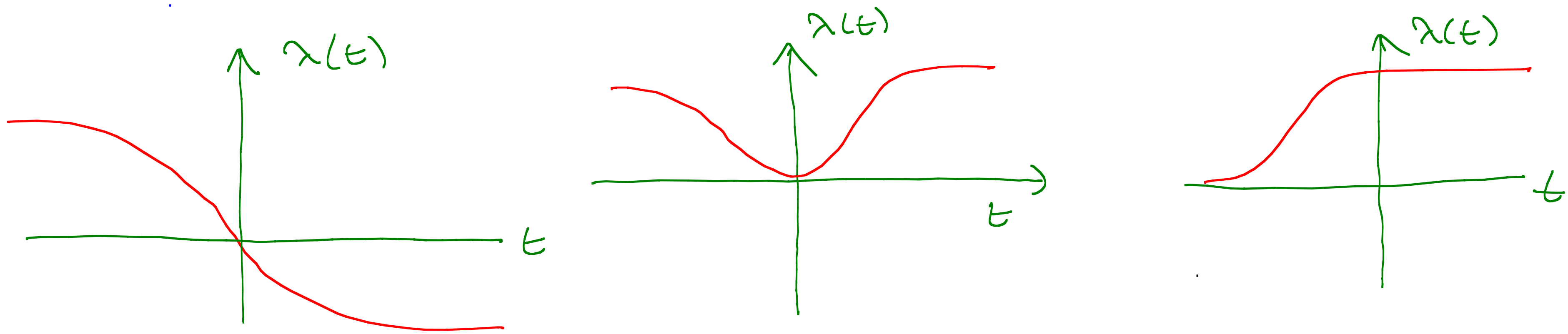
SCALING IN QUANTUM QUENCH & HOLOGRAPHY

SUMIT R. DAS

CONSIDER SOME HAMILTONIAN WITH A TIME DEP. PARAMETER $\lambda(t)$ WHICH INTERPOLATES BETWEEN CONSTANT VALUES



REGARDLESS OF THE SPEED: CALL THIS QUANTUM QUENCH

THIS TALK WILL, IN FACT, DEAL WITH THE QUESTION OF DEPENDENCE OF VARIOUS QUANTITIES ON QUENCH RATE

QUANTUM QUENCH IS PARTICULARLY INTERESTING WHEN THE PROTOCOL INVOLVES A CRITICAL POINT

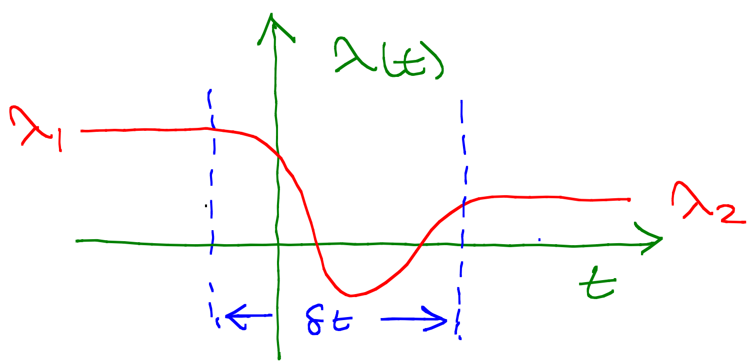
ONE THEN EXPECTS THAT TIME EVOLUTION CARRIES UNIVERSAL SIGNATURES OF THE CRITICAL POINT

OUR SETTING INVOLVES AN ACTION

$$S = S_{CFT} - \int d^{d-1}x \, dt \; \lambda(t) \, \mathcal{O}_\Delta(x,t)$$

Δ : DIMENSION OF RELEVANT OPERATOR $\mathcal{O}_\Delta$

$$\lambda(t) = \lambda_0 \, F(t/\delta t)$$



λ_0 : OVERALL SCALE

$(\delta t)^{-1}$: QUENCH RATE

$F \sim O(1)$ VALUES

UNIVERSAL RESULTS ARE KNOWN IN TWO LIMITS

SLOW QUENCH : KIBBLE ZUREK SCALING

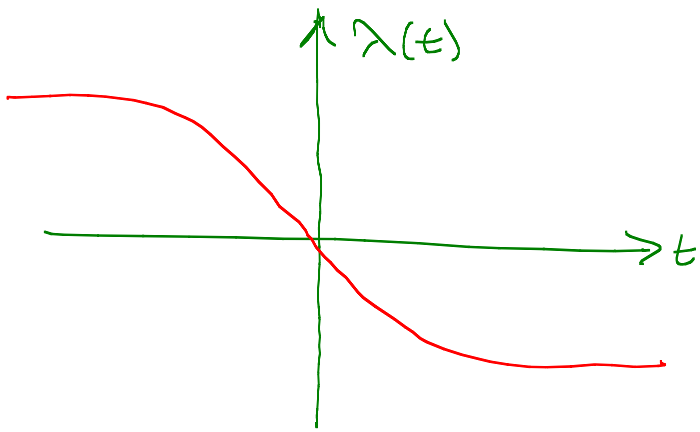
ABRUPT QUENCH : CALABRESE-CARDY

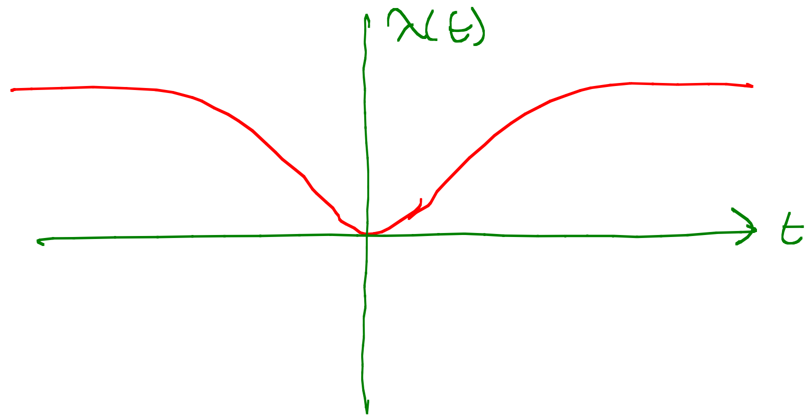
BULK OF THIS TALK : A SCALING RESULT FOR FAST QUENCH

- FIRST FOUND IN HOLOGRAPHY
- NOW SHOWN TO BE A GENERAL FIELD THEORY RESULT

$\boxed{\text{KIBBLE - ZUREK \& HOLOGRAPHY}}$

SUPPOSE THE COUPLING CROSSES A CRITICAL POINT STARTING FROM A GAPPED PHASE

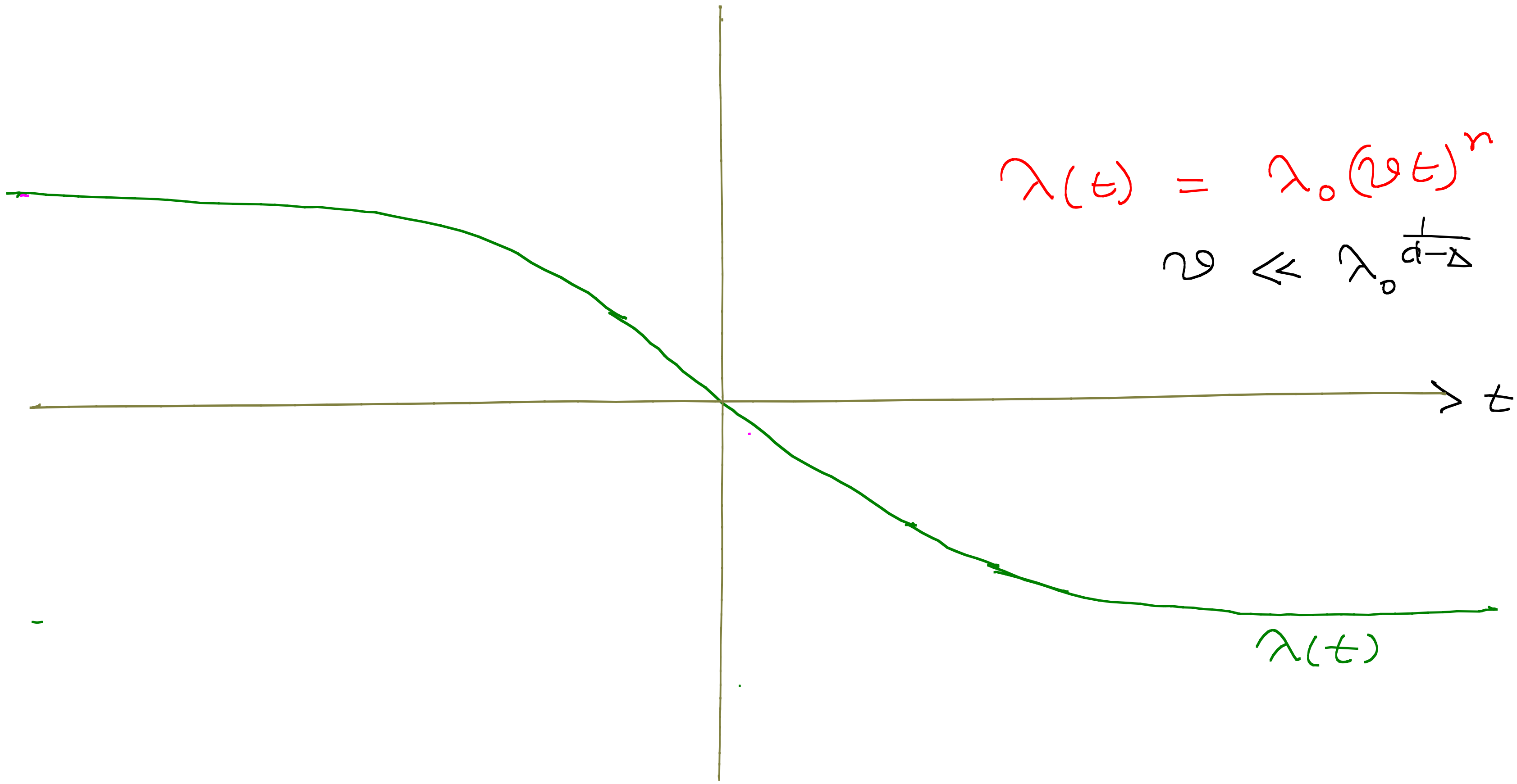


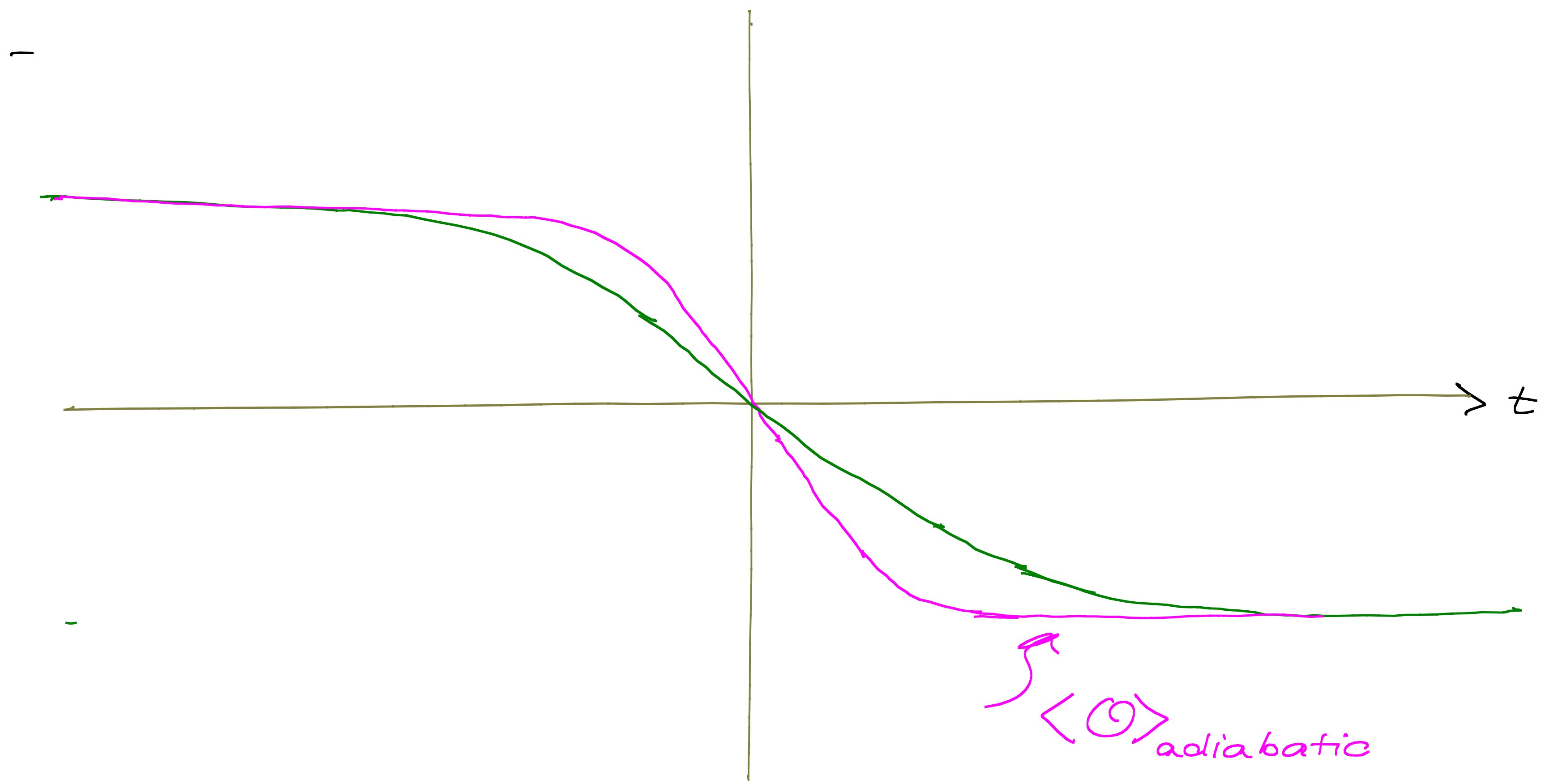


e.g TIME DEPENDENT MAGNETIC FIELD AT $T = T_c$

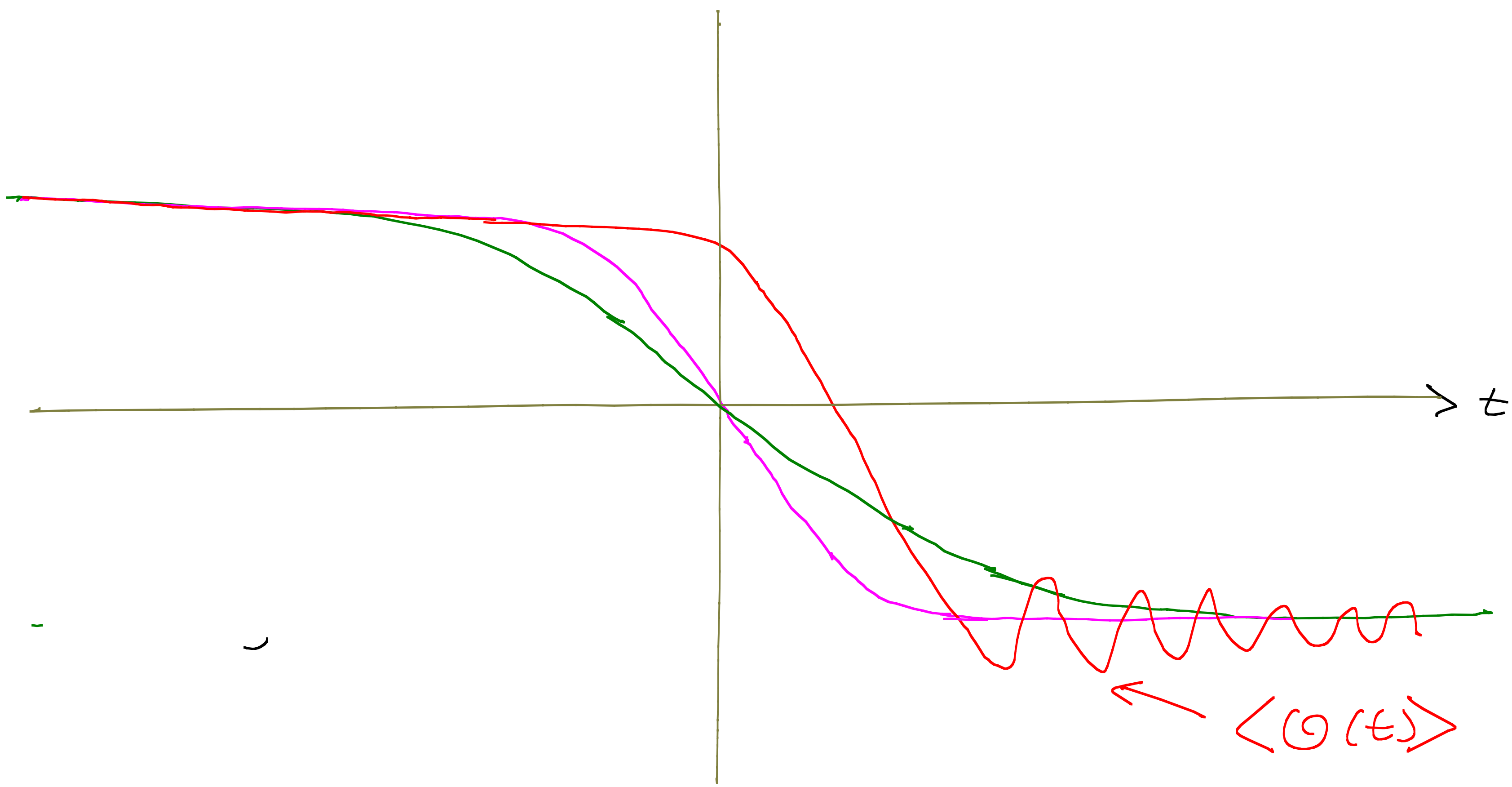
$$\lambda(t) = \lambda_0 (\vartheta t)^n$$

$$\vartheta \ll \lambda_0^{\frac{1}{d-\Delta}}$$

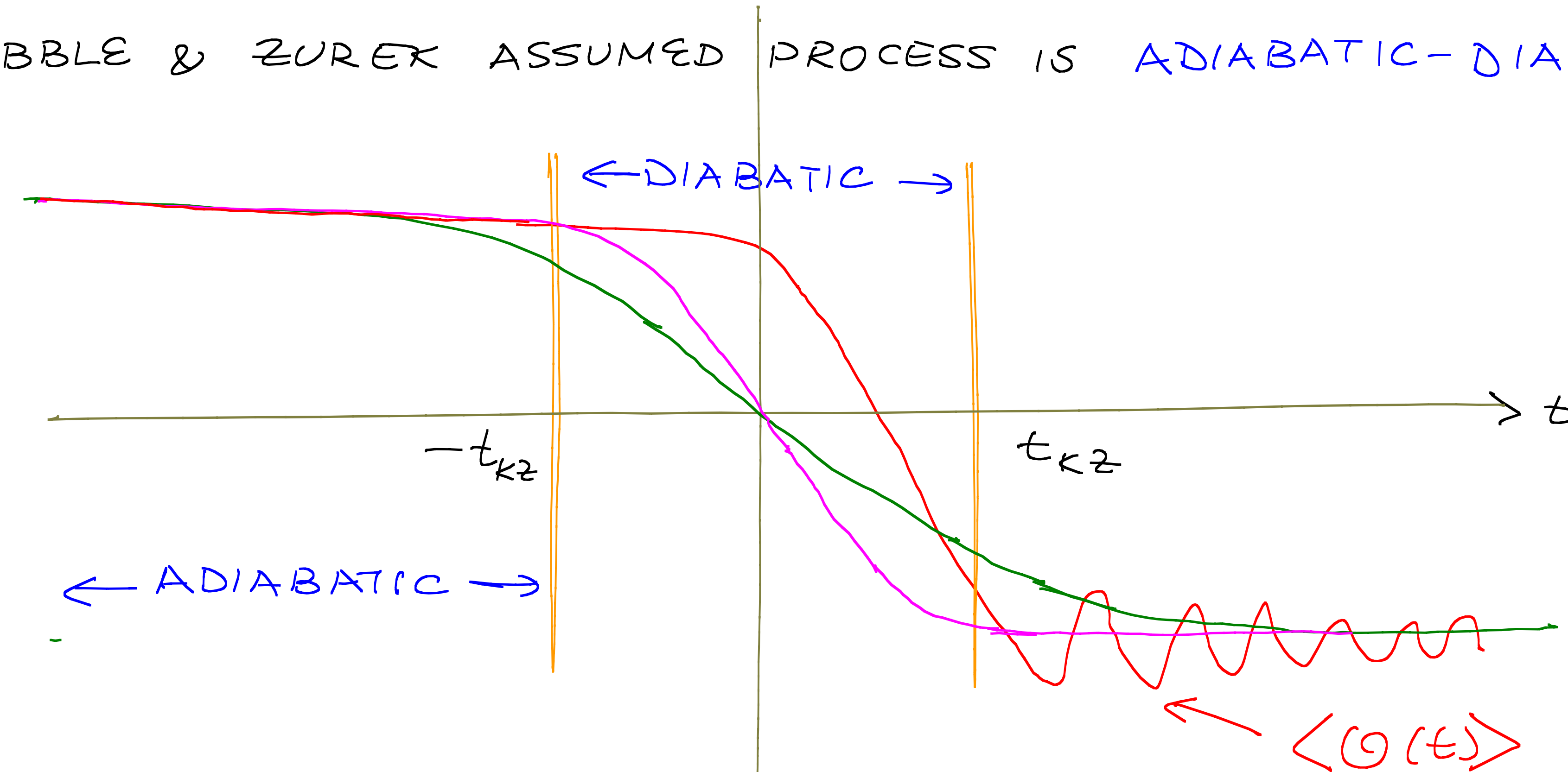




1
t
$\langle \sigma \rangle_{adiabatic}$


t
⟨σ(t)⟩

KIBBLE & ZUREK ASSUMED PROCESS IS ADIABATIC-DIABATIC



ASSUME : INSTANTANEOUS CORRELATION LENGTH ONLY SCALE

$$\boxed{\langle \mathcal{O} \rangle \sim (\lambda_0 v^r)^{\frac{\Delta}{r+d-\Delta}} f\left(t\,(\lambda_0 v^r)^{\frac{1}{r+d-\Delta}}\right)}$$

THIS IS A SPECIAL CASE OF KIBBLE-ZUREK SCALING

UNDERLYING ASSUMPTIONS NOT EASY TO UNDERSTAND FOR STRONGLY COUPLED THEORIES

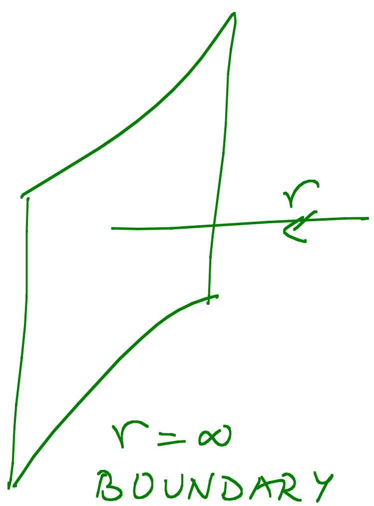
USING HOLOGRAPHY WE CAN GET SOME INSIGHT

P. BASU & S.R.D – JHEP 1201 (2012) 103
P. BASU, D. DAS, S.R.D & T. NISHIOKA – JHEP 1303 (2013) 146
P. BASU, D. DAS, S.R.D & K. SENGUPTA – JHEP 1312 (2013) 070
S.R.D & T. MORITA – JHEP 1501 (2015) 084

HOLOGRAPHIC KIBBLE ZUREK

$$\boxed{S = S_{eFT} + \int dt\, d^{d-1}x\, \lambda(t)\, \mathcal{O}_\Delta(x,t)}$$

SCALAR OPERATOR $\mathcal{O}_\Delta$ DUAL TO $\phi(r,\vec{x},t)$



$$\boxed{\phi(\vec{x},r,t) \xrightarrow[r\to\infty]{} r^{\Delta-d}\left[\lambda(t) + O(1/r^2)\right] + r^{\Delta}\left[A(t) + O(1/r^2)\right]}$$

$$\boxed{\langle \mathcal{O}_\Delta(\vec{x},t)\rangle = A(t)}$$

$\Rightarrow$ SOLVE BULK EQUATIONS WITH A TIME DEP. BOUNDARY CONDITION

STRATEGY

- CONSIDER A BULK THEORY SUCH THAT THE BOUNDARY FIELD THEORY HAS AN ISOLATED CRITICAL POINT (e.g. HOLOGRAPHIC SUPERCONDUCTOR)
- INTRODUCE A TIME DEPENDENT SOURCE FOR THE CORRESPONDING ORDER PARAMETER WHICH CROSSES THIS CRITICAL POINT
- CALCULATE THE RESPONSE

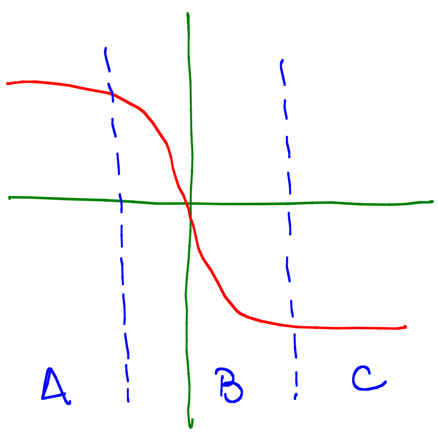
THIS IS EXACTLY WHAT IS DONE TO STUDY THERMALIZATION

- JANIK & PESCHANSKI
- CHESLER & YAFFE
- BHATTACHARYA & MINWALLA
- KOVGEKOV & LIN
- ABAJO-ARRISTA, APRICIO & LOPEZ
- S.R.D, T. NISHIOKA & T. TAKAYANAGI
- BALASUBRAMANIAN et. al

. . . .

THE SPECIAL INGREDIENT IS THE CRITICAL POINT

THE RESULT FROM SEVERAL MODELS WITH HOLOGRAPHIC CRITICAL POINTS



$v \ll \lambda_0^{\frac{1}{d-\Delta}}$

REGION A : ADIABATIC EXPANSION VALID
— POWER SERIES EXPANSION IN v

REGION B : A DIFFERENT EXPANSION IN FRACTIONAL POWERS OF v, v^{α}
— POWER DETERMINED BY CRITICAL EXPONENTS

IN REGION B THE BULK FIELD CAN BE EXPANDED IN EIGENMODES OF THE LINEAR FLUCTUATION OPERATOR $\mathcal{D}$

$$\phi(r,t) = \sum_n \varphi_n(r)\, \xi_n(t).$$

$$\mathcal{D}\varphi_n = \lambda_n \varphi_n$$

AT THE CRITICAL POINT $\lambda = 0$ THE OPERATOR $\mathcal{D}$ HAS A ZERO MODE

TO LOWEST ORDER IN THE ϑ^α EXPANSION THE ZERO MODE DOMINATES THE DYNAMICS

THE EQUATION SATISFIED BY THE ZERO MODE $\xi_0(t)$ HAS A SCALING SOLUTION

$$\phi(r,t) = \varphi_0(r)\, v^{\alpha}\, \bar{\xi}_0(t v^{\beta})$$

THIS IMPLIES THAT THE EXPECTATION VALUE OF THE DUAL OPERATOR SATISFIES

$$\boxed{\langle \mathcal{O}_\Delta(t, v) \rangle = v^{\alpha} f(t v^{\beta})}$$

THIS IS AN ASPECT OF KIBBLE-ZUREK SCALING

CORRECTIONS TO THE LEADING SCALING CAN BE COMPUTED IN AN EXPANSION IN v^{α}

- WE HAVE CONSIDERED GLOBAL QUENCH AND LOOKED AT BULK SOLUTIONS WHICH ARE HOMOGENEOUS IN BOUNDARY SPATIAL COORDINATES

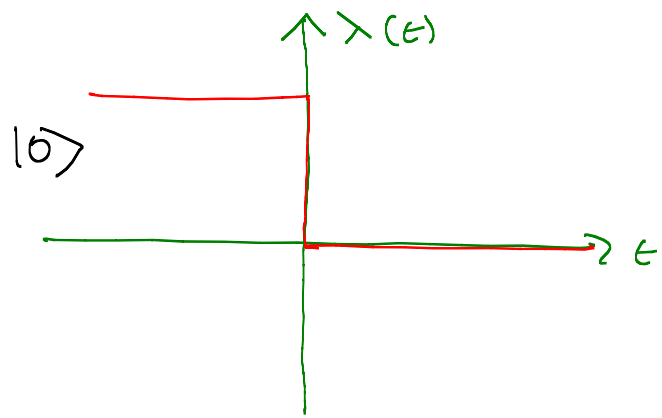
  — THIS IS LEADING ORDER OF $1/N$ IN THE CFT

- CORRECTIONS LEAD TO INHOMOGENEITIES DUE TO FLUCTUATIONS — AND DEFECT FORMATION

  - SONNER, DEL-CAMPO & ZUREK
  - CHESLER, GARCIA-GARCIA & LIU.

ABRUPT QUENCH

IN THE OTHER EXTREME THE COUPLING $\lambda(t)$ CHANGES ABRUPTLY FROM SOME VALUE TO SOME OTHER VALUE, e.g.



GROUND STATE OF THE HAMILTONIAN AT $t<0$ IS THEN A SQUEEZED STATE OF THE HAMILTONIAN AT $t>0$

CALABRESE & CARDY SHOWED THAT THE LONG DISTANCE PROPERTIES OF THIS SPECIAL EXCITED STATE CAN BE CAPTURED BY A SIMPLE STATE IN A BOUNDARY CFT

$$\boxed{|\psi_0\rangle \simeq e^{-\tau_0 H}|B\rangle}$$

CONFORMALLY INVARIANT BOUNDARY STATE

$\tau_0 \sim 1/m_g$ $\quad$ m_g : INITIAL MASS GAP.

IN 1+1 DIMENSIONS THIS LEADS TO A SET OF UNIVERSAL PREDICTIONS e.g. ONE POINT FUNCTION FOR SOME OPERATOR (WHICH IS NOT CONSERVED)

$$\boxed{\langle A \rangle \sim \exp\left(-\frac{\pi x}{2\tau_0} t\right)}$$

x: DIMENSION OF A

$\Rightarrow$ RATIOS OF RELAXATION TIMES UNIVERSAL

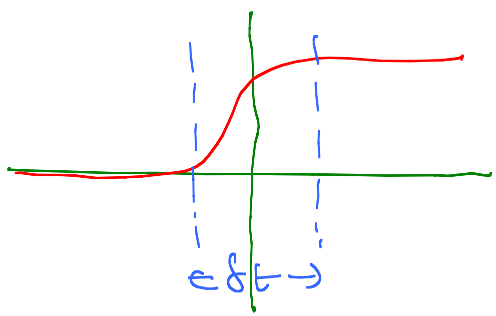
- OTHER RESULTS FOR CORR. FUNCTIONS
- TIME EVOLUTION OF ENTANGLEMENT ENTROPY

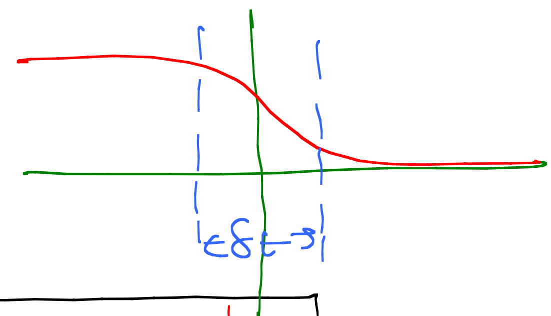
HOLOGRAPHIC REALIZATION : HARTMAN & MALDACENA

HOLOGRAPHIC FAST QUENCH

BUCHEL, LEHNER, MYERS & VAN NIEKERK STUDIED QUENCHES HOLOGRAPHICALLY

$$S = S_{CFT} + \int \lambda(t)\, \mathcal{O}_\Delta(x,t)$$





$$\boxed{\delta t \ll \lambda_0^{-\frac{1}{d-\Delta}}}$$

USING STANDARD TECHNIQUES OF HOLOGRAPHIC RENORMALIZATION

$$\boxed{\begin{aligned} \langle \mathcal{O}_\Delta \rangle_{ren} &\sim (\delta t)^{d-2\Delta} \\ \langle \mathcal{E} \rangle_{ren} &\sim (\delta t)^{d-2\Delta} \end{aligned}}$$

AT TIMES SOON AFTER THE QUENCH

CONSISTENT WITH WARD IDENTITY

$$\boxed{\frac{d\langle \mathcal{E} \rangle}{dt} = \frac{d\lambda}{dt} \langle \mathcal{O}_\Delta \rangle}$$

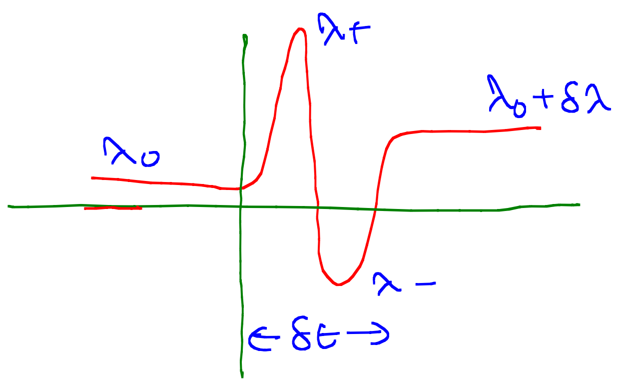
THESE RESULTS ARE SOMEWHAT PUZZLING.

$\langle O_\Delta \rangle \sim (\delta t)^{d-2\Delta} \Rightarrow$ NO SMOOTH LIMIT $\delta t \to 0$ WHEN $d > \Delta > d/2$

NEVERTHELESS THERE IS A PERFECTLY WELL DEFINED ABRUPT QUENCH

COULD THIS BE A SPECIAL FEATURE OF STRONGLY COUPLED FIELD THEORIES WHICH HAVE GRAVITY DUALS ?

WE WILL ARGUE THAT THIS IS A GENERAL RESULT FOR ANY DEFORMED CFT IN ANY NUMBER OF DIMENSIONS



$$\boxed{\delta t \ll \lambda_i^{\frac{1}{\Delta-d}}\,,\ (\lambda_0+\delta\lambda)^{\frac{1}{\Delta-d}}}$$

FOR TIMES $t \lesssim \delta t$

$$\boxed{\langle \mathcal{O}_\Delta \rangle \sim (\delta\lambda)(\delta t)^{d-2\Delta}}$$

FOR TIMES AFTER QUENCH IS OVER

$$\boxed{\langle \mathcal{E} \rangle \sim (\delta\lambda)^2(\delta t)^{d-2\Delta}}$$

S.R.D, D.GALANTE & R.MYERS

PRL 112 (2014) 171601

JHEP 1502 (2015) 167

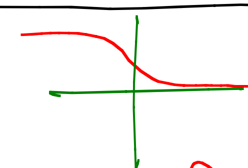
arxiv 1503.xxxx

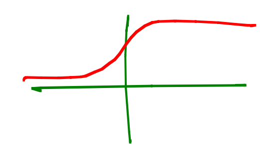
$$\boxed{\text{FAST QUENCH IN FREE FIELDS}}$$

SMOOTH QUENCHES IN SOLVABLE FREE FIELD THEORY WITH TIME DEPENDENT MASS

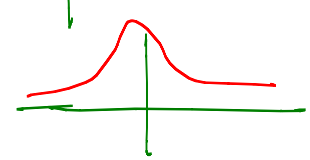
$$\boxed{S = \int d^{d-1}x\, dt\, [(\partial\varphi)^2 - m^2(t)\varphi^2]}$$

$$m^2(t) = A + B\tanh(t/\delta t)$$





$$m^2(t) = m_0^2 + \frac{m^2}{\cosh^2(t/\delta t)}$$



PROBLEM CAN BE EXACTLY SOLVED FOR ALL δt FOR THESE MASS PROFILES

THE (HEISENBERG PICTURE) STATE IS THE "IN" VACUUM

$$\boxed{\phi(\vec{x},t) = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left[a(k)\, u_k(x,t) + h.c. \right]}$$

$$\boxed{u(k) = \frac{1}{\sqrt{2\omega_{in}}}\, {}_2F_1\left(a, b; c; \tfrac{1}{2}\left(1+\tanh\tfrac{t}{\delta t}\right)\right) e^{i\left(\vec{k}\cdot\vec{x} - \omega_+ t - \omega_- \delta t \log\left(2\cosh\frac{t}{\delta t}\right)\right)}}$$

$$a = 1 + i\omega_- \delta t, \quad b = i\omega_- \delta t, \quad c = 1 - i\omega_{in} \delta t$$

$$\omega_{in} = \sqrt{k^2 + m_{in}^2} \qquad \omega_{out} = \sqrt{k^2 + m_{out}^2} \qquad \omega_\pm = \frac{1}{2}\left(\omega_{out} \pm \omega_{in}\right)$$

$$\boxed{u_k \to \frac{1}{\sqrt{2\omega_{in}}} \exp\left[i\left(\vec{k}\cdot\vec{x} - \omega_{in} t\right)\right]} \quad \text{as } t \to -\infty$$

$$\boxed{a_k |0\rangle_{in} = 0}$$

THERE IS ALSO A SET OF "OUT" MODES

$$\phi = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left[b_k v_k + h.c. \right]$$

BOGOLIUBOV :

$$u_k = \alpha_k v_k + \beta_k v_{-k}^*$$

$$\alpha_k = \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(1 - i\omega_{in}\delta t)\,\Gamma(-i\omega_{out}\delta t)}{\Gamma(-i\omega_+\delta t)\,\Gamma(1 - i\omega_+\delta t)}$$

$$\beta_k = \sqrt{\frac{\omega_{out}}{\omega_{in}}} \frac{\Gamma(1 - i\omega_{in}\delta t)\,\Gamma(i\omega_{out}\delta t)}{\Gamma(i\omega_-\delta t)\,\Gamma(1 + i\omega_-\delta t)}$$

THE ONE POINT FUNCTION OF QUENCHED OPERATOR

$$\boxed{{}_{in}\langle 0| \phi^2(x,t) |0\rangle_{in} = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2\omega_{in}} |{}_2F_1|^2}$$

THIS QUANTITY IS OF COURSE UV DIVERGENT

AN EFFICIENT WAY TO RENORMALIZE THIS IS TO REGARD $m^2(t)$ AS A BACKGROUND FIELD

$$S \to S_0(\phi, m^2, g_{\mu\nu}) - S_{ct}(m^2, g_{\mu\nu}, \Lambda)$$

$g_{\mu\nu}$ IS A BACKGROUND METRIC

EXPECTATION VALUES MAY BE COMPUTED FROM

$$Z(m^2, g_{\mu\nu}) = \int \mathcal{D}\phi_x \exp[-iS].$$

$$\langle \phi^2 \rangle_{ren} = -2i \left(\frac{1}{\sqrt{g}} \frac{\delta}{\delta m^2} \log Z \right)_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \Lambda \to \infty}}$$

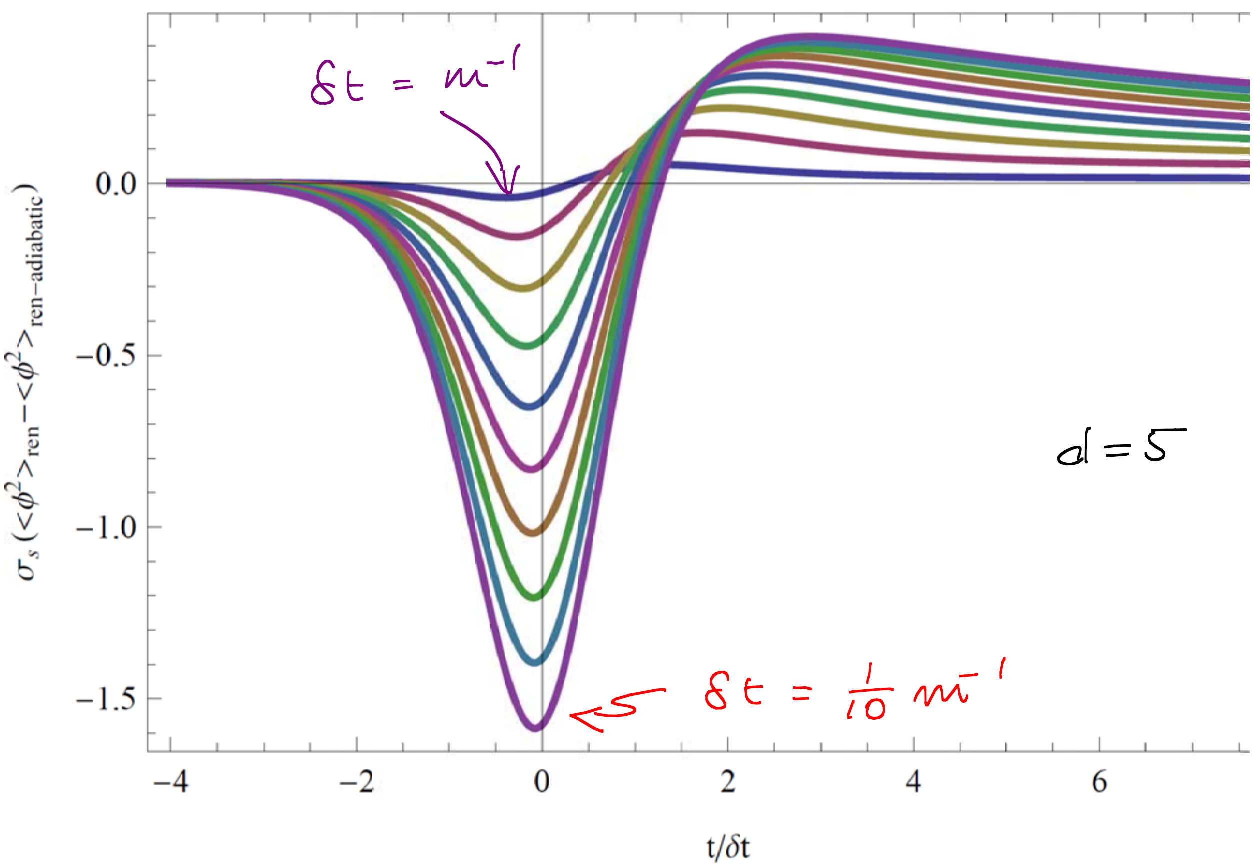
$$\langle T_{\mu\nu} \rangle_{ren} = -2i \left(\frac{1}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} \log Z \right)_{\substack{g_{\mu\nu} = \eta_{\mu\nu} \\ \Lambda \to \infty}}$$

IT TURNS OUT THAT THE COUNTERTERM ACTION MAY BE OBTAINED FROM AN ADIABATIC EXPANSION

e.g. $$\langle 0 | \phi^2 | 0 \rangle_{\text{adiabatic}} = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{2\Omega_k}$$

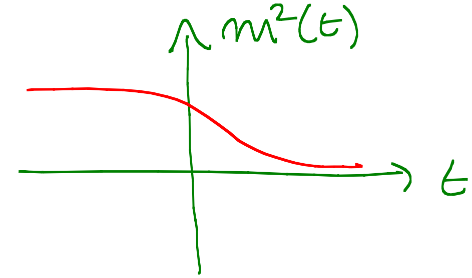
$$\Omega_k(t) = \omega_k(t) - \frac{1}{4\omega_k}\left(\frac{\ddot{\omega}_k}{\omega_k} - \frac{3}{2}\left(\frac{\dot{\omega}_k}{\omega_k}\right)^2\right) + \cdots$$

$$\omega_k(t) \equiv \sqrt{k^2 + m^2(t)}$$

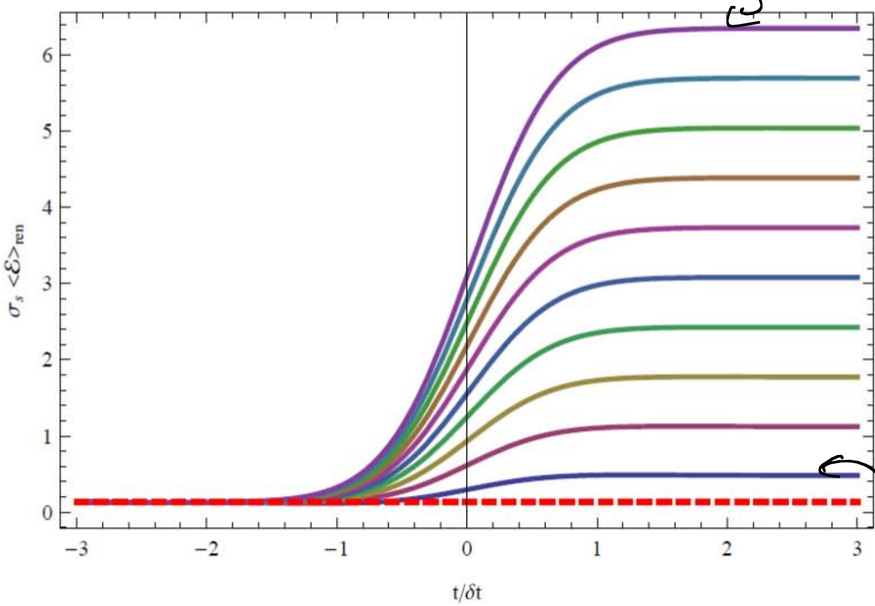


$d = 5$

$$m^2(t) = \frac{1}{2} M_0^2 \left(1 - \tanh \frac{t}{\delta t}\right)$$

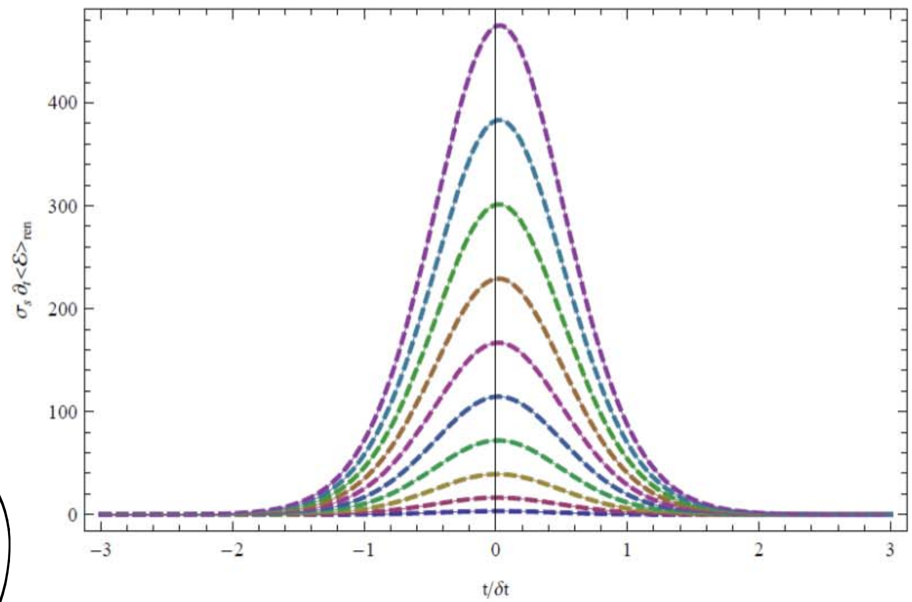


$\delta t = 0.01$



$\delta t = 0.1$

$\langle \varepsilon \rangle$ AS FUNCTION OF $t/\delta t$



$\partial_t \langle \varepsilon \rangle$ AS FUNCTION OF $t/\delta t$

WE CAN NOW CONSIDER FAST QUENCH LIMIT

$$m_0 \, \delta t \ll 1$$

IN THIS LIMIT ONE RECOVERS ANALYTIC EXPRESSIONS

ODD d
$d \geq 5$

$$\boxed{\langle \phi^2 \rangle_{ren} = (-1)^{\frac{d-1}{2}} \frac{\pi}{2^{d-2}} \partial_t^{d-4} m^2 + O(\delta t^{6-d})}$$

$d = 3$

$$\boxed{\langle \phi^2 \rangle_{ren} = -\frac{m_0}{4} - \frac{m_0^2 \, \delta t}{16} \log\left[\frac{1}{2}\left(1 - \tanh \frac{t}{\delta t}\right)\right]}$$

$$\left(m^2(t) = \frac{1}{2} m_0^2 \left(1 - \tanh \frac{t}{\delta t}\right) \right)$$

FOR EVEN d THE RESULTS ARE

$d > 4$

$$\boxed{\langle \phi^2 \rangle_{ren} = \frac{(-1)^{d/2}}{2^{d-3}} \log(\mu \delta t)\, \partial_t^{d-4} m^2(t) + \ldots}$$

$d = 4$

$$\boxed{\langle \phi^2 \rangle_{ren} = \frac{m_0^2}{4} \left(1 + \tanh \frac{t}{\delta t}\right) \log(\mu \delta t) + \ldots}$$

THESE RESULTS IMPLY THAT, UPTO logs

$$\boxed{\langle \phi^2 \rangle \sim m_0^2 (\delta t)^{4-d}}$$

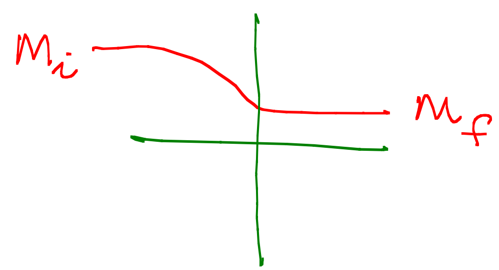
SINCE FOR THIS OPERATOR $\Delta = d-2$

$$\boxed{\langle \phi^2 \rangle \sim m_0^2 (\delta t)^{d-2\Delta}}$$

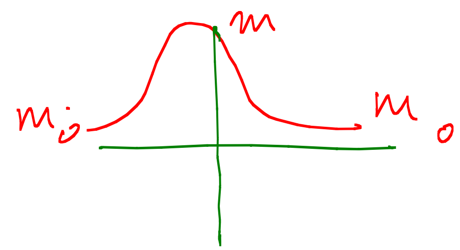
THIS IS EXACTLY AS FOUND IN HOLOGRAPHY

– LOG CORRECTIONS ALSO FOUND THERE

RESULTS GENERALIZE TO GENERIC MASS PROFILES



$m_i \delta t \ll 1 \qquad m_f \delta t \ll 1$



$m_0 \delta t \ll 1 \qquad m \, \delta t \ll 1$

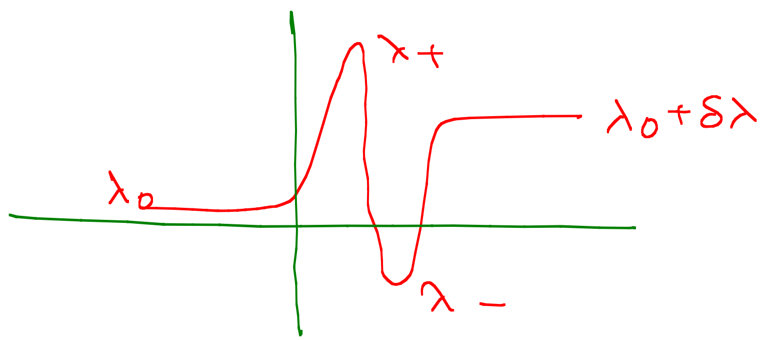
SIMILAR RESULTS FOR DIRAC FERMIONS

THE GENERAL RESULT

CONSIDER NOW A GENERAL INTERACTING THEORY

$$\boxed{S = S_{CFT} + \int d^{d-1}x\, dt\, \lambda(t)\, \mathcal{O}_\Delta(x,t)}$$

LET US NOW COMPUTE $\langle \mathcal{O}_\Delta \rangle$ PERTURBATIVELY FOR



QUENCH STARTS AT $t = 0$

$$\langle \mathcal{O}_\Delta \rangle = \langle \mathcal{O}_\Delta \rangle_{\lambda_0} + \int_0^t dt' \int d^{d-1}x' \, \lambda(t') \, G_R(x-x'; t-t') + \int dt' dt'' dx' dx'' \lambda(t') \lambda(t'') K(x, x', x''; t, t', t'')$$

FIRST TERM IS STANDARD LINEAR RESPONSE

$$G_R(x-x', t-t') \equiv \Theta(t-t') \langle 0 | [\mathcal{O}_\Delta(x,t), \mathcal{O}_\Delta(x',t')] | 0 \rangle_{\lambda_0}$$

WE NOW EXAMINE THIS TERM IN SOME DETAIL

$$\boxed{\int_0^t dt' \int d^{d-1}x' \; \lambda(t') \; G_R(x-x'; t-t')}$$

- (x', t') LIES IN PAST LIGHT CONE OF (x, t)
- IF $t \lesssim \delta t$ THE SPATIAL SEPARATION $|x-x'| \lesssim \delta t$
- HOWEVER IF δt IS MUCH SMALLER THAN ALL OTHER LENGTH SCALES, INCL. $(\lambda_0)^{-\frac{1}{d-\Delta}}$ CORRELATORS AT DISTANCES SHORTER THAN δt ARE BASICALLY CFT CORRELATORS

THUS RENORMALIZED $\langle \mathcal{O}_\Delta \rangle$ DEPENDS ONLY ON $\delta\lambda$ AND δt

IN TERMS OF A DIMENSIONLESS COUPLING

$$g \equiv (\delta\lambda)(\delta t)^{d-\Delta}$$

WE HAVE AN EXPANSION

$$\boxed{\langle \mathcal{O} \rangle - \langle \mathcal{O} \rangle_{\lambda_0} = (\delta t)^{-\Delta}\left[a_1\left(\frac{t}{\delta t}\right) \cdot g + a_2\left(\frac{t}{\delta t}\right) g^2 + \cdots \right]}$$

HOWEVER FAST QUENCH MEANS

$$g \ll 1$$

THUS THE LEADING ANSWER IS

$$\boxed{\delta\langle \mathcal{O}_\Delta \rangle \sim (\delta t)^{-\Delta}\, g\, a_1\!\left(\frac{t}{\delta t}\right) = (\delta\lambda)(\delta t)^{d-2\Delta}\, a_1(t/\delta t)}$$

$\Rightarrow$ "EARLY TIME" QUANTITY HAS THIS UNIVERSAL FORM IN ANY THEORY

— SIMILAR ARGUMENT FOR $\langle \mathcal{E} \rangle_{ren}$

THE ABRUPT LIMIT

- DOES $\delta t \to 0$ LIMIT REPRODUCE ABRUPT QUENCH ?
- TO INVESTIGATE THIS CONSIDER THE EXPLICIT RESULTS FOR FREE BOSONIC FIELDS
- OUR RESULTS ARE FOR RENORMALIZED QUANTITIES
  - THESE ARE RELEVANT ONLY IF THE QUENCH RATE IS MUCH BELOW THE CUTOFF

$$\boxed{\Lambda_{UV}^{-1} \ll \delta t \ll (\delta\lambda)^{-\frac{1}{d-\Delta}}}$$

THIS IS WHY ADIABATIC EXPN. GIVES COUNTERTERMS

AN ABRUPT QUENCH HAS A RATE FASTER THAN ALL SCALES – HAS TO BE OF CUTOFF SCALE

$\Rightarrow$ CLEARLY LOCAL OBSERVABLES IN FAST QUENCH WOULD BE, IN GENERAL, DIFFERENT FROM THOSE IN ABRUPT QUENCH

TO EXAMINE THIS IN MORE DETAIL, LOOK AT UV-FINITE QUANTITIES –

EXCESS ENERGY

ONE SUCH QUANTITY IS THE EXCESS ENERGY DENSITY AT LATE TIMES

$$\Delta \mathcal{E} = \mathcal{E} - \mathcal{E}_{ground}$$

WHERE $\mathcal{E}_{ground}$ IS THE GROUND STATE ENERGY OF THE FINAL HAMILTONIAN

$$\mathcal{E}_{ground} = \frac{1}{2}\int \frac{d^{d-1}k}{(2\pi)^{d-1}} (\omega_{out})$$

THE EXPRESSION FOR THIS IS

$$\boxed{\Delta E = \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \, \omega_{out} \, |\beta_k|^2}$$

WHERE β_k IS THE BOGOLIUBOV COEFFICIENT

USING EXPLICIT EXPRESSIONS IT IS EASY TO CHECK THAT THIS IS FINITE FOR ANY FINITE δt

IN THE FAST QUENCH LIMIT $m\delta t \ll 1$.

$$\boxed{\Delta E_{smooth} \sim m^4 (\delta t)^{4-d} + \text{subleading terms}}$$

FOR $d=2,3$ THIS IS FINITE AS $\delta t \to 0$.
FOR OTHER d THIS BLOWS UP.

CORRESPONDING QUANITY FOR THE ABRUPT QUENCH

$$(\Delta \mathcal{E})_{abrupt} = \begin{cases} \dfrac{m^2}{16\pi} & d=2 \\ \dfrac{m^3}{24\pi} & d=3 \\ m^4 \Lambda_{UV}^{d-4} & d \geq 4. \end{cases}$$

FOR $d=2,3$ FINITE LIMIT OF $\Delta \mathcal{E}$ AGREES WITH $(\Delta \mathcal{E})_{abrupt}$
FOR $d \geq 4$ $\delta t \to 0$ DIVERGENCE $\approx$ UV DIVERGENCE $(\Delta \mathcal{E})_{abrupt}$

CORRELATION FUNCTIONS

ANOTHER CLASS OF UV FINITE QUANTITIES ARE CORRELATION FUNCTIONS AT FINITE SPATIAL SEPARATIONS

$$C(\vec{r},t) \equiv {}_{in}\langle 0|\, \phi(\vec{r},t)\, \phi(0,t)\,|0\rangle_{in}$$

$$\boxed{\text{LATE TIME CORRELATORS}}$$

$$C_{\text{smooth}}(t,r) = \int \frac{[d\vec{k}]}{2k} e^{i\vec{k}\cdot\vec{r}} \left\{ |\alpha_k|^2 + |\beta_k|^2 + \alpha_k \beta_k^* e^{2ikt} + \alpha_k^* \beta_k e^{-2ikt} \right\}$$

$$C_{\text{abrupt}}(t,r) = \int \frac{[d\vec{k}]}{2k} e^{i\vec{k}\cdot\vec{r}} \left\{ \frac{k^2 + m^2 \sin^2(kt)}{k^2 \sqrt{k^2+m^2}} \right\}$$

(δt) IS CONTAINED IN THE BOGOLIUBOV COEFFICIENTS α_k, β_k.

THE INTEGRANDS AGREE IN TWO LIMITS

- $m\delta t \ll 1 \qquad |\vec{k}|\delta t \ll 1$
- $m\delta t \ll 1 \qquad |\vec{k}|/m \gg 1 \qquad |\vec{k}|\delta t = \text{arbitrary}$

THE SECOND IMPLIES THAT THE LEADING SHORT DISTANCE SINGULARITIES OF THE TWO CORRELATORS ARE THE SAME – BUT THE SUBLEADING TERMS ARE NOT

IN FACT, THE SHORT DISTANCE EXPANSION FOR SMOOTH QUENCH CORRELATOR IS

$$C(t,r) \xrightarrow[r\to 0]{} \frac{a_1}{r^{d-2}} + \frac{a_2\, m^2(t)}{r^{d-4}} + \frac{a_3\left(3m^4(t) + \partial_t^2 m^2(t)\right)}{r^{d-6}} + \cdots$$

THESE TERMS ARE IN 1-1 CORRESPONDENCE WITH THE COUNTERTERMS FOR $\langle \phi^2 \rangle$ PREDICTED BY ADIABATIC EXPANSION

NOTE: FOR $d \geq 6$ DIVERGENT TERMS (as $r \to 0$) INVOLVE DERIVATIVES OF MASS.

THE SECOND REGIME WHERE THERE IS AGREEMENT IS $m\delta t \ll 1 \quad |\vec{k}|\delta t \ll 1$

THUS LONG DISTANCE CORRELATORS SHOULD AGREE, SINCE FOR $r \gg \delta t$ WE EXPECT CONTRIBUTIONS FOR $k \lesssim r^{-1} \Rightarrow k \ll (\delta t)^{-1}$

— INDEED THEY DO

$$\boxed{\text{CROSSOVER BEHAVIOR}}$$

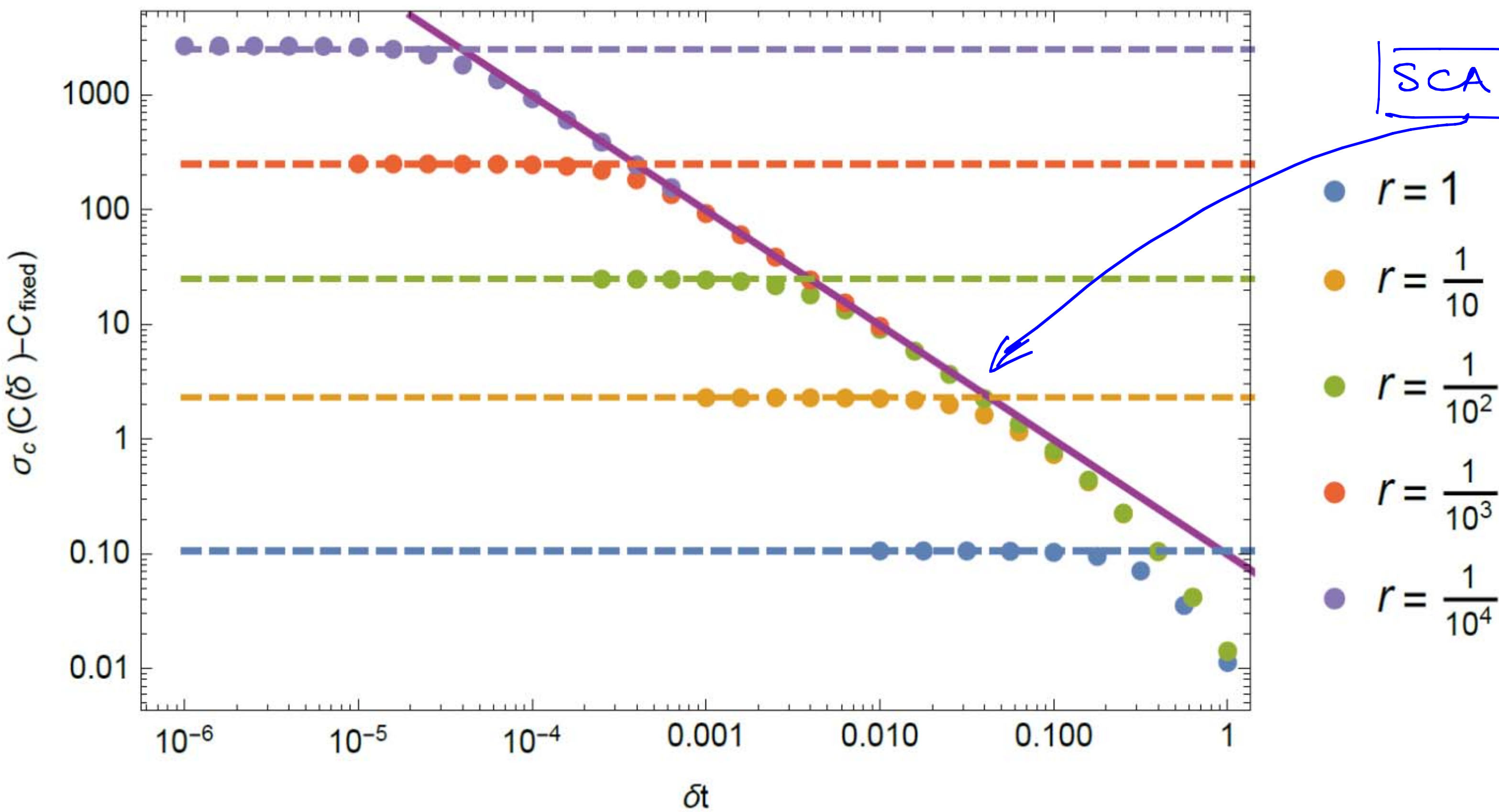
PERHAPS THE MOST INTERESTING BEHAVIOR APPEARS IN CORRELATORS AT EARLY TIMES.

RECALL: THE SCALING OF LOCAL OBSERVABLES APPEAR AT EARLY TIMES AS WELL.

SHOULD EXPECT THAT FOR SOME REGIME OF $|\vec{r}|$ WE SHOULD RECOVER SCALING

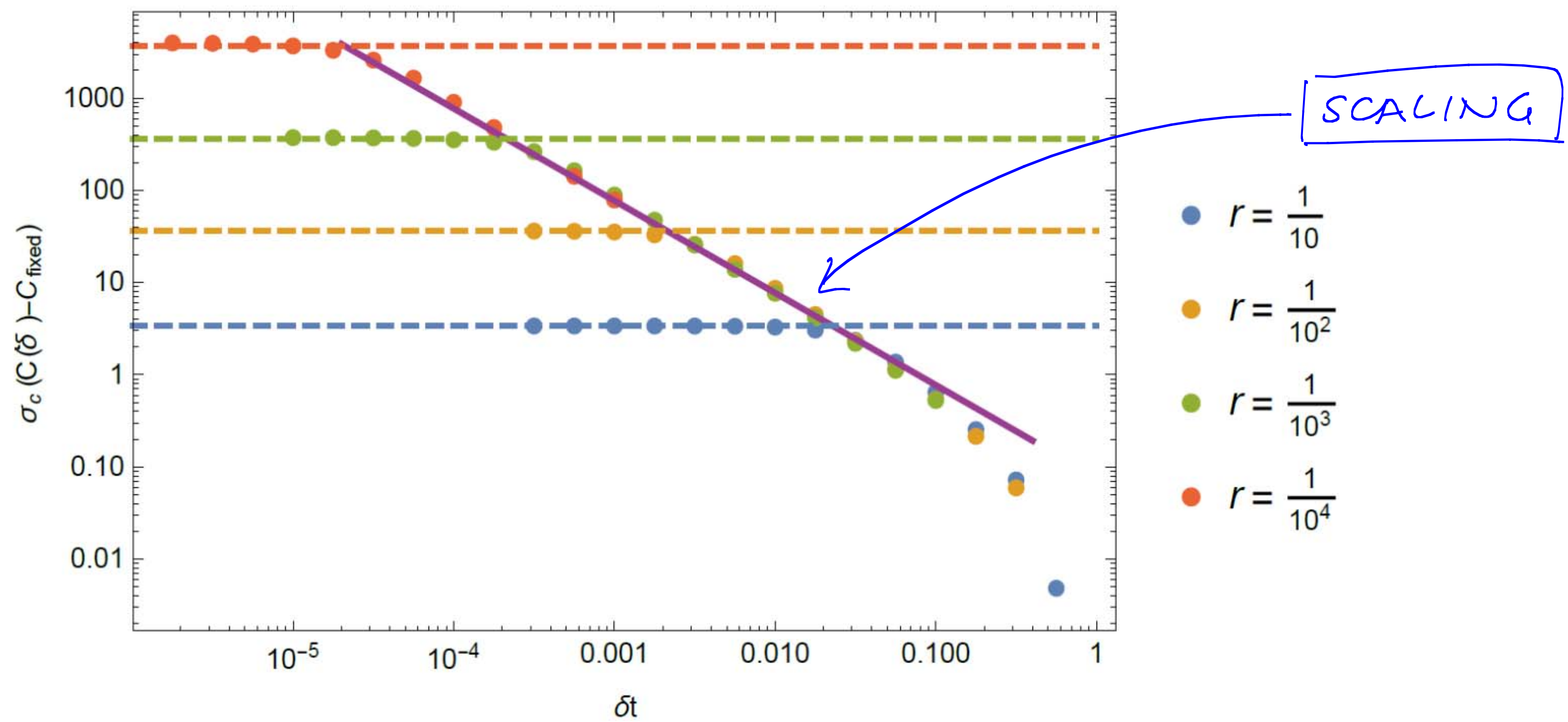
FOR LARGER $|\vec{r}|$ EXPECT SATURATION

$m_0 = 1$ $\tau = t/\delta t = 0$



$\boxed{\text{SCALING}}$

$m = 1 \qquad \tau = t/\delta t = 1/2$



THUS, FOR AN INTERMEDIATE REGIME OF $|\vec{r}|$ WE INDEED SEE UNIVERSAL SCALING

WE EXPECT THE SAME TO HOLD FOR INTERACTING THEORIES AS WELL

A SIMILAR PHYSICS HOLDS FOR LOCAL QUANTITIES IN A THEORY WITH FINITE CUTOFF

OUTLOOK

WE HAVE IDENTIFIED A UNIVERSAL SCALING FOR FAST, SMOOTH QUENCHES IN A SUITABLE REGIME

FOR SLOW QUENCH KIBBLE-ZUREK HOLDS

IN FREE FERMION THEORY WE HAVE BEEN ABLE TO SEE THE INTERPOLATION

THESE ARE RESULTS IN GENERAL INTERACTING FIELD THEORIES – EXPERIMENTALLY OBSERVABLE ?

THANK YOU