Anomalies, Chern-Simons Terms, and Black Hole Entropy

Tatsuo Azeyanagi (ENS)

Based on arXiv:1311.2940, 1407.6364 and to appear
with R. Loganayagam (IAS), G.S. Ng (McGill), M.J. Rodriguez (Harvard)

Workshop “Holographic Methods for Strongly Coupled Systems” @ GGI, Florence Italy, March 20, 2015

Introduction: Anomalies in QFT

Anomalies in QFT

(Quantum) Anomalies in QFT$_{2n}$

Breakdown of symmetries by quantum effect

Interest in this talk

global U(1), gravitational(breakdown of Lorentz sym.), mixed U(1)-gravitational

Anomalies at Zero Temperature

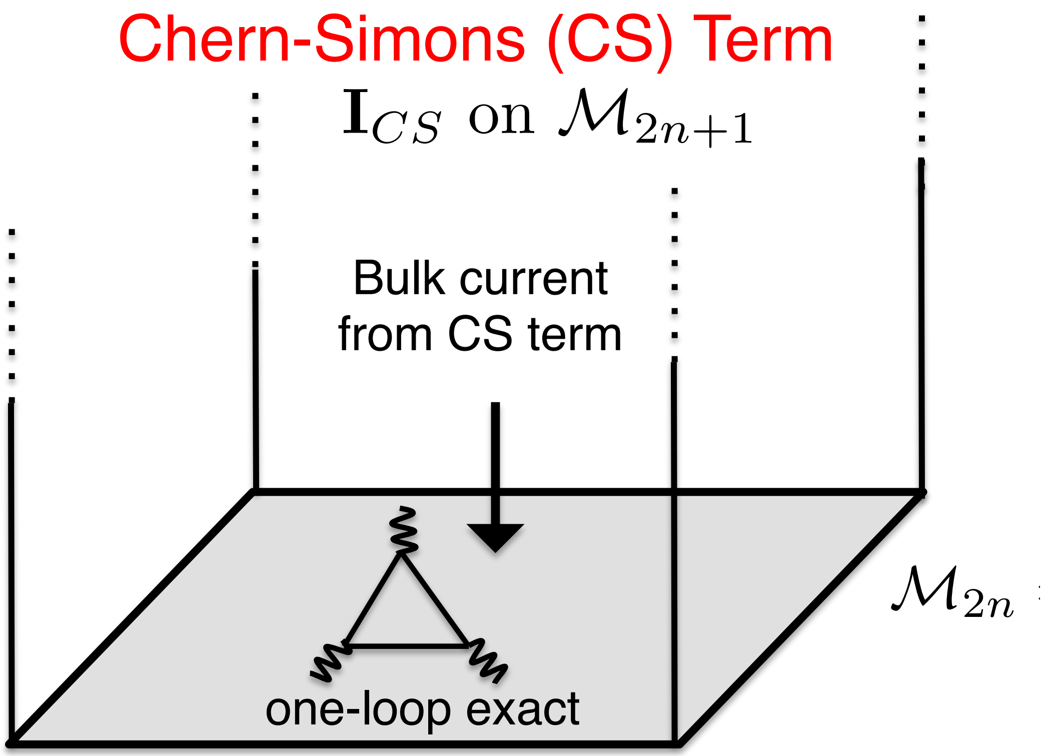
- Adler-Bardeen Theorem

  → Anomalies are one-loop exact

- Systematic study based on Anomaly Inflow Mechanism (next slide)

Anomaly Inflow Mechanism

[Callan-Harvey]



also Bardeen-Zumino current from CS term

Anomalies are classified by Anomaly Polynomials

$$\mathbf{P}_{anom}(\mathbf{F} = d\mathbf{A}, \mathbf{R} = d\mathbf{\Gamma} + \mathbf{\Gamma} \wedge \mathbf{\Gamma}) \equiv d\mathbf{I}_{CS}$$

Simple Examples

Anomaly	Chern-Simons Term $\mathbf{I}_{CS}(\mathbf{A},\mathbf{F},\mathbf{\Gamma},\mathbf{R})$	Anomaly Polynomial $\mathbf{P}_{anom}(\mathbf{F},\mathbf{R}) = d\mathbf{I}_{CS}$
2d U(1)	$\mathbf{A}\wedge\mathbf{F}$	$\mathbf{F}\wedge\mathbf{F}$
2d gravitational	$tr\left(\mathbf{\Gamma}\wedge d\mathbf{\Gamma}+\frac{2}{3}\mathbf{\Gamma}\wedge\mathbf{\Gamma}\wedge\mathbf{\Gamma}\right)$	$tr(\mathbf{R}\wedge\mathbf{R})$
4d U(1)	$\mathbf{A}\wedge\mathbf{F}\wedge\mathbf{F}$	$\mathbf{F}\wedge\mathbf{F}\wedge\mathbf{F}$
4d mixed	$\mathbf{A}\wedge tr(\mathbf{R}\wedge\mathbf{R})$	$\mathbf{F}\wedge tr(\mathbf{R}\wedge\mathbf{R})$
$\vdots$	$\vdots$	$\vdots$

$\mathbf{A} = A_a dx^a$: U(1) potential 1-form

$\mathbf{F} = d\mathbf{A}$: U(1) field-strength 2-form

${\mathbf{\Gamma}^a}_b = \Gamma^a_{bc}\, dx^c$: connection 1-form

${\mathbf{R}^a}_b = d{\mathbf{\Gamma}^a}_b + {\mathbf{\Gamma}^a}_c\wedge{\mathbf{\Gamma}^c}_b$: curvature 2-form

Anomalies at Finite Temperature

Big recent development!

Anomaly-Induced Transport [Son-Surowka, Bhattacharyya et.al. Erdmenger et.al., Torabian-Yee, …]

In hydrodynamic limit,
anomalies generate new type of transports

(example) U(1) current

without anomalies

$$J^\mu \propto u^\mu = \text{fluid velocity}$$

with anomalies

$$(J^\mu)_{anom} \propto V^\mu = \epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}} u_\nu (\partial_{\rho_1} u_{\rho_2}) \cdots (\partial_{\rho_{2n-3}} u_{\rho_{2n-2}})$$

To understand anomaly-induced transports systematically,
let's start with Thermal Helicity

Thermal Helicity

Setup

QFT on $\mathbb{R}^{2n-1,1}$ at finite temperature with global U(1) + Lorentz symmetry $\leftarrow$ Anomalous

T : Temperature μ : U(1) chemical potential

Thermal Helicity (per unit spatial volume)

[Loganayagam]

$$\frac{1}{\mathrm{vol}(\mathbb{R}^{2n-1})}\langle\hat{\mathfrak{L}}_{12}\hat{\mathfrak{L}}_{34}\ldots\hat{\mathfrak{L}}_{2n-3,2n-2}\hat{\mathcal{P}}_{2n-1}\rangle$$

$$\equiv -(n-1)!(2T)^{n-1}\mathfrak{F}[T,\mu]$$

$\hat{\mathfrak{L}}_{2k-1,2k}$: Angular momentum operator on (x^{2k-1}, x^{2k})-plane

$\hat{\mathcal{P}}_{2n-1}$: Translation operator in x^{2n-1}-direction

Computation of Thermal Helicity

Thermal Partition Function → Thermal Helicity

Thermal Partition Function on $\mathbb{R} \times S^{2n-1}$ (radius: R)

$$Z[\Omega] = tr_{\mathbb{R}\times S^{2n-1}} \exp\left(-\frac{H - \mu Q - \sum_{a=1}^{n} \Omega_a L_a}{T}\right)$$

= Generating Functional of Thermal Helicity

$$\frac{1}{\mathrm{vol}(\mathbb{R}^{2n-1})}\langle \hat{\mathfrak{L}}_{12}\hat{\mathfrak{L}}_{34}\dots\hat{\mathfrak{L}}_{2n-3,2n-2}\hat{\mathcal{P}}_{2n-1}\rangle = \lim_{R\to\infty}\frac{T^n}{\mathrm{vol}(S^{2n-1})R}\left[\left(\prod_{a=1}^{n}\frac{\partial}{\partial\Omega_a}\right)\log Z[\Omega]\right]_{\Omega_a=0}$$

$$\left(\begin{array}{ll} \text{Scaling in the flat space limit} & \\ L_k \to \hat{\mathfrak{L}}_{2k-1,2k} \quad (k=1,\cdots,n-1) & \text{('paired directions')} \\ L_n \to R\hat{\mathcal{P}}_{2n-1} & \text{('un-paired direction')} \end{array}\right)$$

Example

Example: 2d CFT with U(1)L x U(1)R

Anomaly Polynomial

$$\mathbf{P}_{anom}(\mathbf{F}_L, \mathbf{F}_R, \mathbf{R}) = (2\pi)\left[-\frac{c_R - c_L}{24}\left(\frac{1}{2(2\pi)^2} tr(\mathbf{R}^2)\right) + k_L\left(\frac{\mathbf{F}_L}{2\pi}\right)^2 - k_R\left(\frac{\mathbf{F}_R}{2\pi}\right)^2\right]$$

Cardy Formula for Entropy + 1st Law $\rightarrow$ $\langle L \rangle$

$$\langle L \rangle = \frac{2\pi R}{1 - R^2\Omega^2}\left(\frac{2R^2\Omega}{1 - R^2\Omega^2}\right)(2\pi)\left[\frac{c_R + c_L}{24} T^2 + k_R\left(\frac{\mu_R}{2\pi}\right)^2 + k_L\left(\frac{\mu_L}{2\pi}\right)^2\right]$$
$$+ \frac{\Omega^{-1} + R^2\Omega}{1 - R^2\Omega^2}\left(\frac{2\pi R^2\Omega}{1 - R^2\Omega^2}\right)(2\pi)\left[\frac{c_R - c_L}{24} T^2 + k_R\left(\frac{\mu_R}{2\pi}\right)^2 - k_L\left(\frac{\mu_L}{2\pi}\right)^2\right]$$

Thermal Helicity

$$\frac{1}{\mathrm{Vol}(\mathbb{R}^1)}\langle \hat{\mathcal{P}} \rangle = \lim_{R\to\infty}\left[\frac{T}{\mathrm{vol}(S^1)R}\frac{\partial}{\partial\Omega}\log Z[\Omega]\right]_{\Omega=0} = (2\pi)\left[\frac{c_R - c_L}{24} T^2 + k_R\left(\frac{\mu_R}{2\pi}\right)^2 - k_L\left(\frac{\mu_L}{2\pi}\right)^2\right]$$

$$= -\mathbf{P}_{anom}(\mathbf{F}_L \to \mu_L, \mathbf{F}_R \to \mu_R, tr(\mathbf{R^2}) \to 2(2\pi T)^2)$$

Relation to Anomaly Polynomial in General?

Replacement Rule for Thermal Helicity

Conjectured by [Loganayagam], [Loganayagam-Surowka]

$$\mathfrak{F}[T,\mu] = \mathbf{P}_{anom}(\mathbf{F} \to \mu, tr(\mathbf{R}^{2k}) \to 2(2\pi T)^{2k})$$

Determined Completely by Anomaly Polynomial

Analysis in General Dimensions

In higher-dim, still manageable in the hydrodynamic limit:

$$\log Z[\Omega] \simeq \log Z_{hydro} = -\int_{S^{2n-1}} \frac{1}{\gamma T} \mathcal{G}^t$$

<u>Gibbs Current</u> $\quad \mathcal{G}^\mu = \cdots + \hat{\mathfrak{F}} V^\mu + \cdots$

$\downarrow$ Integration of Gibbs current for rotating fluid on $\mathbb{R} \times S^{2n-1}$ cf. [Bhattacharrya et. al.]

<u>Partition Function</u> $\quad \log Z_{hydro}$

$\downarrow$ Generating functional (angular velocities in fluid velocity)

<u>Thermal Helicity</u> $\quad \mathfrak{F} = \hat{\mathfrak{F}}$

Thermal Helicity ⇔ Anomaly-Induced Gibbs Current

Replacement Rule for Anomaly-Induced Transport

Stress-Energy Tensor

$$(T_{\alpha\beta})_{anom} = -n\mathfrak{F}[\mu, T](u_\alpha V_\beta + u_\beta V_\alpha) + \cdots$$

U(1) current

$$(J_\alpha)_{anom} = -\frac{\partial \mathfrak{F}}{\partial \mu} V_\alpha + \cdots$$

Entropy current

$$(J^S_\alpha)_{anom} = -\frac{\partial \mathfrak{F}}{\partial T} V_\alpha + \cdots$$

with $V^\mu = \epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}} u_\nu (\partial_{\rho_1} u_{\rho_2}) \cdots (\partial_{\rho_{2n-3}} u_{\rho_{2n-2}})$

$$\mathfrak{F}[T, \mu] = \mathbf{P}_{anom}(\mathbf{F} \to \mu, tr(\mathbf{R}^{2k}) \to 2(2\pi T)^{2k})$$

Determined Completely by Anomaly Polynomial!

Proved by [Jensen-Loganayagam-Yarom]

Short Summary

Replacement Rule
for Anomaly-Induced Transports!

$$\mathfrak{F}[T,\mu] = \mathbf{P}_{anom}(\mathbf{F} \to \mu, tr(\mathbf{R}^{2k}) \to 2(2\pi T)^{2k})$$

Question

Replacement Rule from Gravity Dual?

cf. [Chapman, Neiman, Oz,… Kharzeev, Yee,…
Amado, Landsteiner, Megias, Melgar, Pena-Benitez, …]

Outline

(1) Replacement Rule From Gravity

(2) Replacement Rule and Black Hole Entropy

Replacement Rule From Gravity

Setup

CFT Side

Fluid with non-trivial anomaly-induced transports

→ U(1) charged rotating (conformal) fluid in 2n-dim

Setup on Gravity Side

Theory

- (2n+1)-d Einstein-Maxwell-Chern-Simons theory with negative cosmological const.
- CS Terms: U(1), Gravitational, Mixed
  → Same as those introduced in anomaly inflow

Configuration

U(1) charged rotating black hole (BH) on AdS_{2n+1}

Equations of Motion

EOM

$$R_{ab} - \frac{1}{2}(R - 2\Lambda)g_{ab} = 8\pi G_N[(T_M)_{ab} + (T_{CS})_{ab}]$$

$$\nabla_a F^{ab} = g_{YM}^2 (J_{CS})^a \qquad \Lambda = -\frac{d(d-1)}{2}$$

Maxwell part of stress-energy tensor

$$T_M^{ab} = \frac{1}{g_{YM}}\left(F^{ac}F^b{}_c - \frac{1}{4}F_{cd}F^{cd}\right)$$

CS part of stress-energy tensor and U(1) current

$$(T_{CS})^{ab} = \nabla_c(\Sigma)^{(ab)c} \qquad \star(\mathbf{\Sigma})^b{}_a = (\Sigma)^{cb}{}_a \star dx_c = 2\frac{\partial \mathbf{P}_{anom}}{\partial \mathbf{R}^a{}_b}$$

$$\star\mathbf{J}_{CS} = J_{CS}^c \star dx_c = \frac{\partial \mathbf{P}_{anom}}{\partial \mathbf{F}}$$

Gravity Dual of Anomalous Fluid (1)

<u>Difficulty</u>

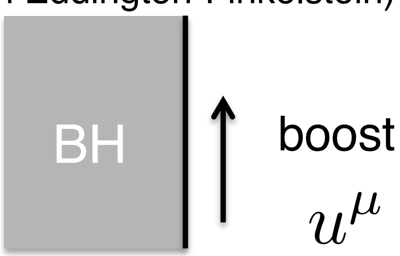
Want AdS charged rotating BHs,
but exact solution is not known for higher dim…

<u>Fluid/Gravity: AdS/CFT in Hydrodynamic Limit</u>

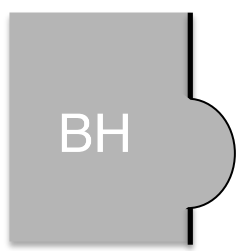
<u>Recipe</u>

[Bhattacharya-Hubeny-Minwalla-Rangamani]

(1) Static AdS BH
(in Eddington-Finkelstein)



(2) $u^\mu, T \to u^\mu(x), T(x)$



metric $g_{\mu\nu}^{(0)}$
(NOT solution)

(3) Derivative exp.
to solve EoM

$$g_{\mu\nu}^{(0)} + g_{\mu\nu}^{(1)} + \ldots$$

Boundary stress-energy tensor & U(1) current = Those for fluid

u^μ = fluid velocity

Gravity Dual of Anomalous Fluid (2)

Detail of Steps

(1) Start with EoM for Einstein-Maxwell theory and charged-AdS BH solution

(2) Carry out fluid/gravity expansion (up to 2nd order)

$$ds^2 = -2u_\mu dx^\mu dr + r^2[-f(r,m,q)u_\mu u_\nu + P_{\mu\nu}]dx^\mu dx^\nu + (\text{2nd order})$$

projection matrix

$$A = \Phi(r,q)u_\mu dx^\mu + \ (\text{2nd order})$$

electric potential

r : radial direction $\quad r_H$: horizon $(f(r_H,m,q)=0)$ $\quad r=\infty$: boundary

(3) Substitute to compute CS contribution to currents

‘Bulk Replacement Rule’

Chern-Simons contributions to bulk currents

→ evaluated directly from the fluid/gravity solution

$$(T_{CS})_{ab}dx^a dx^b = T^{(V)}_{CS}(dr + r^2 f u_\mu dx^\mu)V_\nu dx^\nu + \cdots$$

$$(J_{CS})_a dx^a = J^{(V)}_{CS} V_\mu dx^\mu + \cdots$$

$$\boxed{\begin{array}{c} J^{(V)}_{CS} = \dfrac{1}{r^{2n-3}}\dfrac{d}{dr}\dfrac{\partial \mathbb{G}^{(V)}}{\partial \Phi} \qquad T^{(V)}_{CS} = -\dfrac{1}{2r^{2n-1}}\dfrac{d}{dr}\left(r^2 \dfrac{d}{dr}\dfrac{\partial \mathbb{G}^{(V)}}{\partial \Phi_T}\right) \\ \text{with} \quad \mathbb{G}^{(V)} = \mathbf{P}_{anom}(\mathbf{F} \to \Phi, tr(\mathbf{R}^{2k}) \to 2\Phi_T^{2k}) \end{array}}$$

$$\Phi(r,q) = \frac{q}{r^{2n-2}} \qquad \Phi_T(r,m,q) = \frac{1}{2}r^2\frac{df}{dr} = \frac{1}{2r^{2n-1}}\left[(2n)m - \kappa_q(2n-1)\frac{q^2}{r^{2n-2}}\right]$$

Replacement Rule for Bulk!

Gravity Dual of Anomalous Fluid (3)

Detail of Steps

(1) Start with EoM for Einstein-Maxwell theory and charged-AdS BH solution

(2) Carry out fluid/gravity expansion (up to 2nd order)

(3) Substitute to compute CS contribution to currents

(4) Back reaction to metric & gauge field
→ leading order terms proportional to pseudo-vector

$$ds^2 = -2u_\mu dx^\mu dr + r^2[-f(r,m,q)u_\mu u_\nu + P_{\mu\nu}]dx^\mu dx^\nu + \cdots$$
$$+\underline{g_V(r,m,q)(u_\mu V_\nu + u_\nu V_\mu)dx^\mu dx^\nu} + \cdots$$

$$A = \Phi(r,q)u_\mu dx^\mu + \cdots$$
$$\underline{+a_V(r,m,q)V_\mu dx^\mu} + \cdots$$

CFT Replacement Rule

CFT Replacement Rule

Evaluate currents on a fixed r hypersurface and take $r \to \infty$

$$(J_\alpha)_{anom} = -\lim_{r\to\infty} \frac{\sqrt{-g}}{g_{EM}^2} g_{\alpha\mu}(F^{\mu r})_{anom} = V_\alpha \int_{r_H}^{\infty} dr'(r')^{2n-3} J_{CS}^{(V)}(r')$$

$$= -\left(\frac{\partial \mathbb{G}^{(V)}}{\partial \Phi}\right)_{r=r_H} V_\alpha$$

$$(T_{\alpha\beta})_{anom} = -\lim_{r\to\infty} \frac{r^{2n-2}}{8\pi G_N} (t_{\alpha\beta}^{Brown-York})_{anom}$$

$$= \left(\mathbb{G}^{(V)} - \Phi \frac{\partial \mathbb{G}^{(V)}}{\partial \Phi} - \Phi_T \frac{\partial \mathbb{G}^{(V)}}{\partial \Phi_T}\right)_{r=r_H} (V_\alpha u_\beta + V_\beta u_\alpha)$$

$$= \left(1 - l - \sum_{i=1}^{p} 2k_i\right) \mathbb{G}^{(V)}(r = r_H) = -n\mathbb{G}^{(V)}(r = r_H)$$

(note) $\mathbf{P}_{anom} = \mathbf{F}^l \wedge \prod_{i=1}^{p} tr(\mathbf{R}^{2k_i}) \quad \longrightarrow \quad \mathbb{G}^{(V)} = \Phi^l \prod_{i=1}^{p} 2\Phi_T^{2k_i}$

$2n + 2 = 2l + \sum_{i=1}^{p} 4k_i$

CFT Replacement Rule

CFT Replacement Rule

$$(T_{\alpha\beta})_{anom} = -n\mathbb{G}^{(V)}(r=r_H)(V_\alpha u_\beta + V_\beta u_\alpha)$$

$$(J_\alpha)_{anom} = -\left(\frac{\partial \mathbb{G}^{(V)}}{\partial \Phi}\right)_{r=r_H} V_\alpha$$

At horizon $\Phi_T(r_H) = 2\pi T \qquad \Phi(r_H) = \mu$

$$\longrightarrow \ \mathbb{G}^{(V)}(r=r_H) = \mathbf{P}_{anom}(\mathbf{F}\to\mu, tr(\mathbf{R}^{2k}) \to 2(2\pi T)^{2k})$$

Replacement Rule for CFT !

Comment : Higher Order Term

Metric and gauge field up to 2nd order are enough?

- Anomaly-induced contribution is higher-order in general …

$$V^{\mu} = \epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}} u_\nu (\partial_{\rho_1} u_{\rho_2}) \cdots (\partial_{\rho_{2n-3}} u_{\rho_{2n-2}})$$

→ AdS_7: 2 derivatives, AdS_9: 3 derivatives, …

- Actually, even metric and gauge fields at the 2nd order do not contribute to the (leading order) anomaly-induced transports in any dimensions

→ From the explicit form of the solution up to 2nd order, we can prove this “non-renormalization”!

Comment : Higher Order Term

Sketch of main ideas

- Currents $\sim$ derivatives of anomaly polynomial
  $\rightarrow$ wedge products of $\mathbf{F}$ and $\mathbf{R}$
- Anomaly-induced transport is fixed order in fixed dim
  $\rightarrow$ How to distribute derivatives?

(example) 3-derivative contribution to $\mathbf{F} \wedge \mathbf{F} \wedge \mathbf{F} \wedge \mathbf{F}$

$$\mathbf{A} = \Phi(r, q) u_\mu dx^\mu + \cdots \;\rightarrow\; \mathbf{F}^{(0)} \propto dr \wedge u_\mu dx^\mu$$

$$\mathbf{F}^{(0)} \wedge \mathbf{F}^{(0)} \wedge \mathbf{F}^{(0)} \wedge \mathbf{F}^{(3)} = \mathbf{F}^{(0)} \wedge \mathbf{F}^{(0)} \wedge \mathbf{F}^{(1)} \wedge \mathbf{F}^{(2)} = 0$$

$$\mathbf{F}^{(0)} \wedge \mathbf{F}^{(1)} \wedge \mathbf{F}^{(1)} \wedge \mathbf{F}^{(1)} \neq 0$$

To add higher order terms $\rightarrow$ To add a lot of 0^{th} order terms

- Some exceptions treated by symmetry + explicit form of 2^{nd} order metric and gauge field

Replacement Rule
and Black Hole Entropy

Anomaly-Induced Entropy

Replacement Rule for Entropy Current

Anomaly-induced entropy current $$(J^S_\alpha)_{anom} = -\frac{\partial \mathfrak{F}}{\partial T} V_\alpha + \cdots$$

with $$\mathfrak{F}[T,\mu] = \mathbf{P}_{anom}(\mathbf{F} \to \mu, tr(\mathbf{R}^{2k}) \to 2(2\pi T)^{2k})$$
$$V^\mu = \epsilon^{\mu\nu\rho_1\cdots\rho_{2n-2}} u_\nu (\partial_{\rho_1} u_{\rho_2}) \cdots (\partial_{\rho_{2n-3}} u_{\rho_{2n-2}})$$

Gravity Dual = Black Hole Entropy

- Einstein gravity → Bekenstein-Hawking formula

  [Bekenstein, Hawking]

- Covariant higher-derivative corrections → Wald formula

  $\delta_\chi L_{cov} = \mathcal{L}_\xi L_{cov}$

  [Wald, Lee-Wald, Iyer-Wald]

- Chern-Simons terms → "Tachikawa formula"

  [Tachikwa, Bonora et.al.]

  $\delta_\chi \mathbf{I}_{CS} = \mathcal{L}_\xi \mathbf{I}_{CS} + d(\ldots)$

CS Contribution to BH Entropy → Replacement Rule!

“BH Entropy is Noether Charge”

BH Entropy for Covariant Lagrangian

[Wald, Lee-Wald, Iyer-Wald]

$$\delta_\chi L_{cov} = \mathcal{L}_\xi L_{cov} \qquad \text{(example)}\ R_{abcd}R^{abcd}$$

- Killing vector $\xi = \partial_t + \Omega_H \partial_\phi$ $\longrightarrow$ $d\not\delta \mathbf{Q}_{\text{Noether}} = 0$

  $\not\delta(\ldots)$: cannot written as $\delta(\text{something})$

- 1st law of BH thermodynamics $T_H \delta S = \delta M + \Omega_H \delta J$

$$\longrightarrow \quad \underbrace{\int_\infty \not\delta \mathbf{Q}_{\text{Noether}}}_{\delta M + \Omega_H \delta J} = \underbrace{\int_{Horizon} \not\delta \mathbf{Q}_{\text{Noether}}}_{T_H \delta S}$$

- Correct result for any coordinates & gauges

Noether Procedure

How to construct differential Noether charges?

Point 1. Variation of Lagrangian

$$\delta \mathbf{L}(\phi) = \not{\delta}\mathbf{E} + d(\not{\delta}\mathbf{\Theta})$$

$\not{\delta}(...)$: cannot written as $\delta(\mathrm{something})$

Point 2. Pre-symplectic current

2-form on solution space (not spacetime)

$$d(\mathbf{\Omega}(\delta_1\phi, \delta_2\phi)) = \delta_1(\not{\delta}_2\mathbf{E}) - \delta_2(\not{\delta}_1\mathbf{E})$$

$\vdots$ Construction of on-shell vanishing Noether current …

Point 3. Differential Noether charge

$$d(\not{\delta}\mathbf{Q}_{Noether}) = \mathbf{\Omega}(\delta\phi, \delta_\chi\phi) + (\mathrm{on-shell\ vanishing\ terms})$$

How to integrate by part to get $\mathbf{\Omega}$ and then $\not{\delta}\mathbf{Q}_{Noether}$?

Wald Formalism and Extension

Key Point of Wald Formalism

A prescription for integration by part [Lee-Wald, Iyer-Wald]

$$d(\mathbf{\Omega}(\delta_1\phi, \delta_2\phi)) = \delta_1(\not{\delta}_2\mathbf{E}) - \delta_2(\not{\delta}_1\mathbf{E})$$
$$= d(-\delta_1(\not{\delta}_2\mathbf{\Theta}) + \delta_2(\not{\delta}_1\mathbf{\Theta}))$$

"Lagrangian-Based Prescription"

Extension to CS Term

- Some modification to take into account $\delta_\chi \mathbf{I}_{CS} = \mathcal{L}_\xi \mathbf{I}_{CS} + d(\ldots)$
  (pre-symplectic current is constructed as above) [Tachikawa]

- In 5d and higher, appropriate coordinate & gauge need to be taken to get desirable results … ???
  [Bonora et. al.]

Manifestly Covariant Formalism

Origin of Non-Covariance

$$\not\delta \boldsymbol{\Theta} = \delta \boldsymbol{\Gamma}^b{}_a \left(\frac{\partial \mathbf{I}_{CS}}{\partial \mathbf{R}^a{}_b} \right) + \delta \mathbf{A} \left(\frac{\partial \mathbf{I}_{CS}}{\partial \mathbf{F}} \right) + \cdots$$

$\longrightarrow$ Non-covariant $\boldsymbol{\Omega}$ and then $\not\delta \mathbf{Q}_{Noether}$

Manifestly Covariant Formalism

CS contribution to EoM～derivatives of anomaly polynomials

(example) $\delta \mathbf{A} \wedge tr(\mathbf{R} \wedge \mathbf{R}) = \delta \mathbf{A} \dfrac{\partial \mathbf{P}_{anom}}{\partial \mathbf{F}}$

$\rightarrow$ Integrate by part the defining eq. of pre-symp. current directly

$\longrightarrow$ Covariant $\boldsymbol{\Omega}$ and then $\not\delta \mathbf{Q}_{Noether}$

"EoM-Based Prescription"

Covariant Proof of "Tachikawa's Entropy Formula"

Implication of Our Result

Typical Microstate Counting for Black Hole Entropy

→ "Map to CFT_2 entropy counting" → Cardy Formula

(example) BTZ BH, (near) extremal BHs

Black Holes in higher-dimensional AdS spacetime

- Dual higher-dim CFTs do not have neither infinite dimensional symmetries nor modular invariance

→ Difficult to compute entropy in CFT

cf. supersymmetric index in 4d [Komargodski et.al.]

Our Result + Replacement Rule

By using replacement rule, we can compute CS part of entropy for **higher-dim finite temperature BH from CFT!**

Summary

Anomaly polynomials play crucial roles!

1. BH entropy formula for CS terms
   $\to$ Manifestly covariant formulation

2. Holography for CFT with anomalies at finite temp.
   $\to$ Replacement rule reproduced