Living Dangerously with Low-Energy Supersymmetry

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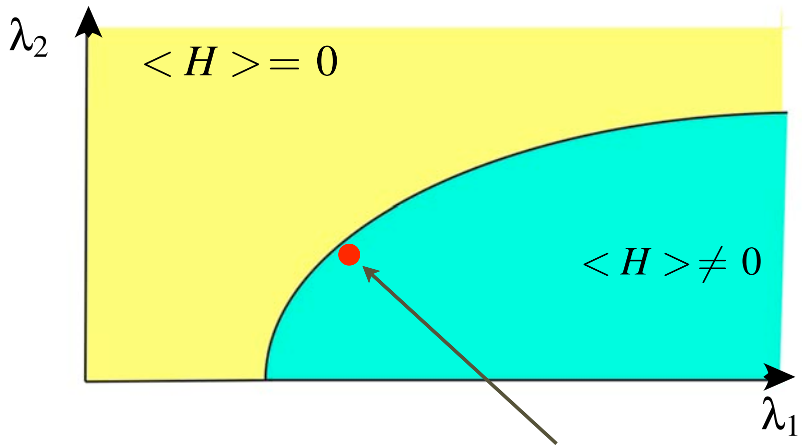
Hierarchy problem & `criticality'

toy phase diagram
in Planck units



In SM critical line not special as symmetry is concerned

Why the SM distance from the critical line is $\frac{m_Z^2}{M_P^2} \sim 10^{-34}$?

- SUSY helps in two ways

$\mu\, H_1 H_2$

- critical line locus of enhanced symmetry PQ $\longrightarrow$ $\mu = 0$

$V \propto (H_1^2 - H_2^2)^4 \sim H^4$

- SUSY broken only by tiny non-perturbative effects $\sim e^{-\frac{1}{g^2}}$

Outline

I. Post-LEP `little criticality' of Supersymmetry

Try to modify MSSM to dynamically account
for this little hierarchy

II. There exists a simple & quantitatively adequate
statistical explanation

Radiative electroweak breaking in hidden sector models

at Planck scale $m_i \sim c_i M_S$

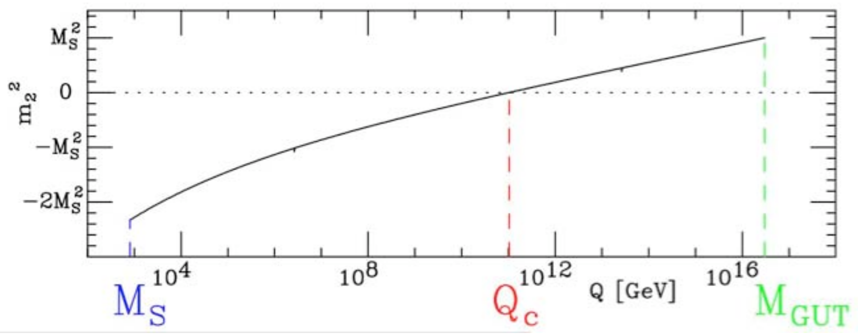
Higgs mass m_2^2 driven negative at RG scale $Q_c = M_P F(\alpha_A, c_i) \ll M_P$

$Q_c = M_P e^{-\frac{1}{\alpha}}$

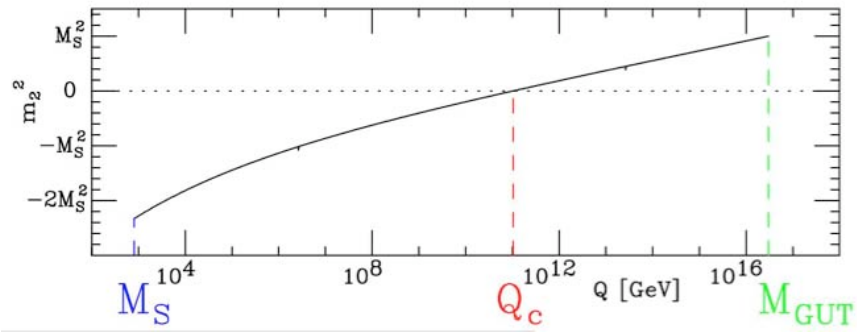
Q_c is parametrically unrelated to M_S

Two possibilities

- $Q_c \ll M_S \ll M_P$ $\quad m_2^2|_{\rm phys} > 0$ electroweak symmetry unbroken
- $M_S \ll Q_c \ll M_P$



$$M_S \ll Q_c \ll M_P$$



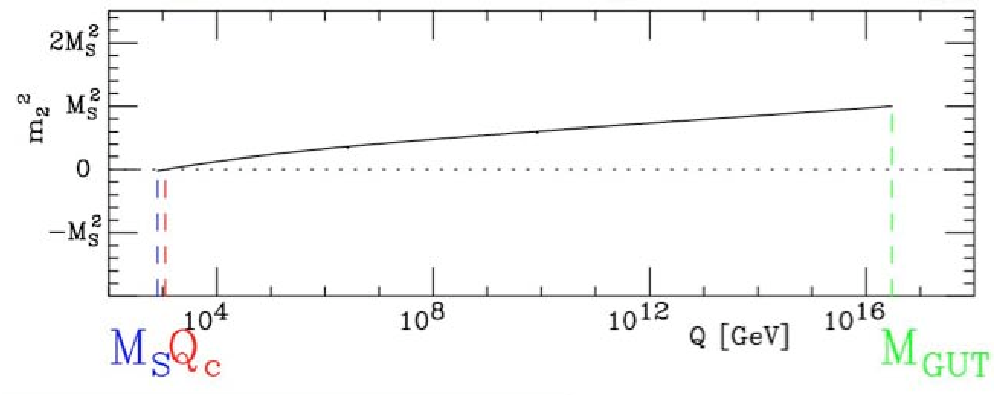
$$m_Z^2 \simeq -2m_2^2|_{\text{phys}} \sim M_S^2$$

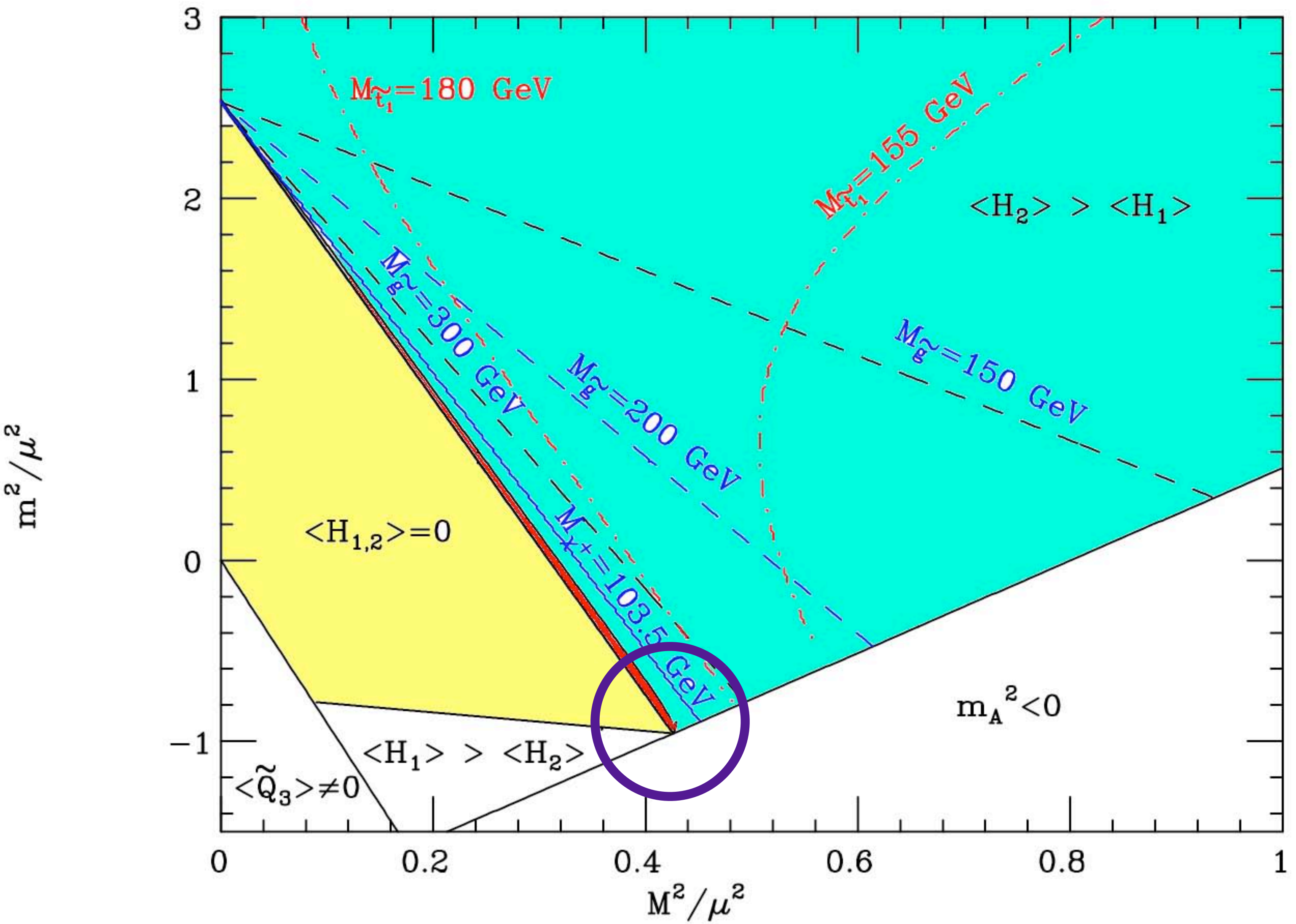
the stops drive m_2^2 RG evolution $\Rightarrow$ $m_Z^2 \sim m_{\tilde{t}}^2$

Expected discovery
at LEP1 & LEP2

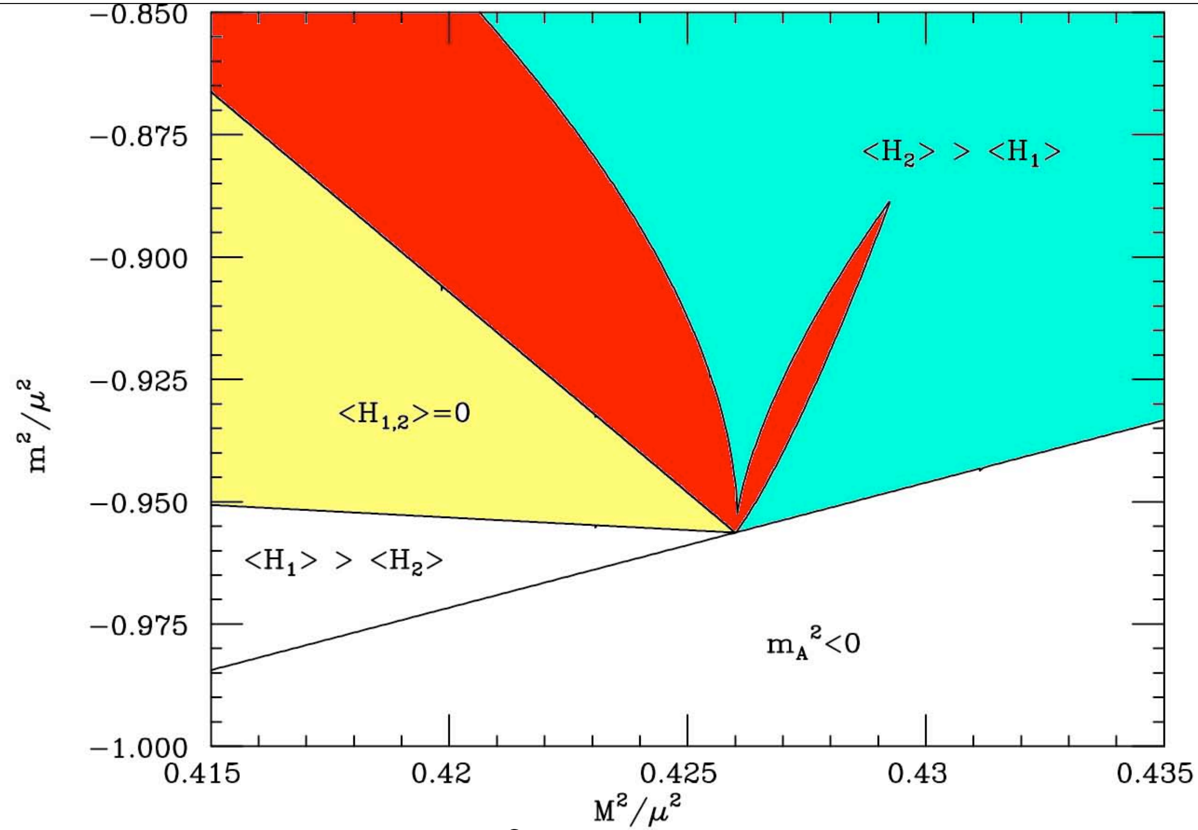
but LEP ... $|m_2^2| \sim m_Z^2 \ll M_S^2$ $m_{\tilde{t}} \gtrsim 500\,\text{GeV}$

Q_c and M_S
nearly coincide !





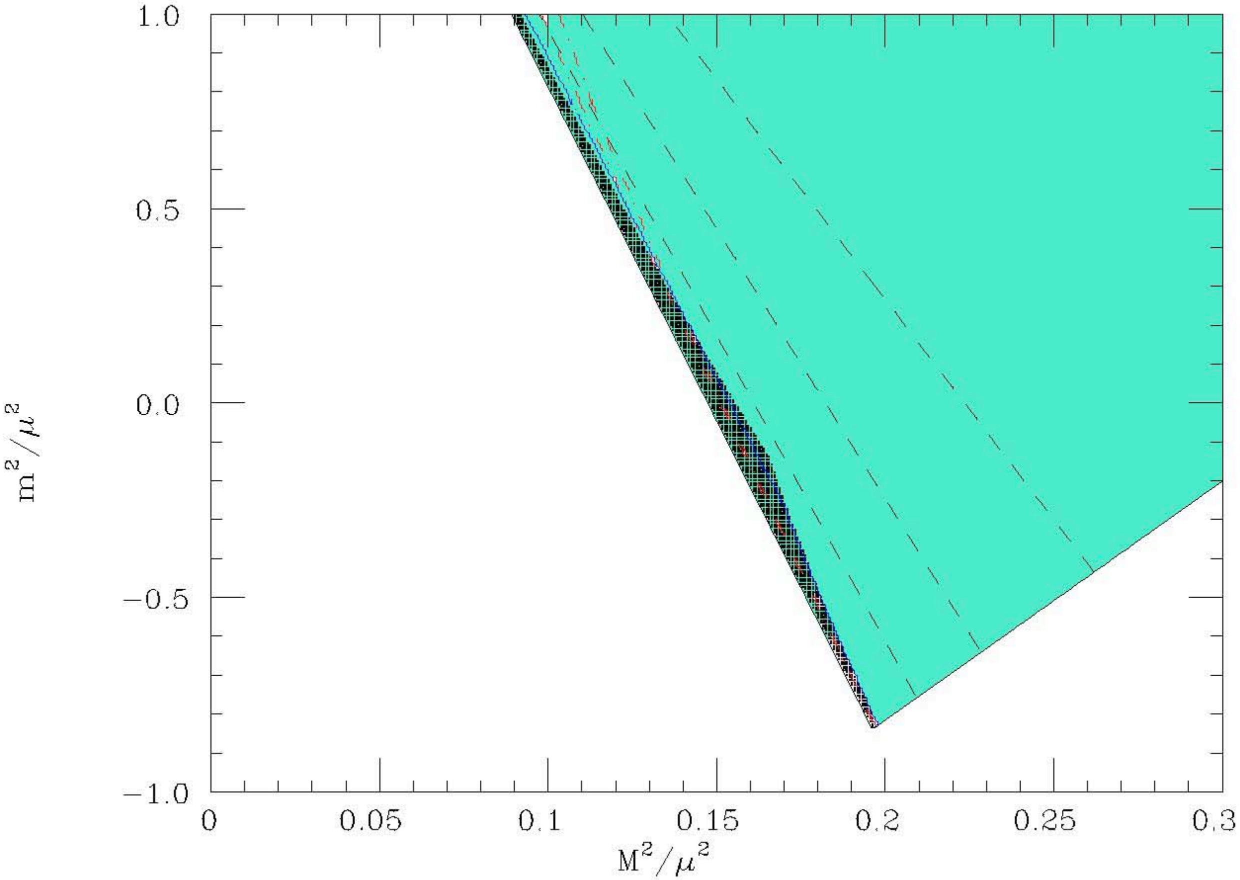
$M_{\tilde{t}_1}$=180 GeV
$M_{\tilde{t}_1}$=155 GeV
<H_2> > <H_1>
$M_{\tilde{g}}$=300 GeV
$M_{\tilde{g}}$=200 GeV
$M_{\tilde{g}}$=150 GeV
M_{χ^+}=103.5 GeV
<$H_{1,2}$>=0
$m_A{}^2$<0
<H_1> > <H_2>
<$\tilde{Q}_3$>≠0
m^2/μ^2
M^2/μ^2



$$\left| Z \to h\, Z \right|^2 \propto \sin^2(\beta-\alpha) = \frac{1}{2}\left[1 + \frac{m_A^2 - (m_Z^2 + \Delta)}{m_H^2 - m_h^2}\right]$$

$$\left| Z \to h\, A \right|^2 \propto \cos^2(\beta-\alpha)$$

stop correction Δ to Higgs masses must be sizeable anyway



1.0
0.5
0.0
−0.5
−1.0
0
0.05
0.1
0.15
0.2
0.25
0.3
m²/μ²
M²/μ²

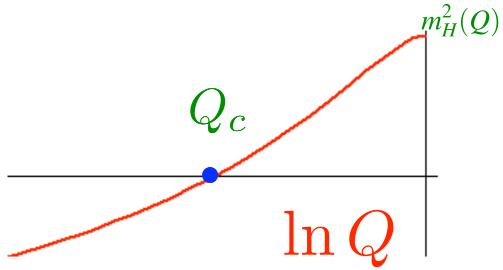
Statistical Criticality

- Assume soft terms are *environmental* parameters

◆ Simplest possibility: only overall scale M_S varies through multiverse

$$Q = M_P \quad \begin{cases} m_i = c_i M_S \\ \alpha_A, \lambda_t, \ldots \end{cases}$$

fixed



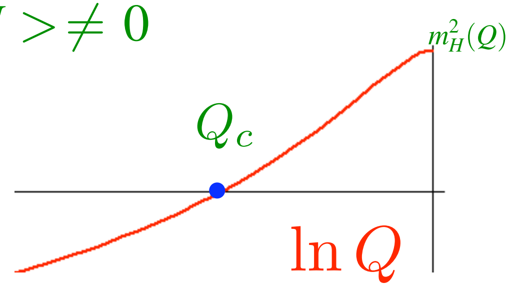
Normal situation: Higgs mass matrix positive definite at Planck scale

$$Q_c = M_P \times F(c_i, \alpha_A, \lambda_t, \ldots)$$

fixed as well

$$
\text{2 cases}
\begin{cases}
M_S > Q_c & \Rightarrow \quad m_H^2|_{phys} > 0 \quad \Rightarrow \quad <H> = 0 \\[2em]
M_S < Q_c & \Rightarrow \quad m_H^2|_{phys} < 0 \quad \Rightarrow \quad <H> \neq 0
\end{cases}
$$

we do not live here!



◆ Chemistry probably exists in region with $\langle H \rangle = 0$

Agrawal, Barr, Donoghue, Seckel 97

◆ $m_W \sim g_W f_\pi \sim 50\,\mathrm{MeV}$ ➡

electroweak sphaleron transitions are active down to $T \sim f_\pi$

at QCD scale any primordial baryon density is very efficiently converted into leptons down to a relic density

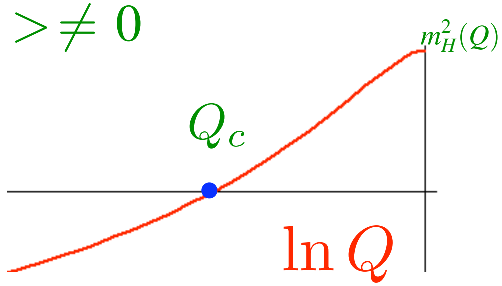
$$\frac{n_B}{n_\gamma} \sim 10^{-18}$$

Arkani-Hamed, Dimopoulos, Kachru 05

2 cases

$$\begin{cases} M_S > Q_c \;\rightarrow\; m_H^2|_{\rm phys} > 0 \;\rightarrow\; \langle H \rangle = 0 \\ M_S < Q_c \;\rightarrow\; m_H^2|_{\rm phys} < 0 \;\rightarrow\; \langle H \rangle \neq 0 \end{cases}$$

we do not live here!



natural expectation $\boxed{M_S \sim Q_c}$

$$m_Z^2(1+\delta) = -2m_2^2(M_S) \simeq -2\frac{dm_2^2}{d\ln Q}\bigg|_{Q=Q_c} \ln\frac{Q_c}{M_S}$$

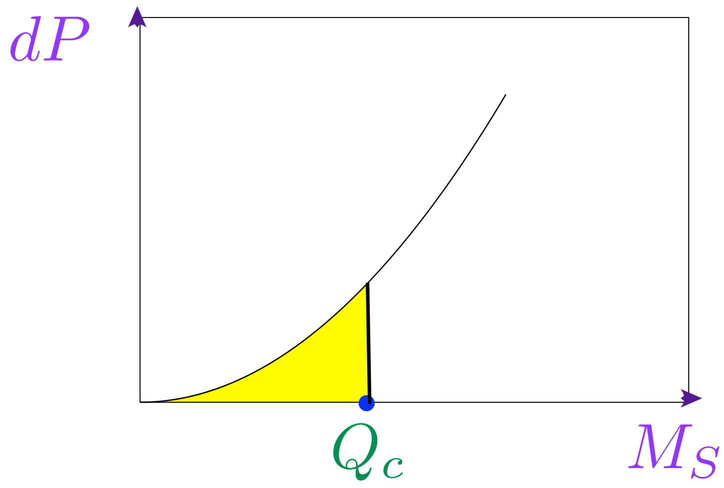
$$= \frac{3}{4\pi^2}\left[\lambda_t^2\left(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + |A_t|^2\right) - \frac{g_1^2}{5}\left(M_1^2+\mu^2\right) - g_2^2\left(M_2^2+\mu^2\right)\right]\ln\frac{Q_c}{M_S}$$

1-loop hierarchy between m_Z^2 and M_S^2

$$dN_{vacua} \propto dM_S^n$$

conditional probability

$$dP(M_S \mid \langle H \rangle \neq 0) = \begin{cases} \dfrac{dM_S^n}{Q_c^n} & M_S < Q_c \\ 0 & M_S > Q_c \end{cases}$$



$$\langle \ln \frac{Q_c}{M_S} \rangle = \frac{1}{n}$$

$$\begin{cases} m_{\tilde{t}_L}^2 \sim m_{\tilde{t}_R}^2 \sim |A_t|^2 \sim M_S^2 \\ \left\langle \dfrac{m_Z^2}{M_S^2} \right\rangle \simeq \dfrac{9\lambda_t^2}{4\pi^2(1+\delta)} \times \dfrac{1}{n} \simeq \dfrac{0.15}{n} \end{cases}$$

- most generic expectation in field theoretic landscape $n = 6$
- n theoretically bounded by total number of vacua $N = 10^{500}$, $n < 30$

$$\boxed{\left\langle \frac{m_Z^2}{M_S^2} \right\rangle}$$ **0.1 to 0.01 but not much less**

- SUSY will look tuned because there are many more vacua with $< H >= 0$ than there are with $< H > \neq 0$

- The amount of apparent tuning is dictated by the speed of RG evolution and is parametrically of order a 1-loop factor

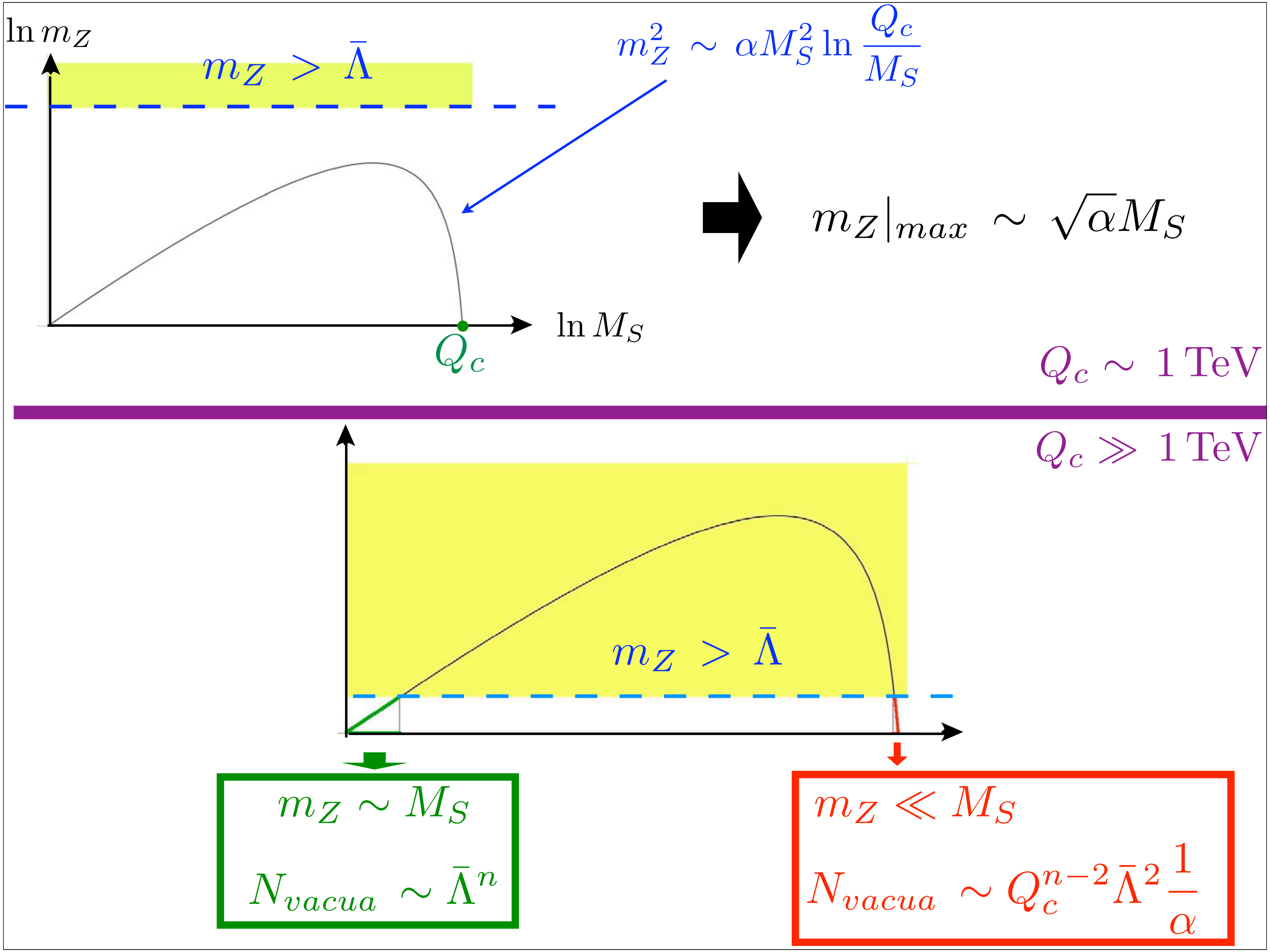
Stability of result under stronger anthropic requests on Fermi scale

I) $\langle H \rangle \neq 0$ $\left(\begin{array}{c}\text{baryons are not}\\ \text{washed out}\end{array}\right)$ Weak Principle

II) $\langle H \rangle < 10^3 \Lambda_{QCD} \equiv \bar{\Lambda}$ $\left(\begin{array}{c}\text{there exists}\\ \text{rich chemistry}\end{array}\right)$ Atomic Principle

Agrawal, Barr, Donoghue, Seckel 97

Arkani-Hamed, Dimopoulos 04



$\ln m_Z$
$m_Z > \bar{\Lambda}$
$m_Z^2 \sim \alpha M_S^2 \ln \frac{Q_c}{M_S}$
$m_Z|_{max} \sim \sqrt{\alpha} M_S$
$\ln M_S$
Q_c
$Q_c \sim 1\,\mathrm{TeV}$
$Q_c \gg 1\,\mathrm{TeV}$
$m_Z > \bar{\Lambda}$
$m_Z \sim M_S$
$N_{vacua} \sim \bar{\Lambda}^n$
$m_Z \ll M_S$
$N_{vacua} \sim Q_c^{n-2} \bar{\Lambda}^2 \frac{1}{\alpha}$

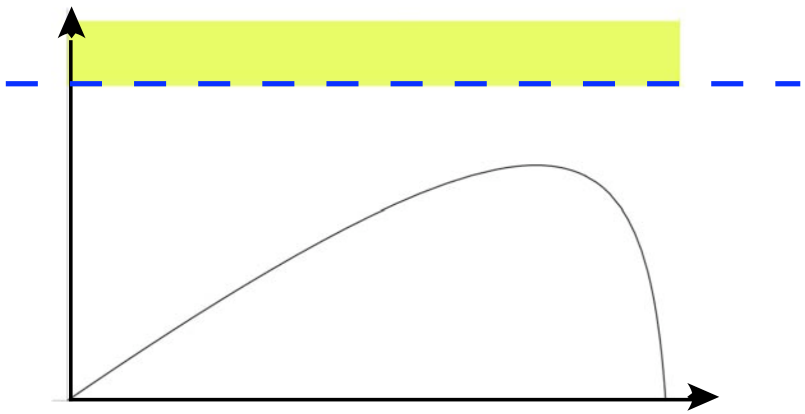
In general also $\bar{\Lambda}$, Q_c will scan

likely with distribution $dN \propto \frac{d\bar{\Lambda}}{\bar{\Lambda}} \frac{dQ_c}{Q_c}$

$$\bar{\Lambda}_{min} < \bar{\Lambda} < \bar{\Lambda}_{max}$$

$$Q_{c\,min} < Q_c < Q_{c\,max}$$

If $\bar{\Lambda}_{max} > Q_{c\,max}$ then the Atomic Principle plays no relevant constraints on vacuum selection



Λ_{QCD} adjusts at no cost as to satisfy the AP

Scanning M_S and μ independently

at Planck scale

- $\mu \equiv \mu_0, \qquad B\mu \equiv b M_S \mu_0$
- every other soft term $\propto M_S$

running mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} \tilde{m}_1^2 + \mu^2 & B\mu \\ B^*\mu & \tilde{m}_2^2 + \mu^2 \end{pmatrix}$$

critical curve

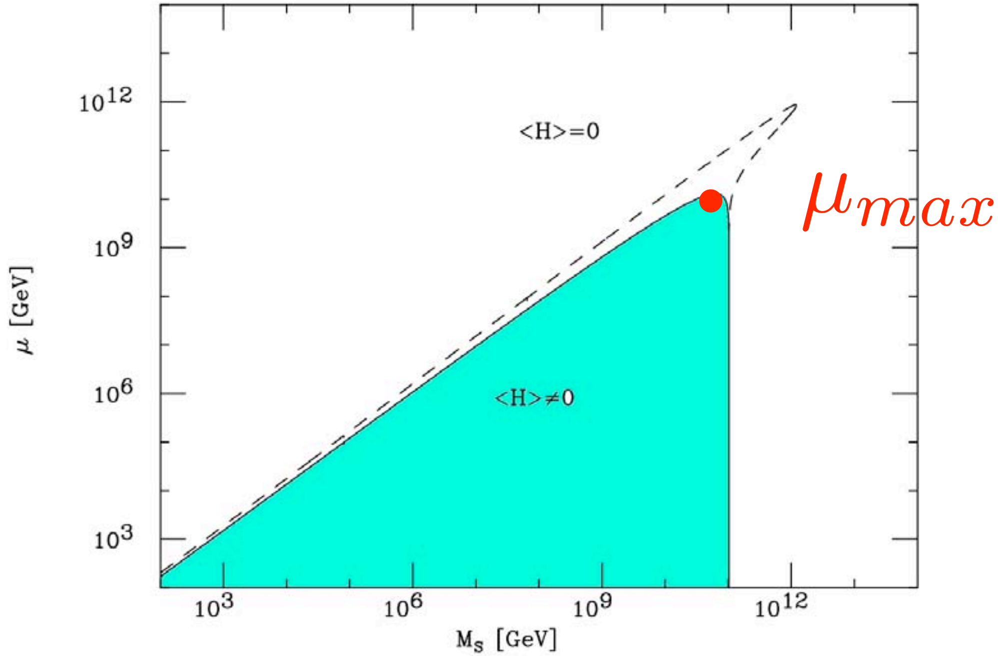
$$\Rightarrow \quad \det \mathcal{M}^2\big|_{Q=M_S} = \overset{O(M_S^4)}{\overset{\uparrow}{\tilde{m}_1^2 \tilde{m}_2^2}} + (\underbrace{\tilde{m}_1^2 + \tilde{m}_2^2 - |B|^2}_{\text{normally } > 0}) \overset{O(M_S^2\mu^2)}{\overset{\uparrow}{\mu^2}} + \mu^4 = 0$$

$$\mu^2 \sim \alpha\, M_S^2 \ln \frac{Q_c}{M_S}$$

Weak Principle simply `solves' the μ problem

$$\mu^2 \sim \alpha\, M_S^2 \,\ln \frac{Q_c}{M_S}$$

$$\mu^2_{max} \sim \alpha Q_c^2$$



$$dN \,\propto\, dM_S^n \, d\mu^m \qquad \left\langle \frac{m_Z^2}{M_S^2} \right\rangle = \frac{\alpha}{n+m} \qquad \left\langle \frac{\mu^2}{M_S^2} \right\rangle = \alpha\, \frac{m}{(n+m)}$$

$$\tan\beta = \frac{m_A^2}{B\mu} \sim \frac{M_S}{\mu} \sim \frac{1}{\sqrt{\alpha}} \sim 10$$

also helps with LEP Higgs mass bound

predict $m_Z^2,\, \mu^2 \ll m_{\tilde{t}}^2,\, m_{\tilde{g}}^2,\, \ldots$

can have well tempered bino-higgsino LSP

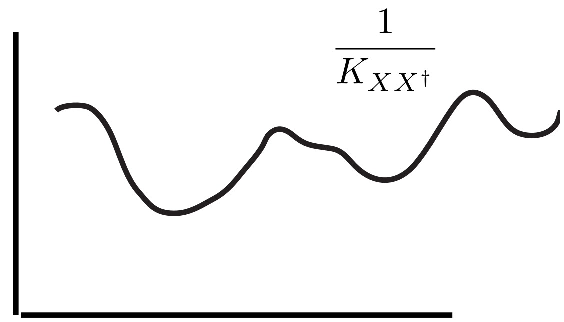
Distribution of SUSY scale

Denef, Douglas 04
Dine, O'Neil, Sun 05

Ex. 1 chiral field $\qquad V = \dfrac{|\partial_X W|^2}{K_{XX^\dagger}}$

I) SUSY $\qquad \partial_X W = 0$

II) ~~SUSY~~ stabilised by $\qquad \dfrac{1}{K_{XX^\dagger}}$



conditions for survival of $\frac{1}{K_{XX^\dagger}}$ minima at non-constant $\partial_X W$

A) $\partial_X^2 W \lesssim \partial_X W$ $\qquad$ must $\exists$ light fermion Goldstino

B) $\partial_X^3 W \lesssim \partial_X W$ $\qquad$ scalar masses not dominated by W

otherwise $\quad m^2_{\mathrm{Re}X} + m^2_{\mathrm{Im}X} = 0$

Ferrara, Girardello, Palumbo 79

$$|M_S| = \frac{\partial W}{M_P}$$

$$\partial^2 W, \partial^3 W \lesssim \partial W \sim M_S$$

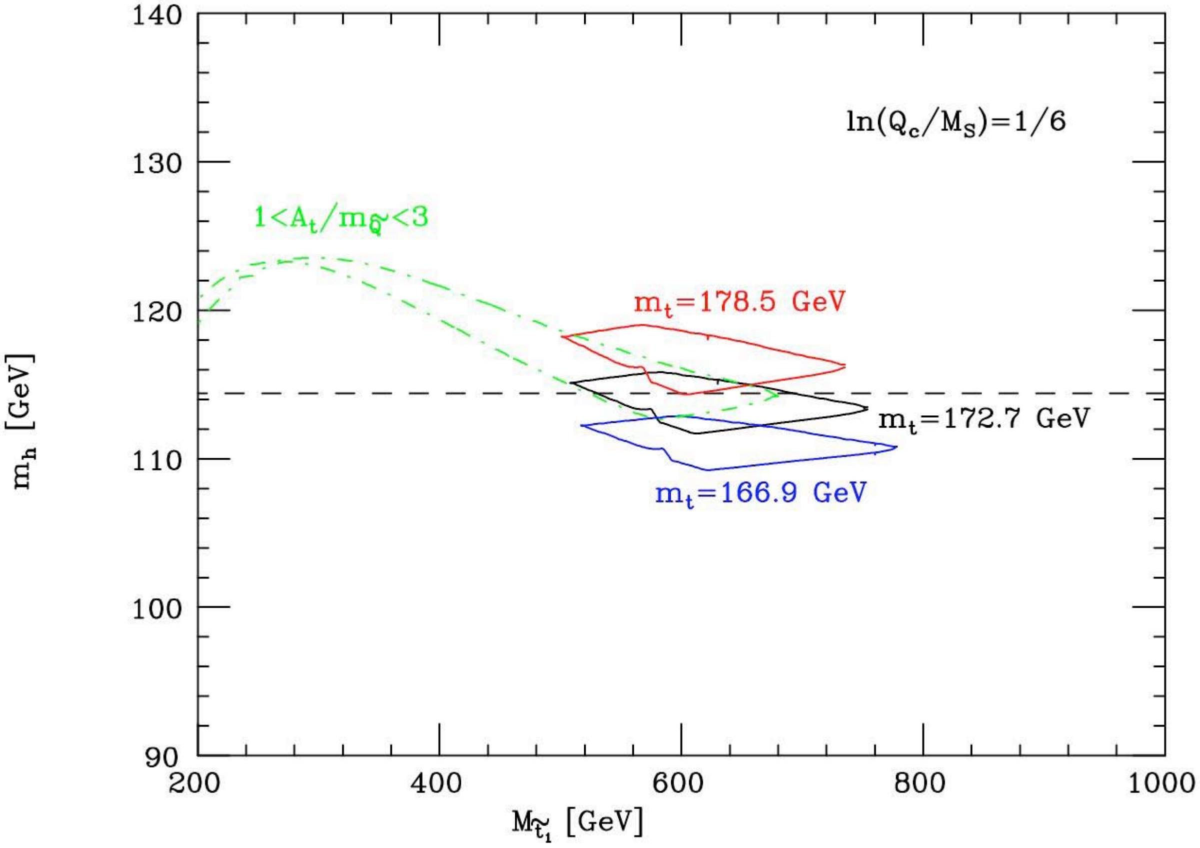
at small M_S must `tune'
3 complex parameters $< M_S$

expect linear scanning at small a_i

$$dN \propto d^2 a_1\, d^2 a_2\, d^2 a_3 \propto dM_S^6$$

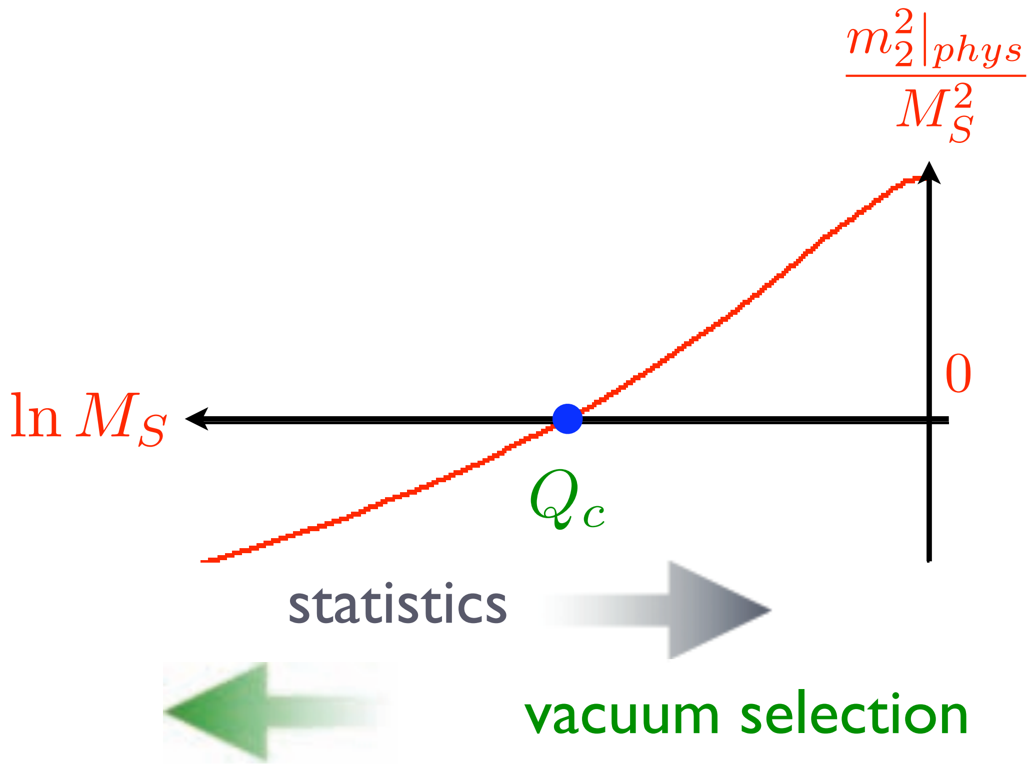
- Can consider case of N fields Ψ_i
  at SUSY breaking point Goldstino must exist
  expect 1 combination X to become light
  integrate out heavy guys and focus on effective theory for X

- What about classically supersymmetric vacua
  that are lifted by non-perturbative IR dynamics?



140
130
120
110
100
90
200
400
600
800
1000
m_h [GeV]
$M_{\tilde{t}_1}$ [GeV]
$\ln(Q_c/M_S)=1/6$
$1<A_t/m_{\tilde{Q}}<3$
m_t=178.5 GeV
m_t=172.7 GeV
m_t=166.9 GeV

LHC will discover MSSM with 1% apparent tuning



$$m_Z^2 \sim \mu^2 \sim \alpha m_{\tilde{t}}^2$$