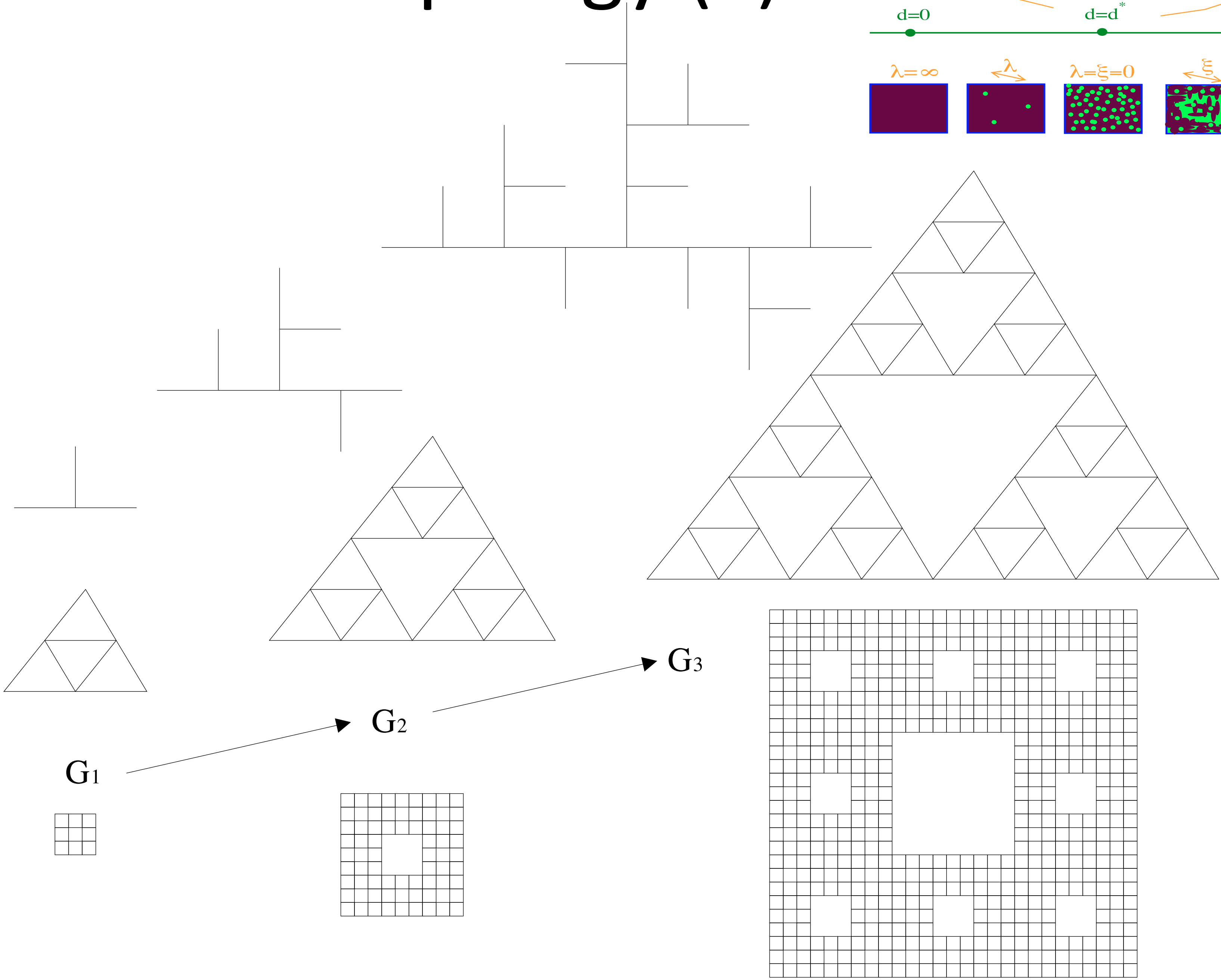
1
0.5
0.3
1
2
3
x
C(t,t_w,d)
C_0(x)
C_d*(x)
d=0.05 , t_w=10
d=0.05 , t_w=42
d=0.05 , t_w=162
d=0.05 , t_w=626
d=0.05 , t_w=2420
d=0.05 , t_w=9358
d=d*
d=0



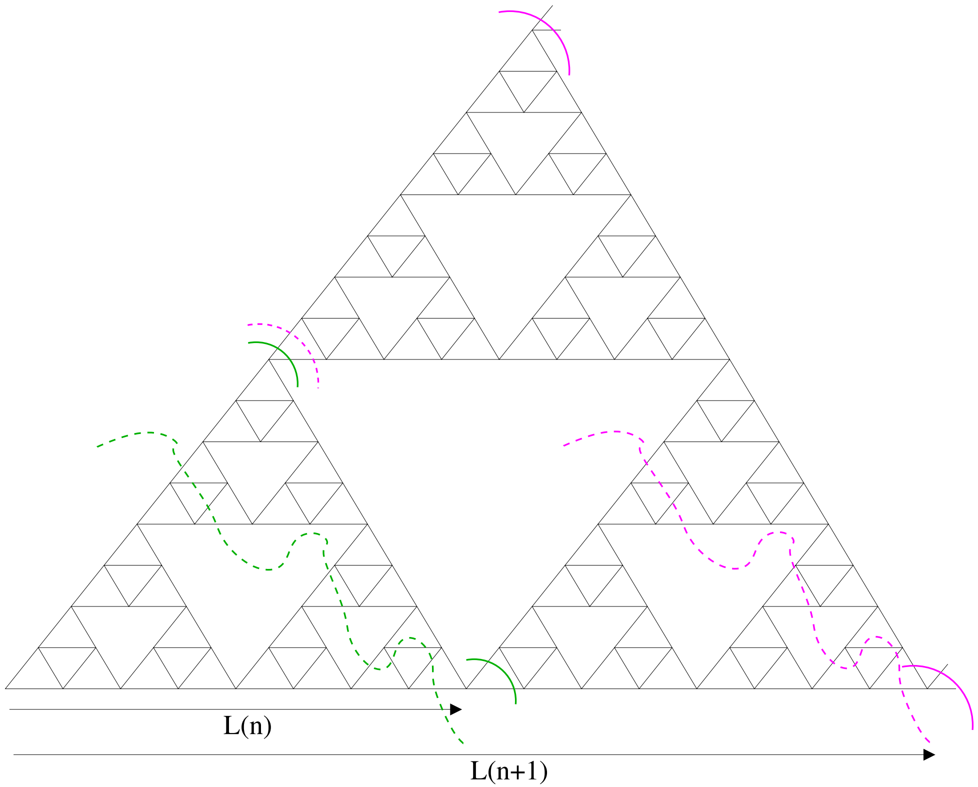
1
0.7
0.5
1
2
3
x
C(t,t_w,d)
d=0.3 , t_w=10
d=0.3 , t_w=42
d=0.3 , t_w=162
d=0.3 , t_w=626
d=0.3 , t_w=2420
C_d*(x)
C_dc(x)
d=d_c
d=d*

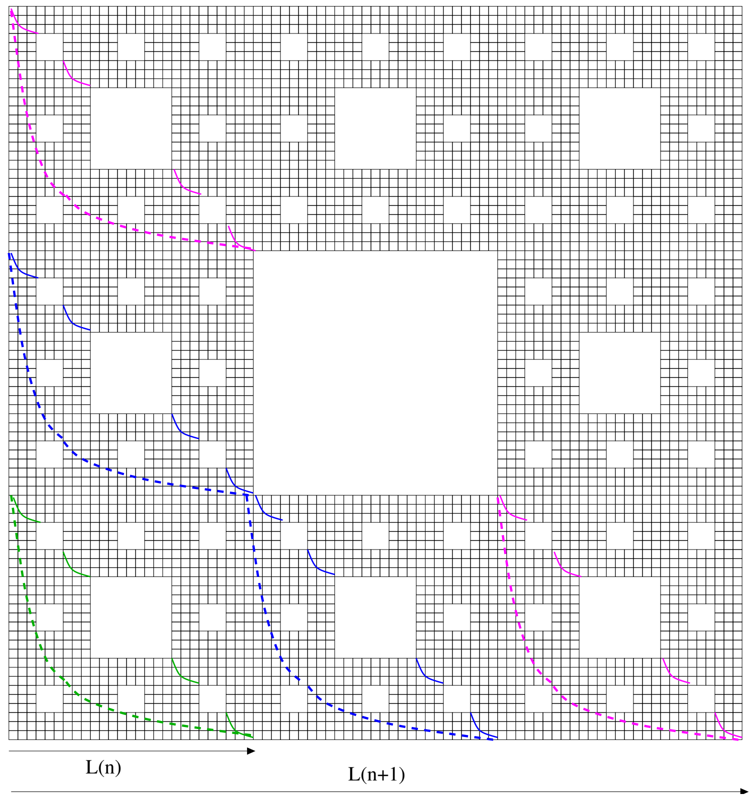
Role of topology (?)





Role of Topology (?)

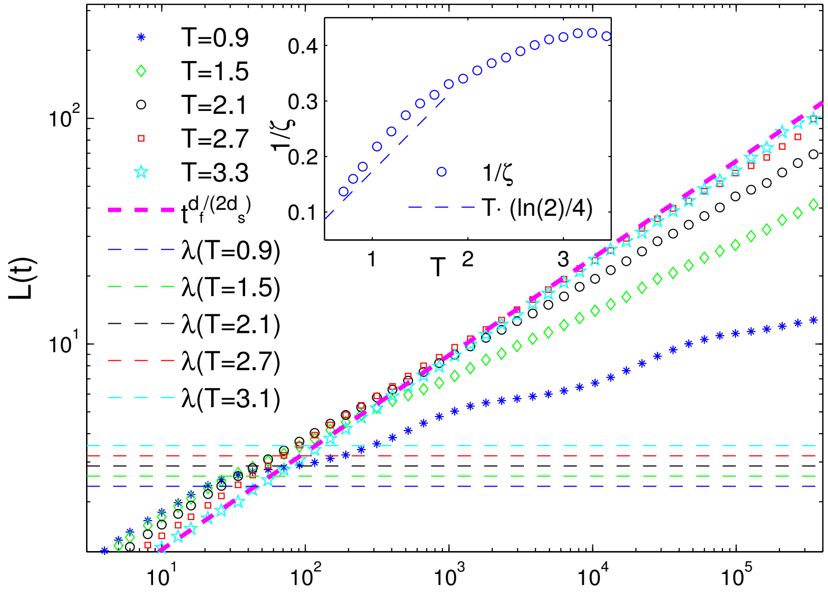


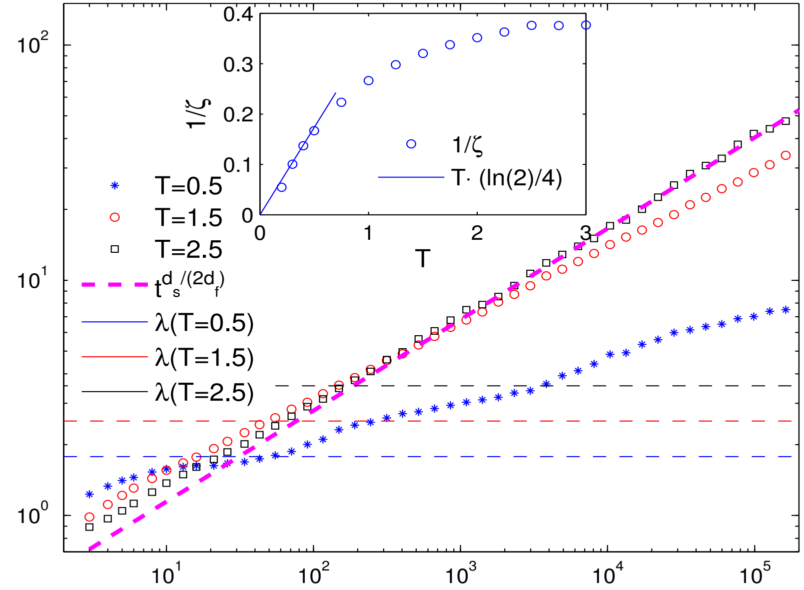


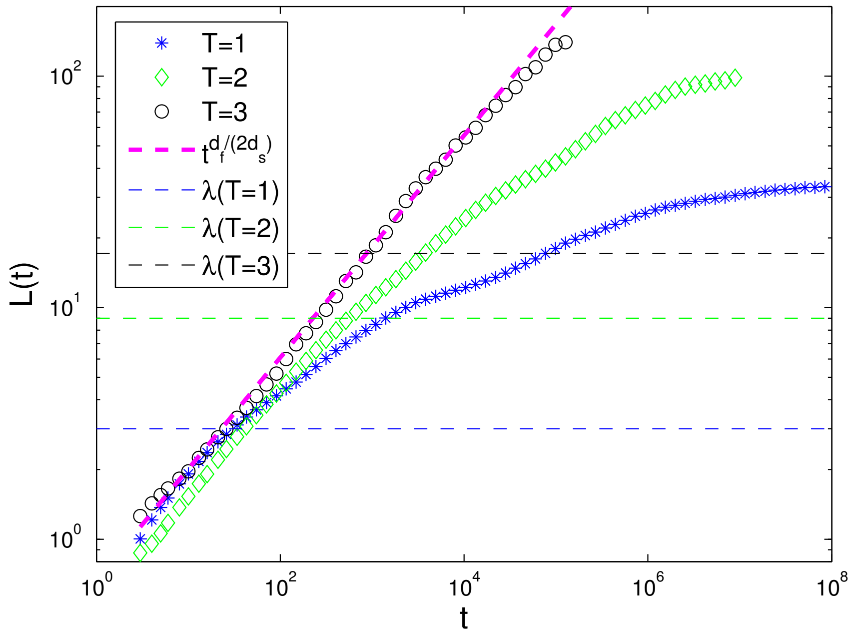
Role of Topology (?)

Gasket

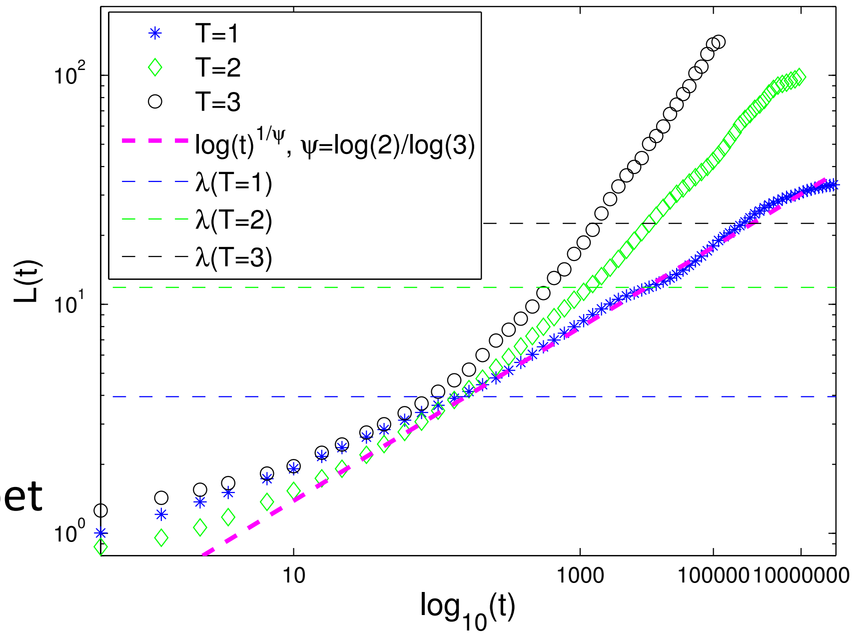
T-frac



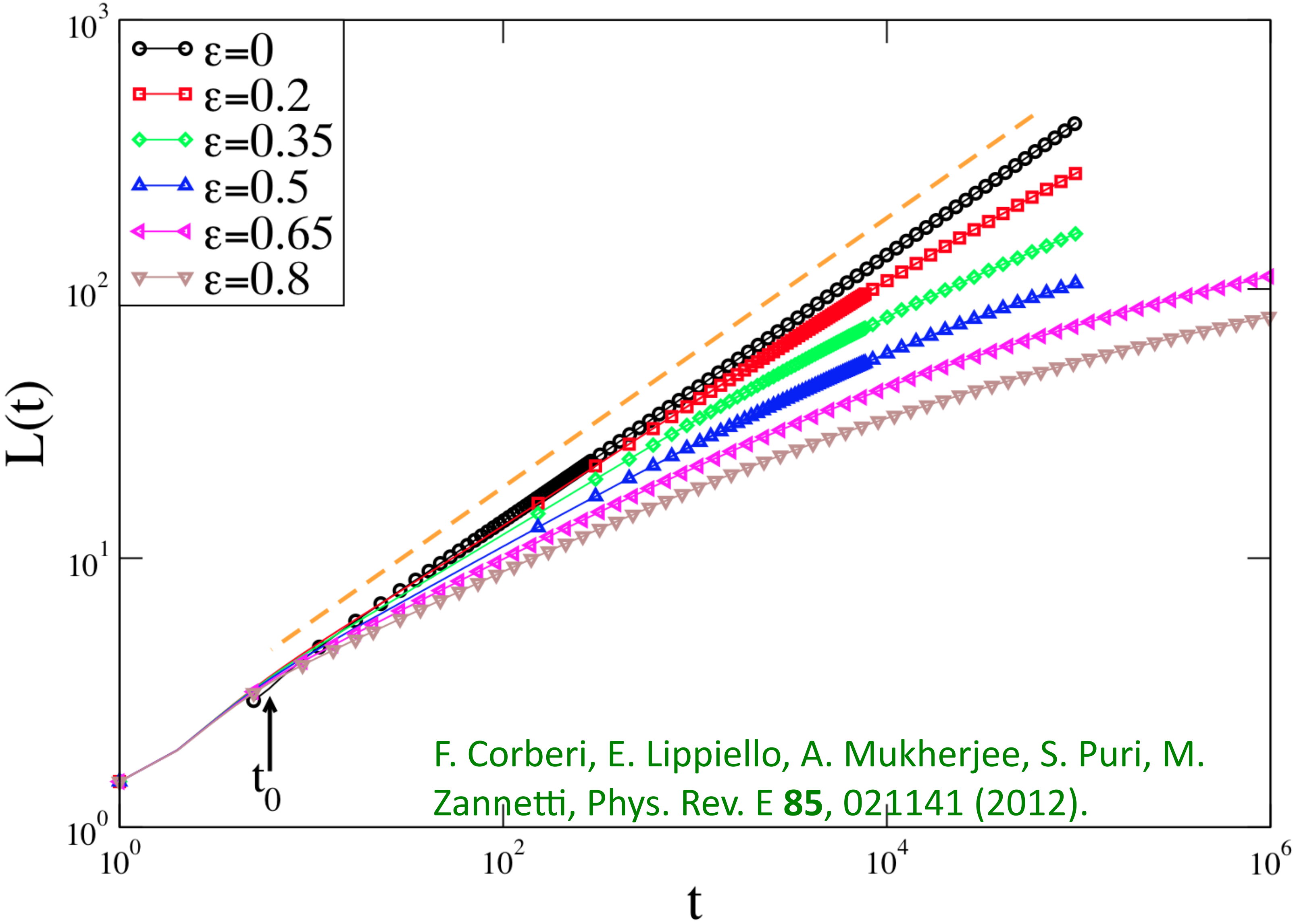




Carpet



Some results (RF d=2)



F. Corberi, E. Lippiello, A. Mukherjee, S. Puri, M. Zannetti, Phys. Rev. E **85**, 021141 (2012).

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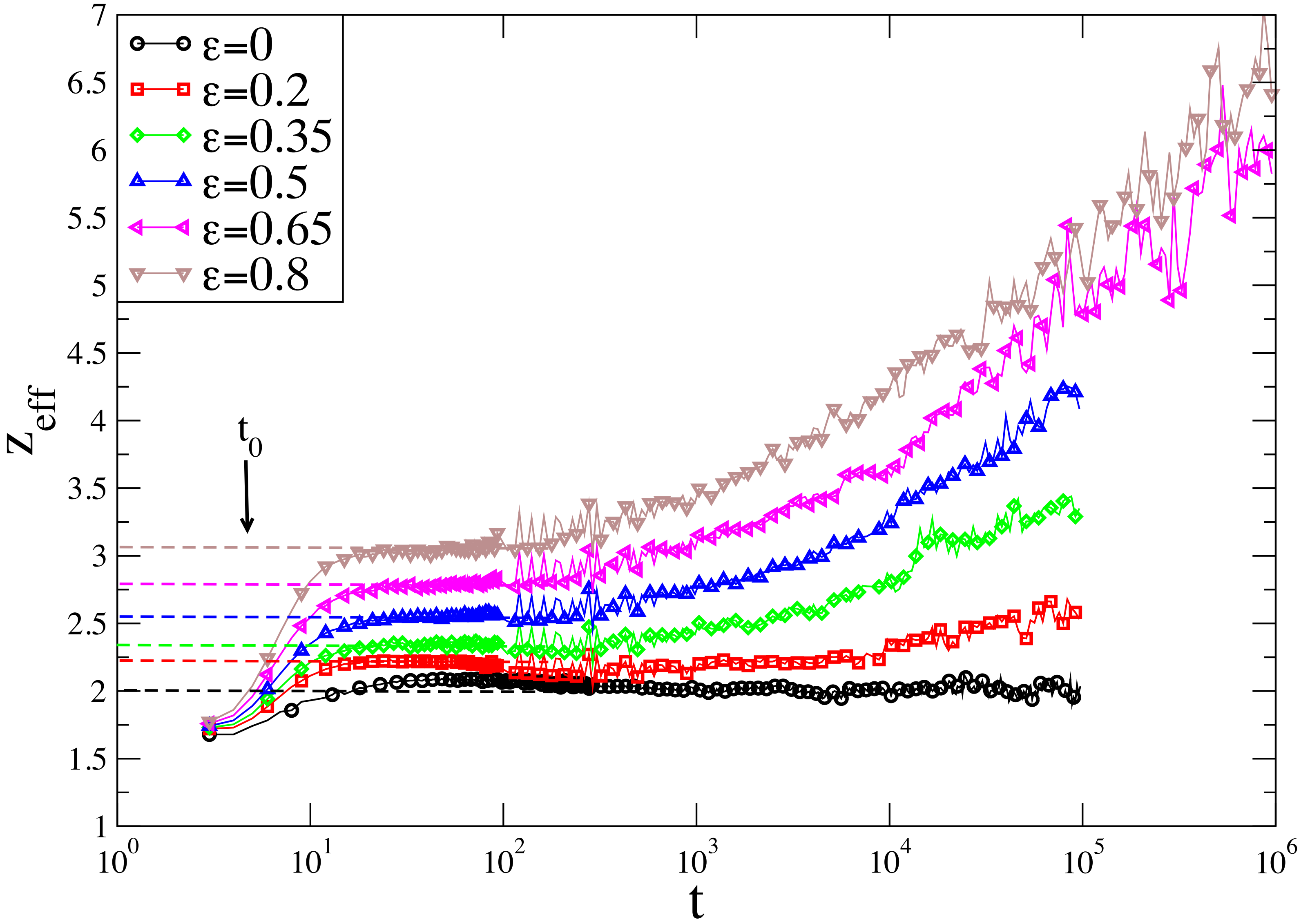
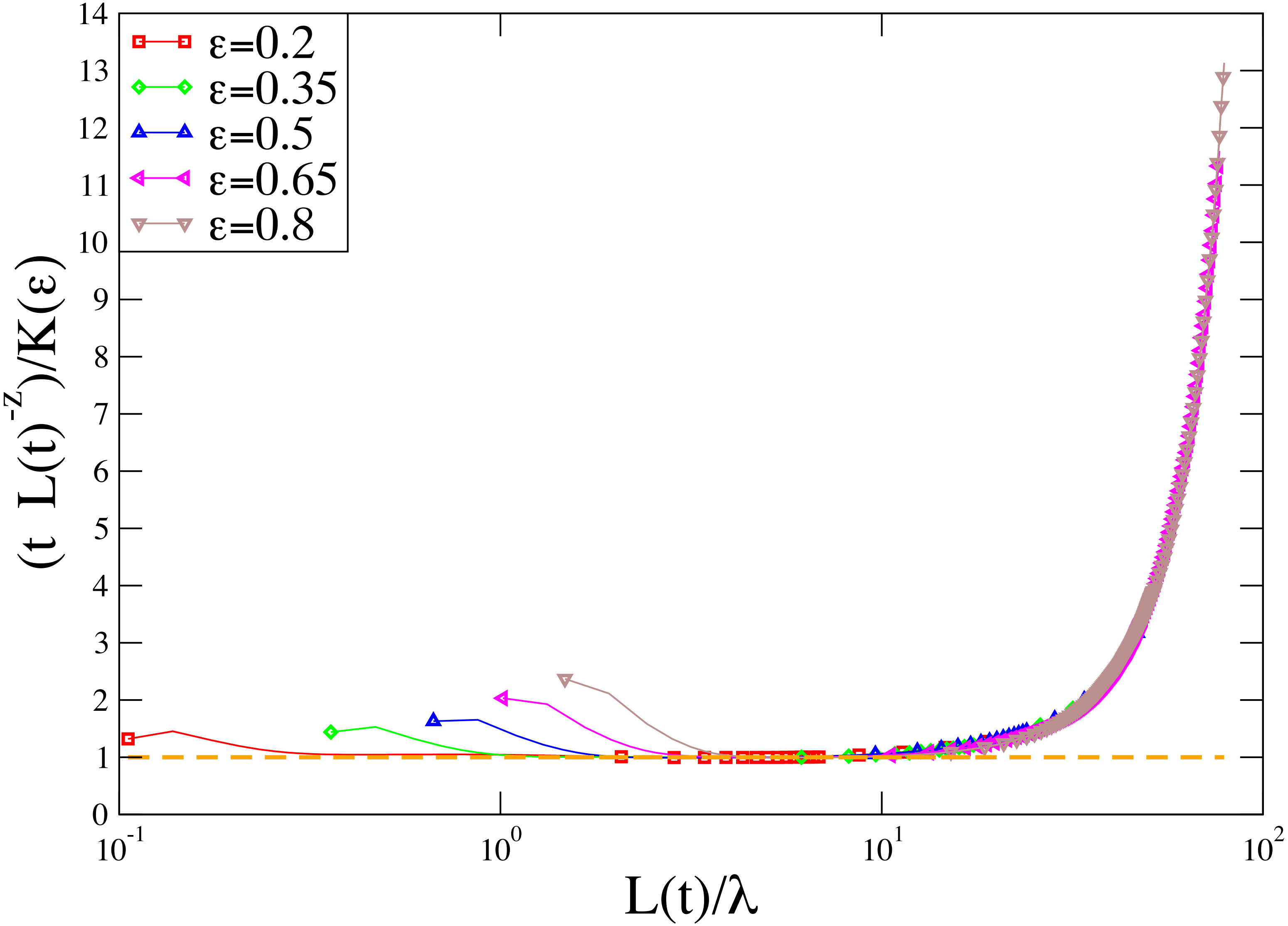
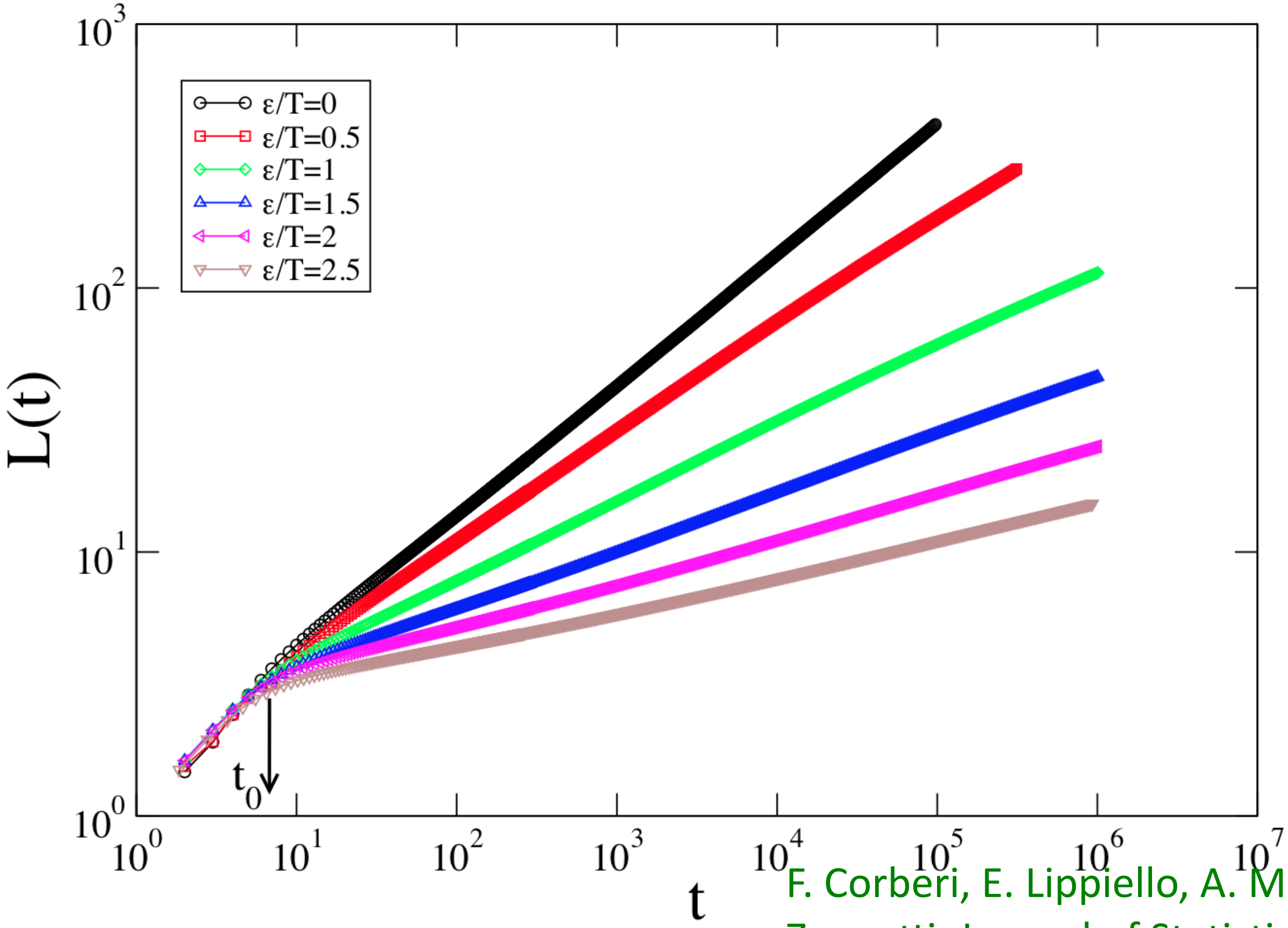


Some results (RF d=2)



Some results (RB d=2)



F. Corberi, E. Lippiello, A. Mukherjee, S. Puri, M. Zannetti, Journal of Statistical Mechanics: Theory and Experiments (2011) P03016.

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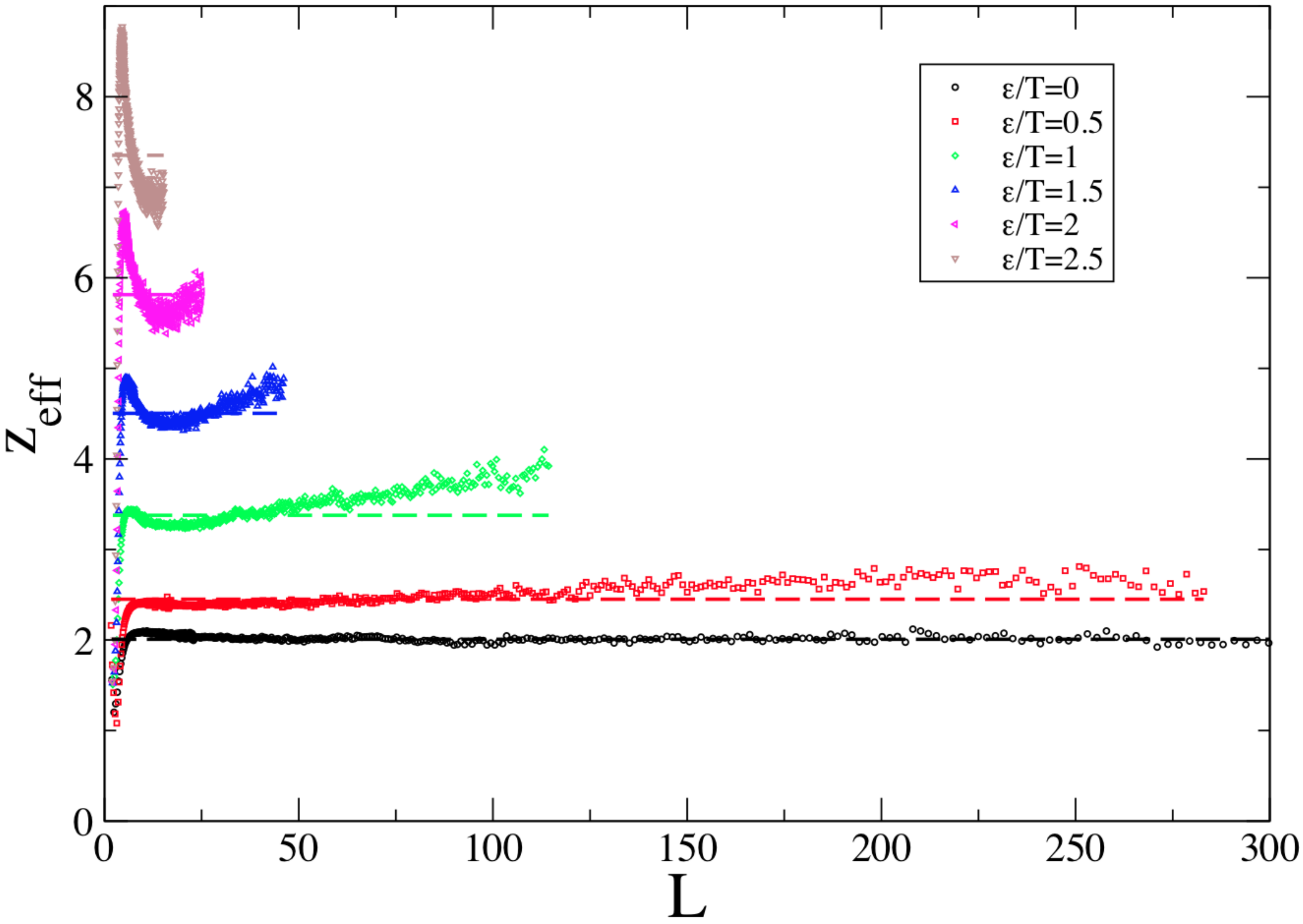

Conclusions

Not concluded !

Thank You !