String-brane scattering

Tidal excitations and time delays

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Outline

We review two topics in the dynamics of strings at high energy, related by a common theme

the Regge behaviour of string amplitudes

- The eikonal operator: its meaning and its derivation
- How string theory avoids potential violations of causality that can occur in gravity theories with higher derivative corrections

Work done in collaboration with Paolo Di Vecchia, Rodolfo Russo and Gabriele Veneziano

Outline

- String-string and string-brane collisions at high energy
- The eikonal operator: covariant vs light-cone dynamics
- Inelastic amplitudes and the Reggeon vertex
- Time delays

String theory at high energy

- There is a large parameter... it may simplify the analysis of the dynamics
- The dynamics remains very interesting
  - States of arbitrary mass and spin
  - Large energy causes a large backreaction of the spacetime
- Well-defined framework: S-matrix, unitarity, UV complete

String-string collisions

Relevant scales

$$\alpha' s \gg 1 \ , \qquad R_g^{d-3} \sim G_d \sqrt{s} \ , \qquad b_T^{d-2} \sim G_d \alpha' s$$

Various possible processes as the impact parameter is varied

- $b \gg b_T \gg R_g$ — elastic scattering
- $b_T \geq b \gg R_g$ — string tidal excitations
- $b < R_g \begin{cases} R_g \ll l_s & \text{creation of closed strings} \\ R_g \gg l_s & \text{formation of a black hole} \end{cases}$

Dynamical effective geometry

String-brane collisions

Relevant scales

$$\alpha' s \gg 1 \ , \qquad \left(\frac{R}{l_s}\right)^{7-p} \sim \ g\,N \ , \qquad b_T^{8-p} \sim \alpha' E R^{7-p}$$

Various possible processes as the impact parameter is varied

- $b \gg b_T \gg R$ elastic scattering
- $b_T \geq b \gg R$ string tidal excitations
- $b < R \begin{cases} R \ll l_s & \text{creation of open strings} \\ & \\ R \gg l_s & \text{infall into the singularity} \end{cases}$

Fixed effective background: extremal p-brane metric

The eikonal operator

Regge limit of the disk amplitude (tree level)

$$\mathcal{A}_1(s,t) \sim \Gamma\left(-\frac{\alpha'}{4}t\right) e^{-i\pi\frac{\alpha' t}{4}} \, (\alpha' s)^{1+\frac{\alpha' t}{4}} \ ,$$

$$s = E^2 \ , \quad t = -(p_1+p_2)^2$$

Grows too fast with energy. Include higher-orders.

Regge limit of the annulus amplitude (one loop)

$$\frac{\mathcal{A}_2(s,t)}{2E} = \frac{i}{2}\int \frac{d^{8-p}\mathbf{k}_1}{(2\pi)^{8-p}} \, \frac{\mathcal{A}_1(s,t_1)}{2E} \, \frac{\mathcal{A}_1(s,t_2)}{2E} \, V_2(t_1,t_2,t)$$

$$t = -\mathbf{q}^2 \ , \qquad t_1 = -\mathbf{k}_1^2 \, , \qquad t_2 = -\mathbf{k}_2^2 \equiv (\mathbf{q}-\mathbf{k})^2$$

Momenta transverse to the brane and the collision axis.

The eikonal operator

Simple operator representation for the two-Reggeon vertex

$$V_2(t_1, t_2, t) = \int_0^{2\pi}\int_0^{2\pi} \frac{d\sigma_1}{2\pi}\frac{d\sigma_2}{2\pi} \langle 0| : e^{i\mathbf{k}_1 X(\sigma_1)} :: e^{i\mathbf{k}_2 X(\sigma_2)} : |0\rangle$$

$X(\sigma)$: closed string position operators at $\tau = 0$.
This structure extends to surfaces with h boundaries

$$\frac{\mathcal{A}_h(s,t)}{2E} \sim \frac{1}{h!}\frac{i^{h-1}}{(2E)^h}\prod_{i=1}^{h-1}\int \frac{d^{8-p}\mathbf{k}_i}{(2\pi)^{8-p}}\, \mathcal{A}_1(s,t_1)..\mathcal{A}_1(s,t_h) V_h(\mathbf{k}_1, \mathbf{k}_2, .., \mathbf{k}_h)$$

$$V_h(\mathbf{k}_1, \ldots, \mathbf{k}_h) = \langle 0| \prod_{i=1}^{h}\int_0^{2\pi}\frac{d\sigma_i}{2\pi} : e^{i\mathbf{k}_i X(\sigma_i)} : |0\rangle$$

The eikonal operator

In impact parameter space

$$\mathcal{A}(E,b) = \int \frac{d^{8-p}\mathbf{q}}{(2\pi)^{8-p}}\, \mathrm{e}^{i\mathbf{b}\cdot\mathbf{q}}\, \mathcal{A}(E,t)$$

we can sum explicitly the series

$$i \sum_{h=1}^{\infty} \frac{\mathcal{A}_h(s,\mathbf{b})}{2E} \sim \langle 0| \left[e^{2i\hat{\delta}(s,b)} - 1 \right] |0\rangle$$

The result is the eikonal operator

$$S(s,b) = e^{2i\hat{\delta}(s,b)} \;,$$

$$2\hat{\delta}(s,b) = \int_0^{2\pi} \frac{d\sigma}{2\pi} \frac{:\mathcal{A}_1\left(s, \mathbf{b}+\mathbf{X}(\sigma)\right):}{2E}$$

Amati, Ciafaloni e Veneziano (1987)
GD, Di Vecchia, Russo e Veneziano (2010).

The eikonal operator

Two main effects

- Deflection of the trajectory $\theta = -\frac{2}{E}\frac{\partial\delta(s,b)}{\partial b}$
- Excitation of the internal degrees of freedom of the string: tidal forces

When $b \gg R \gg l_s\sqrt{\ln(\alpha' s)}$

$$2\,\hat{\delta}(s,\mathbf{b}+\hat{\mathbf{X}}) \sim \frac{1}{2E}\left[\mathcal{A}_1(s,b) + \frac{1}{2}\frac{\partial^2\mathcal{A}_1(s,b)}{\partial b^i \partial b^j}\,\overline{\hat{X}^i\hat{X}^j} + \ldots\right]$$

where $\bar{Q} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\sigma : Q(\sigma) :$
The string position operators are

$$X^i = i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\left(\frac{A_n^i}{n}\mathrm{e}^{in\sigma} + \frac{\bar{A}_n^i}{n}e^{-in\sigma}\right)\ , \quad [A_n^i, A_m^j] = n\delta^{ij}\delta_{n+m,0}$$

The eikonal operator

Existence and form of the eikonal operator deduced from the elastic amplitude. An inclusive sum over the intermediate states.
Natural interpretation: Hilbert space of the string quantized in a light-cone gauge aligned to the collision axis

- special kinematics of the Regge limit
- original derivation
- quantization of the σ-model in the light-cone gauge
  - string-string: Aichelburg-Sexl
  - string-brane: Penrose limit of the extremal p-brane

Natural questions:

- Is it correct?
- Can it be derived from the covariant dynamics?

The eikonal operator

Some interesting features

- Remarkably compact description of the inelastic amplitudes
- the string modes appear as a simple shift of the impact parameter b by the string position operator X
- it does not contain the light-cone modes of the fermionic fields
- Its simple structure encodes the dynamics of both the transverse and longitudinal polarizations of the string states

Let us assume it is correct and study some examples, e.g. the inelastic transitions from a massless string to a massive string.

The eikonal operator

NS sector states created by the action of A^i_{-n} and B^i_{-r}
Characterized by their mass level and $SO(8)$ representation
Polarization tensors ω corresponding to Young diagrams and normalized, $\omega \cdot \omega = 1$.
Work in momentum space

$$W(s,\bar{q}) \;=\; \mathcal{A}(s,t)\int_0^{2\pi}\frac{d\sigma}{2\pi}\;:\mathrm{e}^{i\bar{q}\hat{X}}:\equiv \mathcal{A}(s,t)\sum_{n,m=0}^{\infty}\Delta_{n,m}(\bar{q})\,\bar{\Delta}_{n,m}(\bar{q})$$

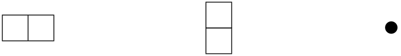
The operators $\Delta_{n,m}$ generate all the transitions between an initial level m and a final level n

$$\Delta_{1,0} \;=\; -\sqrt{\frac{\alpha'}{2}}\,\bar{q}^i\,A^i_{-1}$$

$$\Delta_{2,0} \;=\; \frac{\alpha'}{4}\bar{q}^i\bar{q}^j\,A^i_{-1}A^j_{-1}-\sqrt{\frac{\alpha'}{8}}\bar{q}^i\,A^i_{-2}$$

The eikonal operator

Transitions to the first level



$SO(8)$ representation	Matrix element $\langle \omega \vert \Delta_{10} \vert \epsilon \rangle$
$\lvert\omega^{(2)}\rangle = \omega^{(2)}_{ij} A^i_{-1} B^j_{-\frac{1}{2}} \lvert 0\rangle$	$-\sqrt{\frac{\alpha'}{2}}\, \epsilon^i\, \omega^{(2)}_{ij}\, \bar{q}^j$
$\lvert\omega^{(1,1)}\rangle = \omega^{(1,1)}_{ij} A^i_{-1} B^j_{-\frac{1}{2}} \lvert 0\rangle$	$\sqrt{\frac{\alpha'}{2}}\, \epsilon^i\, \omega^{(1,1)}_{ij}\, \bar{q}^j$
$\lvert\omega^{(0)}\rangle = \frac{1}{\sqrt{8}}\, A^i_{-1} B^i_{-\frac{1}{2}} \lvert 0\rangle$	$-\frac{\sqrt{\alpha'}}{4}\, \epsilon \bar{q}$

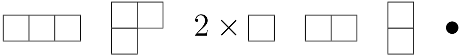
Subleading transition amplitudes to the remaining 64 NS states

$$\lvert\omega^{(1,1,1)}\rangle = \frac{1}{\sqrt{6}} \omega^{(1,1,1)}_{ijk} B^i_{-\frac{1}{2}} B^j_{-\frac{1}{2}} B^k_{-\frac{1}{2}} \lvert 0\rangle \ , \qquad \lvert\omega^{(1)}\rangle = \omega_i B^i_{-\frac{3}{2}} \lvert 0\rangle$$

The eikonal operator

Transitions to the second level

352 states



$SO(8)$ representation	Matrix element
$\vert\omega^{(3)}\rangle = \frac{1}{\sqrt{2}}\omega^{(3)}_{ijk} A^i_{-1} A^j_{-1} B^k_{-\frac{1}{2}} \vert 0\rangle$	$\frac{\alpha'}{\sqrt{8}}\epsilon^i\, \omega^{(3)}_{ijk}\bar{q}^j\bar{q}^k$
$\vert\omega^{(2,1)}\rangle = \sqrt{\frac{2}{3}}\omega^{(2,1)}_{ij;k} A^i_{-1} A^j_{-1} B^k_{-\frac{1}{2}} \vert 0\rangle$	$-\frac{\alpha'}{\sqrt{6}}\epsilon^i\, \omega^{(2,1)}_{jk;i}\bar{q}^j\bar{q}^k$
$\vert\omega^{(2)}\rangle = \frac{1}{\sqrt{2}}\omega^{(2)}_{ij} A^i_{-2} B^j_{-\frac{1}{2}} \vert 0\rangle$	$\frac{\sqrt{\alpha'}}{2}\epsilon^i\, \omega^{(2)}_{ij}\bar{q}^j$
$\vert\omega^{(1,1)}\rangle = \frac{1}{\sqrt{2}}\omega^{(1,1)}_{ij} A^i_{-2} B^j_{-\frac{1}{2}} \vert 0\rangle$	$-\frac{\sqrt{\alpha'}}{2}\epsilon^i\, \omega^{(1,1)}_{ij}\bar{q}^j$
$\vert\omega^{(1)}\rangle = -\frac{\omega_i}{4\sqrt{35}}\left[8A^i_{-1}A^j_{-1}B^j_{-\frac{1}{2}} - A^j_{-1}A^j_{-1}B^i_{-\frac{1}{2}}\right]\vert 0\rangle$	$-\frac{\alpha'}{\sqrt{35}}\left(\epsilon\bar{q}\omega\bar{q} + \frac{\alpha' t}{8}\epsilon\omega\right)$
$\vert\lambda^{(1)}\rangle = \frac{\lambda_i}{4} A^j_{-1} A^j_{-1} B^i_{-\frac{1}{2}} \vert 0\rangle$	$-\frac{\alpha' t}{8}\epsilon\lambda$
$\vert\omega^{(0)}\rangle = \frac{1}{4}\, A^i_{-2} B^i_{-\frac{1}{2}} \vert 0\rangle$	$-\frac{\sqrt{\alpha'}}{4\sqrt{2}}\epsilon\bar{q}$

Longitudinal polarizations

Massless particle

$$p_1 = (E, p\hat{p}_1) \ , \qquad p_1^2 = 0$$

Frame

$$t^\mu \ , \quad \hat{p}_1^\mu \ , \quad \tilde{e}_i^\mu$$

Massive particle

$$p_2 = (E, p\hat{p}_2) \ , \qquad p_2^2 = -m^2$$

Frame

$$\frac{p_2^\mu}{m} \ , \quad v^\mu \ , \quad e_i^\mu$$

Longitudinal vector

$$v^\mu = \frac{p}{m} t^\mu + \frac{E}{m} \hat{p}_2^\mu = \frac{E}{p} \frac{p_2^\mu}{m} - \frac{m}{p} t^\mu$$

Longitudinal polarizations

Decomposition of the momentum transfer $q = p_1 + p_2$

$$q^\mu = \frac{m}{2}\left(1 + \frac{t}{m^2}\right)\left(\frac{p_2^\mu}{m} + v^\mu\right) + \bar{q}^\mu$$

When q is contracted with a massless polarization becomes transverse

$$\epsilon q = \epsilon \bar{q}$$

When q is contracted with a massive polarization we find

$$\zeta q = \zeta \bar{q} + \frac{m}{2}\left(1 + \frac{t}{m^2}\right)\zeta v$$

Therefore

$$\bar{q}^\rho \sim q^\rho - \frac{m}{2}\left(1 + \frac{t}{m^2}\right) v^\rho$$

Longitudinal polarizations

Using transversality and the on-shell condition

$$\eta^{\mu\nu} \sim \delta_\perp^{\mu\nu} \ , \quad \eta^{\rho\sigma} \sim v^\rho v^\sigma + \delta_\perp^{\mu\nu} \ , \quad \eta^{\mu\rho} \sim -\frac{\bar{q}^\mu}{m} v^\rho + \delta_\perp^{\mu\rho}$$

μ, ν massless and ρ, σ massive polarization indexes

$$\eta^{\mu\rho} = -\frac{p_2^\mu}{m}\frac{p_2^\rho}{m} + v^\mu v^\rho + e_i^\mu e_i^\rho \sim v^\mu v^\rho + e_i^\mu e_i^\rho \sim -\frac{\bar{q}^\mu}{m} v^\rho + e_i^\mu e_i^\rho$$

since

$$\epsilon v = -\frac{E}{p}\frac{\epsilon q}{m} \sim -\frac{\epsilon q}{m}$$

Light-cone basis

$$\sqrt{2} e^+ = -\hat{t} + \hat{p}_2 \ , \qquad \sqrt{2} e^- = -\hat{t} - \hat{p}_2$$

Product with a massive polarization

$$\sqrt{2} e^+ \ = \ -\left(1 + \frac{E}{p}\right)\hat{t} + \frac{p_2}{p} \sim \left(1 + \frac{E}{p}\right)\left(\frac{p}{m} v + \frac{E}{m^2} p_2\right) \sim \frac{2E}{m} v$$

The Reggeon vertex operator

Regge limit: $\alpha' s \gg 1$, $\alpha' t$ fixed

Interactions mediated by the exchange of the leading Regge trajectory in the t-channel

Regge behaviour $\qquad \mathcal{A}(s,t) \sim (\alpha' s)^{a(t)}$

Elegant description in terms of an effective string state: the Reggeon

$$\mathcal{A}(s,t) \sim \Pi_R^{D_p} \, C_{12R} \, \bar{C}_{12R} \ .$$

- Factorized form for the four (two) point amplitudes
- Process independent propagator (tadpole)
- Evaluation of three-point couplings

The Reggeon vertex operator

Derivation of the Reggeon vertex operator

Ademollo, Bellini, Ciafaloni (1989)

Brower, Polchinski, Strassler, Tan (2006)

- Regge limit $\sim$ limit of short worldsheet distances
- Identification of the dominant intermediate states
- Sum over intermediate states $\rightarrow$ single local operator

Vertex operator for a generic external state $(S, \bar{S})$

$$\mathcal{V}_{(S,\bar{S})} \sim \epsilon_{\mu_1 \ldots \mu_k} V_S^{\mu_1 \ldots \mu_k} \quad \bar{\epsilon}_{\nu_1 \ldots \nu_l} V_{\bar{S}}^{\nu_1 \ldots \nu_l}$$

$$V_S = \mathrm{Pol}\left[\partial^r X^\mu, \partial^s \psi^\nu\right] \, e^{ipX}$$

$$\epsilon_{\mu_1 \ldots \mu_k} \epsilon^{\mu_1 \ldots \mu_k} = \bar{\epsilon}_{\nu_1 \ldots \nu_l} \bar{\epsilon}^{\nu_1 \ldots \nu_l} = 1$$

The Reggeon vertex operator

Main dependence on the Mandelstam variables from the correlation of the exponential part of the vertex operators

$$e^{-\frac{\alpha' t}{4}\ln|z|^2-\frac{\alpha' s}{4}\ln|1-z|^2} \quad\rightarrow\quad z\alpha' s\ \sim\ 1$$

Factorization of the amplitude in the t-channel

$$A_{12}\sim\sum_l\int d^2z\ (z\bar{z})^{l-1-\frac{\alpha' t}{4}}\left\langle \mathcal{V}^{(-1,-1)}_{(S_1,\bar{S}_1)}\,\mathcal{V}^{(0,0)}_{(S_2,\bar{S}_2)}\mathcal{V}^{(-1,-1)}_{l,n_l,\bar{n}_l}\right\rangle_{\mathcal{S}}\left\langle \mathcal{V}^{(-1,-1)}_{l,n_l,\bar{n}_l}\right\rangle_{D_p}$$

Three-point coupling, propagator, tadpole

Count the factors of E in the three-point couplings

The Reggeon vertex operator

- OPE of $\partial^r X^\mu$

$$\sqrt{2}\partial^r X^+ \, e^{ip_1 X_1} \sim \alpha' E \, \partial_z^{r-1} \left(\frac{1}{z-w} \right) e^{ip_1 X_1}$$

$$\sqrt{2}\partial^r X^+ \, \partial^s X^\rho \sim -\alpha' \frac{E}{m} \, v^\rho \, \partial_z^{r-1} \partial_w^{s-1} \left(\frac{1}{z-w} \right)$$

- OPE of $\partial^r \psi^\mu$

$$\sqrt{2}\partial^r \psi^+ \, \partial^s \psi^\rho \sim \frac{2E}{m} \, v^\rho \, \partial_z^r \partial_w^s \left(\frac{1}{z-w} \right)$$

Leading intermediate states

$$Q_l \sim \psi^+ \left(\partial X^+ \right)^l e^{-iqX}$$

The leading Regge trajectory

The Reggeon vertex operator

Perform the sum and integrate over z

$$\mathcal{A}_{12} \sim \Pi_R^{D_p} \; C_{S_1,S_2,R} \, \bar{C}_{\bar{S}_1,\bar{S}_2,\bar{R}}$$

Three-point coupling with the Reggeon

$$C_{S_1,S_2,R} = \left\langle V_{S_1}^{(-1)} V_{S_2}^{(0)} V_R^{(-1)} \right\rangle$$

Reggeon tadpole

$$\Pi_R^{D_p} = \mathcal{A}_1(s,t) = \frac{\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})} R_p^{7-p} \, \Gamma\left(-\frac{\alpha' t}{4}\right) \mathrm{e}^{-i\pi\frac{\alpha' t}{4}} (\alpha' s)^{1+\frac{\alpha' t}{4}}$$

Reggeon vertex operator (picture (-1))

$$V_R^{(-1)} = \frac{\psi^+}{\sqrt{\alpha'} E} \left(\sqrt{\frac{2}{\alpha'}} \frac{i\partial X^+}{\sqrt{\alpha'} E} \right)^{\frac{\alpha' t}{4}} \mathrm{e}^{-iqX} \quad .$$

The Reggeon vertex operator

The Reggeon vertex is a superconformal primary of dimension one half in the high-energy limit

Picture zero

$$V_R^{(0)} = \left[-\frac{2}{\alpha'}\frac{\partial X^+ \partial X^+}{\alpha' E^2} - iq\psi\frac{\psi^+ \partial X^+}{\alpha' E^2} - \frac{\alpha' t}{4}\frac{\psi^+ \partial \psi^+}{\alpha' E^2}\right] \left(\sqrt{\frac{2}{\alpha'}}\frac{i\partial X^+}{\sqrt{\alpha'}E}\right)^{\frac{\alpha' t}{4}-1} \mathrm{e}^{-iqX_L}$$

Reggeon three-point couplings

$$C_{S_1,S_2,R} = \epsilon_{\mu_1\ldots\mu_r}\ \zeta_{\nu_1\ldots\nu_s}\ T_{S_1,S_2,R}^{\mu_1\ldots\mu_r;\nu_1\ldots\nu_s}$$

The Regge limit of the inelastic amplitudes

Transitions from the ground state

$$\mathcal{A}_{g,(S,\bar{S})} = \Pi_R^{D_p} C_{g,S,R} \bar{C}_{g,\bar{S},R}$$

Let us start from the elastic amplitude. The massless vertex in the -1 picture is

$$V_g^{\mu} = \psi^{\mu} \mathrm{e}^{ipX} \, \mathrm{e}^{-\varphi}$$

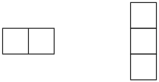
and we find

$$T_{g,g,R}^{\mu;\rho} = \eta^{\mu\rho}$$

$$\mathcal{A}_{g,g} = \Pi_R^{D_p} \epsilon_{\mu\nu} \zeta^{\mu\nu}$$

The Regge limit of the inelastic amplitudes

First massive level, NS sector: 128 bosonic physical states.
S_2 (44 components) and A_3 (84 components)



The corresponding vertex operators in the -1 picture are

$$
\begin{aligned}
V_{S_2}^{\rho\alpha} &= i\sqrt{\frac{2}{\alpha'}}\,\psi^\rho \partial X^\alpha \mathrm{e}^{ipX}\,\mathrm{e}^{-\varphi} \\
V_{A_3}^{\rho\alpha\gamma} &= \frac{1}{\sqrt{3!}}\,\psi^\rho\psi^\alpha\psi^\gamma \mathrm{e}^{ipX}\,\mathrm{e}^{-\varphi}
\end{aligned}
$$

The Regge limit of the inelastic amplitudes

Transition $g \to S_2$

$$V_{S_2}^{\rho\alpha} = i\sqrt{\frac{2}{\alpha'}}\ \psi^\rho \partial X^\alpha \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi}$$

$$\begin{aligned} T^{\mu;\rho\alpha}_{g,S_2,R} &= -\sqrt{\frac{\alpha'}{2}}\left[\eta^{\mu\rho}\left(q^\alpha - \frac{2}{\alpha'}\left(1+\frac{\alpha' t}{4}\right)\frac{v^\alpha}{m}\right)\right. \\ &+ \left.\frac{q^\mu}{m}v^\rho\left(q^\alpha - \frac{t}{2m}v^\alpha\right)\right] \end{aligned}$$

Using transverse tensors

$$T^{\mu;\rho\alpha}_{g,S_2,R} = -\sqrt{\frac{\alpha'}{2}}\left[\delta_\perp^{\mu(\rho}\bar{q}^{\alpha)} + \frac{q^\mu}{2}v^\rho v^\alpha\right]$$

The Regge limit of the inelastic amplitudes

Transition g $\to A_3$

$$V_{A_3}^{\rho\alpha\gamma} = \frac{1}{\sqrt{3!}}\,\psi^\rho\psi^\alpha\psi^\gamma \mathrm{e}^{ipX}\,\mathrm{e}^{-\varphi}$$

$$T_{g,A_3,R}^{\mu;\rho\alpha\gamma} = \frac{\sqrt{6}}{m}\eta^{\mu[\rho}q^\alpha v^{\gamma]}$$

Using transverse tensors

$$T_{g,A_3,R}^{\mu;\rho\alpha\gamma} = \frac{\sqrt{6}}{m}\delta_\perp^{\mu[\rho}\bar{q}^\alpha v^{\gamma]}$$

The Regge limit of the inelastic amplitudes

From $SO(9)$ to $SO(8)$

$$\zeta^{S} \mapsto \sum \zeta^{S,(n_1,n_2,\ldots,n_r)} \ .$$

For S_2

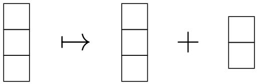
$$\square\!\square \ \mapsto \ \square\!\square \ + \ \square \ + \ \bullet$$

$$\zeta^{S_2,(2)}_{\rho\alpha} = \omega^{(2)}_{\rho\alpha} \ , \quad \zeta^{S_2,(1)}_{\rho\alpha} = \sqrt{2}\,\omega_{(\rho}v_{\alpha)} \ , \quad \zeta^{S_2,(0)}_{\rho\alpha} = \frac{1}{3\sqrt{8}}\left(-\delta^{\rho\alpha}_{\perp} + 8v^{\rho}v^{\alpha}\right)$$

$$\begin{aligned} T^{\mu,\rho\alpha}_{g,S_2,R}\ \zeta^{S_2,(2)}_{\rho\alpha} &= -\sqrt{\frac{\alpha'}{2}}\delta^{\mu\rho}_{\perp}\omega^{(2)}_{\rho\alpha}\bar{q}^{\alpha} \ , \\ T^{\mu,\rho\alpha}_{g,S_2,R}\ \zeta^{S_2,(1)}_{\rho\alpha} &= 0 \ , \\ T^{\mu,\rho\alpha}_{g,S_2,R}\ \zeta^{S_2,(0)}_{\rho\alpha} &= -\frac{\sqrt{\alpha'}}{4}\bar{q}^{\mu} \ . \end{aligned}$$

The Regge limit of the inelastic amplitudes

For A_3



$$\zeta^{A_3,(1,1,1)}_{\rho\alpha\gamma} = \omega^{(1,1,1)}_{\rho\alpha\gamma} \ , \qquad \zeta^{A_3,(1,1)}_{\rho\alpha\gamma} = \sqrt{3}\,\omega^{(1,1)}_{[\rho\alpha}v_{\gamma]} \ .$$

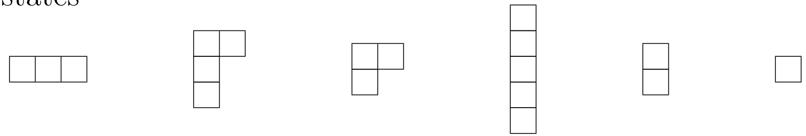
$$\begin{aligned} T^{\mu,\rho\alpha\gamma}_{g,A_3,R}\ \zeta^{A_3,(1,1,1)}_{\rho\alpha\gamma} &= 0 \ , \\ T^{\mu,\rho\alpha\gamma}_{g,A_3,R}\ \zeta^{A_3,(1,1)}_{\rho\alpha\gamma} &= \sqrt{\frac{\alpha'}{2}}\delta^{\mu\rho}_{\perp}\omega^{(1,1)}_{\rho\alpha}\bar{q}^{\alpha} \ . \end{aligned}$$

States in the first massive level that can be excited

$$S = \zeta^{\mathcal{S}_2,(2)}S_2 \ , \qquad A = \zeta^{\mathcal{A}_3,(1,1)}A_3 \ , \qquad I = \zeta^{\mathcal{S}_2,(0)}S_2$$

A total of 64 degrees of freedom. Agreement with the light-cone gauge.

Second massive level, NS sector: 1152 bosonic physical states



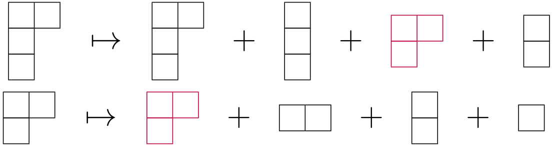
$$
\begin{aligned}
V_Z^{\rho\alpha\gamma} &= -\frac{1}{\sqrt{2}}\frac{2}{\alpha'}\partial X^\rho \partial X^\alpha \psi^\gamma \, \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \ , \\
V_Y^{\rho\alpha\gamma\omega} &= i\sqrt{\frac{3}{8}}\sqrt{\frac{2}{\alpha'}}\partial X^{(\rho}\psi^{\alpha)}\psi^\gamma\psi^\omega \, \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \ , \\
V_U^{\rho\alpha\gamma} &= -\frac{1}{\sqrt{6}}\left[\frac{2}{\alpha'}\partial X^\rho \partial X^\alpha \psi^\gamma + 2\partial\psi^{(\rho}\psi^{\alpha)}\psi^\gamma\right] \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \ , \\
V_X^{\rho\alpha\gamma\omega\xi} &= \frac{1}{\sqrt{5!}}\,\psi^\rho\psi^\alpha\psi^\gamma\psi^\omega\psi^\xi \, \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \ ,
\end{aligned}
$$

$$
V_V^{\gamma\omega} = \frac{2}{5}\frac{i}{\sqrt{7\alpha'}}\left[\left(\hat{\eta}_{\rho\alpha}\partial X^\rho\psi^\alpha\right)\psi^\gamma\psi^\omega - \frac{7}{2}\left(\partial^2 X^\gamma\psi^\omega - 2\partial X^\gamma\partial\psi^\omega\right)\right] \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \ ,
$$

$$
V_W^\gamma = \frac{\hat{\eta}_{\alpha\rho}}{8\sqrt{22}}\left[-\frac{2}{\alpha'}\partial X^\gamma\partial X^\alpha\psi^\rho + 5\frac{2}{\alpha'}\partial X^\rho\partial X^\alpha\psi^\gamma + 11\partial\psi^\rho\psi^\alpha\psi^\gamma\right] \mathrm{e}^{ipX}\, \mathrm{e}^{-\varphi} \ .
$$

The Regge limit of the inelastic amplitudes

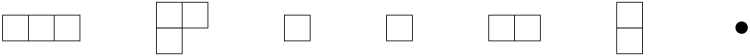
An example of a degenerate representation



$$H_1 = \frac{1}{2}\zeta^{Y,(2,1)}\,Y - \frac{\sqrt{3}}{2}\zeta^{U,(2,1)}\,U \ , \quad C_{g,H_1,R} = 0$$

$$H_2 = \frac{\sqrt{3}}{2}\zeta^{Y,(2,1)}\,Y + \frac{1}{2}\zeta^{U,(2,1)}\,U \ , \quad C_{g,H_2,R} = -\frac{\alpha'}{\sqrt{6}}\epsilon^{\alpha}\omega^{(2,1)}_{\rho\gamma;\alpha}\bar{q}^{\rho}\bar{q}^{\gamma}$$

States in the second massive level that can be excited



A total of 352 degrees of freedom. Agreement with the light-cone gauge.

The Regge limit of the inelastic amplitudes

In general

- d_r: multiplicity of the representation r of $SO(8)$
- c_r: number of independent couplings
- $(d_r - c_r)$: number of decoupled states

Can we derive the simple form of the eikonal operator from the covariant dynamics?
We can write the phase of the eikonal operator as follows

$$W_R(s,q) = 4E\hat{\delta}(s,q) = \Pi_R^{D_p} \sum_{i,\bar{i},j,\bar{j}} C_{(S_i,S_{\bar{i}}),(S_j,S_{\bar{j}}),R} \left|S_i, S_{\bar{i}}\right\rangle \left\langle S_j, S_{\bar{j}}\right|$$

Choose a suitable basis: DDF operators

- straightforward enumeration of the physical states
- elementary couplings to the Reggeon

The Regge limit of the inelastic amplitudes

DDF operators for the NS sector

$$A_{-n,j} = -i \oint_0 dz \; (e_j)_\mu \left(\partial X^\mu + in(k\psi)\psi^\mu\right) \mathrm{e}^{-inkX(z)}$$

$$B_{-r,j} = i \oint_0 dz \; (e_j)_\mu \left(\partial X^\mu \, (k\psi) - \psi^\mu(k\partial X) + \frac{\psi^\mu}{2}(k\psi)\frac{(k\partial\psi)}{(k\partial X)}\right) \frac{\mathrm{e}^{-irkX(z)}}{(ik\partial X)^{\frac{1}{2}}}$$

$k \sim e^+$. They do not depend on X^+, ψ^+. In the Regge limit

$$V_R^{(0)}(z) \;\sim\; \left(\sqrt{\frac{2}{\alpha'}}\frac{i\partial X^+(z)}{\sqrt{\alpha'}E}\right)^{\frac{\alpha' t}{4}+1} \mathrm{e}^{-iqX(z)}$$

$$A_{-n,j}(z) \;\sim\; -i\sqrt{\frac{2}{\alpha'}} \oint_z dw \; (e_j)_\mu \partial X^\mu \mathrm{e}^{-inkX}$$

$$B_{-r,j}(z) \;\sim\; -i \oint_z dw \; (e_j)_\mu \psi^\mu (ik\partial X)^{\frac{1}{2}} \mathrm{e}^{-irkX}$$

The Regge limit of the inelastic amplitudes

In the DDF basis the operator W_R acts as follows

- it is the identity on the $B_{-r,i}$
- it replaces the $A_{-n,i}$ with $\sqrt{\frac{\alpha'}{2}}\epsilon\bar{q}$ or $-\sqrt{\frac{\alpha'}{2}}\zeta\bar{q}$
- it imposes the constraint
  $\sum n_{a_1} - \sum n_{a_2} + \sum \bar{n}_{b_1} - \sum \bar{n}_{b_2} = 0$

It is then given by

$$W_R(s,q) \;\; = \;\; \mathcal{A}(s,t) \int_0^{2\pi} \frac{d\sigma}{2\pi} \; : \mathrm{e}^{i\bar{q}X} :$$

Covariant derivation of the operator $\hat{\delta}(s,t)$.
Additional derivation: Regge limit of the Green-Schwarz three-string vertex in the light-cone gauge.

Time delays

Time delays in weakly-coupled gravity theories with higher derivative corrections
Camanho, Edelstein, Maldacena and Zhiboedov (2014)

- Phase shift $\delta(E,b)$ related to the graviton three-point coupling
- Time delay

$$\Delta t = 2\partial_E \delta(E,b)$$

sensitive to modifications of the three-point coupling induced by the higher derivative corrections

In general dimensionality three possible structures

$$\begin{aligned} C_R &= [(\epsilon_1\epsilon_2)(\epsilon_3 p_1) + (\epsilon_1\epsilon_2)(\epsilon_3 p_1) + (\epsilon_1\epsilon_2)(\epsilon_3 p_1)]^2 \\ C_{R^2} &= [(\epsilon_1\epsilon_2)(\epsilon_3 p_1) + (\epsilon_1\epsilon_2)(\epsilon_3 p_1) + (\epsilon_1\epsilon_2)(\epsilon_3 p_1)]\,(\epsilon_1 p_2)(\epsilon_2 p_3)(\epsilon_3 p_1) \\ C_{R^3} &= [(\epsilon_1 p_2)(\epsilon_2 p_3)(\epsilon_3 p_1)]^2 \end{aligned}$$

Time delays

They could be generated by an action of the form

$$S = \frac{1}{16\pi G} \int d^d x \sqrt{-g} \left(R + l_2^2 R^2 + l_4^4 R^3 + \ldots \right)$$

$$R^2 \equiv R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2 \; , \quad R^3 \equiv R^{\mu\nu\rho\sigma} R_{\rho\sigma\alpha\beta} R^{\alpha\beta\mu\nu}$$

In the Regge limit

$$\begin{aligned} C_R &\sim \epsilon_{ij}\, \zeta_{ij} \\ C_{R^2} &\sim \epsilon_{ij}\, \zeta_{ik}\, \bar{q}^j\, \bar{q}^k \\ C_{R^3} &\sim \epsilon_{ij}\, \zeta_{kl}\, \bar{q}^i\, \bar{q}^j\, \bar{q}^k\, \bar{q}^l \end{aligned}$$

Polarization dependent time delay

$$\Delta t = 2\partial_E \delta(E, b) = (\Delta t)_R \left(1 \pm c_2 \frac{l_2^4}{b^4} \pm c_4 \frac{l_4^8}{b^8} \right)$$

Potential causality violations

Time delays

- The problem cannot be fixed at tree level by adding particles with spin less than two
- The problem cannot be fixed at tree level by adding a finite number of higher spin particles

String theory provides a possible (unique?) solution: Regge behaviour at tree level

- Bosonic string: both R^2 and R^3
- Heterotic string: R^2
- Type II string: no corrections

Time delays

High-energy string-brane scattering for the bosonic string.
Tree-level amplitude

$$\langle \zeta | 4E\hat{\delta}(s,t)|\epsilon\rangle = \mathcal{A}_1(s,t)\mathrm{Pol}(\epsilon,\zeta,\bar{q})$$

$$\mathcal{A}_1(s,t) = \Gamma\left(-1-\frac{\alpha'}{4}t\right) e^{-i\pi\left(1+\frac{\alpha' t}{4}\right)} (\alpha' s)^{1+\frac{\alpha' t}{4}}$$

$$\mathrm{Pol} = \left[\epsilon_{ij}\,\zeta_{ij} - \frac{\alpha'}{2}\left(\epsilon_{ij}\,\zeta_{ik}\,\bar{q}^j\,\bar{q}^k + \epsilon_{ji}\,\zeta_{ki}\,\bar{q}^j\,\bar{q}^k\right) + \frac{\alpha'^2}{4}\epsilon_{ij}\,\zeta_{kl}\,\bar{q}^i\,\bar{q}^j\,\bar{q}^k\,\bar{q}^l\right]$$

Field theory limit, graviton pole

$$2\delta(E,q) \sim \frac{E}{\bar{q}^2}\left(\delta_{ij} - \frac{\alpha'}{2}\bar{q}_i\bar{q}_j\right)\left(\delta_{kl} - \frac{\alpha'}{2}\bar{q}_k\bar{q}_l\right)$$

Time delays

In the impact parameter space

$$2\delta(E,q) \sim K_0 \left(\delta_{ij}\delta_{kl} - \frac{\alpha'}{b^2} (\delta_{ij}\Pi_{kl} + \delta_{kl}\Pi_{ij}) + \frac{{\alpha'}^2}{b^4}\Pi_{ijkl} \right)$$

where

$$\Pi_{ij} = \delta_{ij} - \hat{b}_i\hat{b}_j \ , \qquad \hat{b}_i = \frac{b_i}{b}$$

$$\Pi_{ijkl} = 3\delta_{(ij}\delta_{kl)} - (d+2)\,6\,\delta_{(ij}\hat{b}_k\hat{b}_{l)} - (d+2)(d+4)\hat{b}_i\hat{b}_j\hat{b}_k\hat{b}_l$$

Time advances for the components $G_{a_1\hat{j}}$ and $G_{\hat{i}\hat{\mathbf{b}}}$ of the metric and $B_{a_1\hat{j}}$ and $B_{\hat{i}\hat{j}}$ of the Kalb-Ramond field.

Time delays

Let us take into account the Regge behaviour

$$(\alpha' s)^{\frac{\alpha' t}{4}} e^{-i\pi \frac{\alpha' t}{4}} = e^{\frac{\alpha' t}{4}(\log \alpha' s - i\pi)}$$

Fourier transform

$$\mathcal{A}(s,b) \sim M\left(\frac{22-p}{2}, \frac{24-p}{2}, -\frac{b^2}{\alpha' \log \alpha' s}\right)$$

At small b

$$\delta(s,b) \sim \frac{E}{(\log \alpha' s)^{\frac{22-p}{2}}}\left(\frac{1}{22-p} - c_1 \frac{b^2}{\alpha' \log \alpha' s} + \dots\right)$$

Positive time delay for all possible choices of the polarizations

Time delays

In Type II string the three-graviton vertex is not modified

There are polarization dependent time delays for higher spin particles

For instance at level one we have states transforming in the tensor product of two symmetric traceless tensors of rank two

The couplings of the massive rank four symmetric traceless tensor $\epsilon_{\mu\nu\rho\sigma}$ are

$$\epsilon_{ijkl}\zeta_{ijkl} - \alpha' \epsilon_{ijkl}\zeta_{ijkh}\bar{q}^l\bar{q}^h + \frac{\alpha'^2}{4}\epsilon_{ijkl}\zeta_{ijrs}\bar{q}^k\bar{q}^l\bar{q}^r\bar{q}^s$$

Conclusions

We reviewed two aspects of high-energy string dynamics

- Regge behaviour at tree level: single Reggeon exchange
- Eikonal operator: all-order resummation of multi-Reggeon exchanges

Useful framework to address several problems in quantum gravity and to study the structure and symmetries of string theory

- emergence of an effective geometry from the scattering data
- consistent interactions between states of arbitrary mass and spin
- existence of a unitary S-matrix for high-energy collisions
- microscopic description of the infall of a particle into a singularity