DISCRETE SYMMETRIES OF LEPTON MIXING ANGLES

[based on hep-ph/0504165 and hep-ph/0512103 with Guido Altarelli]

Ferruccio Feruglio
Padova

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Where do we go from the Standard Model ?

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Lepton Mixing Angles

[Fogli, Lisi, Marrone, Palazzo 0506083]
[Schwetz 0510331]

$[2\sigma \quad \text{errors}\,(95\%\ \text{C.L.})]$

$\sin^2\vartheta_{23} = 0.44\,(1^{+0.41}_{-0.22})$ $\quad \sin^2\vartheta_{13} = 0.9^{+2.3}_{-0.9}\times 10^{-2}$ $\quad \sin^2\vartheta_{12} = 0.314\,(1^{+0.18}_{-0.15})$

$\vartheta_{23} = (41.6^{+10.4}_{-5.7})^0 \quad [2\sigma]$

[Hall, Murayama, Weiner 2000
De Gouvea, Murayama 0301050]

different viewpoints: - angles are all generically large [anarchy]
- angles reflect an underlying order

$\vartheta_{23} = 45^0$ $\qquad \vartheta_{13} = 0$ $\qquad \sin^2\vartheta_{12} = \frac{1}{3} \qquad \vartheta_{12} = 35.3^0$

[Harrison, Perkins and Scott (HPS) mixing pattern]

$\vartheta_{12} = (34.1\,^{+1.7}_{-1.6})^0 \quad [1\sigma]$

not a bad 1st order approximation!

θ_{12} right within $1\sigma \approx 2^0 \leq 0.04$ rad $\approx \lambda^2$, where λ=0.22
errors on θ_{23} and θ_{13} are still large…

future [< 10 yr] precision/sensitivity on θ_{23} and θ_{13} down to about λ^2
could confirm HPS mixing pattern

$\vartheta_{13} \approx \delta\vartheta_{23} \approx \lambda^2 \approx 0.04 \div 0.05\ \text{rad}\,(2.1^0 \div 2.9^0)$

[Gonzalez-Garcia, Maltoni, Smirnov 0408170]

$\sin^2\theta_{23}$

$\delta(\sin^2\theta_{23})$ reduced by future LBL experiments from $\nu_\mu \to \nu_\mu$ disappearance channel

$$\vartheta_{23} \approx \frac{\pi}{4}$$

$$P_{\mu\mu} \approx 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

$$\delta\vartheta_{23} \approx \frac{\sqrt{\delta P_{\mu\mu}}}{2}$$

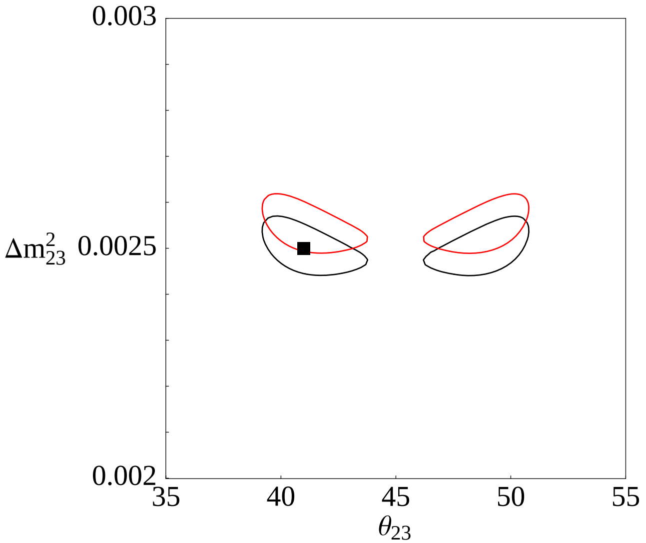
i.e. a small uncertainty on $P_{\mu\mu}$ leads to a large uncertainty on θ_{23}

- no substantial improvements from conventional beams
- superbeams (e.g. T2K in 5 yr of run)

$$\delta P_{\mu\mu} \approx 0.01$$

$$\delta\vartheta_{23} \approx 0.05\,\text{rad} \leftrightarrow 2.9^0$$

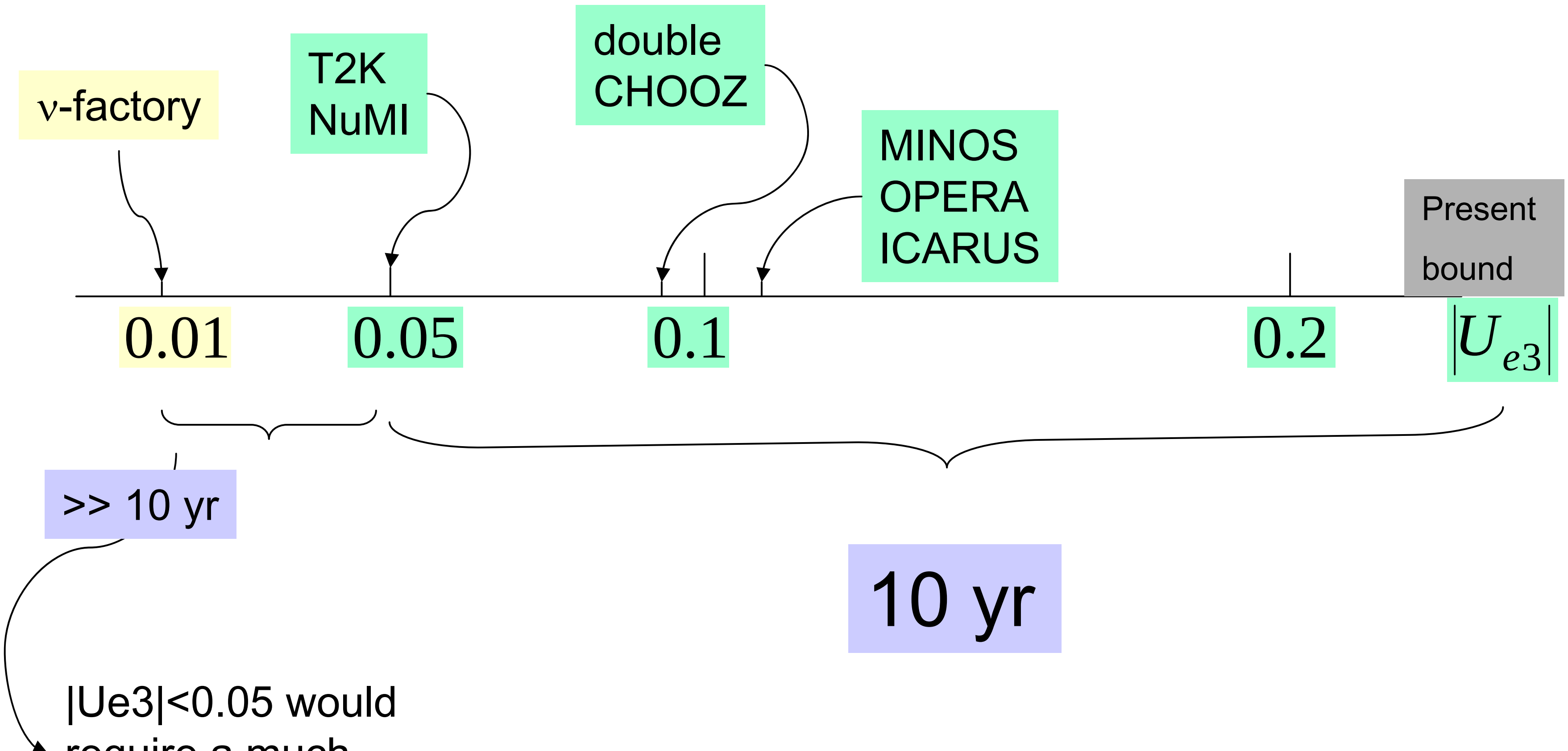
improvement by about a factor 2



T2K-1
90% CL
black = normal hierarchy
red = inverted hierarchy
true value 41^0
[courtesy by Enrique Fernandez]

sin θ_{13}

a similar sensitivity is expected on θ_{13} (U_{e3}=sin θ_{13})



|Ue3|<0.05 would require a much longer timescale

[Donini, Meloni, Rigolin 0506100]

If future data will confirm HPS down to about λ^2 precision

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O(\lambda^2)$$

quite symmetric!
also called
``tribimaximal''

reminiscent of

$$\pi^0 = \frac{|uu\rangle - |dd\rangle}{\sqrt{2}} \qquad \eta = \frac{|uu\rangle + |dd\rangle - 2|ss\rangle}{\sqrt{6}} \qquad \eta' = \frac{|uu\rangle + |dd\rangle + |ss\rangle}{\sqrt{3}}$$

theoretical challenges:
- how to derive HPS from a model?
more in general
- how to achieve exactly maximal θ_{23}
(eventually modified by small, $O(\lambda^2)$, corrections)?

θ_{23} maximal from flavour symmetries ?

an obstruction: $\vartheta_{23} = 45^0$ can never arise in the limit of an **exact realistic** symmetry

charged lepton mass matrix:

$$m_l = m_l^0 + \delta m_l^0$$

symmetric limit

symmetry breaking effects: vanishing when flavour symmetry F is **exact**

realistic symmetry:

(1) $\left|\delta m_l^0\right| < \left|m_l^0\right|$

(2) m_l^0 has rank ≤1

$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

ϑ_{12}^e undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan \vartheta_{23}^0 = \tan \vartheta_{23}^\nu \cos \vartheta_{12}^e + \left(\frac{\tan \vartheta_{13}^\nu}{\cos \vartheta_{23}^\nu} \right) \sin \vartheta_{12}^e$$

undetermined

$$\vartheta_{23} = 45^0$$

determined entirely by breaking effects
(different, in general, for ν and e sectors)

requirements for a model based on a SB flavour symmetry

- spontaneous symmetry breaking → vacuum alignment problem $\langle\varphi_\nu\rangle, \langle\varphi_e\rangle, \ldots$

should have specific magnitudes and relative directions in flavour space.

(1) alignment should be natural

no ad-hoc relations: desired VEVs from most general V in a finite region of parameter space

(2) alignment not spoiled by sub-leading terms

in HPS

$$\vartheta_{13} = \underbrace{0}_{\text{leading order}} + \underbrace{a_1 \frac{\langle\varphi\rangle}{\Lambda} + a_2 \frac{\langle\varphi\rangle^2}{\Lambda^2} + \ldots}$$

$$\vartheta_{23} = \underbrace{\frac{\pi}{4}}_{\text{leading order}} + \underbrace{b_1 \frac{\langle\varphi\rangle}{\Lambda} + b_2 \frac{\langle\varphi\rangle^2}{\Lambda^2} + \ldots}_{\text{from higher-dimensional operators compatible with gauge and flavour symmetries}}$$

from higher-dimensional operators compatible with gauge and flavour symmetries

often $\frac{\langle\varphi\rangle}{\Lambda} \approx \lambda$

then $a_1 = b_1 = 0$ needed

leading order

(3) alignment compatible with mass hierarchies

$$\frac{m_e}{m_\tau}, \quad \frac{m_\mu}{m_\tau}$$

should vanish in the limit of exact symmetry

an example: spontaneously broken A_4 symmetry

[other approaches: Grimus&Lavoura 0310050; King&Ross 0512313; Burgess&Matias 0508156]

[Ma, Rajasekaran 2001; Babu, Ma, Valle 2003; Hirsch, Romao, Skandage, Valle, Villanova de Moral 2003; Ma 0409075]

A_4 - group of even permutation of four objects
- subgroup of SO(3) leaving a tetrahedron invariant

it has 12 elements that can all be generated starting from 2 of them

$$S \quad T$$

``presentation'' $\quad S^2 = (ST)^3 = T^3 = 1$

S generates a Z_2 subgroup : G_S
T generates a Z_3 subgroup : G_T

$$A_4 = \{1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2, T^2ST\}$$

A_4 representations :

1	$S=1$	$T=1$
$1'$	$S=1$	$T=\omega^2$
$1''$	$S=1$	$T=\omega$

$$\omega \equiv e^{i\frac{2\pi}{3}}$$

$$3 \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

basic structure (lepton sector)

[AF hep-ph/0504165 and hep-ph/0512103]

	l	e^c	μ^c	τ^c	h_u	h_d	φ_T	φ_S	ξ_i
A_4	3	1	1''	1'	1	1	3	3	1

matter fields (l, e^c, μ^c, τ^c) Higgses (h_u, h_d) A_4 breaking sector (φ_T, φ_S, ξ_i)

SU(2)xU(1)xA_4 invariant Lagrangian:

$$L = y_e e^c (\varphi_T l) + y_\mu \mu^c (\varphi_T l)' + y_\tau \tau^c (\varphi_T l)'' + x_a \xi (ll) + x_b (\varphi_S ll) + V(\varphi_T, \varphi_S, \xi) + \ldots$$

( ) denotes an A_4 singlet,…

powers of $\left(\frac{h_{u,d}}{\Lambda}\right)$ have been set to 1

[Λ is the cutoff]

higher dimensional operators in $1/\Lambda$ expansion

under appropriate conditions (SUSY + Z_3) minimization of V leads to

$$\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle &= u \end{aligned}$$

$[v_T, v_S, u \leq \Lambda]$

$[\langle h_{u,d} \rangle = v_{u,d} << v_T, v_S, u]$

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda}\right)$$

charged fermion masses

$$m_f = y_f v_d \left(\frac{v_T}{\Lambda}\right)$$

free parameters as in the SM at this level

$$m_\nu = \begin{pmatrix} a+\frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & \frac{2}{3}b \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$a \equiv 2x_a \frac{u}{\Lambda}$$

$$b \equiv 2x_b \frac{v_S}{\Lambda}$$

2 complex parameters in ν sector (overall phase unphysical)

|a|, |b|, $\Delta \equiv$arg(a)-arg(b)

mixing angles entirely from ν sector:

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O\left(\frac{VEV}{\Lambda}\right)$$

independent from
|a|, |b|, $\Delta \equiv$arg(a)-arg(b) !!

from higher dimensional operators

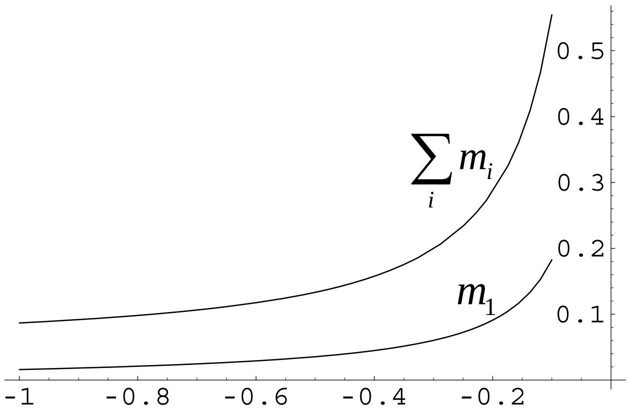
other predictions:

ν masses: $m_1 = |a+b|\dfrac{v_u^2}{\Lambda}$ $m_2 = |a|\dfrac{v_u^2}{\Lambda}$ $m_3 = |a-b|\dfrac{v_u^2}{\Lambda}$

$m_2 > m_1$ ⇨ $-1 \le \cos\Delta < -\left|\dfrac{b}{2a}\right|$ ⇨ ν spectrum always of **normal hierarchy type**

$$\left|\frac{b}{2a}\right| \approx \begin{cases} 1 & \text{[almost hierarchical spectrum]} \\ 0 & \text{[almost degenerate spectrum]} \end{cases}$$

$r \equiv \dfrac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \dfrac{1}{35}$ requires a (moderate) tuning



$\cos\Delta$

$$m_1 \ge 0.017 \ \text{eV}$$
$$\sum_i m_i \ge 0.09 \ \text{eV}$$
$$|m_3|^2 = |m_{ee}|^2 + \frac{10}{9}\Delta m^2_{atm}\left(1 - \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

$m_\tau = y_\tau v_d \left(\dfrac{v_T}{\Lambda}\right)$

$y_\tau < 4\pi$

⇨

$\dfrac{v_T}{\Lambda} > 0.002(0.02)$

$\tan\beta = 2.5(30)$

$\left[\tan\beta = \dfrac{v_u}{v_d}\right]$

assuming all VEVs of the same order corrections to masses and mixing angles can kept below λ^2

quark masses

simple and good first order approximation:

	q	u^c	c^c	t^c	d^c	s^c	b^c
A_4	3	1	1''	1'	1	1''	1'

same assignment as in the lepton sector

quark mass matrices diagonal in the leading order
mixing matrix V_{CKM}=1

unfortunately:
corrections induced by higher dimensional operators:
negligibly small

additional sources of A_4 breaking are needed in the quark sector

relation to the modular group

modular group PSL(2,Z): linear fractional transformation

complex variable

$$\boxed{z \to \frac{a\,z+b}{c\,z+d}} \qquad \begin{array}{l} a,b,c,d \in Z \\ ad-bc=1 \end{array}$$

discrete, infinite group generated by two elements

$$\boxed{\underbrace{z \to -\frac{1}{z}}_{S}} \qquad \boxed{\underbrace{z \to z+1}_{T}}$$

obeying

$$\boxed{S^2 = (ST)^3 = 1}$$

the modular group is present everywhere in string theory

[any relation to string theory approaches to fermion masses?]

Ibanez; Hamidi, Vafa; Dixon, Friedan, Martinec, Shenker; Casas, Munoz; Cremades, Ibanez, Marchesano; Abel, Owen

A_4 is a finite subgroup of the modular group and

$$\boxed{A_4 = \frac{PSL(2,Z)}{H}} \quad \Rightarrow \quad \boxed{\begin{array}{l}\text{representations of } A_4 \text{ are} \\ \text{representations of PSL(2,Z)}\end{array}}$$

infinite discrete normal subgroup of PSL(2,Z)

A_4 as a leftover of Poincare symmetry in D>4

[Altarelli,F,Lin 2006]

D dimensional Poincare symmetry ⟺ usually broken by compactification down to 4 dimensions

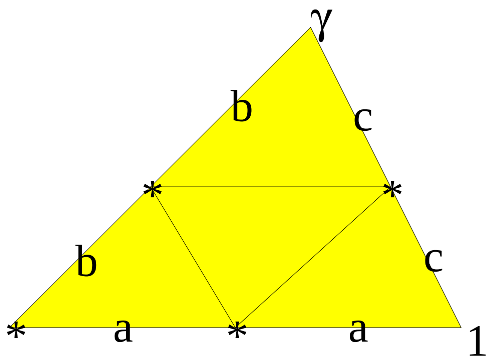
a discrete subgroup of the (D-4) euclidean group can survive in specific geometries

Example: D=6

2 dimensions compactified on T^2/Z_2

$$z \to z+1$$
$$z \to z+\gamma$$
$$z \to -z$$

four fixed points



$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

regular tetrahedron invariant under

$$S: \quad z \to z + \frac{1}{2}$$
$$T: \quad z \to \gamma^2 z$$

$$S^2 = T^3 = (ST)^3 = 1$$

conclusion

mixing in the lepton sector is well described by the HPS pattern

$\vartheta_{23} = 45^0$ $\vartheta_{13} = 0$ $\sin^2 \vartheta_{12} = \frac{1}{3}$

errors on θ_{23} and θ_{13} are still large and future data are needed to confirm HPS at the λ^2 level

most of existing models predict $\left|\frac{\pi}{4} - \vartheta_{23}\right| >> \lambda^2$

only in ``special'' models this condition is violated. If based on a SB flavour symmetry, special models should give rise to a

natural vacuum alignment

preserved by high-order effects

with a structure compatible with the observed charged fermion hierarchy

Here: an existence proof based on the discrete group A_4

vacuum alignment and stability is non-trivial

the neutrino spectrum is of normal hierarchy type and the relations

$$|m_3|^2 = |m_{ee}|^2 + (10/9)\Delta m^2_{atm}\left(1 - \Delta m^2_{sol} / \Delta m^2_{atm}\right)$$

$$m_1 > 0.017 \text{ eV} \qquad \sum_i m_i > 0.09 \text{ eV}$$

are predicted

low-energy parameters

ν masses

[3 light active ν]

$$m_1, m_2, m_3$$

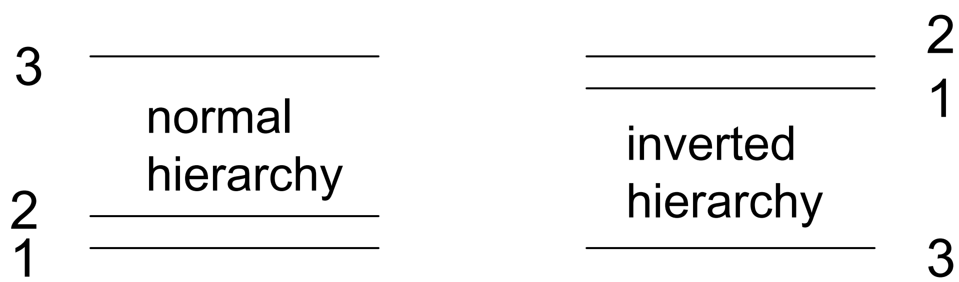
order $m_1 < m_2$

$$\Delta m^2_{21} < \left|\Delta m^2_{32}\right|, \left|\Delta m^2_{31}\right| \qquad [\Delta m^2_{ij} \equiv m_i^2 - m_j^2]$$

i.e. 1 and 2 are, by definition, the closest levels

two possibilities:

$$\left.\begin{array}{l} \Delta m^2_{21} = 7.9\,(1 \pm 0.09) \times 10^{-5} \quad \mathrm{GeV}^2 \\ \left|\Delta m^2_{31}\right| = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3} \quad \mathrm{GeV}^2 \end{array}\right\} \text{ at } 2\,\sigma$$



Mixing matrix (analogous to V_{CKM})

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ -c_{12}s_{13}c_{23} + s_{12}s_{23}e^{-i\delta} & -s_{12}s_{13}c_{23} - c_{12}s_{23}e^{-i\delta} & c_{13}c_{23} \end{pmatrix} \times \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}}$$

$$c_{12} \equiv \cos\vartheta_{12}, \ldots$$

- only if ν are Majorana
- drops in oscillations

(3) alignment and mass hierarchies

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda}\right)$$

charged fermion masses are already diagonal

$$m_e << m_\mu << m_\tau$$

easily reproduced by U(1) flavour symmetry

$$\left.\begin{array}{lll} Q(e^c) = 4 & Q(\mu^c) = 2 & Q(\tau^c) = 0 \\ Q(l) = 0 & & \end{array}\right\}$$

compatible with A_4

$$Q(\vartheta) = -1 \qquad \langle\vartheta\rangle \neq 0$$

$$\Downarrow$$

$$y_e \approx \frac{\langle\vartheta\rangle^4}{\Lambda^4} \qquad y_\mu \approx \frac{\langle\vartheta\rangle^2}{\Lambda^2} \qquad y_\tau \approx 1$$