The Super-little Higgs

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Outline

- The fine tuning of SUSY and little Higgs theories – why and how to combine them?
- The simplest little Higgs and SUSY
- A beautiful (old) SU(6) GUT model and a super-little Higgs
- Higgs potential and quartic coupling
- Phenomenology
    - Gauge coupling unification
    - Extra fermions
    - LHC
- Conclusions

The Fine Tuning of SUSY

- SUSY solves hierarchy – no quadratic divergences
- But log divergences are present
- General Higgs potential of MSSM:

$$V(H_1, H_2) = (m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2$$
$$-B\mu(H_1 H_2 + \text{h.c.}) + \frac{g^2}{2}(H_1^\dagger \vec{\tau} H_1 + H_2^\dagger \vec{\tau} H_2)^2 +$$
$$\frac{g'^2}{2}(H_1^\dagger H_1 - H_2^\dagger H_2)^2$$

- EWSB can happen only due to soft SUSY breaking terms

$$M_Z^2 = 2\left(\frac{m_{H_1}^2 - m_{H_2}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2\right)$$

- For large tan β (needed for Higgs mass) and neglecting μ

$$M_Z^2 \sim -2m_{H_2}^2$$

- Leading expression $$M_Z^2 = \frac{3y_t^2}{2\pi^2} m_{\tilde{t}}^2 \ln \frac{f}{m_{\tilde{t}}}$$

- One loop suppressed vs. stop mass, but usually $f \gg m_{\tilde{t}}$

- Log compensates loop suppression, need fine tuning to ensure f~TeV

Goal here: find a rationale why f~TeV, and thus avoid fine tuning of SUSY

- Simplest possibility: Higgs a pseudo-Goldstone boson of symmetry broken at scale f
- This idea already used in SUSY GUTs to solve D-T splitting

Fine-tuning of Little Higgs

- Little Higgs: realistic model for Higgs as PGB
- Aim: to raise cutoff of SM to ~ 10 TeV to solve little hierarchy
- But: Higgs does NOT look like generic PGB!

$$V(h) = \underbrace{0 \cdot |h|^2 + 0 \cdot |h|^4}_{} + \underbrace{f^4 \cos^n(|h|/f)}_{}$$

Tree-level vanishes Due to PGB nature

Generic PGB pot.

- Both mass and quartic generated at one loop: <h> ~f
- Does not raise cutoff $\Lambda = 4\pi$ f
- Little higgs introduces Collective symmetry breaking

$$m^2 = 0 \cdot \Lambda^2 + \frac{\mathcal{O}(g^2, \lambda_t^2)}{(4\pi)^2} f^2 \qquad \lambda_h = \mathcal{O}(g^2, \lambda_t^2)$$

- Higgs VEV now <h> ~ f/4π

Fine-tuning of Little Higgs

- But: many new states at the f~TeV scale
- Generically large corrections to EWPO’s

- In the end usually need f~4-5 TeV to avoid conflict

- Possible way out: T-parity (Cheng &Low) – will not use here

MSSM: $$m^2 \sim \frac{\mathcal{O}(g^2,\lambda_t^2)}{(4\pi)^2} m^2_{\text{soft}} \ln \frac{\Lambda}{m_{\text{soft}}}$$

Problem: large log, EWPT not a problem due to R-parity

LH: $$m^2 \sim \frac{\mathcal{O}(g^2,\lambda_t^2)}{(4\pi)^2} f^2 \ln \frac{f}{m_{\text{soft}}}$$

Problem: EWPT

Problems complementary

Super-little Higgs:

$$m^2 \sim \frac{\mathcal{O}(g^2,\lambda_t^2)}{(4\pi)^2} m^2_{\text{soft}} \ln \frac{f}{m_{\text{soft}}}$$

If we take m_{soft} ~ few 100 GeV (usual SUSY bound)

f ~ 4-5 TeV (EWP bound on LH, cuts off log)

$$\langle h \rangle \sim \frac{m_{soft}}{4\pi}[\log(\frac{f}{m_{soft}})]^{\frac{1}{2}} \sim \mathcal{O}(100GeV)$$

Higgs VEV super-little!

The simplest little Higgs

Schmaltz; Schmaltz & Kaplan

- Extend SU(2)xU(1) to SU(3)xU(1)
- Use two sets of triplets H_1,H_2 to break SU(3)xU(1) ▸ SU(2)xU(1)
- If no H_1 H_2 -type terms, global symmetry breaking pattern

$$\begin{array}{ccc} SU(3)_{H_1} & \times & SU(3)_{H_2} \\ \downarrow \langle H_1 \rangle & & \downarrow \langle H_2 \rangle \\ SU(2) & & SU(2) \end{array}$$

- Two sets of Goldstones, one set eaten, one set remains as physical pseudo-Goldstone boson (PGB)

$$\Pi = \begin{pmatrix} & & h_1 \\ & & h_2 \\ h_1^* & h_2^* & \end{pmatrix} \quad H_1 = e^{i\Pi/f} \begin{pmatrix} \\ \\ f \end{pmatrix} \quad H_2 = e^{-i\Pi/f} \begin{pmatrix} \\ \\ f \end{pmatrix}$$

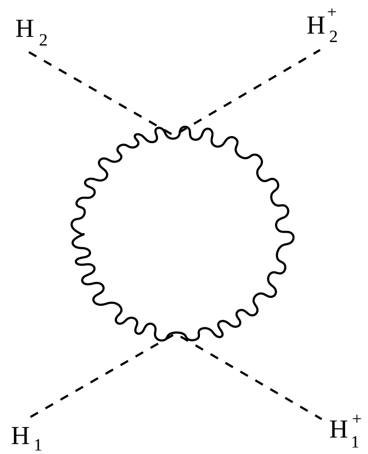
Collective breaking

Arkani-Hamed, Cohen, Georgi

- Gauging of diagonal SU(3) explicitely breaks global sym.
- Symmetry breaking terms:

$$|gA_\mu H_1|^2 + |gA_\mu H_2|^2$$

- If either coupling turned off: larger global symmetry intact
- Any diagram contributing to Higgs mass has to involve both
- Lowest vertex:



SUSY and little Higgs: a difficult marriage

- Make it supersymmetric: $H_1 \to H_1, \bar{H}_1$
- Two sets of chiral SF's $H_2 \to H_2, \bar{H}_2$
- Generic VEVs and parameterization:

$$H_1 = e^{i\Pi F_2/F_1 F}(0,0,f_1/\sqrt{2}), \qquad \bar{H}_1 = (0,0,\bar{f}_1/\sqrt{2})e^{-i\Pi F_2/F_1 F}$$
$$H_2 = e^{i\Pi F_1/F_2 F}(0,0,f_2/\sqrt{2}), \qquad \bar{H}_2 = (0,0,\bar{f}_2/\sqrt{2})e^{-i\Pi F_1/F_2 F}$$

- But D-terms necessarily break global symmetry at tree-level:

$$V_D \in \frac{g^2}{8}\left(|H_1^\dagger \cdot H_2|^2 - |\bar{H}_1 \cdot H_2|^2 - |\bar{H}_2 \cdot H_1|^2 + |\bar{H}_2^\dagger \cdot \bar{H}_1|^2\right) =$$
$$\frac{g^2}{8}(f_1^2 - \bar{f}_1^2)(f_2^2 - \bar{f}_2^2)\cos^2\left[\frac{\sqrt{G^\dagger G}}{F}\left(\frac{F_1}{F_2} - \frac{F_2}{F_1}\right)\right]$$

- Tree-level Goldstone mass if $f_1 \neq \bar{f}_1$ or $f_2 \neq \bar{f}_2$
- VEVs need to be supersymmetric, how to ensure?

Early attempt global symmetry only: Birkedal, Chacko, Gaillard; Pokorski et al.

•Possibility #1:

Add a Z_2 symmetry in one of the H sectors

Berezhiani, Chankowski, Falkowski, Pokorski;
Roy, Schmaltz

•Possibility #2:

Choose a gauge representation that ensures SUSY VEV

$$H_2, \bar{H}_2 \to \Sigma = \begin{pmatrix} w & & h_1 \\ & w & h_2 \\ h_1^* & h_2^* & -2w \end{pmatrix}$$

- D-term issue automatically resolved
- Global sym. breaking pattern:

$$\begin{array}{ccc} SU(3)_\Sigma & \times & SU(3)_H \times U(1)_X \\ \downarrow & & \downarrow \\ SU(2) \times U(1) & & SU(2) \times U(1) \end{array}$$

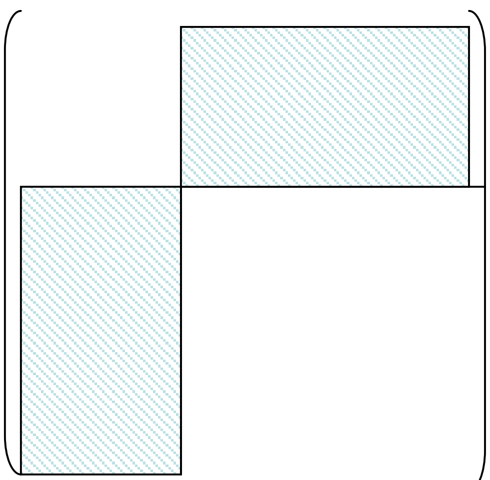
A beautiful old model

Berezhiani, Dvali;
Barbieri, Dvali, Strumia, Hall,
Berezhiani, Randall, C.C.

- SU(6) GUT theory, with Higgs sector Σ, H
- SU(6)xSU(6) global symmetry

$$\langle\Sigma\rangle = w \begin{bmatrix} 4 & & & & & \\ & 4 & & & & \\ & & -2 & & & \\ & & & -2 & & \\ & & & & -2 & \\ & & & & & -2 \end{bmatrix}$$

$$\langle H \rangle = \langle \bar{H} \rangle = (0,0,0,0,0,f)$$



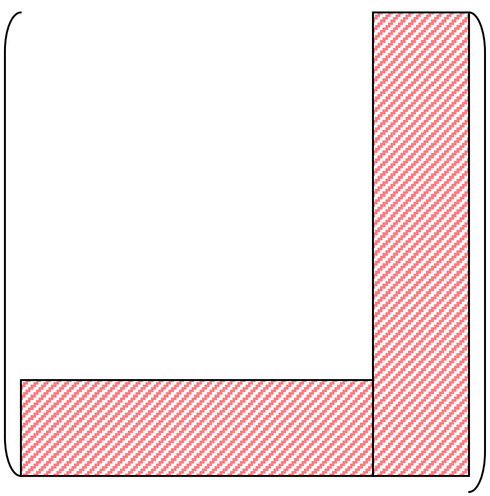
SU(6) → SU(4)xSU(2)xU(1)

A beautiful old model

- SU(6) GUT theory, with Higgs sector Σ, H
- SU(6)xSU(6) global symmetry

$$\langle \Sigma \rangle = w \begin{bmatrix} 4 & & & & & \\ & 4 & & & & \\ & & -2 & & & \\ & & & -2 & & \\ & & & & -2 & \\ & & & & & -2 \end{bmatrix}$$

$$\langle H \rangle = \langle \bar{H} \rangle = (0,0,0,0,0,f)$$



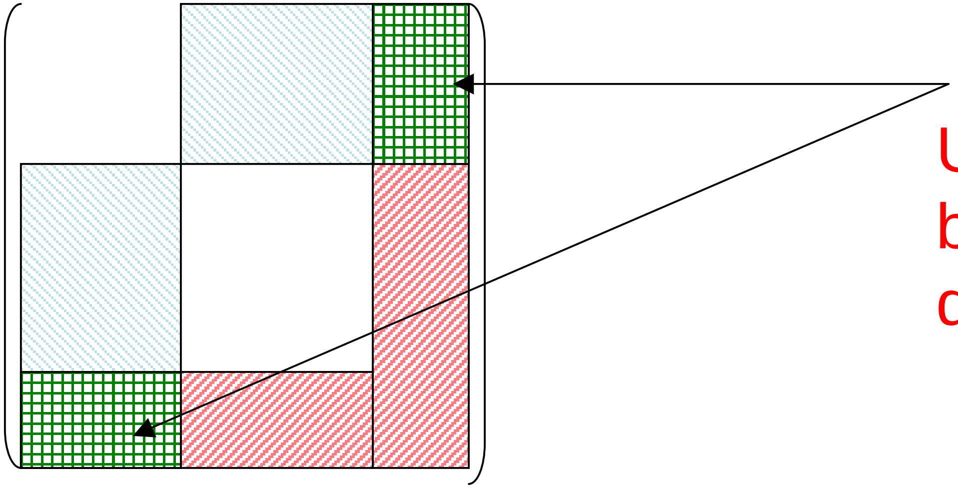
SU(6) → SU(5)

A beautiful old model

- SU(6) GUT theory, with Higgs sector Σ, H
- SU(6)xSU(6) global symmetry

$$\langle \Sigma \rangle = w \begin{bmatrix} 4 & & & & & \\ & 4 & & & & \\ & & -2 & & & \\ & & & -2 & & \\ & & & & -2 & \\ & & & & & -2 \end{bmatrix}$$

$$\langle H \rangle = \langle \bar{H} \rangle = (0,0,0,0,0,f)$$



Uneaten Goldstone boson: one complex doublet

Fermion sector of the SU(6) model

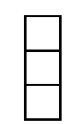
Barbieri, Dvali, Strumia,
Berezhiani, Hall

- SU(5): $3\times(10+\bar{5})$
- SU(6): $3\times(15+\bar{6}+\bar{6}')$ need to extend, more chiral fields?

- But Yukawa coupling: $15_i^{ab}\bar{H}_a\bar{6}_{b\ j}$

- After VEV gives mass to 3x(5+$\bar{5}$)
- Chiral matter content that of SU(5) MSSM

To get natural top Yukawa coupling

- Unusual representation in SU(6): 20, three-index antisym. 
- Self-adjoint (anomaly free), but no mass term: $20^{abc}20^{def}\epsilon_{abcdef}=0$
- Under SU(5): $20\rightarrow 10+\bar{10}$

- Renormalizable Yukawa couplings involving 20:

$$\lambda_1 20^{abc} H^d 15^{ef} + \lambda_2 20^{abc} \Sigma_e^d 20^{efg} \epsilon_{abcdfg}$$

Exchanges a 10 from 15 with a 10 from 20

Produces an order one top Yukawa coupling 10 10 H

- Automatically has the collective breaking pattern: need both couplings to generate top mass

The matter content of the super-little model

- Decompose SU(6) to SU(3)xSU(3)xU(1)
- Automatically anomaly free, flavors universal
- Top Yukawa via collective breaking
- One set of PGB doublet

	$SU(3)_c$	$SU(3)_L$	$\mathrm{U}(1)_X$
H	1	3	$+1/3$
$\bar{H}$	1	$\bar{3}$	$-1/3$
Σ	1	8	0
$2 \times D_{1,2,3}$	$\bar{3}$	1	$+1/3$
$2 \times L_{1,2,3}$	1	$\bar{3}$	$-1/3$
$U_{1,2,3}$	$\bar{3}$	1	$-2/3$
$E_{1,2,3}$	1	$\bar{3}$	$+2/3$
$Q_{1,2,3}$	3	3	0
Q'	$\bar{3}$	3	$-1/3$
$\bar{Q}'$	3	$\bar{3}$	$+1/3$

The Higgs sector

- The superpotential (with λ'< 0.01 to ensure global sym.)

$$W_{\rm Higgs} = \frac{M}{2}{\rm Tr}\Sigma^2 + \frac{\lambda}{3}{\rm Tr}\Sigma^3 + S(\lambda'' H\bar{H} - M'^2) + \lambda' \bar{H}\Sigma H$$

- VEVs

$$\langle\Sigma\rangle = {\rm diag}\,(w/2, w/2, -w), \qquad \langle H\rangle = (0,0,f/\sqrt{2}), \qquad \langle\bar{H}\rangle = (0,0,\bar{f}/\sqrt{2})$$

- Goldstones: ($F^2 = (f^2+\bar{f}^2)/2$, $V^2 = F^2 + 9w^2$)

$$\begin{aligned} H &= \exp\left(i\Pi\frac{3w}{FV}\right)\langle H\rangle, \\ \bar{H} &= \langle\bar{H}\rangle\exp\left(-i\Pi\frac{3w}{FV}\right) \\ \Sigma &= \exp\left(-i\Pi\frac{F}{3wV}\right)\langle\Sigma\rangle\exp\left(i\Pi\frac{F}{3wV}\right) \end{aligned}$$

•Pion matrix:

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0_2 & H_u \\ H_d^t & 0 \end{pmatrix}$$

•Goldstone vs. sGoldstone

$$G \equiv (H_u + H_d^\dagger)/\sqrt{2} \qquad \tilde{G} \equiv (H_u - H_d^\dagger)/\sqrt{2}$$

•Need to make sure VEV is along Goldstone direction (sGoldstone NOT protected by global symmetry)

The top sector

- To ensure that matter content is that of MSSM

$$W_{\text{matter}} = \alpha_{ij} Q_i \bar{H} D_j + \beta_{ij} E_i \bar{H} L_j$$

- Will use trick from SU(6) model to get O(1) top Yukawa:

$$20 \rightarrow (\bar{3}, 3)_{-\frac{1}{3}} + (3, \bar{3})_{\frac{1}{3}} + \text{singlets}$$

$(\bar{3}, 3)_{-\frac{1}{3}} \uparrow Q'$ $\qquad$ $(3, \bar{3})_{\frac{1}{3}} \uparrow \bar{Q}'$

- Superpotential for top Yukawa:

$$W_{\text{top}} = \lambda_1 \bar{Q}' \Sigma Q' + \lambda_2 \bar{Q}' H U + \lambda_3 Q H Q'$$

- Has collective form: need all three couplings to generate top Yukawa

•Heavy top partners and top Yukawa:

$$M_{T_1}^2 = \lambda_1^2 w^2 + \frac{1}{2}\lambda_2^2 f^2, \qquad M_{T_{2,3}}^2 = \frac{1}{4}\left(\lambda_1^2 w^2 + 2\lambda_3^2 f^2\right)$$

$$y_t = \frac{f^2\sqrt{F^2+9w^2}\lambda_1\lambda_2\lambda_3}{F\sqrt{2(2w^2\lambda_1^2+f^2\lambda_2^2)(w^2\lambda_1^2+2f^2\lambda_3^2)}}$$

Electroweak precision constraints

- Little Higgs models usually tightly constrained, need T-parity
- SUSY models usually have R-parity (or matter parity)

Which one?

- T-parity does not commute with SU(3)xU(1): Z' T-even
- Constraint from Z' exchange: F>3 TeV
- If w too small: SU(2) breaking VEV partly in triplet: w>0.5 TeV

Assume F>3 TeV, w>0.5 TeV, and impose usual R-parity

Higgs potential

- D-terms do not give significant contribution to mass or quartic

$$\frac{(m_H^2 - m_{\bar{H}}^2)(f^2 - \bar{f}^2)}{(f^2 + \bar{f}^2)^2} w^2 G^2 \quad \text{for} \quad f \gg w, m_H, m_{\bar{H}}$$

- But this is NOT enough: need to make sure VEV is actually along the Goldstone direction (NOT sGoldstone)

- Soft breaking terms + D-terms introduce mass and mixing

$$V_{soft} = m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + m_\Sigma^2 \mathrm{Tr} \Sigma^\dagger \Sigma$$

- Mixing matrix: $\begin{pmatrix} 0 & m_{G\tilde{G}}^2 \\ m_{G\tilde{G}}^2 & m_{\tilde{G}}^2 \end{pmatrix}$

- The mixing $m_{G\tilde{G}}^2 \sim m_H^2 - m_{\bar{H}}^2$ can be suppressed by

$$m_\Sigma \sim \text{TeV} \gg m_H, m_{\bar{H}}$$

The Higgs quartic

- Now we achieved the Higgs VEV along Goldstone $\tan\beta \sim 1$
- No tree-level quartic from D-terms along this direction
- Top induced potential:

Mass term as expected super-little

$$m^2 \simeq -\frac{3}{8\pi^2} y_t^2 (m_{\tilde{t}}^2 - m_t^2)(2 \ln \frac{m_{T_2}}{m_{\tilde{t}}} + 1 + 2c \ln \frac{m_{T_1}}{m_{T_2}})$$

Quartic too small to exceed 115 GeV Higgs mass

$$\lambda \simeq \frac{3y_t^4}{8\pi^2} (\ln \frac{m_{\tilde{t}}}{m_t} + \frac{A_t^2}{2m_{\tilde{t}}^2}(1 - \frac{A_t^2}{12m_{\tilde{t}}^2}))$$

Need a tree-level quartic!

A tree-level quartic

- Need operator that gives quartic but no mass to Goldstone
- Notoriously difficult in simplest little Higgs already
- Could try to get an NMSSM-type superpot. term SG^2

First try: $S\bar{H}\Sigma H$

- No good because $\bar{H}\Sigma H = -\frac{f^2 w}{2} + \frac{V^2}{12w}|G|^2$ contains both a mass and a quartic
- Need to absorb VEV of operator: "sliding singlet"

Second try: $S(S' + \bar{H}\Sigma H)$

- VEV absorbed since $\langle S' \rangle = -\langle \bar{H}\Sigma H \rangle$ but also FULL Goldstone dependence absorbed

Third try:

$$\frac{1}{\Lambda}(SH + \frac{1}{\Lambda}S'\Sigma H)(\bar{S}\bar{H} + \Sigma\bar{H})$$

Similar to Roy and Schmaltz

- “collective sliding singlet”
- VEV’s will cancel: no mass
- Goldstone dependence remains: O(1) tree-level quartic
- Operator could be generated via exchange of heavy triplets

EOM:

$$\bar{S}\bar{H}H + \bar{H}\Sigma H = 0,$$
$$\bar{S}\bar{H}\Sigma H + \bar{H}\Sigma^2 H = 0$$

Goldstone expansion:

$$\bar{S}f^2 + (-wf^2 + V^2|G|^2/6w) = 0$$
$$\bar{S}(-wf^2 + V^2|G|^2/6w) + (f^2w^2 - V^2|G|^2/6) = 0$$

Other possibilities for generating the quartic

•Use MSSM quartic, but may still be too small (as MSSM has generically hard time getting a heavy higgs)

(Berezhiani, Chankowski, Falkowski, Pokorski)

•Can be implemented here as well, need extra triplets in Σ sector, and w□f,$\bar{f}$

•Most recently: supersymmetric version of twin Higgs idea

(Chacko, Goh,Harnik)

•Falkowski, Pokorski, Schmaltz: VEV still along $\tan\beta = 1$

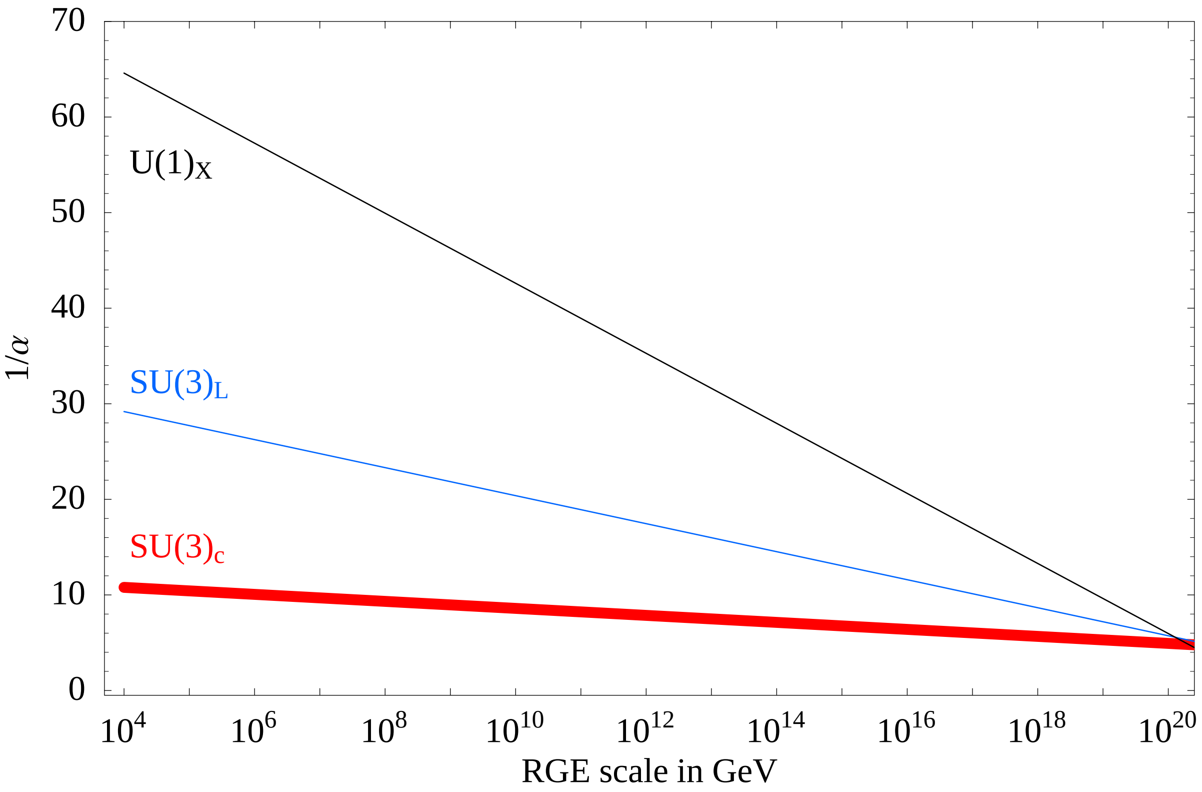
•Still need 4 singlets, though superpotential less complicated

•Chang, Hall, Weiner: use sopersoft D-term breaking

•In the end Higgs mass suppression comes down to controlling soft masses to a scalar and a triplet

Grand Unification?

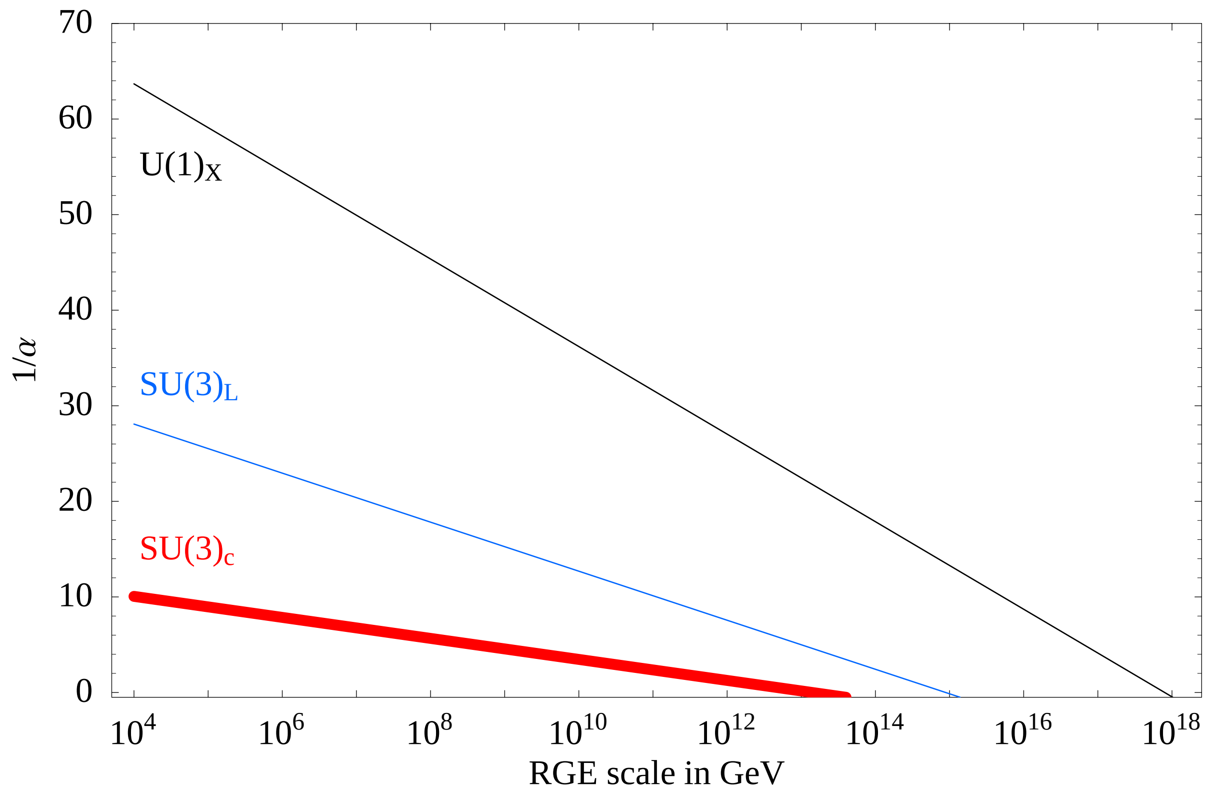
The good news:



The beta functions are such that unification would happen at a high scale (~Planck) with the minimal fermion matter content

Grand Unification?

The bad news:



- Adding the matter needed for generating the top Yukawa will introduce Landau pole before unification
- Seiberg duality, duality cascade a la Klebanov, Strassler?
- Unifies into string theory on warped throat? Meaning of betas?

New particles at LHC

- At low energies model=MSSM with m ~few 100 GeV
- R-parity conservation:traditional SUSY searches apply
- Around f~TeV: lot of new states: little partners+their superpartners

Gauge bosons

- Z’: from EWP m>1.7 TeV. Should be cleanly visible to multi-TeV range (can be singly produced)
- W’: m>1.5 TeV, but couplings for single production v/f suppressed

Heavy top partners

- Expected in 2-3 TeV range (if f close to lower bound)
- LHC reach ~ 2-2.5 TeV

Other additional fermions

- Main distinction from usual 3-3-1 charge assignment:

  Anomaly free
  Generation independent

- Singlet leptons extended into SU(3) triplets (rather than singlets)

  - Vectorlike SU(2) singlet quakrs from Q,D’: O(f) mass
  - Vectorlike SU(2) doublet leptons from E,L’: O(f) mass
  - Two light SU(2) singlet “sterile neutrino” from L,L’

    - Light (no renormalizable mass)
    - Not completely sterile (SU(3) interactions)
    - Can add full SU(6) states to give them O(f) mass

Conclusions

- SUSY models could still be natural if Higgs super-little:

$$m_h^2 \sim \frac{g^2}{16\pi^2} m_{\text{soft}}^2 \ln \frac{f}{m_{\text{soft}}}$$

- Generic SUSY little Higgs models will NOT have this property:
  - D-terms induce tree-level mass
  - Non-Goldstones can be dominant for EWSB
- Simple model based on SU(6) SUSY GUT → SU(3)xSU(3)xU(1)
- Higgs sector: $\Sigma + H, \bar{H}$
  - Anomaly free, generation independent charges
  - No Goldstone mass from D-terms
  - Collective top Yukawa
  - Need acrobatics for quartic
  - Unification?
- New particles at LHC:
  - MSSM few 100 GeV
  - W', Z', T, extra fermions: few TeV
  - Sterile neutrinos