Subtraction method for QCD jet cross sections

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in collaboration with

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NEW SZÉCHENYI PLAN

LHCphenonet

Outline

- The problem and our goals
- Our method: recipe for a general subtraction scheme at any order in perturbation theory
- Main difficulty: integrating the counter terms
- Light in the tunnel: cancellation of poles
- Application
- Conclusions

Problem

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$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m}$$
$$\equiv \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m}$$

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- the three contributions are separately divergent in d = 4 dimensions:
  - in σ^{RR} kinematical singularities as one or two partons become unresolved yielding ϵ-poles at $O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$ after integration over phase space, no explicit ϵ-poles
  - in σ^{RV} kinematical singularities as one parton becomes unresolved yielding ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$ after integration over phase space + explicit ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$
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How to combine to obtain finite cross section?

personal opinion: general solution is not yet available

Approaches

- Sector decomposition

  Anastasiou, Melnikov, Petriallo et al 2004-
- Antennae subtraction

  Gehrmann, Gehrmann-De Ridder, Glover et al 2004-
- q_T-subtraction

  S. Catani, M. Grazzini et al 2007-
- Sector-improved phase space for real radiation (STRIPPER)

  Czakon et al 2010-
- Completely Local Subtractions for Fully Differential Predictions at NNLO (Colorful NNLO)

  Somogyi, TZ et al 2005-
- For details see: NNLO Ante Portas (LHCPhenonet Summer School in Hungary, June 2014)

  http://www.lhcphenonet.eu/debrecen2014/

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- ✓ completely general construction (valid in any order of perturbation theory)

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- ✓ fully local counter-terms (efficiency and mathematical rigor)
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- ✓ completely general construction (valid in any order of perturbation theory)
- ✓ option to constrain subtraction near singular regions (important check)

Recipe

Structure

of subtractions is governed by the jet functions

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

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RR,A_2 regularizes doubly-unresolved limits

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
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RR,A_1 regularizes singly-unresolved limits

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RR,A12 removes overlapping subtractions

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Use known ingredients

- Universal IR structure of QCD (squared) matrix elements
  - ϵ-poles of one-loop amplitudes:

$$|\mathcal{M}_m^{(1)}(\{p\})\rangle = -\frac{1}{2}\boldsymbol{I}_1^{(0)}(\epsilon;\{p\})|\mathcal{M}_m^{(0)}(\{p\})\rangle + \mathrm{O}(\varepsilon^0)$$

$$\boldsymbol{I}_1^{(0)}(\epsilon) = \frac{\alpha_\mathrm{s}}{2\pi}\sum_i\left[\frac{1}{\epsilon}\gamma_i - \frac{1}{\epsilon^2}\sum_{k\neq i}\boldsymbol{T}_i\cdot\boldsymbol{T}_k\left(\frac{4\pi\mu^2}{s_{ik}}\right)^\epsilon\right]$$

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  - ϵ-poles of two-loop amplitudes:

$$|\mathcal{M}_m^{(2)}(\{p\})\rangle = -\frac{1}{2}\left(\boldsymbol{I}_1^{(0)}(\epsilon;\{p\})|\mathcal{M}_m^{(1)}(\{p\})\rangle + \boldsymbol{I}_1^{(1)}(\epsilon;\{p\})|\mathcal{M}_m^{(0)}(\{p\})\rangle\right) + \mathrm{O}(\varepsilon^0)$$

S. Catani 1998, G. Sterman, M.E.Tejeda-Yeomans 2003, S. Moch, M. Mitov 2007

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- Universal IR structure of QCD (squared) matrix elements
  - ϵ-poles of one- and two-loop amplitudes
  - soft and collinear factorization of QCD matrix elements

tree-level 3-parton splitting, double soft current:

J.M. Campbell, E.W.N. Glover 1997, S. Catani, M. Grazzini 1998
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- Extension over whole phase space using momentum mappings (not unique):

$$\{p\}_{n+s} \longrightarrow \{\tilde{p}\}_n$$

Momentum mappings

$$\{p\}_{n+s} \longrightarrow \{\tilde{p}\}_n$$

- implement exact momentum conservation
- recoil distributed democratically

  $\Rightarrow$ can be generalized to any number s of unresolved partons
- different mappings for collinear and soft limits
  - collinear limit $p_i || p_r$: $\{p\}_{n+1} \xrightarrow{\mathsf{C}_{ir}} \{\tilde{p}\}_n^{(ir)}$
  - soft limit $p_s \to 0$: $\{p\}_{n+1} \xrightarrow{\mathsf{S}_s} \{\tilde{p}\}_n^{(s)}$

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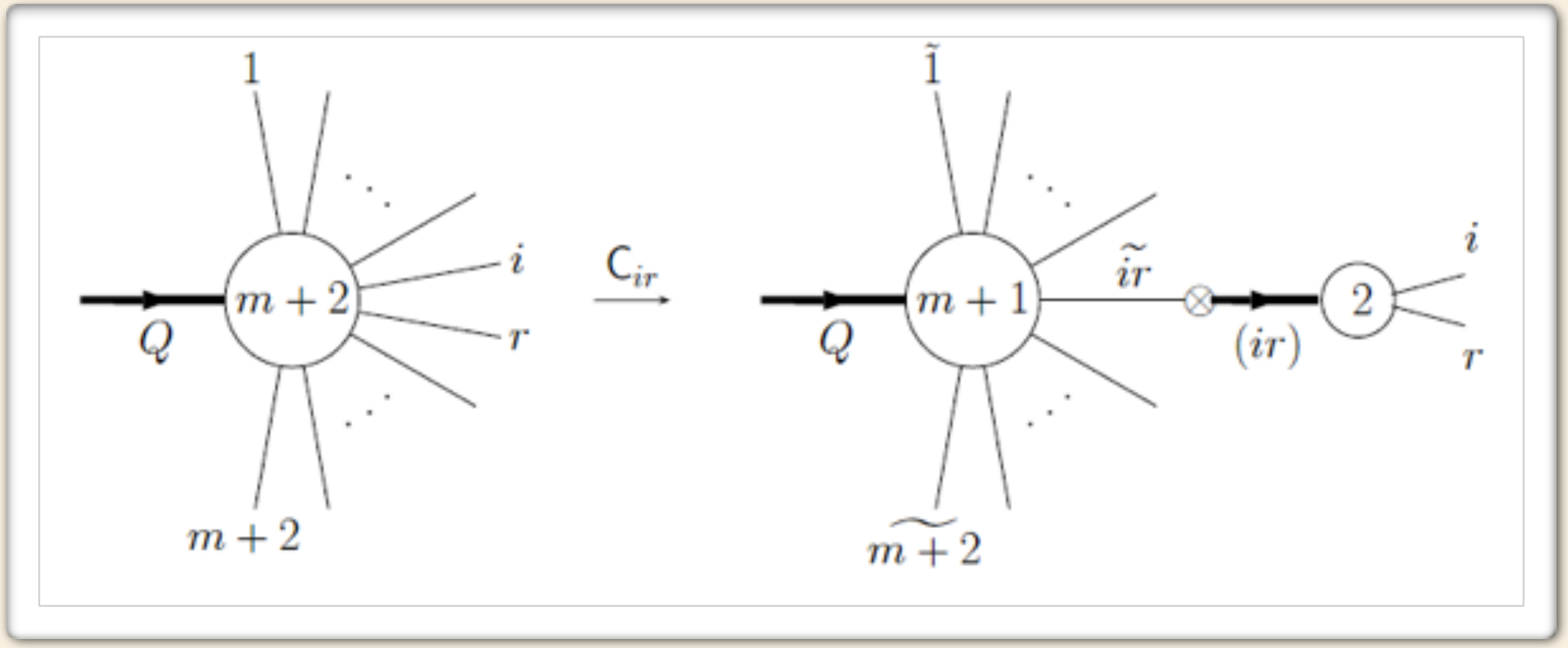

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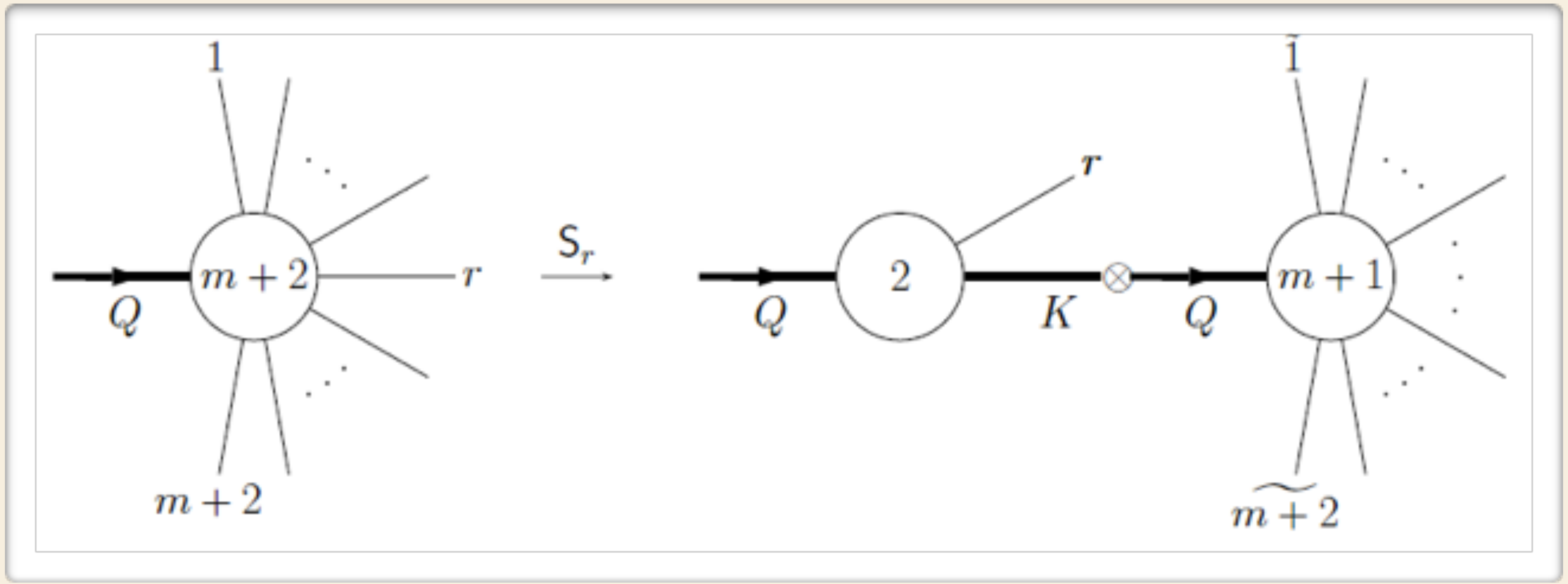

Momentum mappings

define subtractions

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Regularized RR and RV contributions

can now be computed by numerical

Monte Carlo integrations

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$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043

G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042

Z. Nagy, G. Somogyi, ZT hep-ph/0702273

Difficulty

Integrated approximate xsections

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} \boxed{+ \int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2}} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} \boxed{+ \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right]} \right\} J_m$$

After integrating over unresolved momenta & summing over unresolved flavors, the subtraction terms can be written as products of insertion operators (in color space) and lower point cross sections:

$$\int_p \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_p} = \boldsymbol{I}^{(0)}_p(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma^{\mathrm{B}}_n$$

Integrated approximate xsections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR,A}_p} = \int_p \left[\mathrm{d}\phi_{m+2}(\{p\}) \sum_R \mathcal{X}_R(\{p\})\right]$$

$$= \int_p \left[\mathrm{d}\phi_n(\{\tilde{p}\}^{(R)})[\mathrm{d}p_p^{(R)}] \sum_R \left(8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\right)^p \mathit{Sing}_R(p_p^{(R)}) \otimes |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2\right]$$

$$= \left(8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\right)^p \sum_R \left[\int_p [\mathrm{d}p_p^{(R)}]\mathit{Sing}_R(p_p^{(R)})\right] \otimes \mathrm{d}\phi_n(\{\tilde{p}\}^{(R)})|\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

$$= \underbrace{\left(8\pi\alpha_{\mathrm{s}}\mu^{2\epsilon}\right)^p \sum_R \left[\int_p [\mathrm{d}p_p^{(R)}]\mathit{Sing}_R(p_p^{(R)})\right]}_{\boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon)} \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

the integrated counter-terms $[X]_R \propto \int_p [\mathrm{d}p_p^{(R)}]\mathit{Sing}_R(p_p^{(R)})$ are

independent of the process & observable

$\Rightarrow$ need to compute only once (admittedly cumbersome, though)

Summation over unresolved flavors

- integrated counter-terms $[X]_{f_i\ldots}$ carry flavor indices of unresolved patrons

  ⇒ need to sum over unresolved flavors:

  technically simple, though tedious, result can be summarized in flavor-summed integrated counter-terms

P. Bolzoni, G. Somogyi, ZT arXiv:0905.4390

- symbolically:

$$\left(X^{(0)}\right)_{f_i\ldots}^{(j,l)\ldots} = \sum [X^{(0)}]_{f_k\ldots}^{(j,l)\ldots}$$

- and precisely, for instance, two-flavor sum:

$$\sum_{\{m+2\}} \frac{1}{S_{\{m+2\}}} \sum_t \sum_{k \neq t} [X_{kt}^{(0)}]_{f_k f_t}^{(\ldots)} \equiv \sum_{\{m\}} \frac{1}{S_{\{m\}}} \left(X_{kt}^{(0)}\right)^{(\ldots)}$$

Computing the integrals

- Use algebraic and symmetry relations to reduce to a basic set $\Rightarrow$ MI's (but no IBP was used)
- two strategies:

Computing the integrals

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- two strategies:

1. write phase space using angles and energies
2. angular integrals in terms of MB representations
3. resolve ϵ-poles by analytic continuation
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5. evaluate parametric integrals of pole coefficients in terms of multiple polylogs, optional: simplify result

Status of integrals

Int	status
$\mathcal{I}^{(k)}_{1\mathcal{C},0}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},3}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},4}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},5}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},7}$	✔
$\mathcal{I}_{1\mathcal{C},8}$	✔

Int	status
$\mathcal{I}_{1\mathcal{S},0}$	✔
$\mathcal{I}_{1\mathcal{S},1}$	✔
$\mathcal{I}_{1\mathcal{S},2}$	$(m > 3)$ ✘
$\mathcal{I}^{(k)}_{1\mathcal{S},3}$	✔
$\mathcal{I}_{1\mathcal{S},4}$	✔
$\mathcal{I}_{1\mathcal{S},5}$	✔
$\mathcal{I}_{1\mathcal{S},6}$	✔
$\mathcal{I}_{1\mathcal{S},7}$	✔

Int	status
$\mathcal{I}_{1\mathcal{CS},0}$	✔
$\mathcal{I}_{1\mathcal{CS},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{CS},2}$	✔
$\mathcal{I}_{1\mathcal{CS},3}$	✔
$\mathcal{I}_{1\mathcal{CS},4}$	✔

Int	status
$\mathcal{I}^{(k,l)}_{12\mathcal{C},1}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},3}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},5}$	$m = 2$: ✔/✘
$\mathcal{I}^{(k)}_{12\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},7}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},8}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},9}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},10}$	✔

Int	status
$\mathcal{I}_{2\mathcal{S},1}$	✔
$\mathcal{I}_{2\mathcal{S},2}$	✔
$\mathcal{I}_{2\mathcal{S},3}$	✔
$\mathcal{I}_{2\mathcal{S},4}$	✔
$\mathcal{I}_{2\mathcal{S},5}$	✔
$\mathcal{I}_{2\mathcal{S},6}$	✔
$\mathcal{I}_{2\mathcal{S},7}$	✔
$\mathcal{I}_{2\mathcal{S},8}$	✔
$\mathcal{I}_{2\mathcal{S},9}$	✔
$\mathcal{I}_{2\mathcal{S},10}$	✔
$\mathcal{I}_{2\mathcal{S},11}$	✔
$\mathcal{I}_{2\mathcal{S},12}$	✔
$\mathcal{I}_{2\mathcal{S},13}$	✔
$\mathcal{I}_{2\mathcal{S},14}$	✔
$\mathcal{I}_{2\mathcal{S},15}$	✔
$\mathcal{I}_{2\mathcal{S},16}$	✔
$\mathcal{I}_{2\mathcal{S},17}$	✔
$\mathcal{I}_{2\mathcal{S},18}$	✔
$\mathcal{I}_{2\mathcal{S},19}$	✘
$\mathcal{I}_{2\mathcal{S},20}$	✔
$\mathcal{I}_{2\mathcal{S},21}$	✔
$\mathcal{I}_{2\mathcal{S},22}$	✔
$\mathcal{I}_{2\mathcal{S},23}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{S},1}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},3}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},5}$	✔
$\mathcal{I}_{12\mathcal{S},6}$	✔
$\mathcal{I}_{12\mathcal{S},7}$	✔
$\mathcal{I}_{12\mathcal{S},8}$	✔
$\mathcal{I}_{12\mathcal{S},9}$	✔
$\mathcal{I}_{12\mathcal{S},10}$	✘
$\mathcal{I}_{12\mathcal{S},11}$	✘
$\mathcal{I}_{12\mathcal{S},12}$	✔
$\mathcal{I}_{12\mathcal{S},13}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{CS},1}$	✔
$\mathcal{I}_{12\mathcal{CS},2}$	✔
$\mathcal{I}_{12\mathcal{CS},3}$	✔

Int	status
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},1}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},2}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},3}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},4}$	✘
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},5}$	✘
$\mathcal{I}^{(k,l)}_{2\mathcal{C},6}$	✔

Int	status
$\mathcal{I}^{(k)}_{2\mathcal{CS},1}$	✘
$\mathcal{I}^{(k)}_{2\mathcal{CS},2}$	✘
$\mathcal{I}^{(k)}_{2\mathcal{CS},3}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},4}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},5}$	✔

✓: pole coefficients are known analytically, finite numerically

✗: pole coefficients are known up to $O(\epsilon^{-1})$, rest numerically

Structure of insertion operators

recall general form for Born sections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_p} = \boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

Insertion operators involve all possible color connections with given number of unresolved patrons with kinematic coefficients

for 1 unresolved parton on tree SME $|M^{(0)}|^2$:

$$\boldsymbol{I}_1^{(0)}(\{p\}_{m+1};\epsilon) = \frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon \sum_i \left[\mathrm{C}_{1,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_1^{(0),(i,k)} \boldsymbol{T}_i \boldsymbol{T}_k\right]$$

kinematic functions contain poles starting from $O(\epsilon^{-2})$ for collinear and from $O(\epsilon^{-1})$ for soft

G. Somogyi, ZT hep-ph/0609041

Structure of insertion operators

recall general form for Born sections

$$\int_p \mathrm{d}\sigma^{\mathrm{RR,A}_p} = \boldsymbol{I}_p^{(0)}(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma_n^{\mathrm{B}}$$

for 2 unresolved patrons on tree SME $|M^{(0)}|^2$:

$$\boldsymbol{I}_2^{(0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \Bigg\{ \sum_i \left[\mathrm{C}_{2,f_i}^{(0)} \boldsymbol{T}_i^2 + \sum_k \mathrm{C}_{2,f_i f_k}^{(0)} \boldsymbol{T}_k^2\right] \boldsymbol{T}_i^2$$

$$+ \sum_{j,l} \left[\mathrm{S}_2^{(0),(j,l)} C_{\mathrm{A}} + \sum_i \mathrm{CS}_{2,f_i}^{(0),(j,l)} \boldsymbol{T}_i^2\right] \boldsymbol{T}_j \boldsymbol{T}_l$$

$$+ \sum_{i,k,j,l} \mathrm{S}_2^{(0),(i,k)(j,l)} \{\boldsymbol{T}_i \boldsymbol{T}_k, \boldsymbol{T}_j \boldsymbol{T}_l\} \Bigg\}$$

the iterated doubly-unresolved has the same color structure, kinematic coefficients differ

G. Somogyi et al arXiv:0905.4390, arXiv:1301.3504, arXiv:1301.3919

Structure of insertion operators

general form at one loop

$$\int_1 \mathrm{d}\sigma_{m+1}^{\mathrm{RV},\mathrm{A}_1} = \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon) \otimes \mathrm{d}\sigma_m^{\mathrm{V}} + \boldsymbol{I}_1^{(1)}(\{p\}_m;\epsilon) \otimes \mathrm{d}\sigma_m^{\mathrm{B}}$$

for 1 unresolved parton on loop SME $|M^{(1)}|^2$:

$$\boldsymbol{I}_1^{(1)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2}\right)^\epsilon\right]^2 \sum_i \Bigg[\mathrm{C}_{1,f_i}^{(1)} C_{\mathrm{A}} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_1^{(1),(i,k)} C_{\mathrm{A}} \boldsymbol{T}_i \boldsymbol{T}_k + \sum_{\substack{k,l\\k\neq l}} \mathrm{S}_1^{(1),(i,k,l)} \sum_{a,b,c} f_{abc} T_i^a T_k^b T_l^c\Bigg]$$

present for m > 3 (four or more hard patrons)

only non-abelian contributions

G. Somogyi, ZT arXiv:0807.0509

Structure of insertion operators

singly-unresolved integrated singly unresolved:

$$\int_1 \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} = \left[\frac{1}{2} \left\{ \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon), \boldsymbol{I}_1^{(0)}(\{p\}_m;\epsilon) \right\} + \boldsymbol{I}_{1,1}^{(0,0)}(\{p\}_m;\epsilon) \right] \otimes \mathrm{d}\sigma_m^{\mathrm{B}}$$

with only non-abelian contributions on iterated I:

$$\boldsymbol{I}_{1,1}^{(0,0)}(\{p\}_m;\epsilon) = \left[\frac{\alpha_{\mathrm{s}}}{2\pi} S_\epsilon \left(\frac{\mu^2}{Q^2} \right)^\epsilon \right]^2 \sum_i \left[\mathrm{C}_{1,1,f_i}^{(0,0)} C_{\mathrm{A}} \boldsymbol{T}_i^2 + \sum_k \mathrm{S}_{1,2}^{(0,0),(i,k)} C_{\mathrm{A}} \boldsymbol{T}_i \boldsymbol{T}_k \right]$$

kinematic functions contain poles starting from $O(\epsilon^{-3})$ only

G. Somogyi, ZT arXiv:0807.0509

Structure of insertion operators

- the color structures are independent of the precise definition of subtractions (momentum mappings), only subleading coefficients of ϵ-expansion in kinematic functions may depend
- we computed all insertion operators (defined in our subtraction scheme) up to $O(\epsilon^{-2})$ for arbitrary m

Light in the tunnel

Cancellation of poles

- we checked the cancellation of the leading and first subleading poles (defined in our subtraction scheme) for arbitrary m
- for m=2,
  - the insertion operators are independent of the kinematics (momenta are back-to-back, so MI's are needed at the endpoints only)
  - color algebra is trivial: $\boldsymbol{T}_1\boldsymbol{T}_2 = -\boldsymbol{T}_1^2 = -\boldsymbol{T}_2^2 = -C_{\mathrm{F}}$
- so can demonstrate the cancellation of poles

Example: H→b$\bar{\text{b}}$ at μ = m$_H$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ d\sigma_m^{\rm VV} + \int_2 \left[d\sigma_{m+2}^{\rm RR,A_2} - d\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[d\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 d\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$\begin{aligned} d\sigma^{\rm VV}_{H\to b\bar{b}} = \left(\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\right)^2 d\sigma^{\rm B}_{H\to b\bar{b}} \Bigg\{ & \frac{2C_{\rm F}^2}{\epsilon^4} + \left(\frac{11 C_{\rm A} C_{\rm F}}{4} + 6C_{\rm F}^2 - \frac{C_{\rm F} n_{\rm f}}{2}\right)\frac{1}{\epsilon^3} \\ & + \left[\left(\frac{8}{9} + \frac{\pi^2}{12}\right) C_{\rm A} C_{\rm F} + \left(\frac{17}{2} - 2\pi^2\right) C_{\rm F}^2 - \frac{2 C_{\rm F} n_{\rm f}}{9}\right]\frac{1}{\epsilon^2} \\ & + \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2}\right) C_{\rm A} C_{\rm F} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3\right) C_{\rm F}^2 + \frac{65 C_{\rm F} n_{\rm f}}{108}\right]\frac{1}{\epsilon} \Bigg\} \end{aligned}$$

C. Anastasiou, F. Herzog, A. Lazopoulos arXiv:0111.2368

$$\begin{aligned} \sum \int d\sigma^{\rm A} = \left(\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\right)^2 d\sigma^{\rm B}_{H\to b\bar{b}} \Bigg\{ & \frac{-2C_{\rm F}^2}{\epsilon^4} + \left(-\frac{11 C_{\rm A} C_{\rm F}}{4} - 6C_{\rm F}^2 + \frac{C_{\rm F} n_{\rm f}}{2}\right)\frac{1}{\epsilon^3} \\ & + \left[\left(-\frac{8}{9} - \frac{\pi^2}{12}\right) C_{\rm A} C_{\rm F} + \left(-\frac{17}{2} + 2\pi^2\right) C_{\rm F}^2 + \frac{2 C_{\rm F} n_{\rm f}}{9}\right]\frac{1}{\epsilon^2} \\ & + \left[- 3.36424 C_{\rm A} C_{\rm F} + 22.9414 C_{\rm F}^2 - 0.601852 C_{\rm F} n_{\rm f}\right]\frac{1}{\epsilon} \Bigg\} \end{aligned}$$

Example: H→b$\bar{\text{b}}$ at μ = m$_H$

$$\sigma_m^{\rm NNLO} = \int_m \left\{ d\sigma_m^{\rm VV} + \int_2 \left[d\sigma_{m+2}^{\rm RR,A_2} - d\sigma_{m+2}^{\rm RR,A_{12}} \right] + \int_1 \left[d\sigma_{m+1}^{\rm RV,A_1} + \left(\int_1 d\sigma_{m+2}^{\rm RR,A_1} \right)^{\rm A_1} \right] \right\} J_m$$

$$d\sigma_{H\to b\bar{b}}^{\rm VV} = \left(\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\right)^2 d\sigma_{H\to b\bar{b}}^{\rm B} \left\{ \frac{2C_{\rm F}^2}{\epsilon^4} + \left(\frac{11 C_{\rm A} C_{\rm F}}{4} + 6C_{\rm F}^2 - \frac{C_{\rm F} n_{\rm f}}{2} \right) \frac{1}{\epsilon^3} \right.$$
$$+ \left[\left(\frac{8}{9} + \frac{\pi^2}{12} \right) C_{\rm A} C_{\rm F} + \left(\frac{17}{2} - 2\pi^2 \right) C_{\rm F}^2 - \frac{2 C_{\rm F} n_{\rm f}}{9} \right] \frac{1}{\epsilon^2}$$
$$\left. + \left[3.36429 C_{\rm A} C_{\rm F} - 22.9430 C_{\rm F}^2 + 0.601851 \right] \frac{1}{\epsilon} \right\}$$

C. Anastasiou, F. Herzog, A. Lazopoulos arXiv:0111.2368

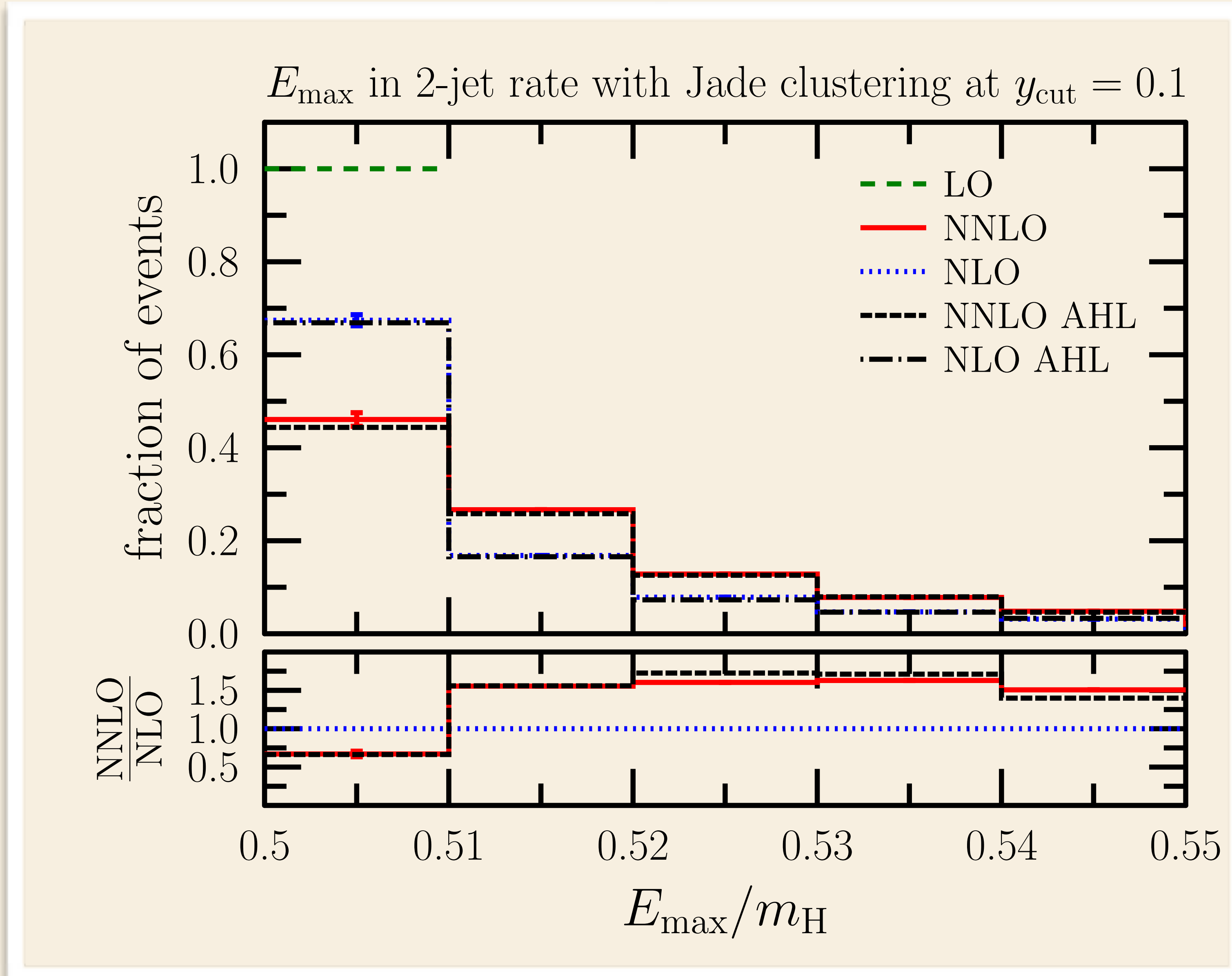
$$\sum \int d\sigma^{\rm A} = \left(\frac{\alpha_{\rm s}(\mu^2)}{2\pi}\right)^2 d\sigma_{H\to b\bar{b}}^{\rm B} \left\{ \frac{-2C_{\rm F}^2}{\epsilon^4} + \left(-\frac{11 C_{\rm A} C_{\rm F}}{4} - 6C_{\rm F}^2 + \frac{C_{\rm F} n_{\rm f}}{2} \right) \frac{1}{\epsilon^3} \right.$$
$$+ \left[\left(-\frac{8}{9} - \frac{\pi^2}{12} \right) C_{\rm A} C_{\rm F} + \left(-\frac{17}{2} + 2\pi^2 \right) C_{\rm F}^2 + \frac{2 C_{\rm F} n_{\rm f}}{9} \right] \frac{1}{\epsilon^2}$$
$$\left. + \left[-3.36424 C_{\rm A} C_{\rm F} + 22.9414 C_{\rm F}^2 - 0.601852 C_{\rm F} n_{\rm f} \right] \frac{1}{\epsilon} \right\}$$

Message:

the method works, try to apply

Application

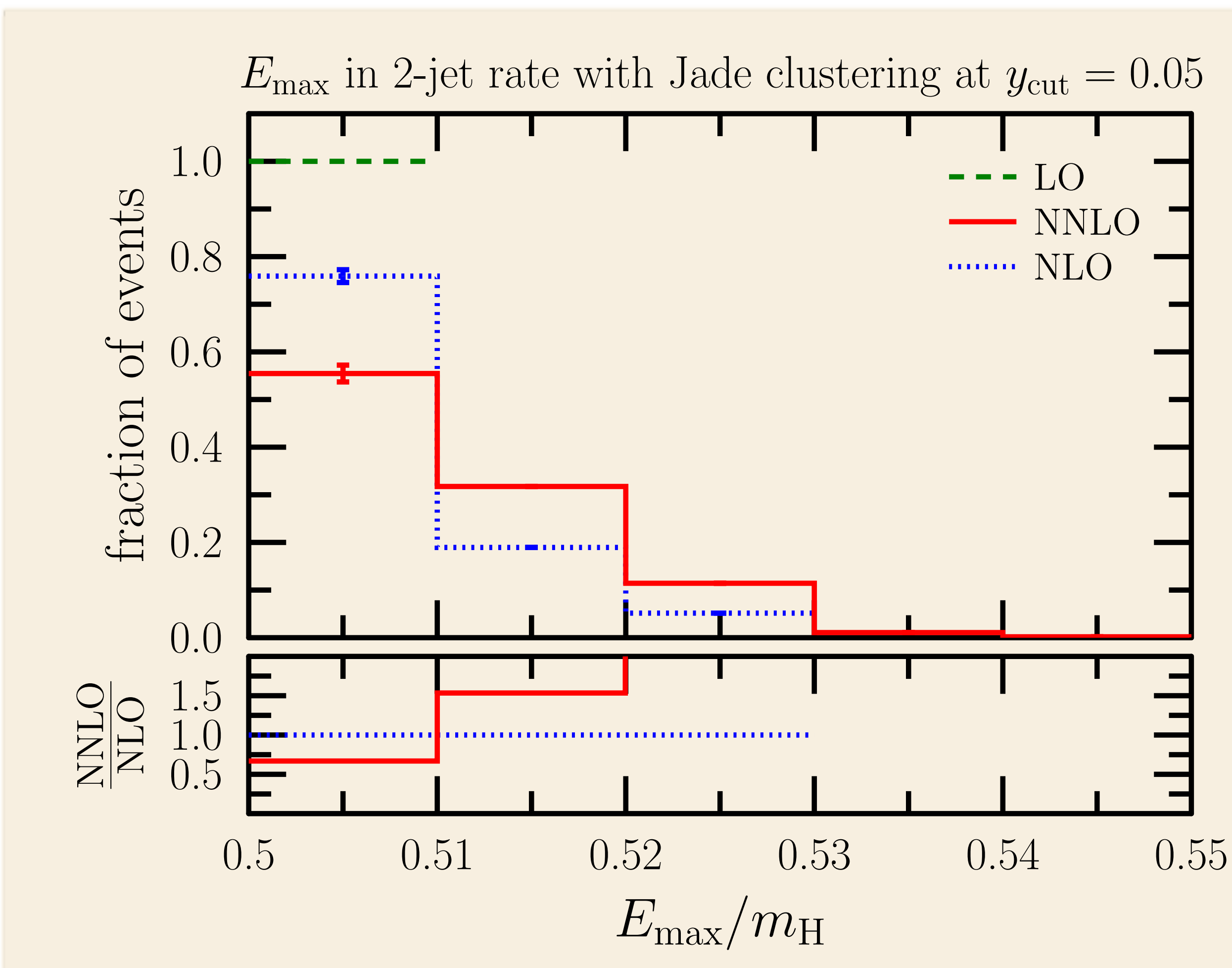
Example: $H \to b\bar{b}$ at $\mu = m_H$

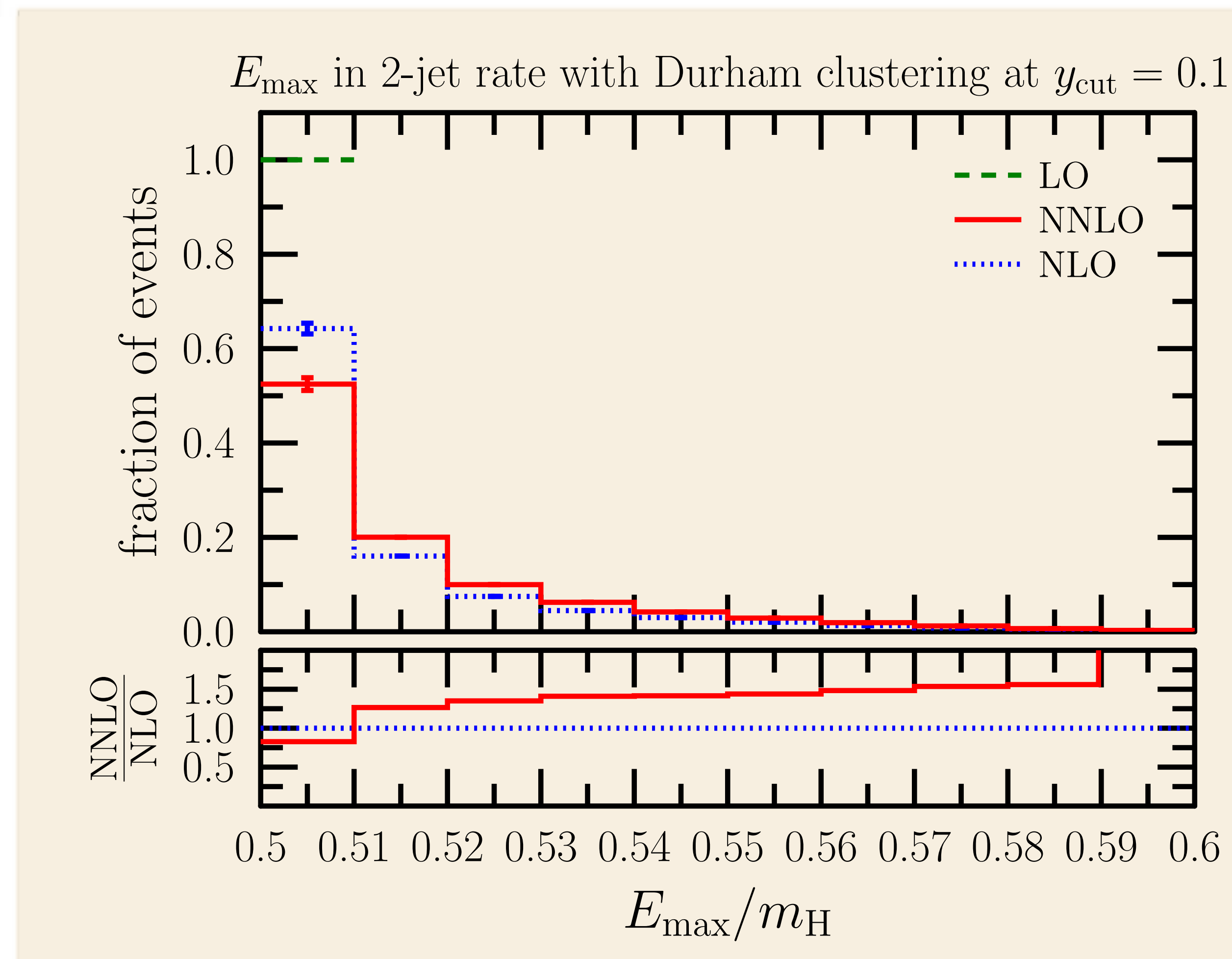


Energy spectrum of the leading jet in the rest frame of the Higgs boson. Jets are clustered using the JADE algorithm with $y_{cut} = 0.1$

AHL = C. Anastasiou, F. Herzog, A. Lazopoulos arXiv:0111.2368

Example: $H \to b\bar{b}$ at $\mu = m_H$





Energy spectrum of the leading jet in the rest frame of the Higgs boson.

left: jets are clustered using the JADE algorithm with $y_{cut} = 0.05$

right: jets are clustered using the Durham algorithm with $y_{cut} = 0.1$

Conclusions

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- ✓ Subtractions are
  - ✓ fully local
  - ✓ exact and explicit in color (using color state formalism)

Conclusions

- ✓ Defined a general subtraction scheme for computing NNLO fully differential jet cross sections (presently only for processes with no colored particles in the initial state)
- ✓ Subtractions are
  - ✓ fully local
  - ✓ exact and explicit in color (using color state formalism)
- ✓ Demonstrated the cancellation of ϵ-poles for m=2
- ✓ First application: Higgs-boson decay into a b-quark pair