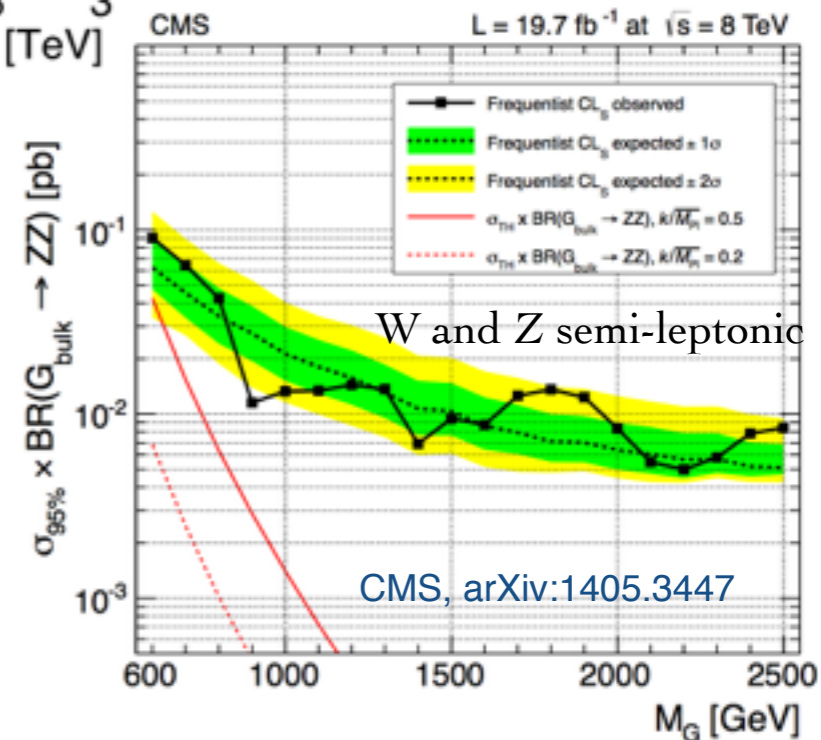
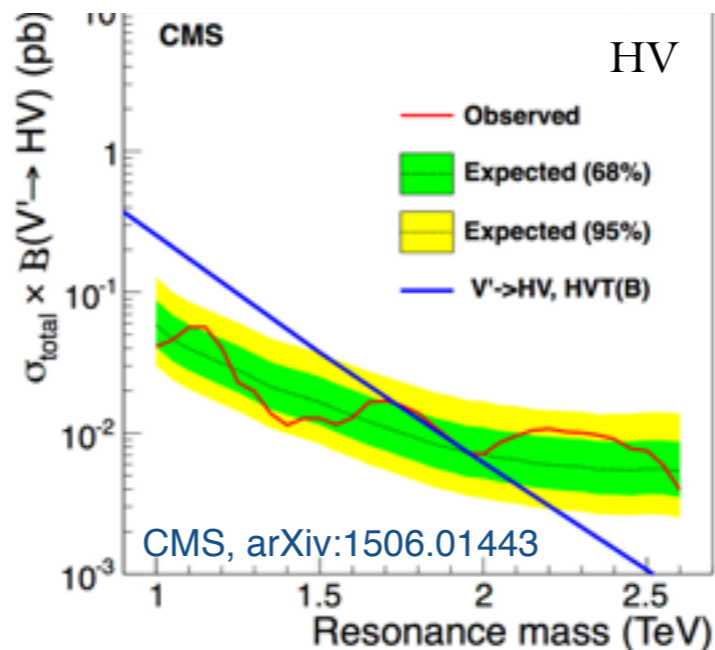
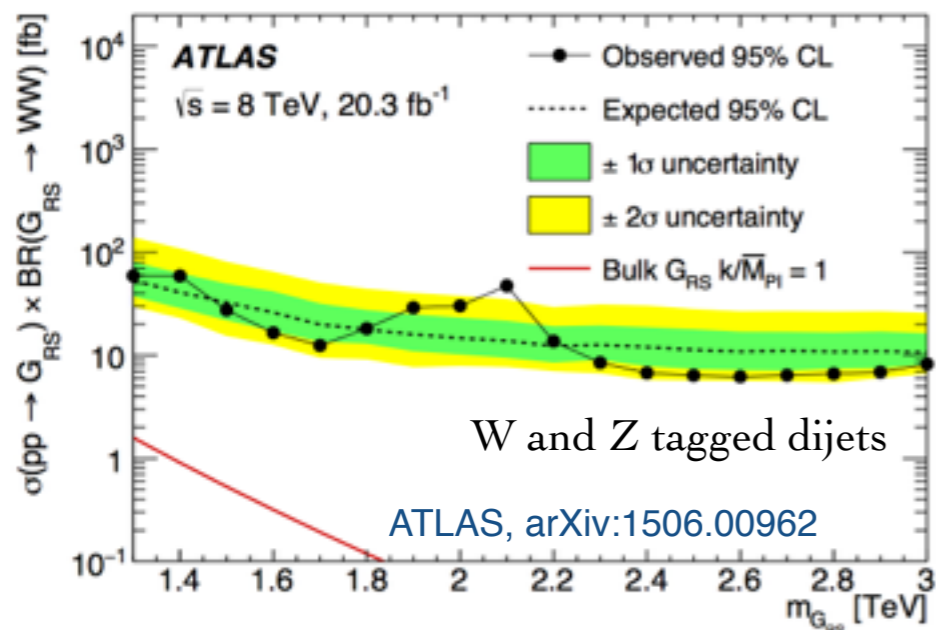
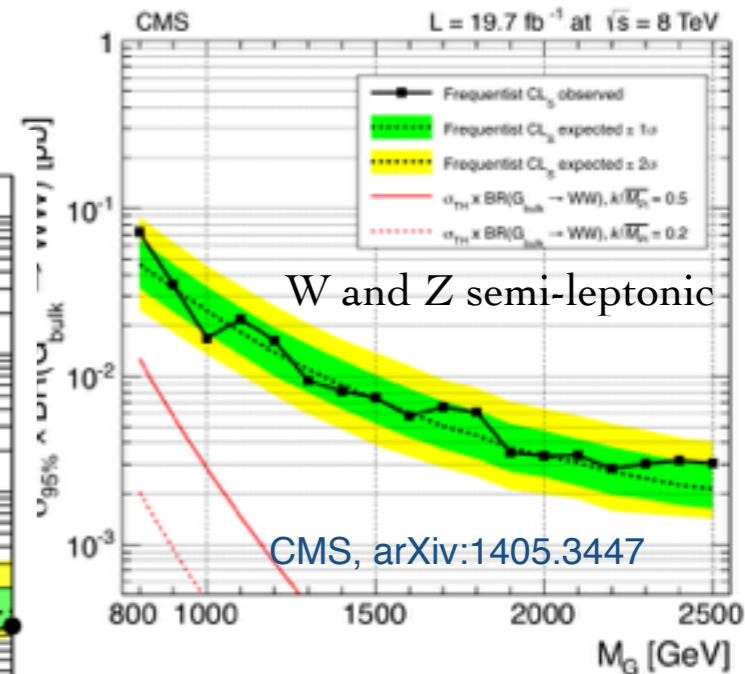
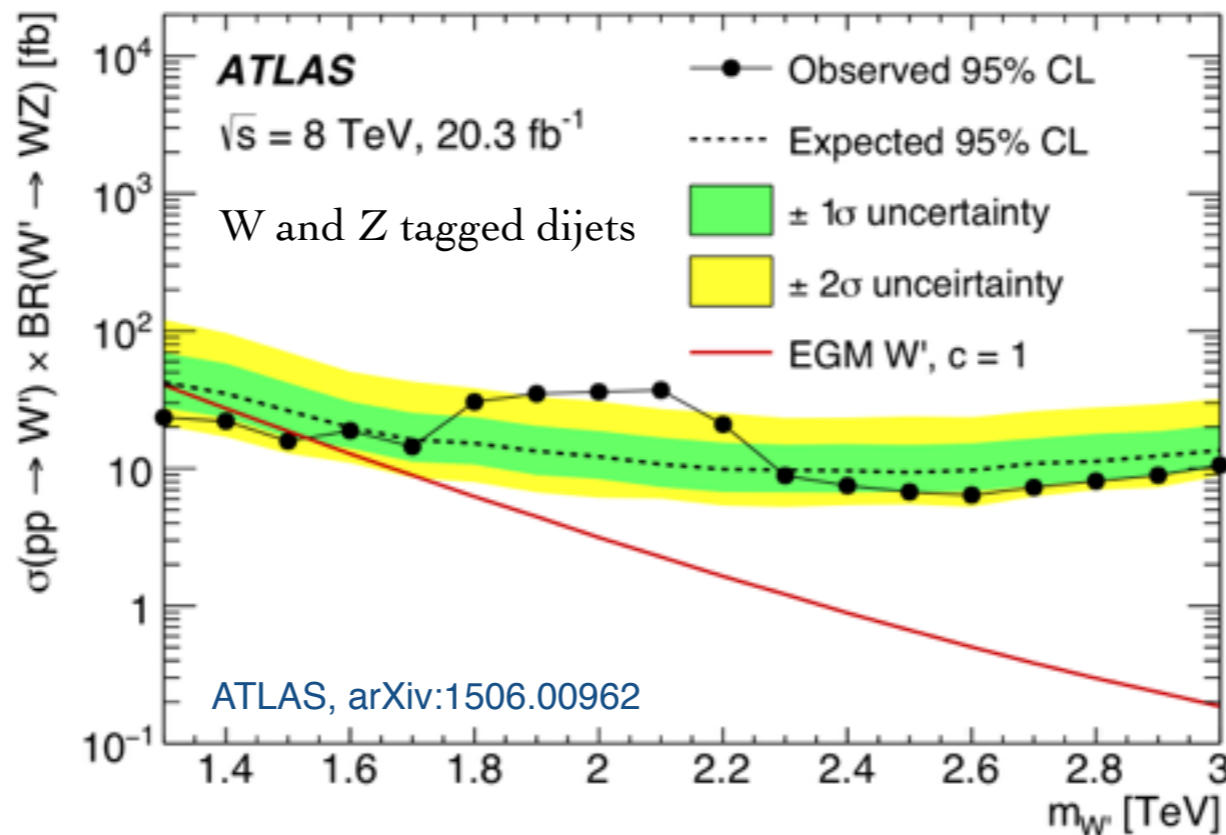
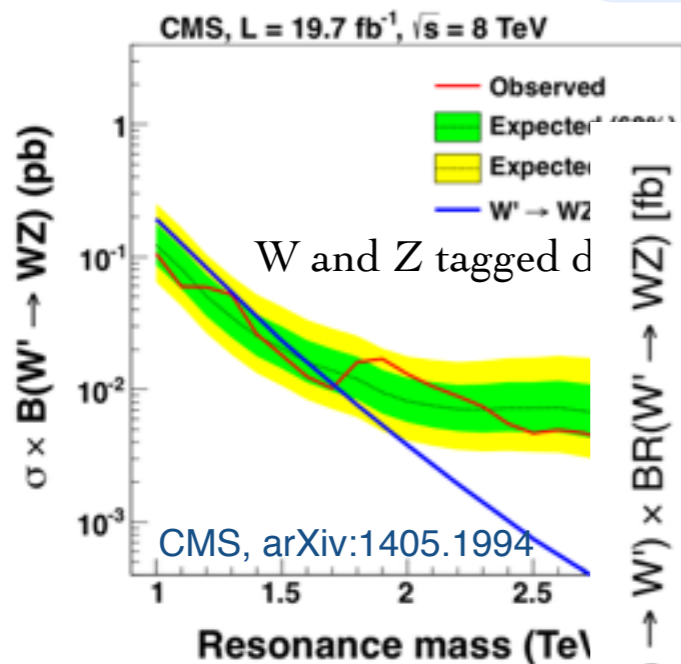


# Composite heavy vector triples in the ATLAS di-boson excess and at future colliders

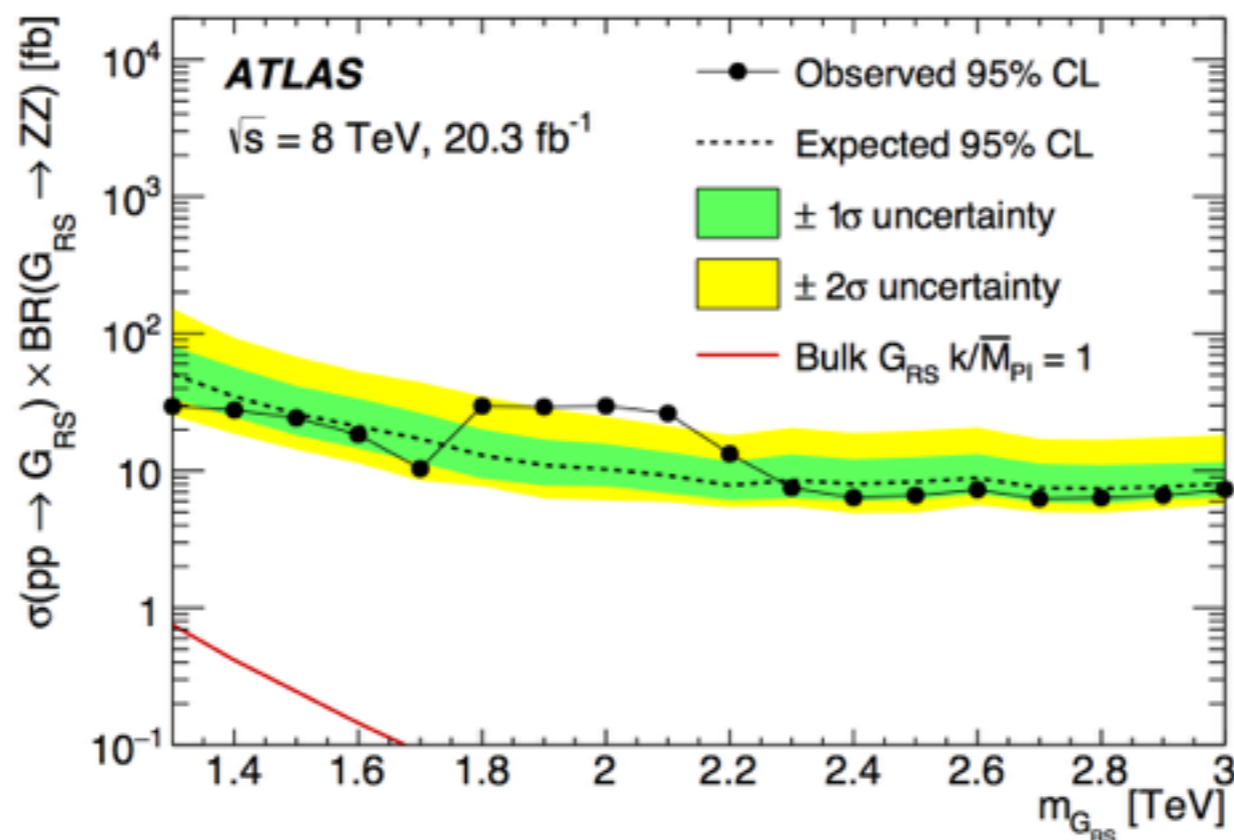
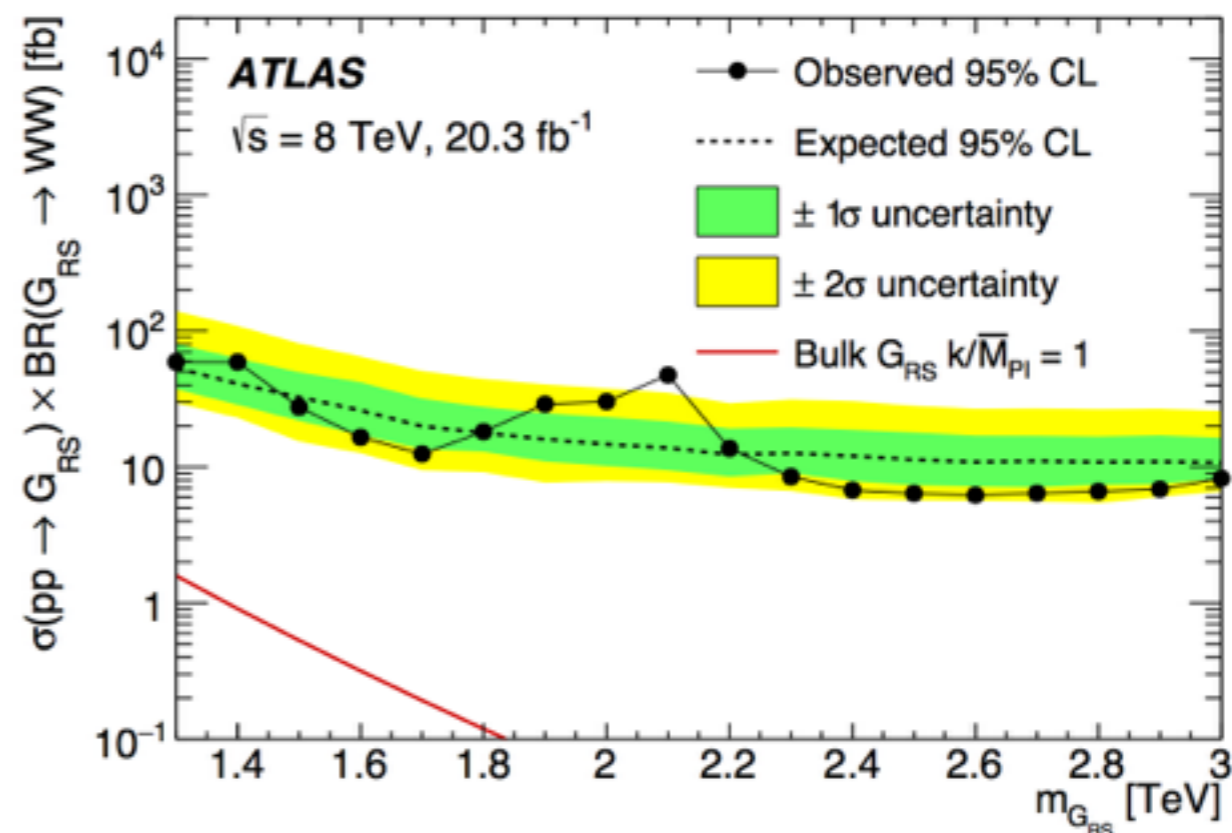
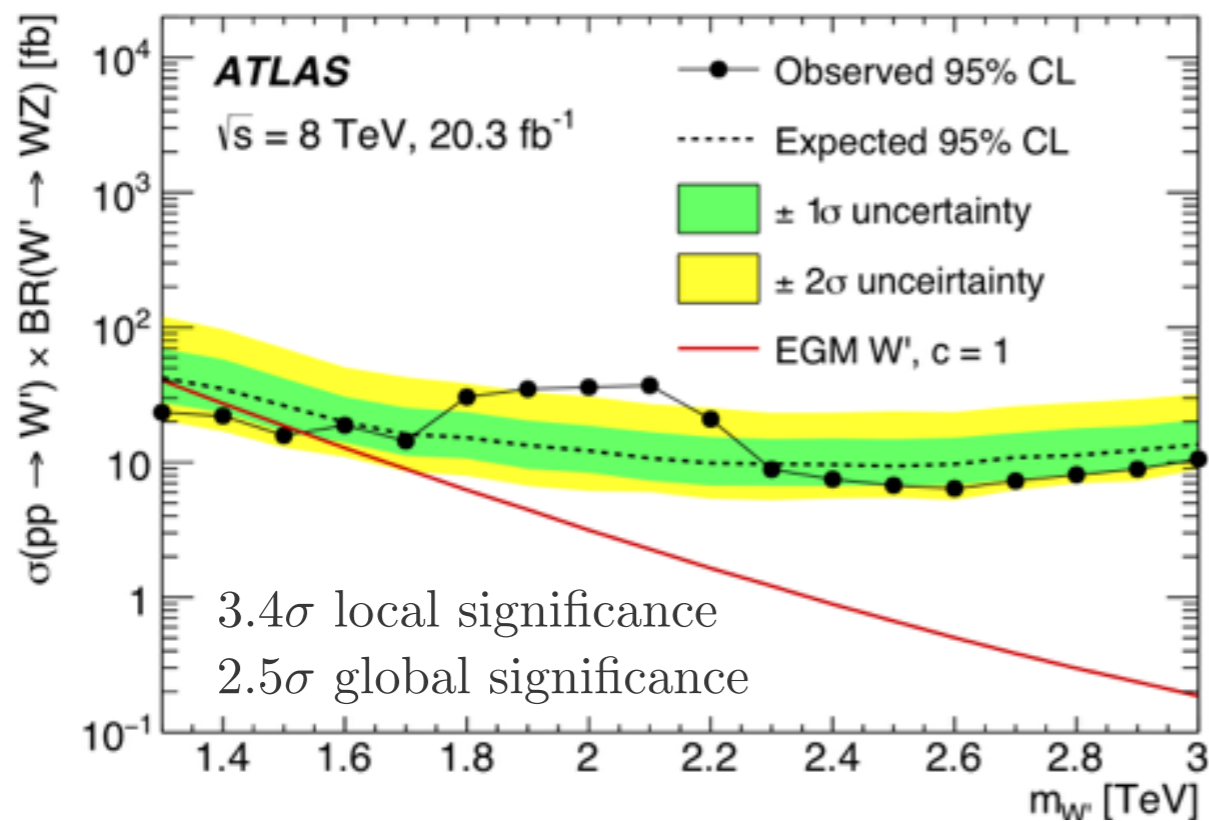
Andrea Thamm  
JGU Mainz

in collaboration with R. Torre and A. Wulzer  
based on arXiv: 1506.08688 and 1502.01701

# Di-boson excess?



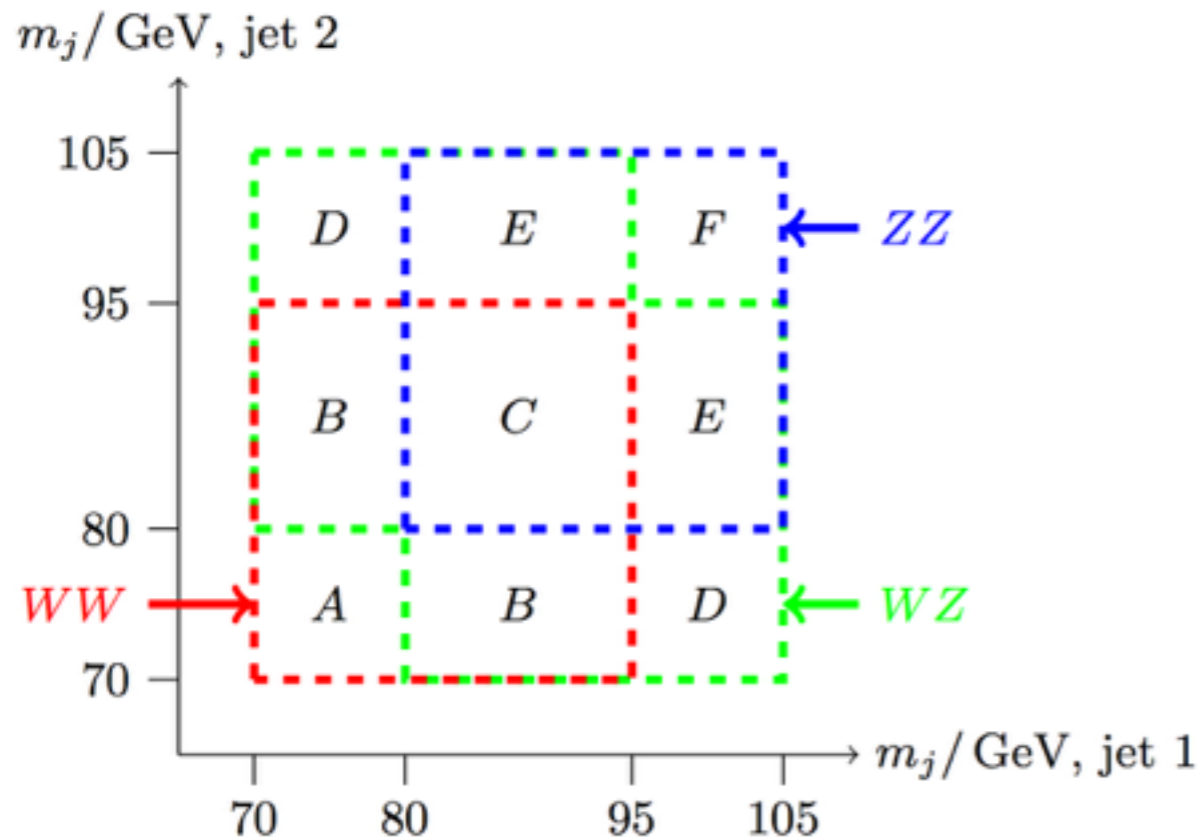
# Di-boson excess?



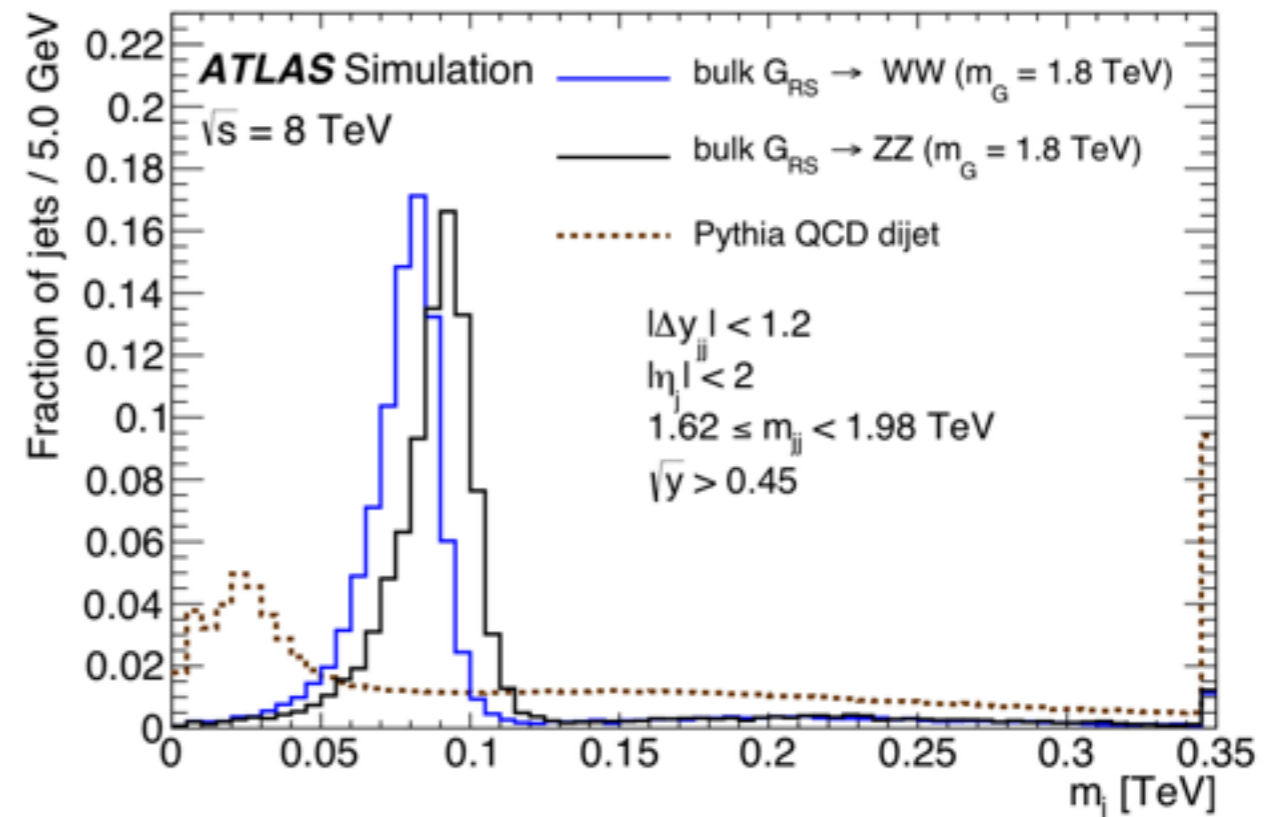
[ATLAS, arXiv:1506.00962]

# Tagging efficiencies

- W-fat jet:  $69.4 \text{ GeV} < m < 95.4 \text{ GeV}$
- Z-fat jet:  $79.8 \text{ GeV} < m < 105.8 \text{ GeV}$



[Allanach, Gripaos, Sutherland: arXiv:1507.01638]

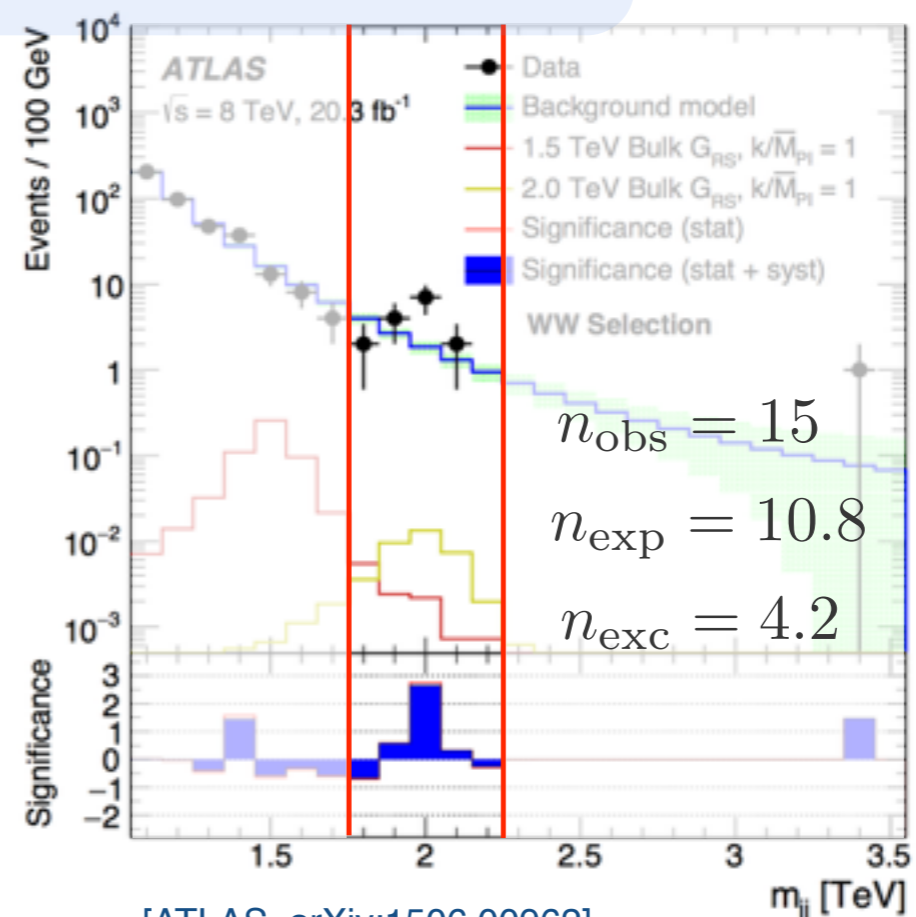
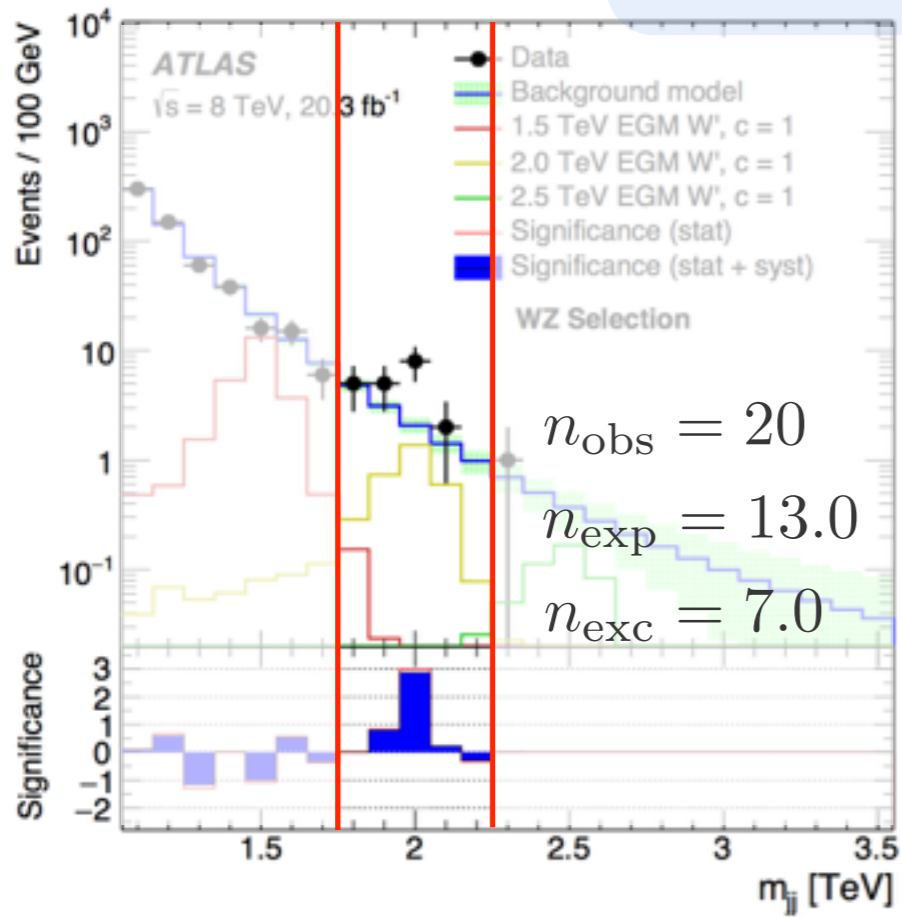


[ATLAS, arXiv:1506.00962]

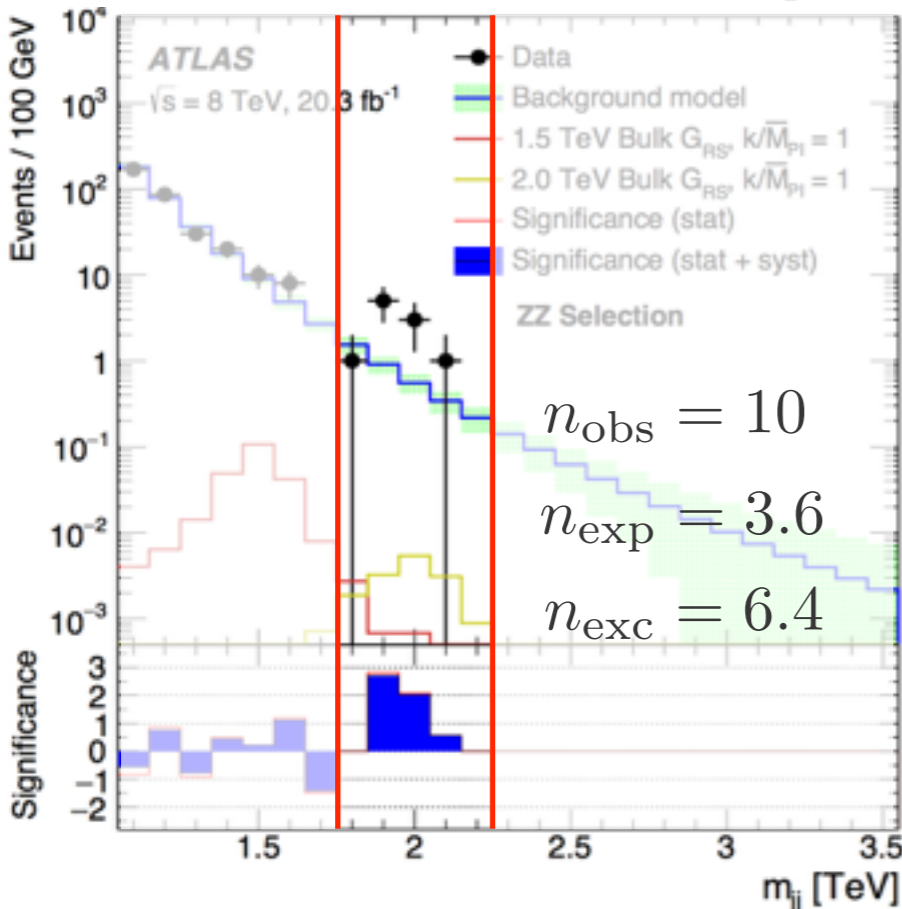
- efficiency of jet invariant mass cuts

$$\begin{bmatrix} \epsilon_{WW \rightarrow WW} & \epsilon_{WZ \rightarrow WW} \\ \epsilon_{WW \rightarrow WZ} & \epsilon_{WZ \rightarrow WZ} \\ \epsilon_{WW \rightarrow ZZ} & \epsilon_{WZ \rightarrow ZZ} \end{bmatrix} = \begin{bmatrix} 0.51 & 0.42 \\ 0.48 & 0.57 \\ 0.21 & 0.32 \end{bmatrix}$$

# Excess events



[ATLAS, arXiv:1506.00962]



$$S_{WZ} = 7.0^{+3.8}_{-2.6}$$

$$S_{WW} = 4.2^{+3.2}_{-2.0}$$

$$S_{ZZ} = 6.4^{+3.6}_{-2.4}$$

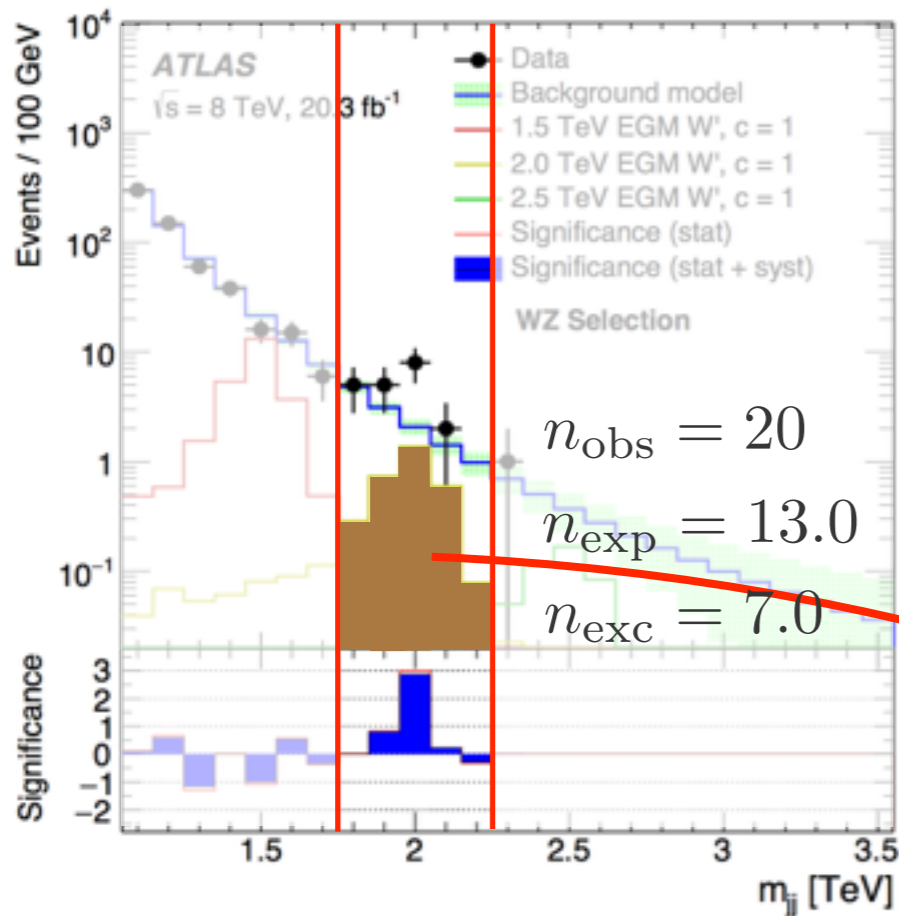
Big statistical uncertainties:

combined fit only by ATLAS

lack information on the correlation of the big systematic uncertainties

We extract the signal CS from a single channel and compare with the others

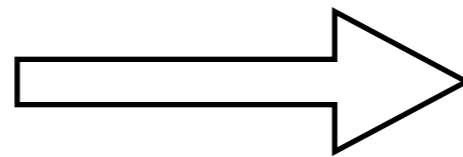
# Signal cross section



$$\text{BR}_{WZ \rightarrow \text{had}} \approx 0.47$$

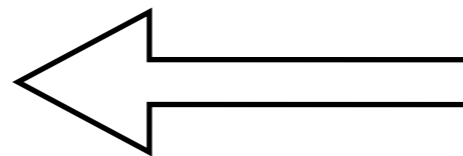
$m$ [TeV]	$\Gamma_{W'}$ [GeV]	$\Gamma_{G_{RS}}$ [GeV]	$W' \rightarrow WZ$		$G_{RS} \rightarrow WW$		$G_{RS} \rightarrow ZZ$	
			$\sigma \times \text{BR}$ [fb]	$f_{10\%}$	$\sigma \times \text{BR}$ [fb]	$f_{10\%}$	$\sigma \times \text{BR}$ [fb]	$f_{10\%}$
1.3	47	76	19.1	0.83	0.73	0.85	0.37	0.84
1.6	58	96	6.04	0.79	0.14	0.83	0.071	0.84
2.0	72	123	1.50	0.72	0.022	0.83	0.010	0.82
2.5	91	155	0.31	0.54	0.0025	0.78	0.0011	0.78
3.0	109	187	0.088	0.31	0.00034	0.72	0.00017	0.71

$$\frac{(\sigma \times \text{BR})_{\text{ATLAS}}}{\text{BR}_{WZ \rightarrow \text{had}}} = 3.17 \text{ fb}$$



3.4 events

$$\sigma_{W'} \times \text{BR}_{W' \rightarrow WZ} = 6.5^{+5.1}_{-4.1} \text{ fb}$$



$$S_{WZ} = 7.0^{+3.8}_{-2.6}$$

Heavy vector triples

# Heavy vector triples

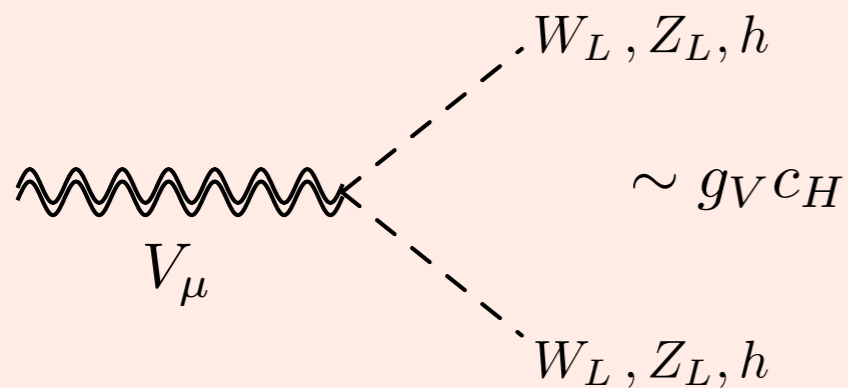
- among the most well motivated particles
- appear in composite Higgs models but also in weakly coupled theories
- associated to the EW gauge symmetry
- consider a 3 of  $SU(2)_L$



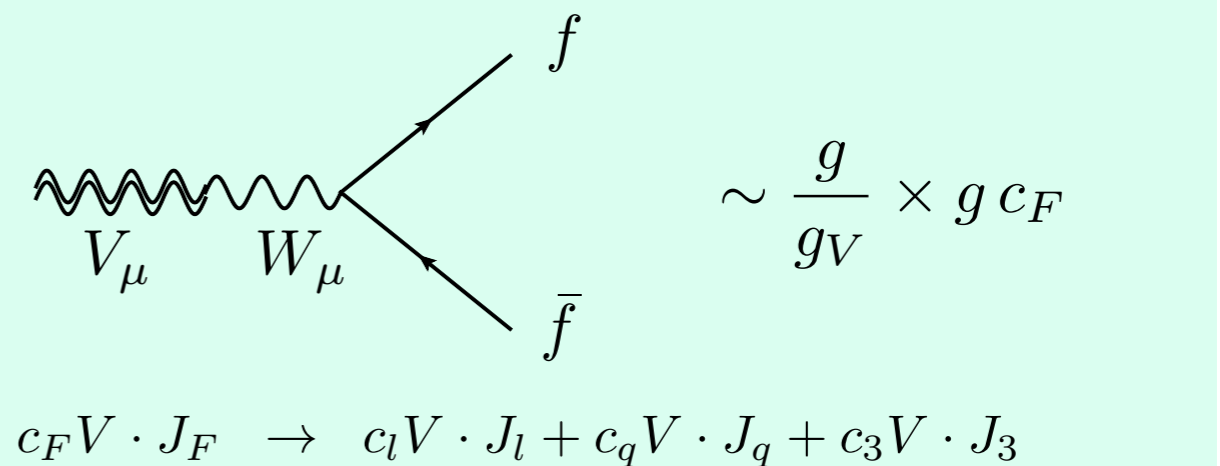
# Phenomenological Lagrangian

$$\begin{aligned}
 \mathcal{L}_V = & -\frac{1}{4} D_{[\mu} V_{\nu]}^a D^{[\mu} V^{\nu] a} + \frac{m_V^2}{2} V_\mu^a V^{\mu a} & V = (V^+, V^-, V^0) \\
 & + i g_V c_H V_\mu^a H^\dagger \tau^a \overleftrightarrow{D}^\mu H + \frac{g^2}{g_V} c_F V_\mu^a J_F^{\mu a} \\
 & + \frac{g_V}{2} c_{VVV} \epsilon_{abc} V_\mu^a V_\nu^b D^{[\mu} V^{\nu] c} + g_V^2 c_{VVHH} V_\mu^a V^{\mu a} H^\dagger H - \frac{g}{2} c_{VW} \epsilon_{abc} W^{\mu\nu a} V_\mu^b V_\nu^c
 \end{aligned}$$

## Coupling to SM Vectors



## Coupling to SM fermions

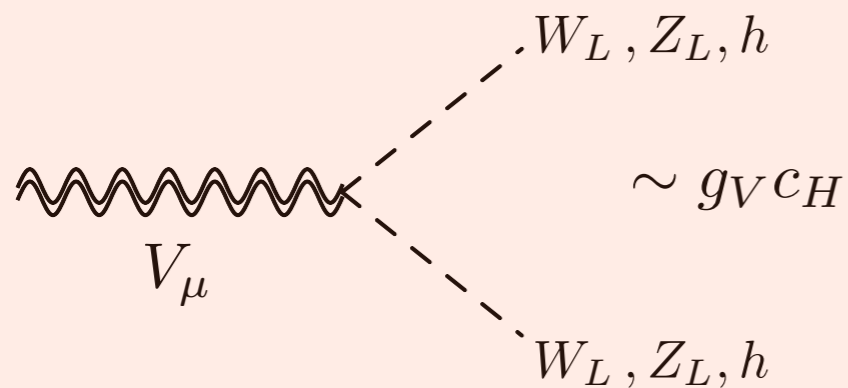


$$J_F^{\mu a} = \sum_f \bar{f}_L \gamma^\mu \tau^a f_L$$

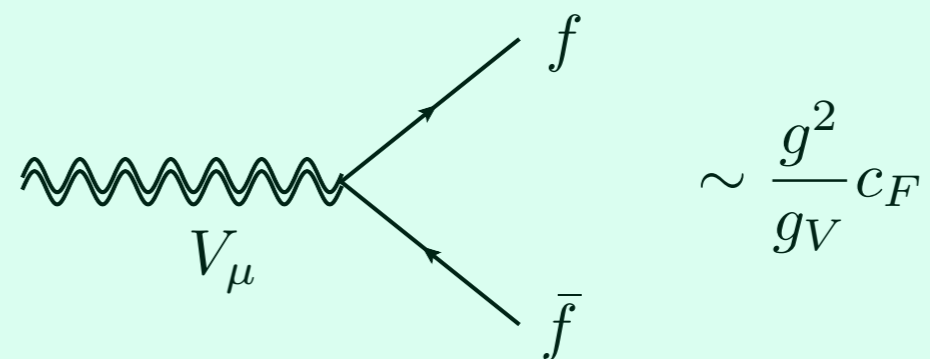
# Phenomenological Lagrangian

$$\begin{aligned}
 \mathcal{L}_V = & -\frac{1}{4} D_{[\mu} V_{\nu]}^a D^{[\mu} V^{\nu]} a + \frac{m_V^2}{2} V_\mu^a V^{\mu a} & V = (V^+, V^-, V^0) \\
 & + i g_V c_H V_\mu^a H^\dagger \tau^a \overleftrightarrow{D}^\mu H + \frac{g^2}{g_V} c_F V_\mu^a J_F^{\mu a} \\
 & + \frac{g_V}{2} c_{VVV} \epsilon_{abc} V_\mu^a V_\nu^b D^{[\mu} V^{\nu]} c + g_V^2 c_{VVHH} V_\mu^a V^{\mu a} H^\dagger H - \frac{g}{2} c_{V VW} \epsilon_{abc} W^{\mu\nu a} V_\mu^b V_\nu^c
 \end{aligned}$$

## Coupling to SM Vectors



## Coupling to SM fermions





$$J_F^{\mu a} = \sum_f \bar{f}_L \gamma^\mu \tau^a f_L$$

$$c_F V \cdot J_F \rightarrow c_l V \cdot J_l + c_q V \cdot J_q + c_3 V \cdot J_3$$

# Phenomenological Lagrangian

$$\begin{aligned}
 \mathcal{L}_V = & -\frac{1}{4} D_{[\mu} V_{\nu]}^a D^{[\mu} V^{\nu] a} + \frac{m_V^2}{2} V_\mu^a V^{\mu a} & V = (V^+, V^-, V^0) \\
 & + i g_V c_H V_\mu^a H^\dagger \tau^a \overleftrightarrow{D}^\mu H + \frac{g^2}{g_V} c_F V_\mu^a J_F^{\mu a} \\
 & + \frac{g_V}{2} c_{VVV} \epsilon_{abc} V_\mu^a V_\nu^b D^{[\mu} V^{\nu] c} + g_V^2 c_{VVHH} V_\mu^a V^{\mu a} H^\dagger H - \frac{g}{2} c_{VW} \epsilon_{abc} W^{\mu\nu a} V_\mu^b V_\nu^c
 \end{aligned}$$

- Couplings among vectors
- do not contribute to  $V$  decays
- do not contribute to single production
- only effects through (usually small)  $VW$  mixing
-  irrelevant for phenomenology  only need  $(c_H, c_F)$

# Phenomenological Lagrangian

$$\begin{aligned}
 \mathcal{L}_V = & -\frac{1}{4} D_{[\mu} V_{\nu]}^a D^{[\mu} V^{\nu]}{}_a + \frac{m_V^2}{2} V_\mu^a V^{\mu a} & V = (V^+, V^-, V^0) \\
 & + i g_V c_H V_\mu^a H^\dagger \tau^a \overleftrightarrow{D}^\mu H + \frac{g^2}{g_V} c_F V_\mu^a J_F^{\mu a} \\
 & + \frac{g_V}{2} c_{VVV} \epsilon_{abc} V_\mu^a V_\nu^b D^{[\mu} V^{\nu]}{}_c + g_V^2 c_{VHH} V_\mu^a V^{\mu a} H^\dagger H - \frac{g}{2} c_{VW} \epsilon_{abc} W^{\mu\nu a} V_\mu^b V_\nu^c
 \end{aligned}$$

Weakly coupled model

$g_V$  typical strength of V interactions

$$g_V \sim g \sim 1$$

$c_i$  dimensionless coefficients

$$c_H \sim -g^2/g_V^2 \quad \text{and} \quad c_F \sim 1$$

Strongly coupled model

$$1 < g_V \leq 4\pi$$

$$c_H \sim c_F \sim 1$$

# Production rates

- DY and VBF production

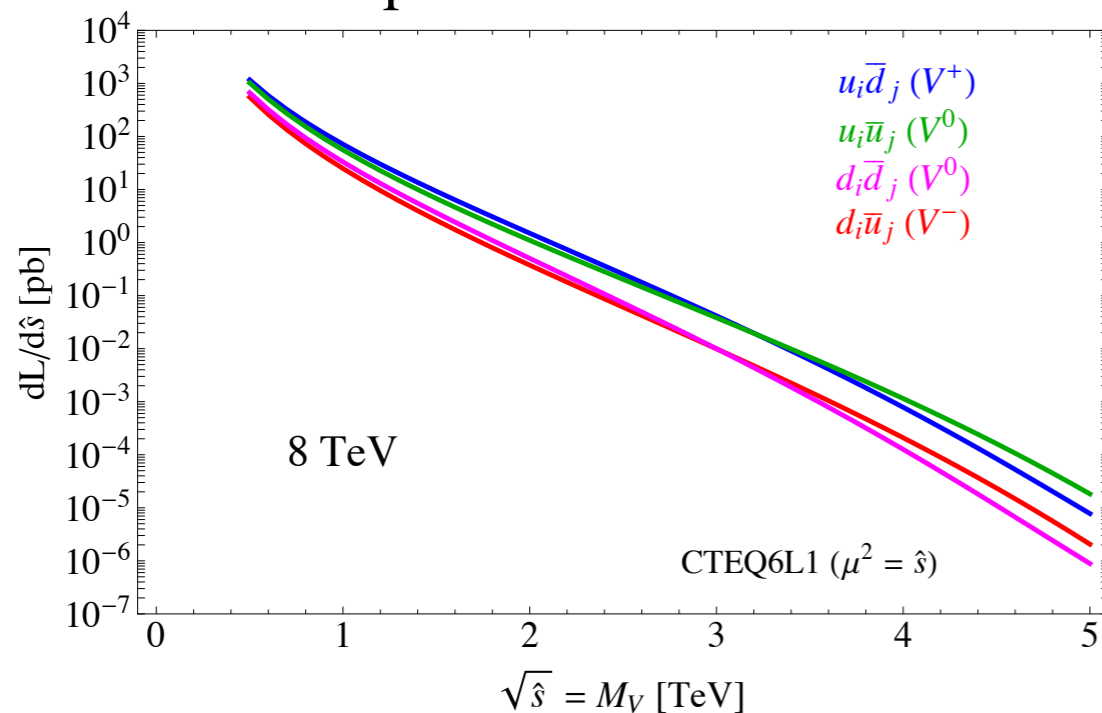
$$\sigma_{DY} = \sum_{i,j \in p} \frac{\Gamma_{V \rightarrow ij}}{M_V} \frac{4\pi^2}{3} \frac{dL_{ij}}{d\hat{s}} \Big|_{\hat{s}=M_V^2}$$

$$\sigma_{VBF} = \sum_{i,j \in p} \frac{\Gamma_{V \rightarrow W_L i W_L j}}{M_V} 48\pi^2 \frac{dL_{W_L i W_L j}}{d\hat{s}} \Big|_{\hat{s}=M_V^2}$$

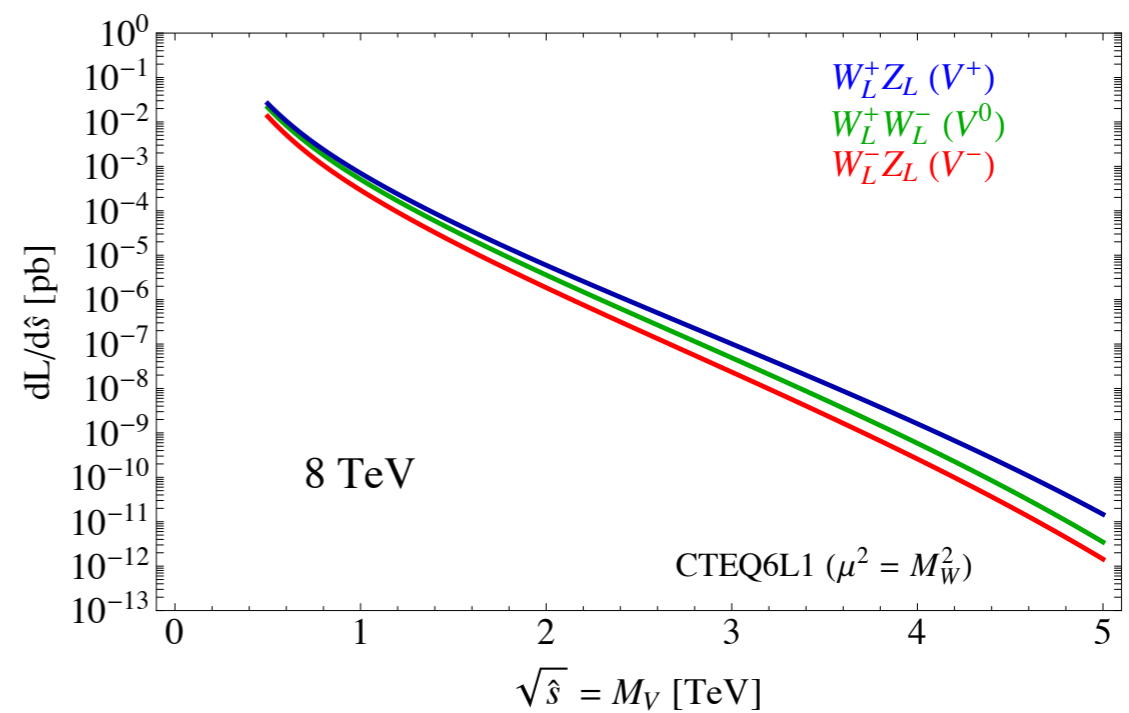
model dependent
model independent

- can compute production rates analytically!
- easily rescale to different points in parameter space

quark initial state



vector boson initial state



# Decay widths

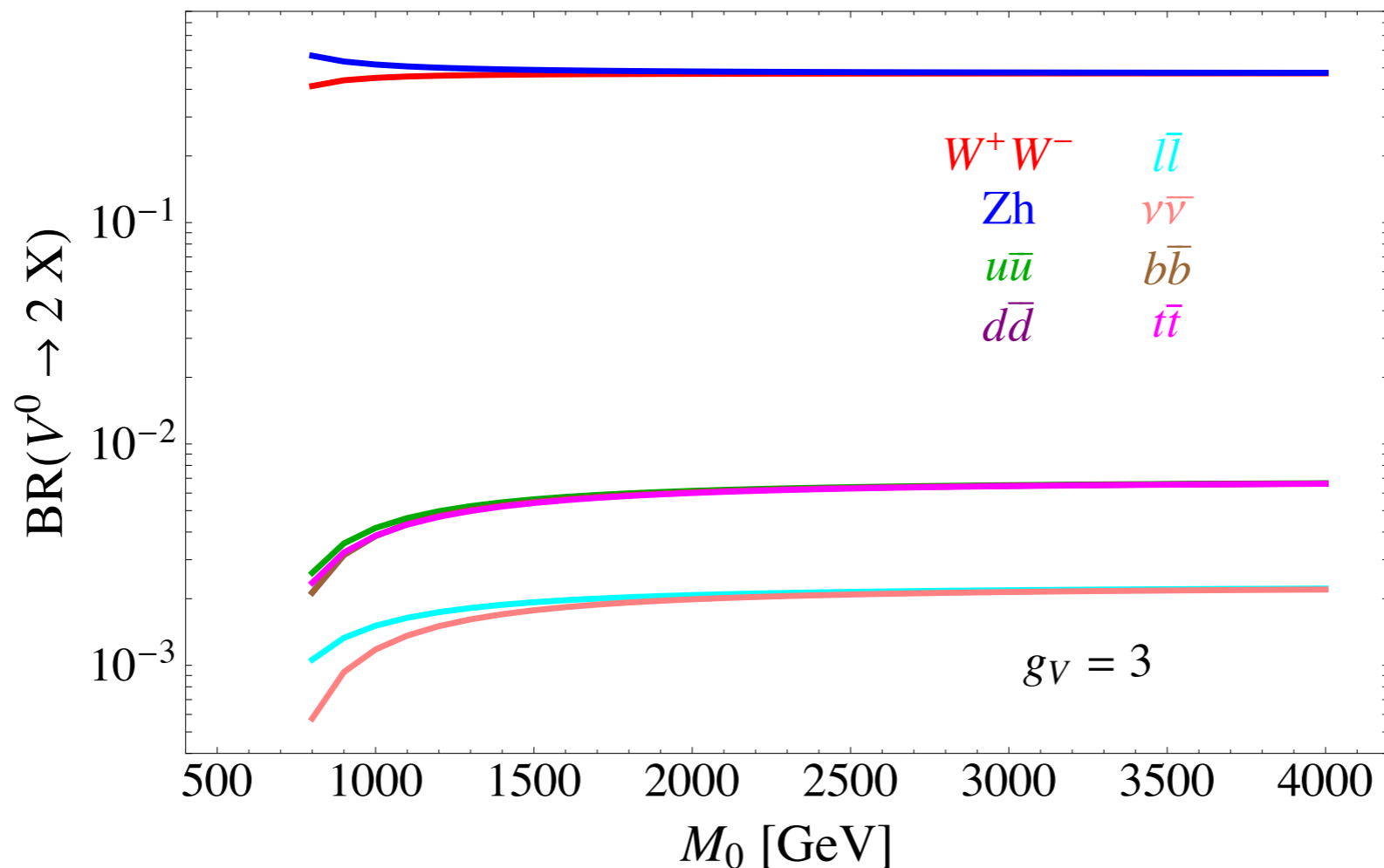
- relevant decay channels: di-lepton, di-quark, di-boson

$$\Gamma_{V_{\pm} \rightarrow f\bar{f}'} \simeq 2\Gamma_{V_0 \rightarrow f\bar{f}} \simeq N_c[f] \left( \frac{g^2 c_F}{g_V} \right)^2 \frac{M_V}{96\pi},$$

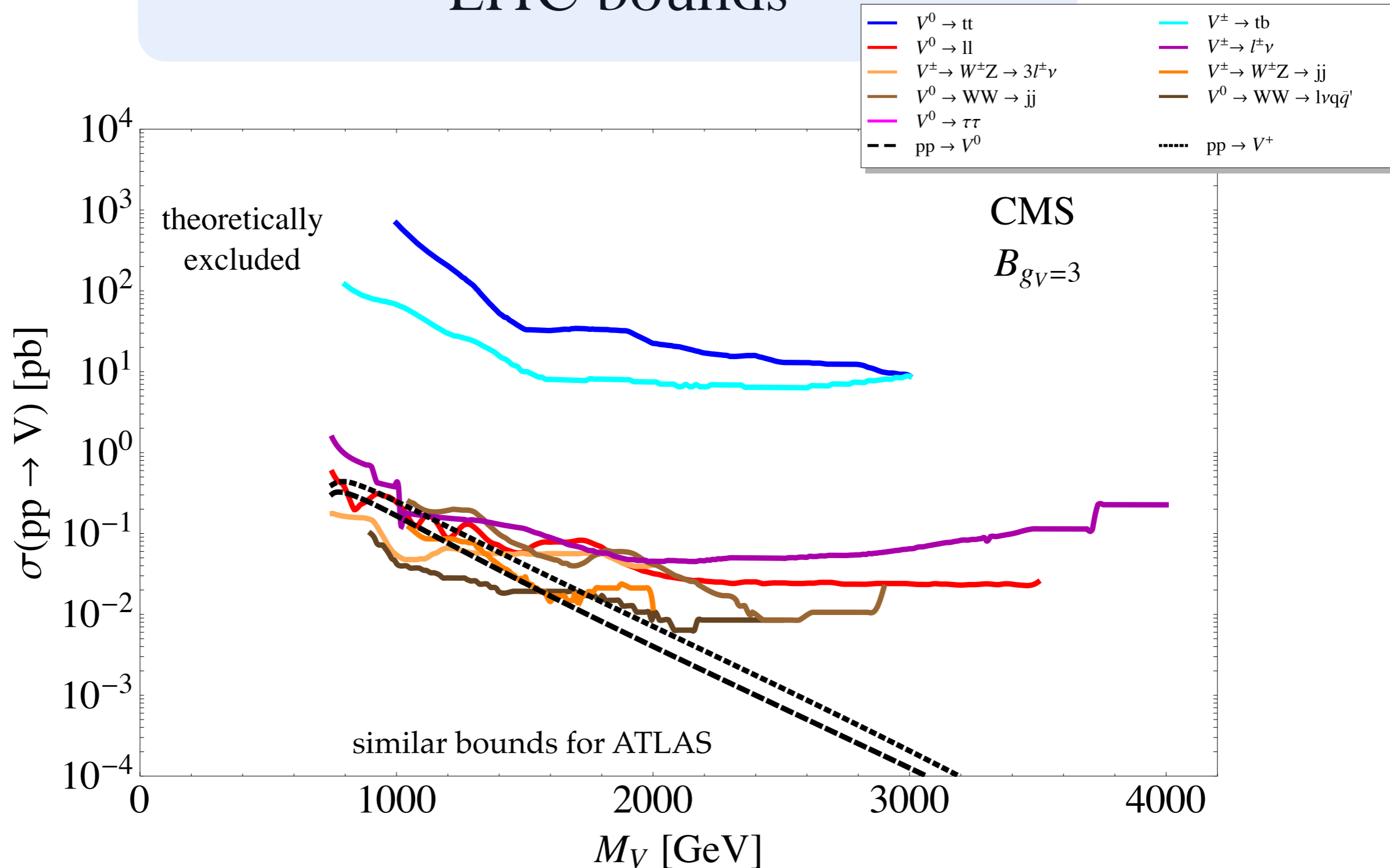
$$\Gamma_{V_0 \rightarrow W_L^+ W_L^-} \simeq \Gamma_{V_{\pm} \rightarrow W_L^{\pm} Z_L} \simeq \frac{g_V^2 c_H^2 M_V}{192\pi} [1 + \mathcal{O}(\zeta^2)]$$

$$\Gamma_{V_0 \rightarrow Z_L h} \simeq \Gamma_{V_{\pm} \rightarrow W_L^{\pm} h} \simeq \frac{g_V^2 c_H^2 M_V}{192\pi} [1 + \mathcal{O}(\zeta^2)]$$

$$g_V c_H \simeq -g_V, \quad g^2 c_F / g_V \simeq g^2 / g_V$$



# LHC bounds



- excluded for masses  $< 1.5$  TeV, unconstrained for larger  $g_V$
- di-boson most stringent
- in excluded region  $G_F$ ,  $m_Z$  not reproduced

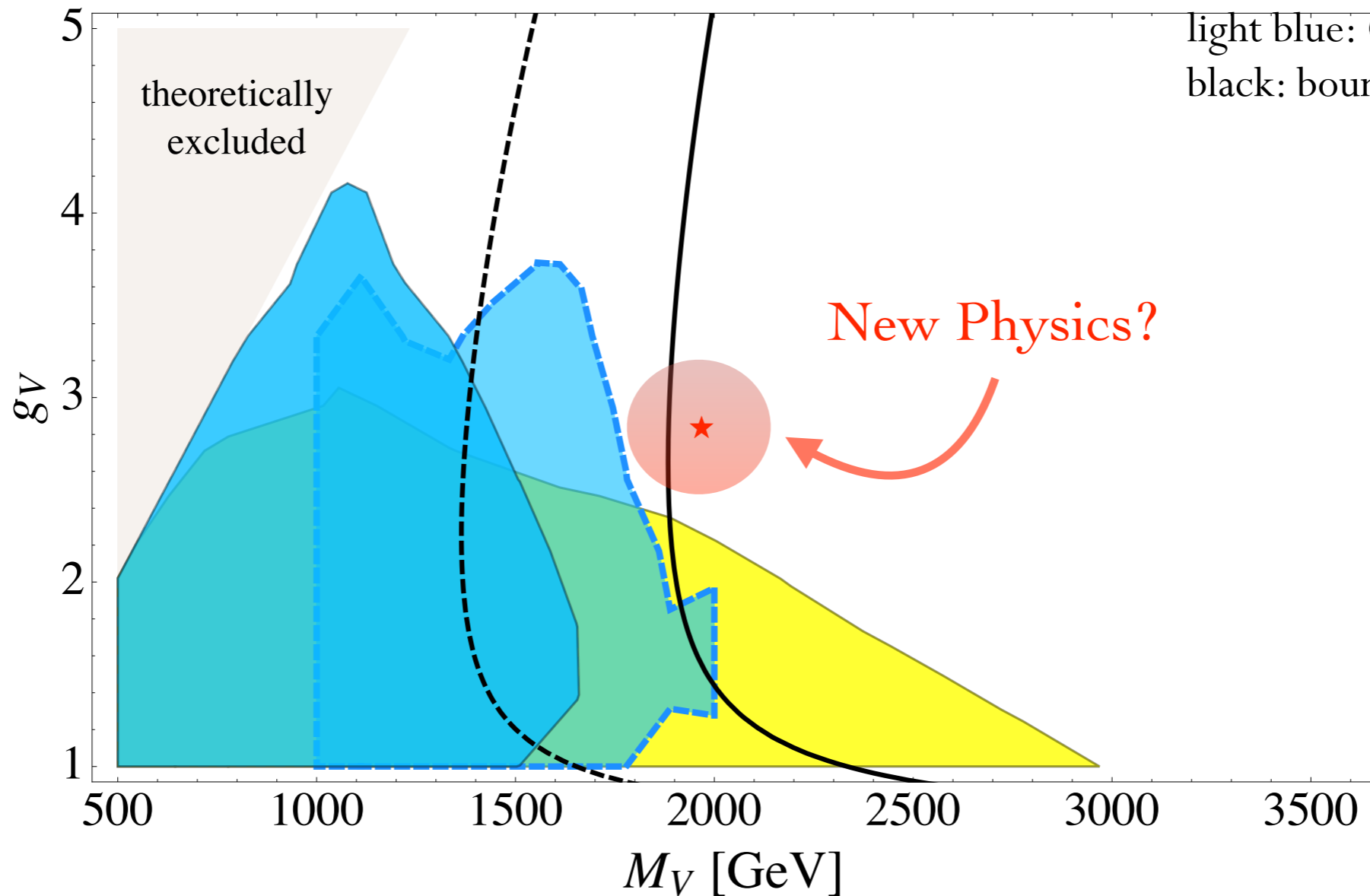
Heavy vector triples in the di-boson excess



# LHC bounds

- experimental limits converted into  $(M_V, g_V)$  plane

[Pappadopulo, Thamm, Torre, Wulzer, arXiv:1402.4431]



- similar exclusions at low  $g_V$ , leptonic final state dominates
- very different for larger coupling
- weaker limits if decay to top partners open

[Greco, Liu: arXiv:1410.2883]

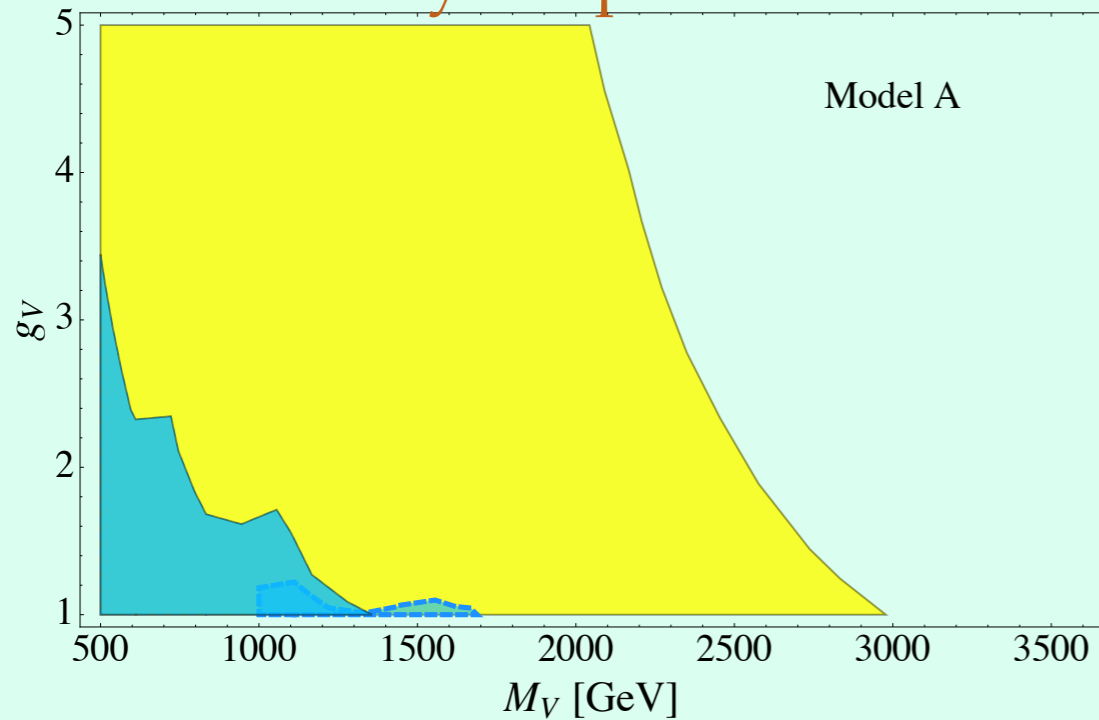
[Chala, Juknevich, Perez, Santiago :arXiv:1411.1771]

# LHC bounds

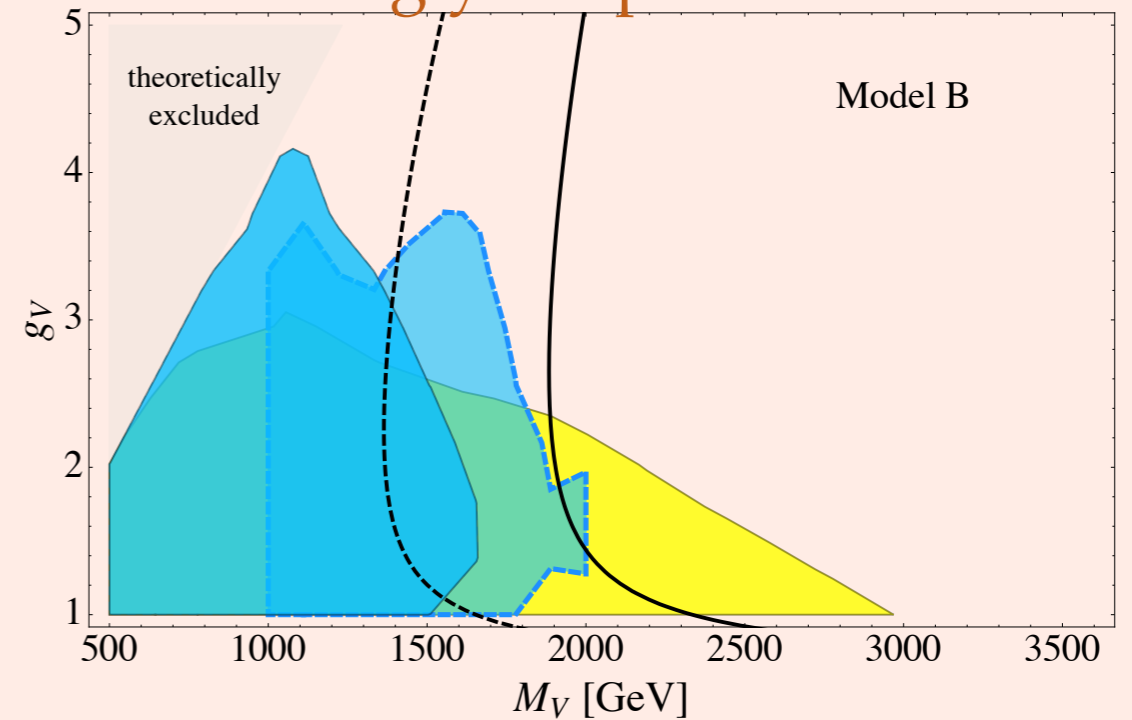
- compare with weakly coupled vectors

yellow: CMS  $l^+\nu$  analysis  
dark blue: CMS  $WZ \rightarrow 3l\nu$   
light blue: CMS  $WZ \rightarrow jj$   
black: bounds from EWPT

## Weakly coupled model



## Strongly coupled model



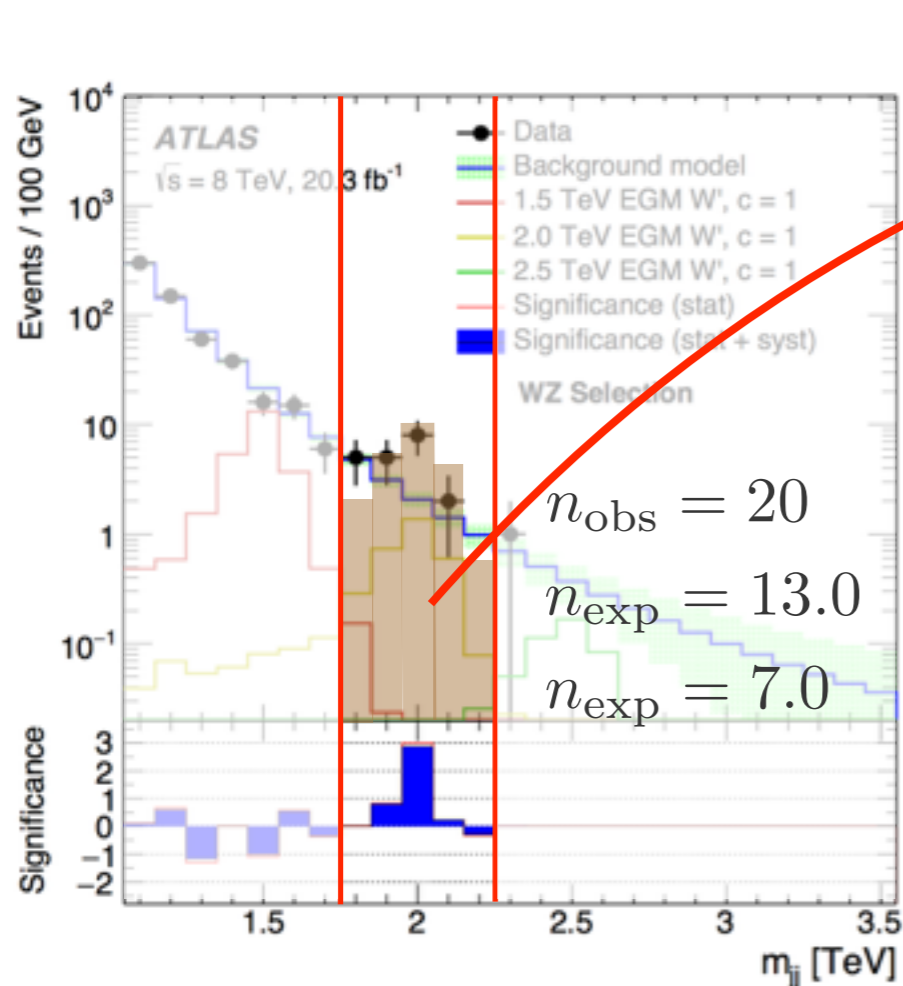
- strongly coupled vectors have weaker bounds

# HVT signal cross section

- neutral and charged components contribute to the various selection regions

$$S_{WZ} = \mathcal{L} \times \mathcal{A} \times [(\sigma \times \text{BR})_{V\pm} \text{BR}_{WZ \rightarrow \text{had}} \epsilon_{WZ \rightarrow WZ} + (\sigma \times \text{BR})_{V0} \text{BR}_{WW \rightarrow \text{had}} \epsilon_{WW \rightarrow WZ}]$$

- Once we fix the mass there is only one parameter  $g_V$



$$S_{WZ} = 7.0^{+3.8}_{-2.6}$$

[Thamm, Torre, Wulzer, arXiv:1506.08688]

$m_V$ [TeV]	$g_V$	$(\sigma \times \text{BR})_{V\pm}$ [fb]	$(\sigma \times \text{BR})_{V0}$ [fb]
1.8	$3.95^{+1.65}_{-0.88}$	4.51	2.04
1.9	$3.37^{+1.63}_{-0.83}$	4.63	2.09
2.0	$2.81^{+1.54}_{-0.82}$	4.79	2.16

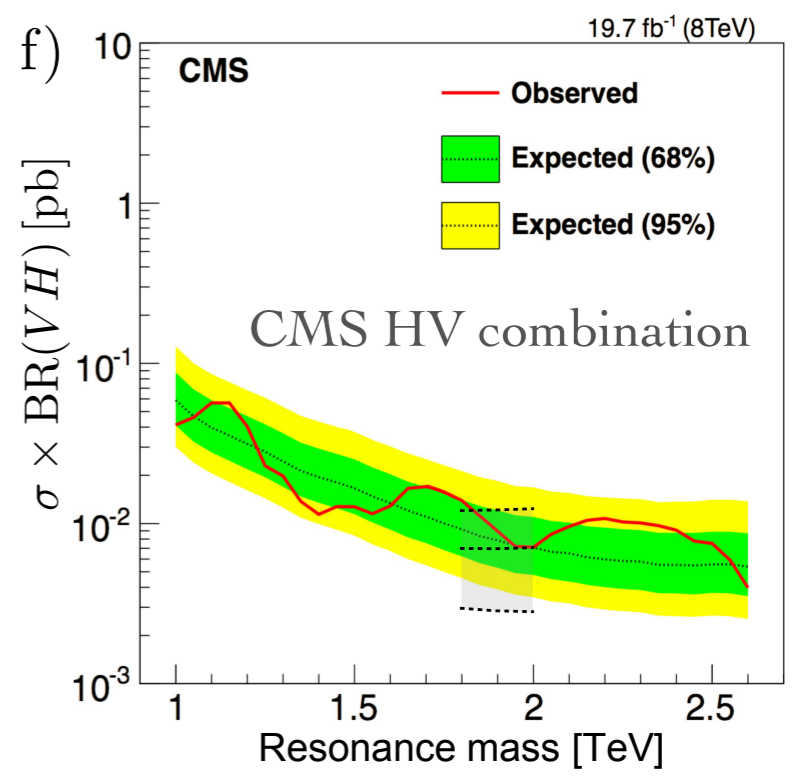
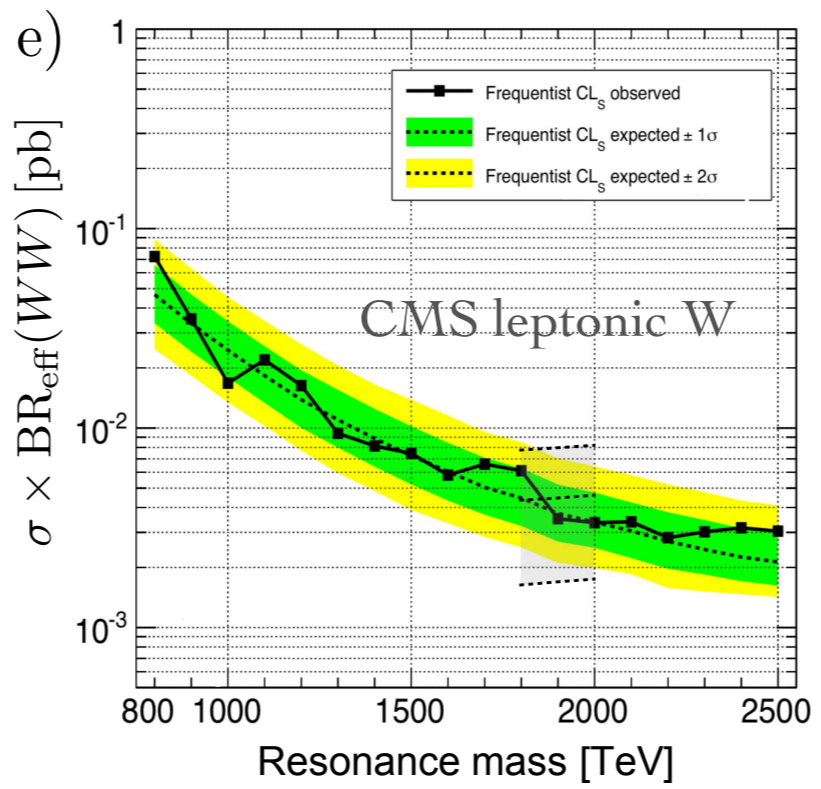
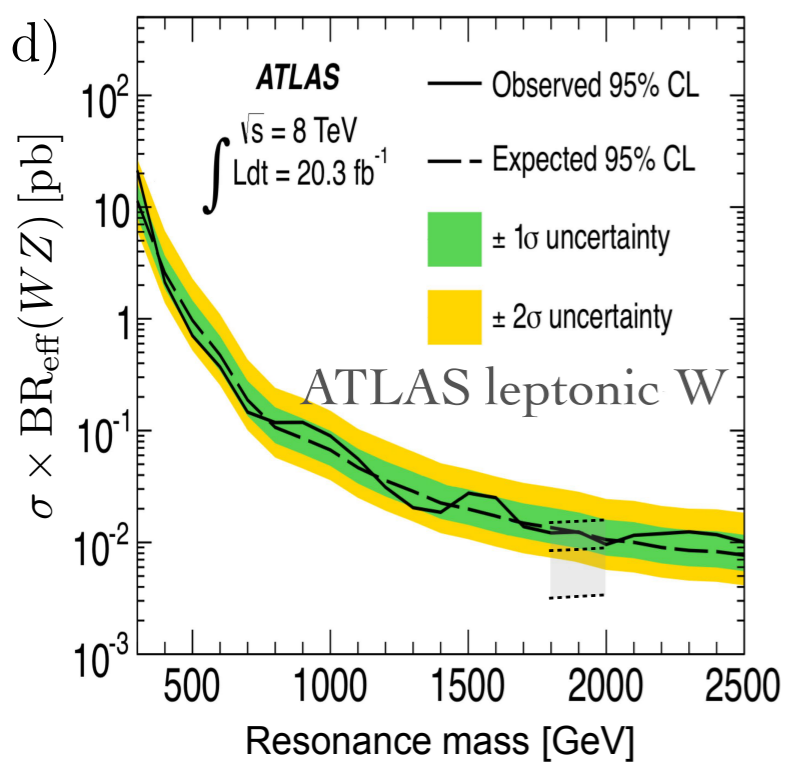
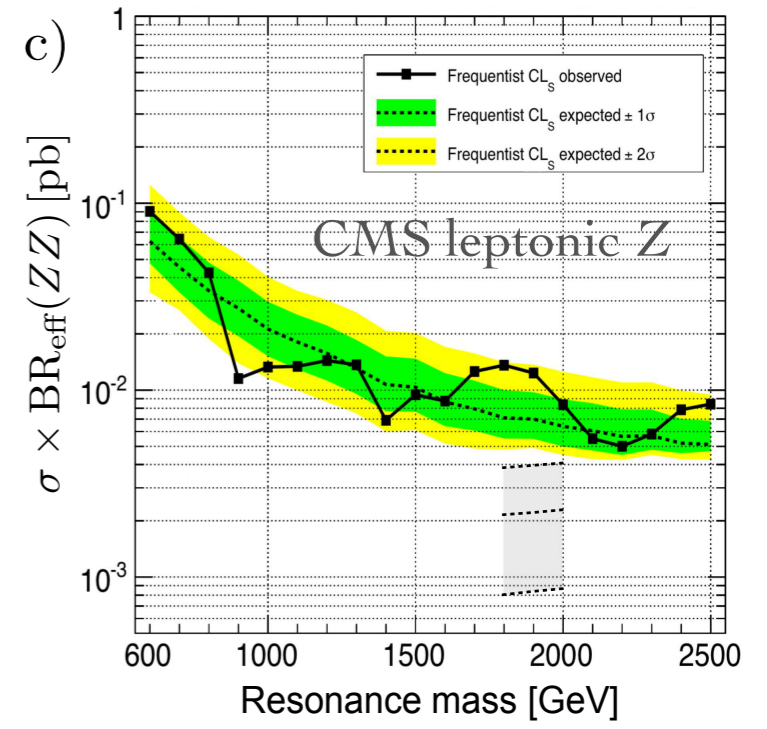
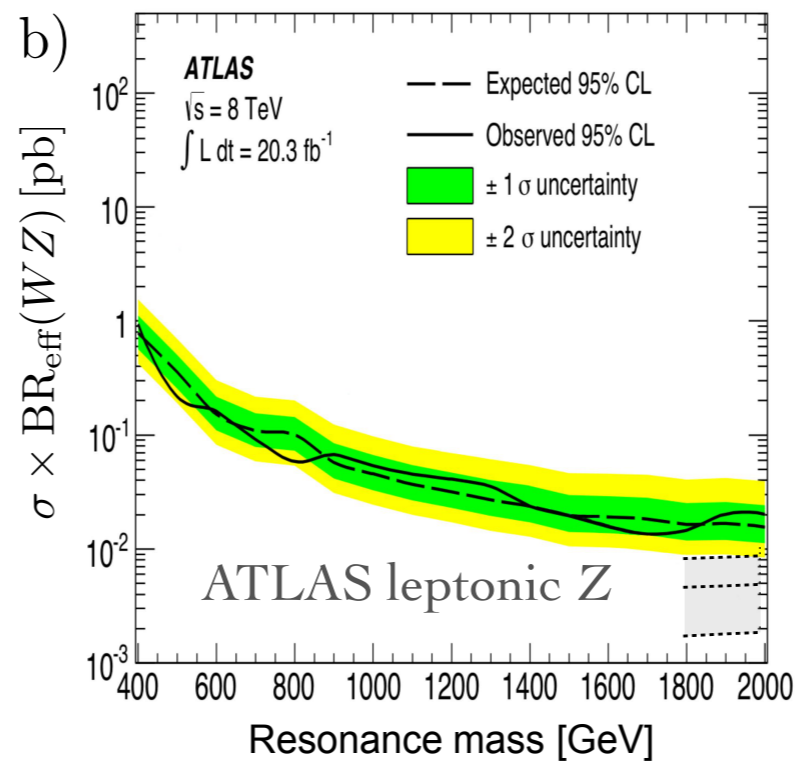
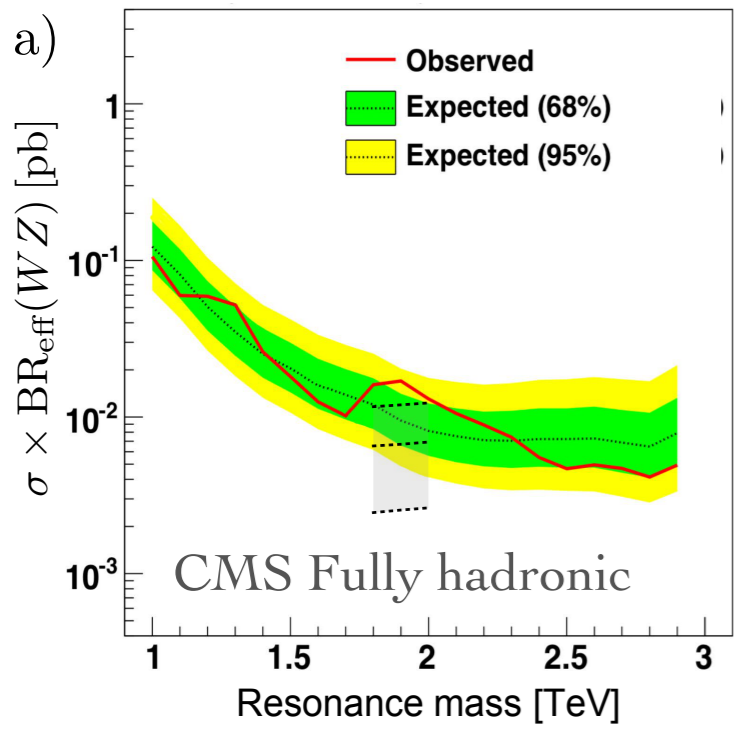
$$S_{WW} \in [2.2, 10.3]$$

$$S_{WW} = 4.2^{+3.2}_{-2.0}$$

$$S_{ZZ} \in [1.4, 6.6]$$

$$S_{ZZ} = 6.4^{+3.6}_{-2.4}$$

# Compatibility with other searches



# Conclusion I

- perfectly agrees with some channels
- could maybe even explain some small excesses
- maybe slight tension in other channels
- maybe this is exactly what we expect?

# Heavy vector triples at future colliders

# Composite Higgs models at future colliders

# Limit extrapolation

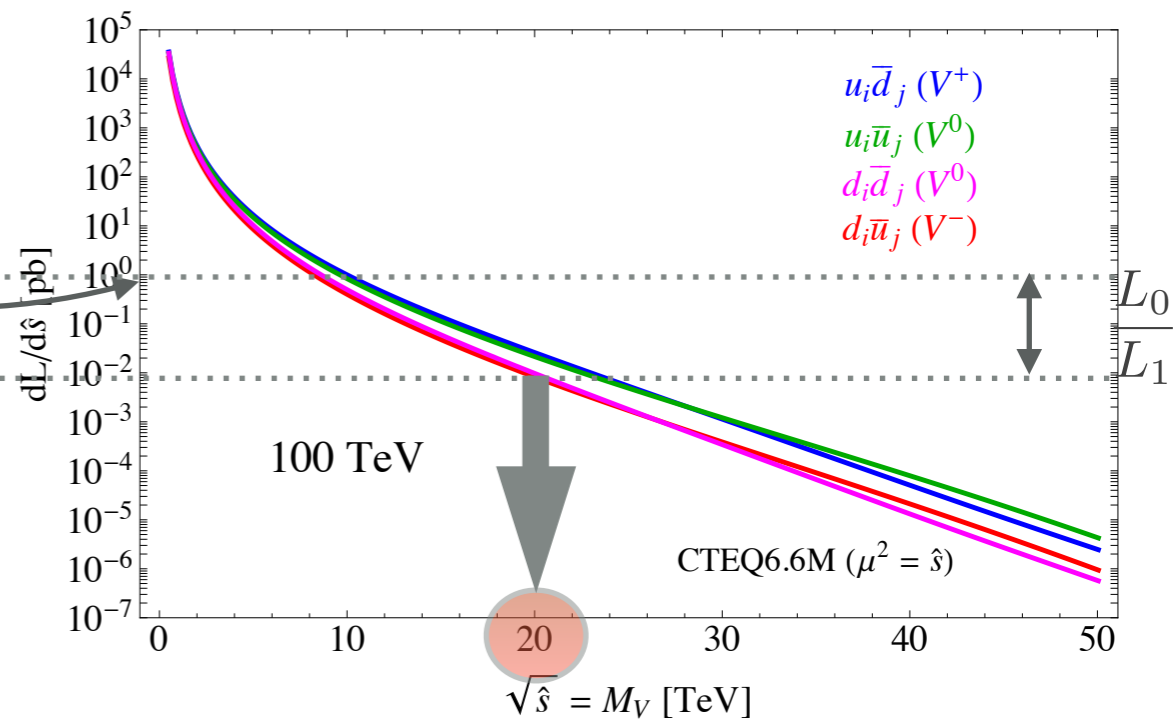
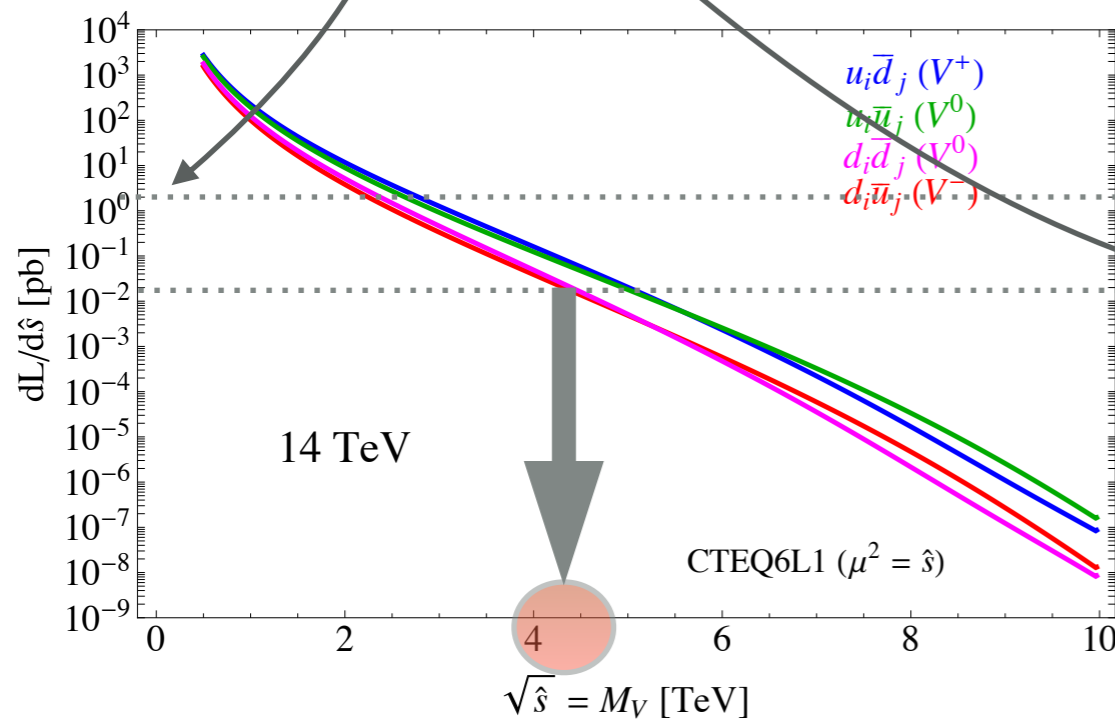
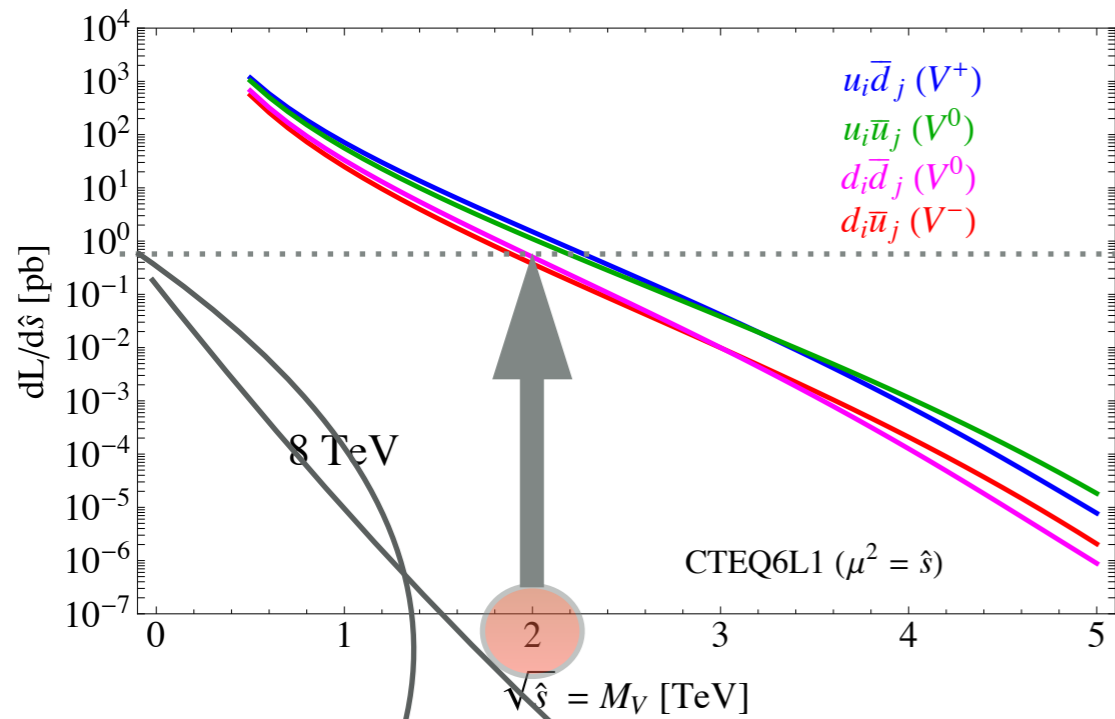
assume: excluded signal is only a function of number of

background events

background rescales with parton luminosities

$$B(s, L, m_\rho) \propto L \cdot \sum_{\{i,j\}} \int d\hat{s} \frac{1}{\hat{s}} \frac{d\mathcal{L}_{ij}}{d\hat{s}}(\sqrt{\hat{s}}; \sqrt{s}) [\hat{s} \hat{\sigma}_{ij}(\hat{s})]$$

identify relevant background process



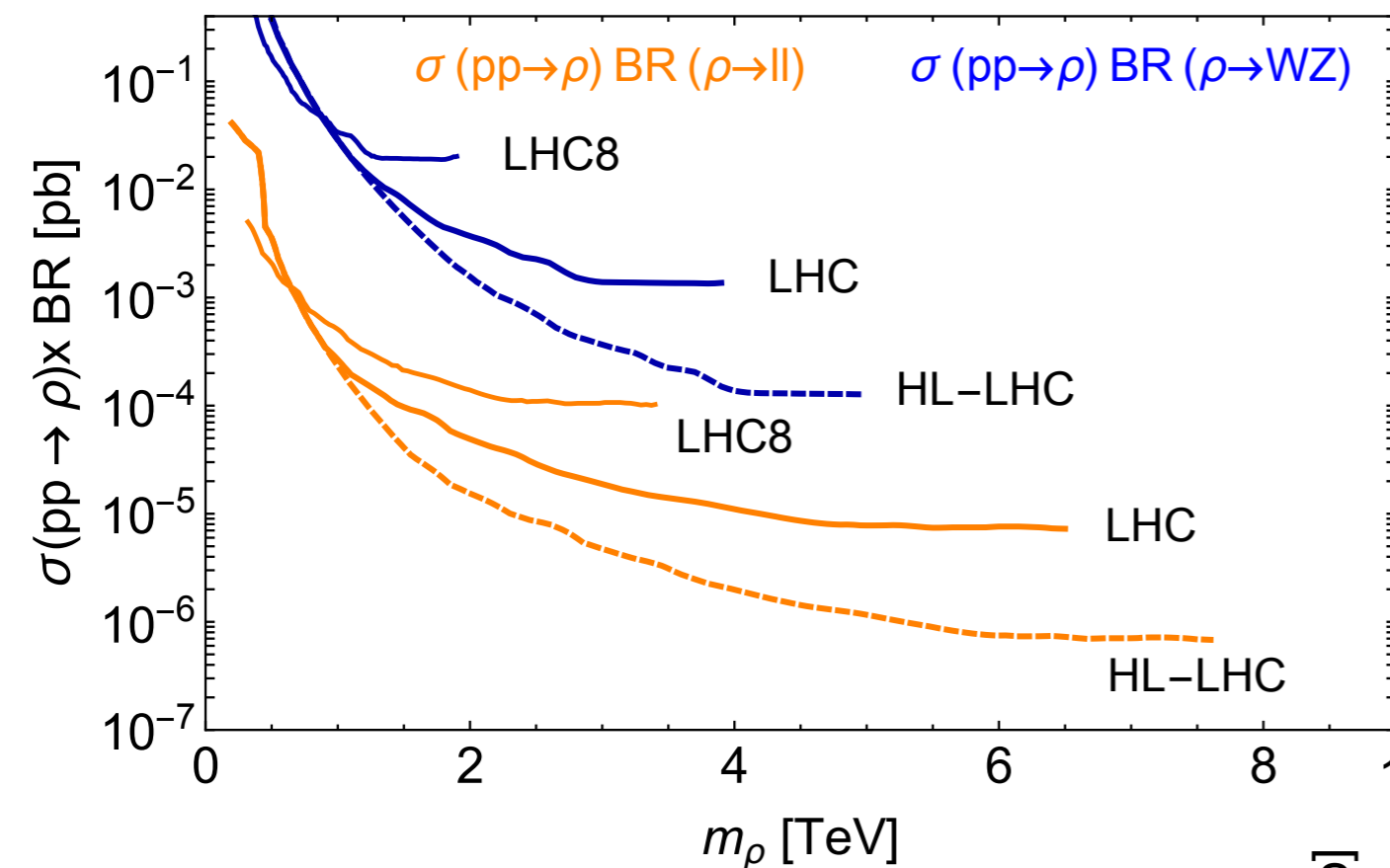


# Limit extrapolation - assumptions

- limit only driven by background for a cut-and-count experiment of events within narrow window
- shape analyses depend on background and signal kinematical distributions
- however, no large deviations expected

# Limit extrapolation

current 8 TeV LHC limits and extrapolated bounds



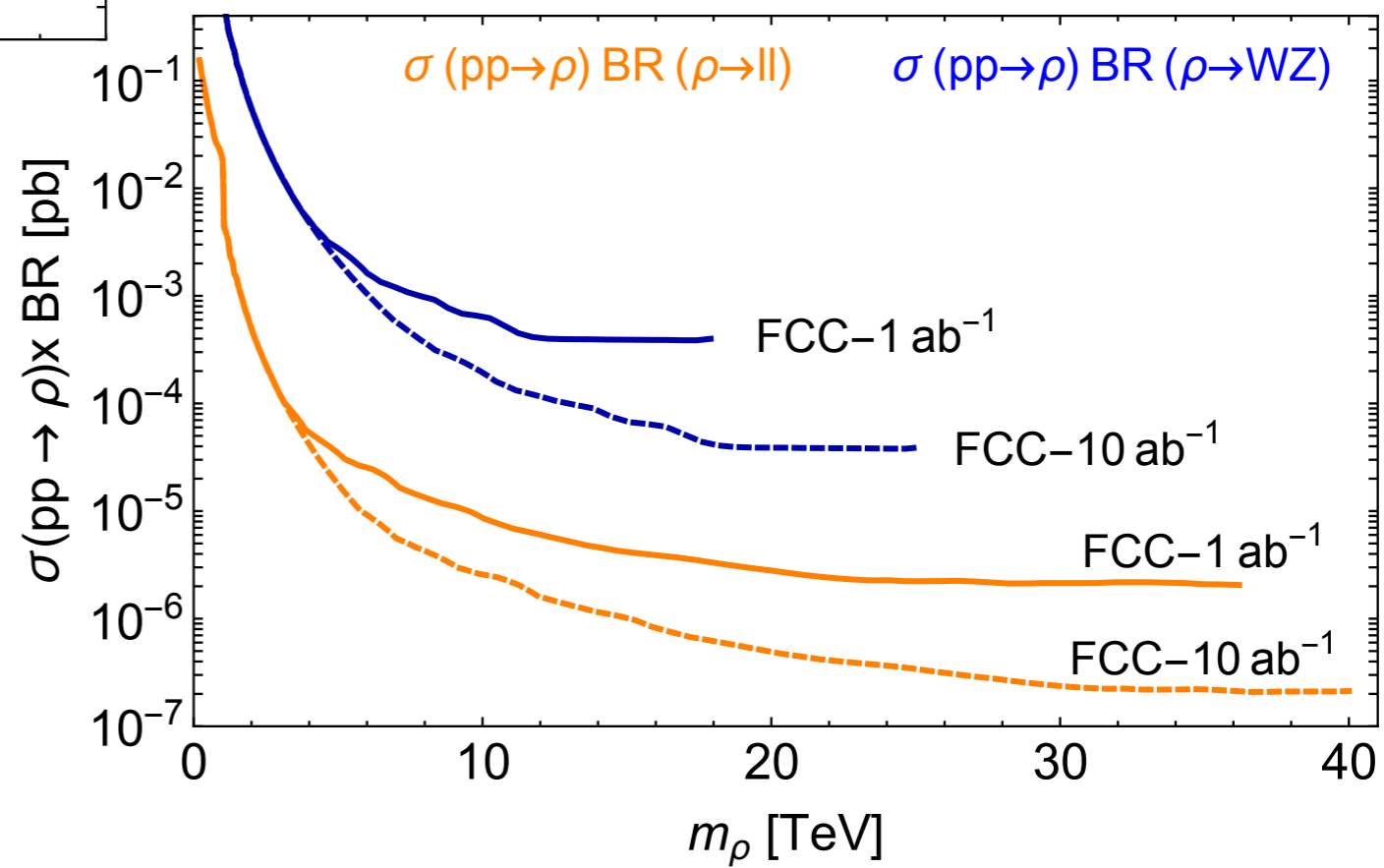
CMS search for

- opposite sign di-leptons
- fully leptonic WZ

[CMS-PAS-EXO-12-061]  
[ATLAS 1405.4123]

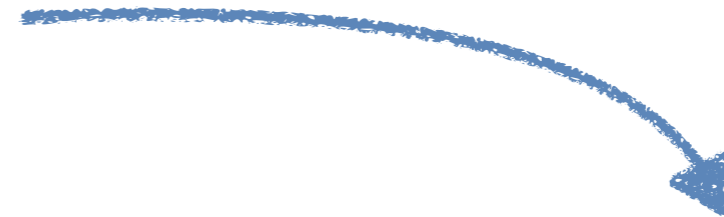
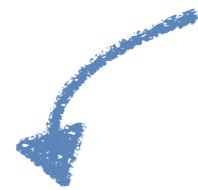
[CMS 1407.3476]  
[ATLAS 1406.4456]

- constant at large masses  
(zero background events)
- too conservative bounds at low masses



# Composite Higgs Model

- predicts direct and indirect effects



- production of EW vector resonances (here consider 3 of  $SU(2)_L$ )

[Pappadopulo, Thamm, Torre, Wulzer: 1402.4431]

- production of top partners (mass controls generation of Higgs potential and fine-tuning, very model dependent)

[Matsedonskyi, Panico, Wulzer: 1409.0100]

- modification of Higgs couplings (predictable in a fairly model-independent way)

$$a = g_{WW_h} = \sqrt{1 - \xi}$$

- EWPT (sensitive to effects only computable in specific models)
- Flavour

- for illustration focus on minimal composite Higgs model

- parameter space:

$m_\rho$

$g_\rho$

$$\xi = \frac{g_\rho^2}{m_\rho^2} v^2$$

# Minimal Composite Higgs

assume global symmetry:  $SO(5)/SO(4)$

breaking scale  $f > v$

Higgs emerges as a pseudo-NG boson

$$\mathcal{L} = \frac{1}{2} (\partial_\mu h)^2 - V(h) + \frac{v^2}{4} \text{Tr} (D_\mu \Sigma^\dagger D^\mu \Sigma) \left( 1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + b_3 \frac{h^3}{v^3} + \dots \right)$$

$$V(h) = \frac{1}{2} m_h^2 h^2 + d_3 \left( \frac{m_h^2}{2v} \right) h^3 + d_4 \left( \frac{m_h^2}{8v^2} \right) h^4 + \dots$$

$$a = \sqrt{1 - \xi}$$

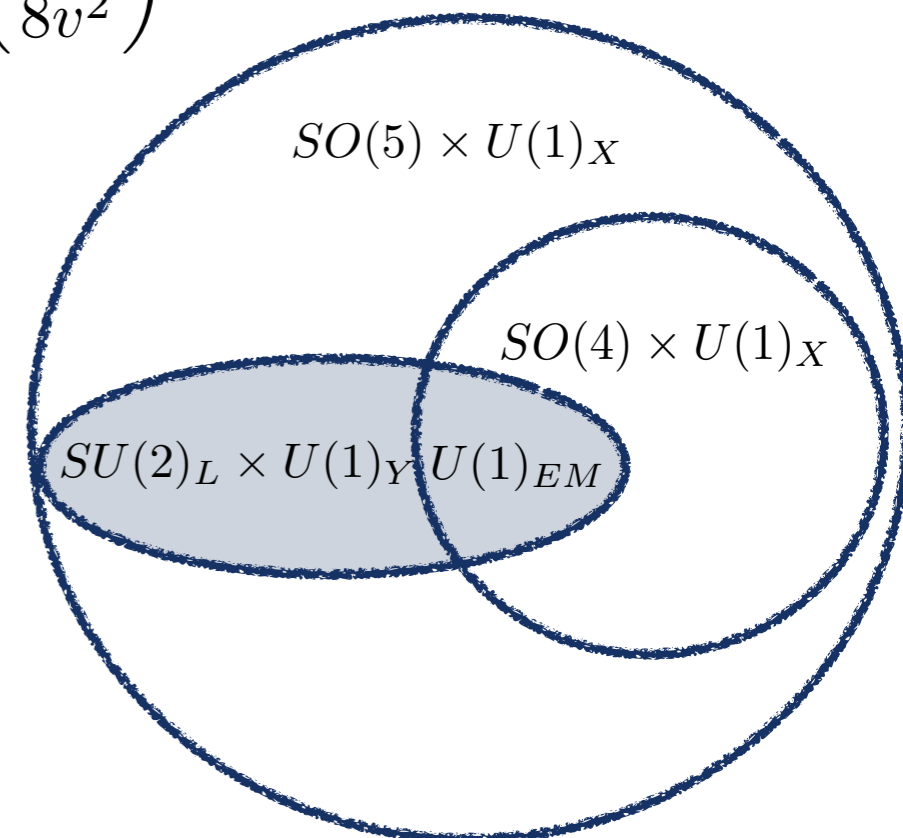
$$b = 1 - 2\xi$$

$$b_3 = -\frac{4}{3} \xi \sqrt{1 - \xi}$$

$$d_3^{(4)} = \sqrt{1 - \xi}$$

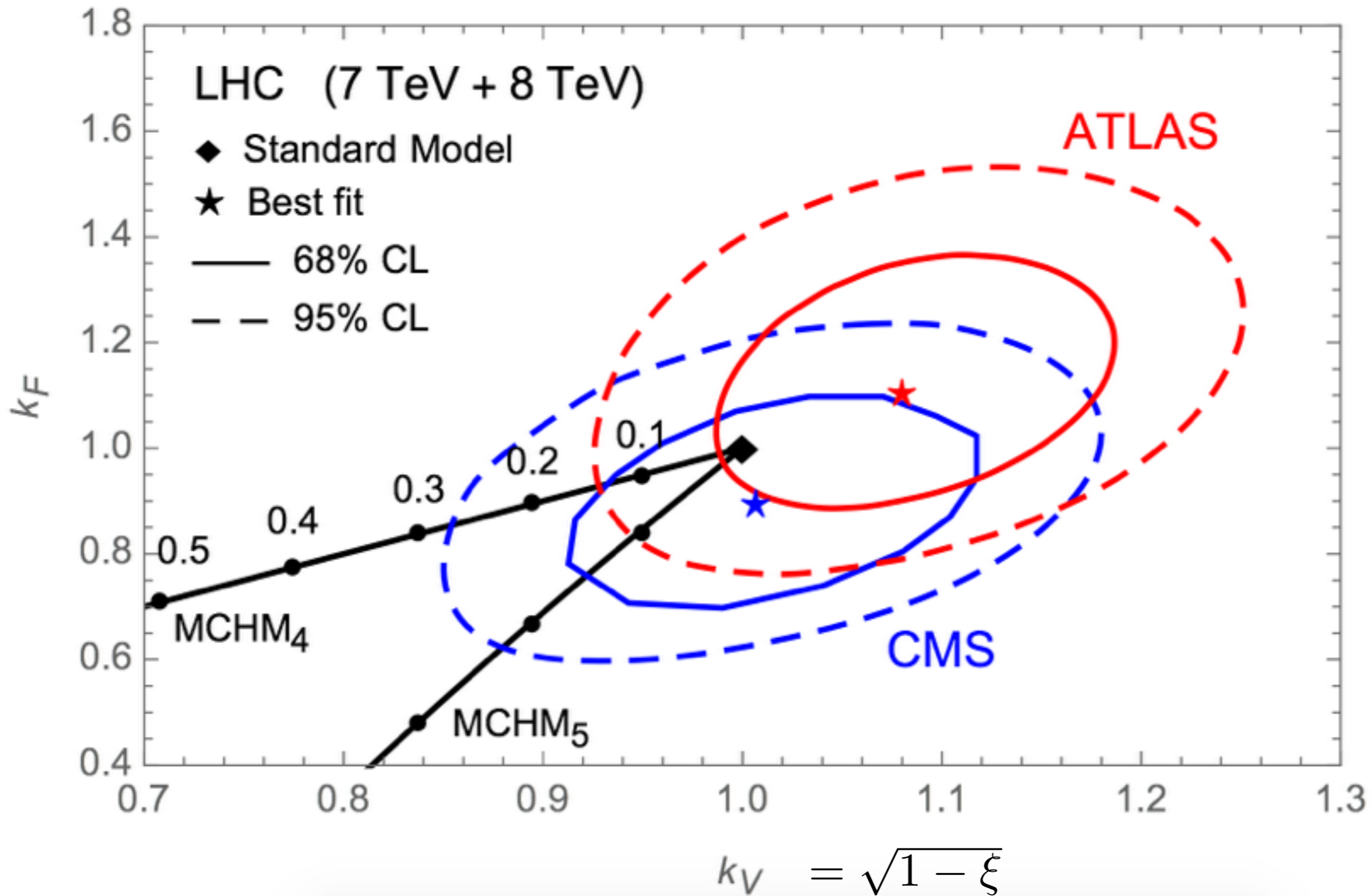
where  $\xi = \frac{v^2}{f^2}$

[Contino, Nomura, Pomarol: hep-ph/0306259]  
 [Agashe, Contino, Pomarol: hep-ph/0412089]  
 [Agashe, Contino: hep-ph/0510164 ]  
 [Contino, Da Rold, Pomarol: hep-ph/0612048]  
 [Barbieri, Bellazzini, Rychkov, Varagnolo: hep-ph/0706.0432]



Higgs couplings receive corrections of order  $\xi$

# Indirect measurements



expected  
LHC reach:  
 $\xi = 0.1$

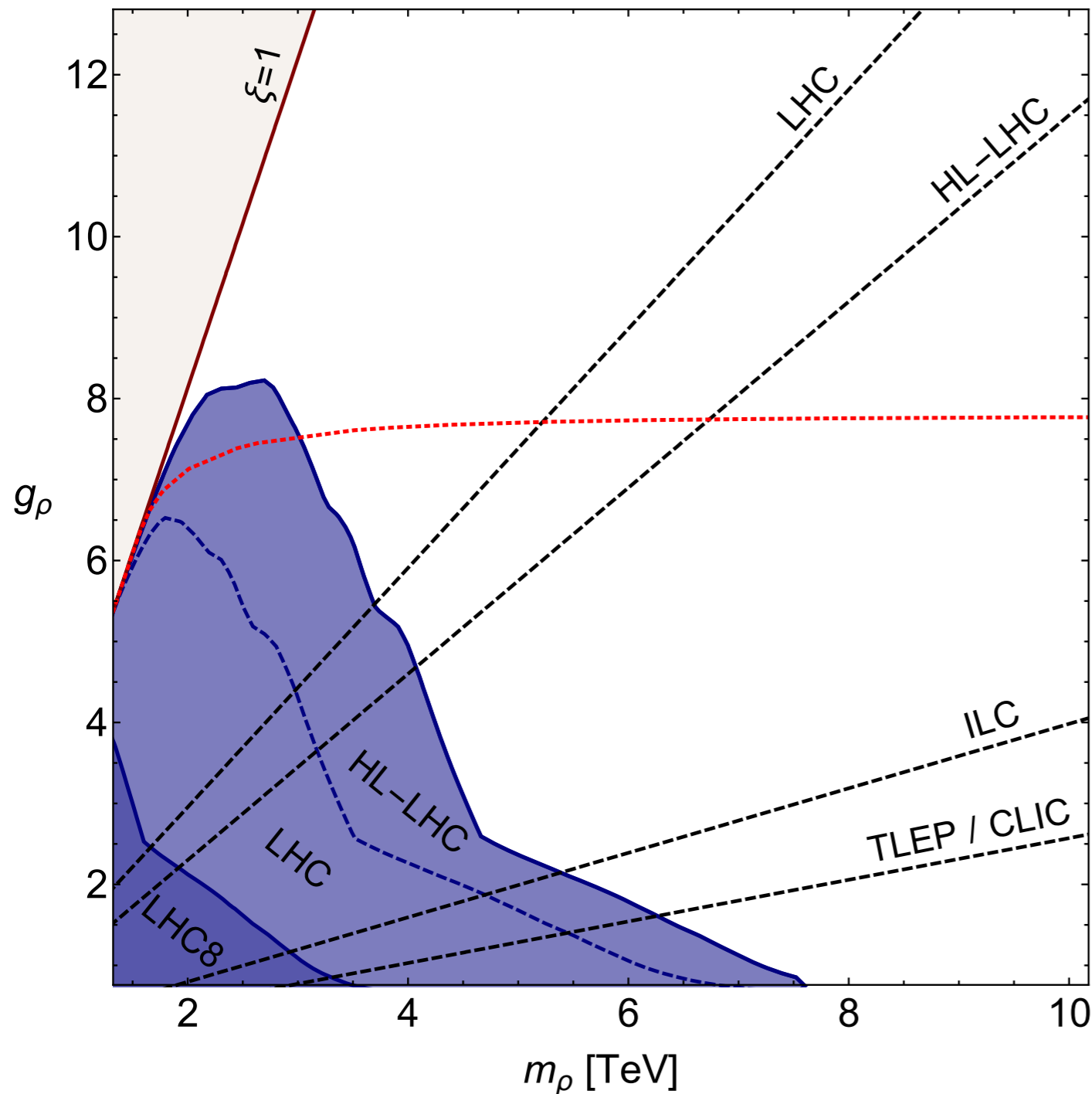
$$\text{MCHM}_4: k_F = \sqrt{1 - \xi}$$

$$\text{MCHM}_5: k_F = \frac{1 - 2\xi}{\sqrt{1 - \xi}}$$

# Indirect measurements

Collider	Energy	Luminosity	$\xi [1\sigma]$
LHC	14 TeV	300 fb <sup>-1</sup>	6.6 – 11.4 × 10 <sup>-2</sup>
LHC	14 TeV	3 ab <sup>-1</sup>	4 – 10 × 10 <sup>-2</sup>
ILC	250 GeV + 500 GeV	250 fb <sup>-1</sup> 500 fb <sup>-1</sup>	4.8-7.8 × 10 <sup>-3</sup>
CLIC	350 GeV + 1.4 TeV + 3.0 TeV	500 fb <sup>-1</sup> 1.5 ab <sup>-1</sup> 2 ab <sup>-1</sup>	2.2 × 10 <sup>-3</sup>
TLEP	240 GeV + 350 GeV	10 ab <sup>-1</sup> 2.6 ab <sup>-1</sup>	2 × 10 <sup>-3</sup>

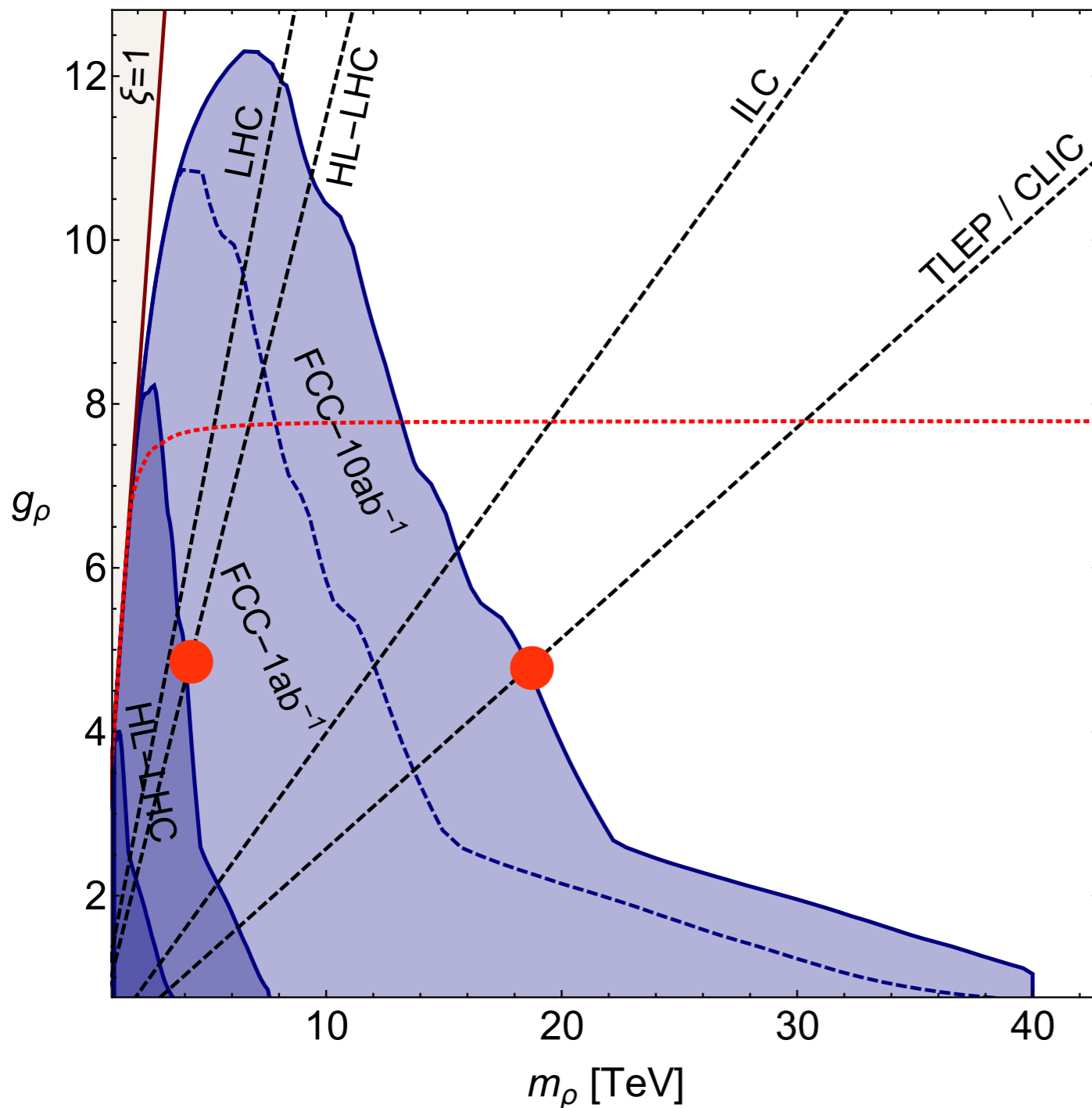
# Results in $(m_\rho, g_\rho)$



95% C.L.

- theoretically excluded  $\xi \leq 1$
- LHC8 at 8 TeV with  $20 \text{ fb}^{-1}$
- LHC at 14 TeV with  $300 \text{ fb}^{-1}$
- HL-LHC at 14 TeV with  $3 \text{ ab}^{-1}$
- di-leptons more sensitive for small  $g_\rho$
- di-boson more sensitive for large  $g_\rho$
- increase in  $\sqrt{s}$  : improves mass reach
- increase in L: improves  $g_\rho$  reach
- resonances too broad for large  $g_\rho$

# Results in $(m_\rho, g_\rho)$

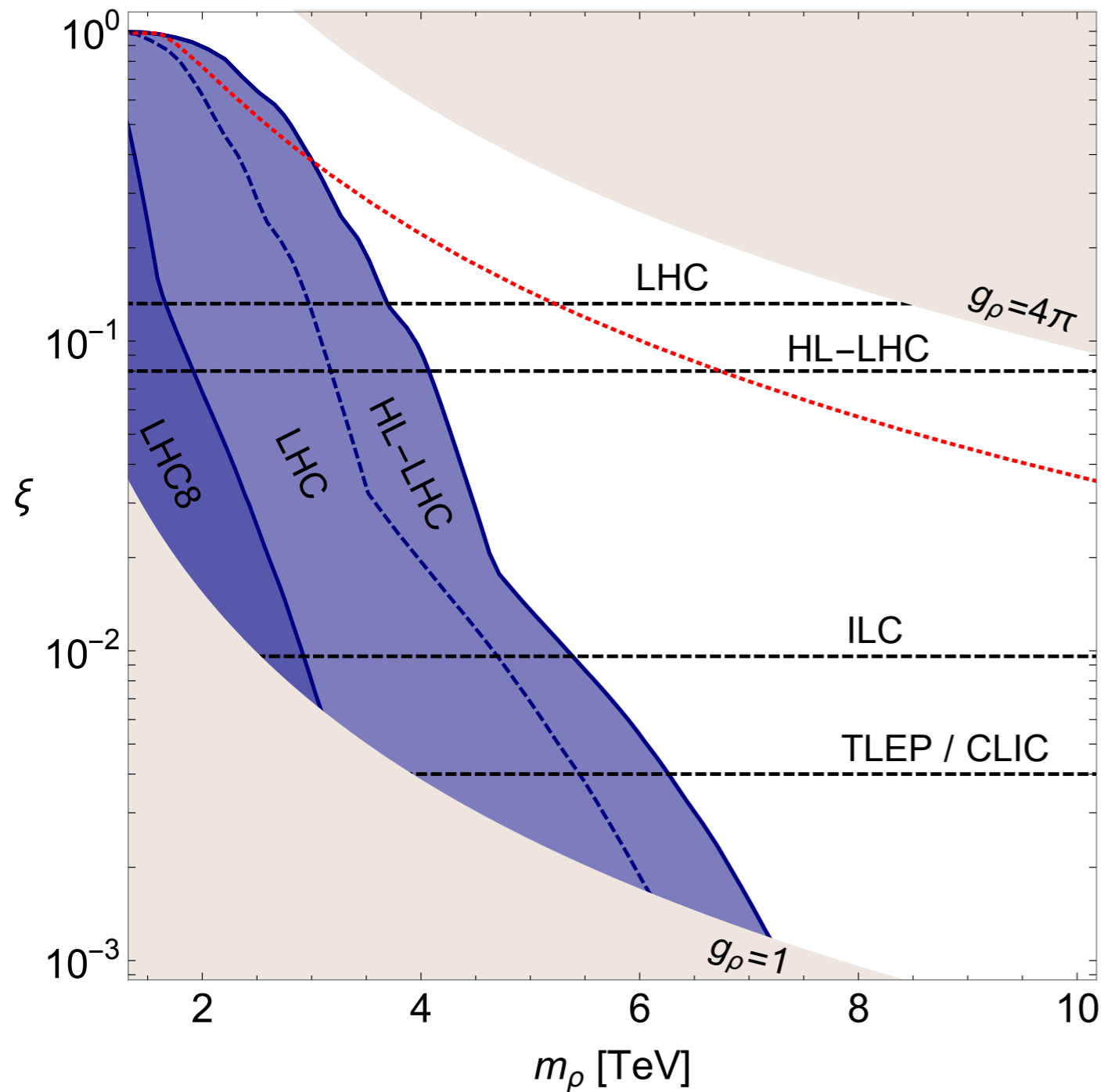


95% C.L.

- theoretically excluded  $\xi \leq 1$
- LHC8 at 8 TeV with 20 fb<sup>-1</sup>  
HL-LHC at 14 TeV with 3 ab<sup>-1</sup>
- direct: more effective for small  $g_\rho$   
ineffective for large  $g_\rho$
- indirect: more effective for large  $g_\rho$



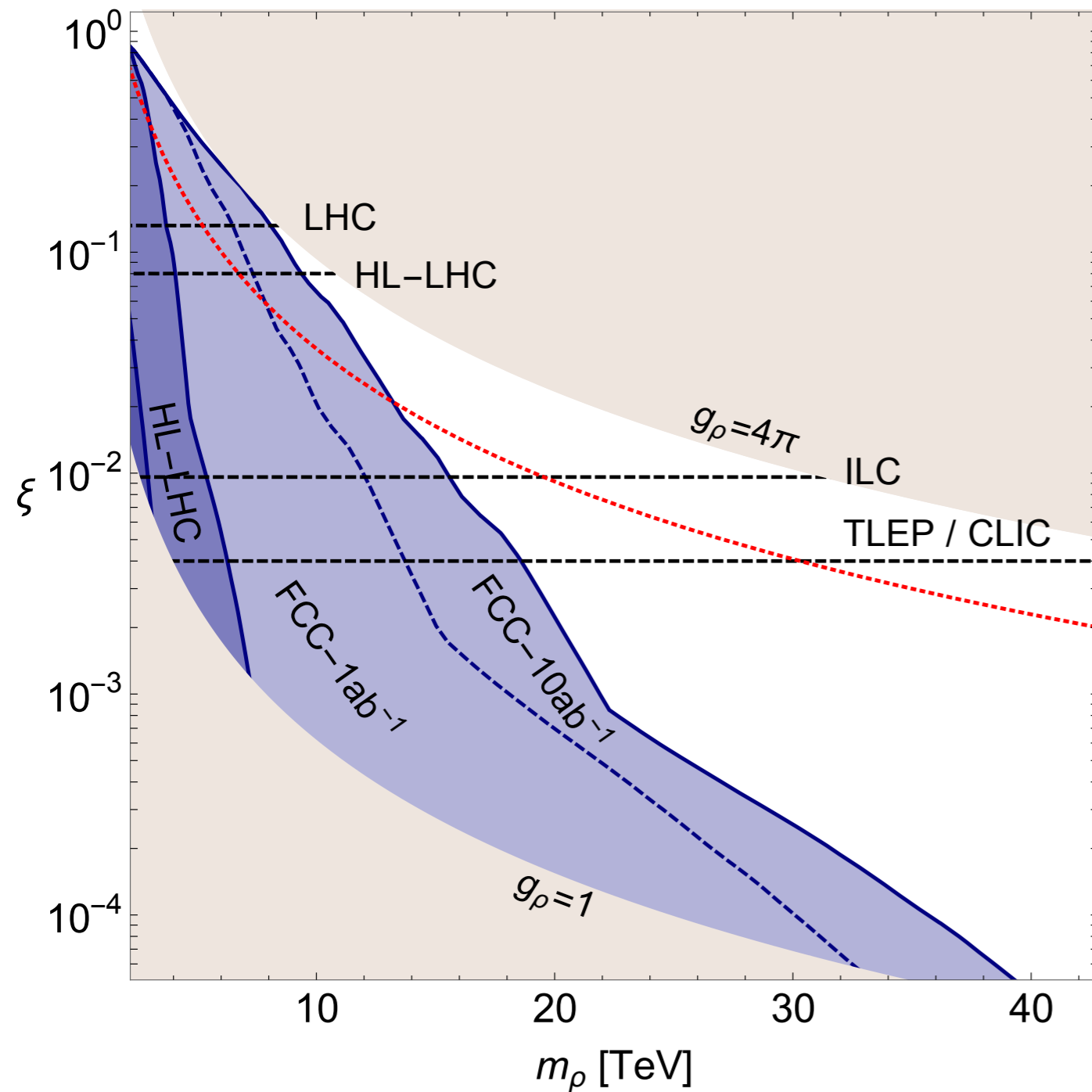
# Results in $(m_\rho, \xi)$



- theoretically excluded  $1 \leq g_\rho \leq 4\pi$
- LHC8 at 8 TeV with  $20 \text{ fb}^{-1}$   
LHC at 14 TeV with  $300 \text{ fb}^{-1}$   
HL-LHC at 14 TeV with  $3 \text{ ab}^{-1}$

95% C.L.

# Results in $(m_\rho, \xi)$



95% C.L.

- theoretically excluded  $1 \leq g_\rho \leq 4\pi$
- LHC8 at 8 TeV with  $20 \text{ fb}^{-1}$   
HL-LHC at 14 TeV with  $3 \text{ ab}^{-1}$

# Conclusions

- CH is a very compelling framework
- many ways to look for it:
  - direct: vector resonance and top partners
  - indirect: coupling modifications
- excess: maybe exactly what a resonance at the edge of discovery should look like?
- learn a lot from LHC RunII
- ... and if not, then at a future collider!

Backup

# Limit extrapolation

Input: experimental bounds on  $\sigma \times \text{BR}$  at  $\sqrt{s_0} = 8 \text{ TeV}$  with  $L_0 \simeq 20 \text{ fb}^{-1}$  for various search channels



- extrapolate limits to different proton-proton collider at  $\sqrt{s}$  and  $L$
- driven by number of background events in a small invariant mass window around the resonance peak

$$\frac{\Delta \hat{s}}{m_\rho^2} = 10\%$$

$$B(s, L, m_\rho) = B(s_0, L_0, m_\rho^0)$$

output

same limit on number of signal events

- excluded cross section at the equivalent mass

$$[\sigma \times \text{BR}](s, L; m_\rho) = \frac{L_0}{L} \cdot [\sigma \times \text{BR}](s_0, L_0; m_\rho^0)$$

# Limit extrapolation - equivalent mass

- extraction of equivalent mass

[Thamm, Torre, Wulzer: 1502.01701]

$$B(s, L, m_\rho) = B(s_0, L_0, m_\rho^0)$$

- number of background events within window  $\hat{s} \in [m_\rho^2 - \Delta\hat{s}/2, m_\rho^2 + \Delta\hat{s}/2]$

$$B(s, L, m_\rho) \propto L \cdot \sum_{\{i,j\}} \int d\hat{s} \frac{1}{\hat{s}} \frac{d\mathcal{L}_{ij}}{d\hat{s}}(\sqrt{\hat{s}}; \sqrt{s}) [\hat{s}\hat{\sigma}_{ij}(\hat{s})]$$

partonic cross-section  
contributing to background

- partonic cross section: SM process much above SM masses

$$[\hat{s}\hat{\sigma}_{ij}(\hat{s})] \simeq c_{ij} \rightarrow \text{constant}$$

- parton luminosities constant within small integration limit

$$B(s, L, m_\rho) \propto \frac{\Delta\hat{s}}{m_\rho^2} \cdot L \cdot \sum_{\{i,j\}} c_{ij} \frac{d\mathcal{L}_{ij}}{d\hat{s}}(m_\rho; \sqrt{s})$$

- equating backgrounds

$$\sum_{\{i,j\}} c_{ij} \frac{d\mathcal{L}_{ij}}{d\hat{s}}(m_\rho; \sqrt{s}) = \frac{L_0}{L} \sum_{\{i,j\}} c_{ij} \frac{d\mathcal{L}_{ij}}{d\hat{s}}(m_\rho^0; \sqrt{s_0})$$

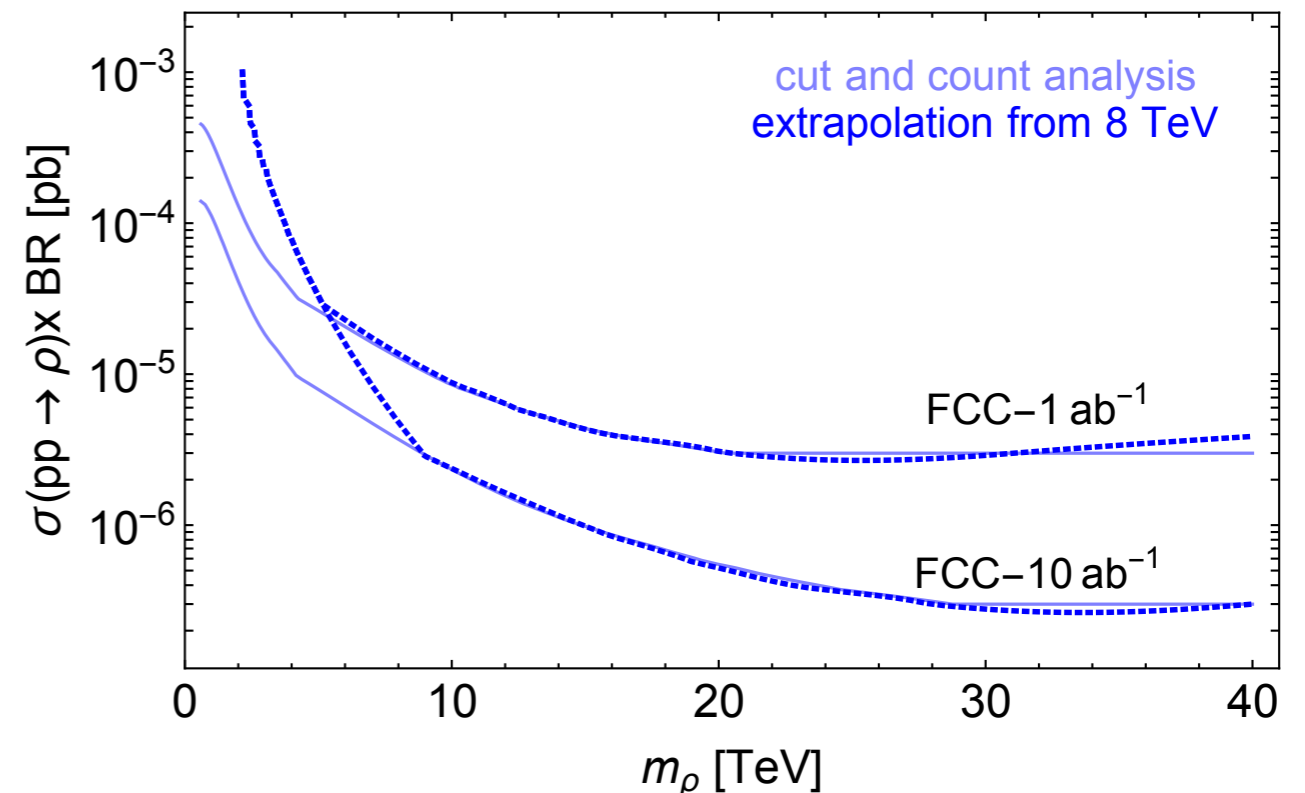
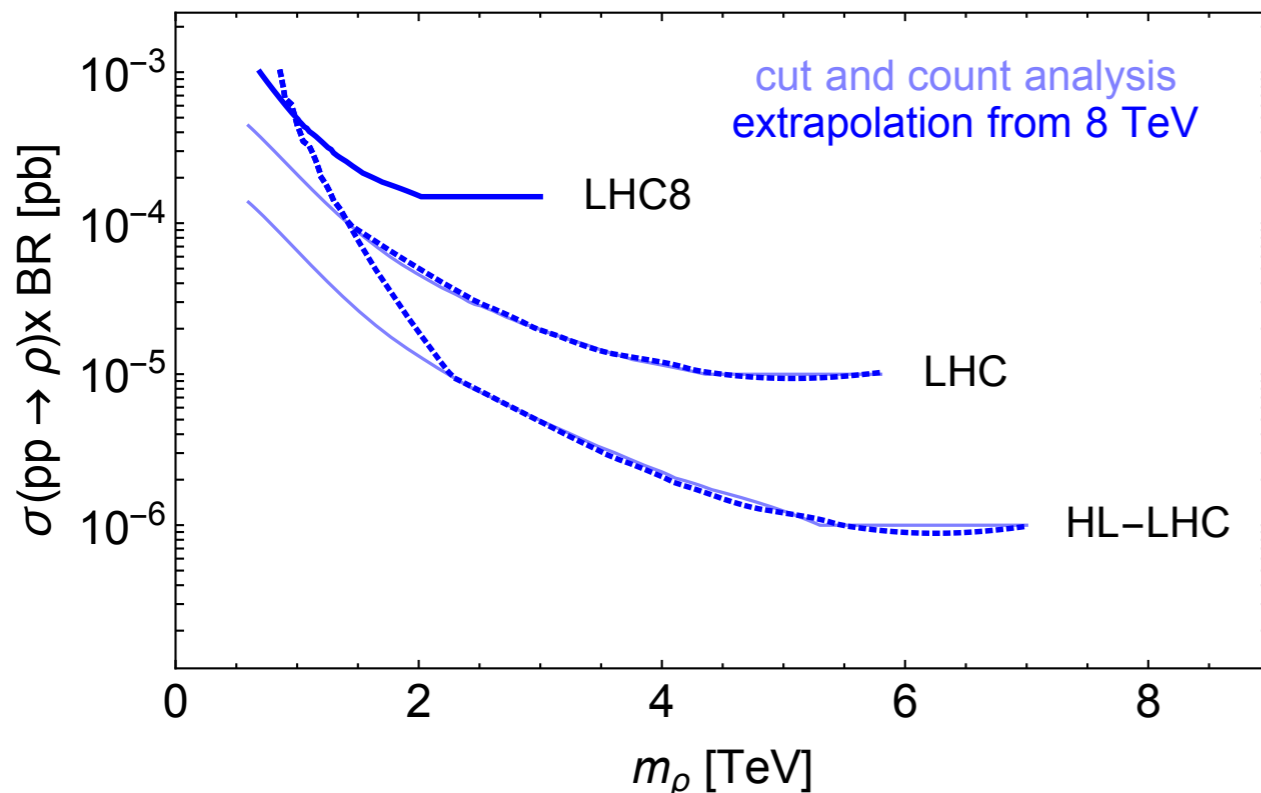
# Limit extrapolation - equivalent mass

$$\sum_{\{i,j\}} c_{ij} \frac{d\mathcal{L}_{ij}}{d\hat{s}}(m_\rho; \sqrt{s}) = \frac{L_0}{L} \sum_{\{i,j\}} c_{ij} \frac{d\mathcal{L}_{ij}}{d\hat{s}}(m_\rho^0; \sqrt{s_0})$$

- need relevant background process and parton luminosities
- sum drops for single partonic initial state
- otherwise linear combination of parton luminosities weighted by  $c_{ij}$

# Limit extrapolation - equivalent mass

- Subtlety at low masses:
  - lowest mass point of 8 TeV limit determined by sensitivity of specific analysis
- arbitrary lowest equivalent mass depending on luminosity
- smoothly raise luminosity of future collider
- extrapolated limit is the strongest at each mass
- low-mass limit conservative, not optimal





# EWPT

- set some of strongest constraints on CH models
- incalculable UV contributions can relax constraints

$$\Delta\hat{S} = \frac{g^2}{96\pi^2}\xi \log\left(\frac{\Lambda}{m_h}\right) + \frac{m_W^2}{m_\rho^2} + \alpha \frac{g^2}{16\pi^2}\xi,$$
$$\Delta\hat{T} = -\frac{3g'^2}{32\pi^2}\xi \log\left(\frac{\Lambda}{m_h}\right) + \beta \frac{3y_t^2}{16\pi^2}\xi$$

tree level exchange of vector resonances

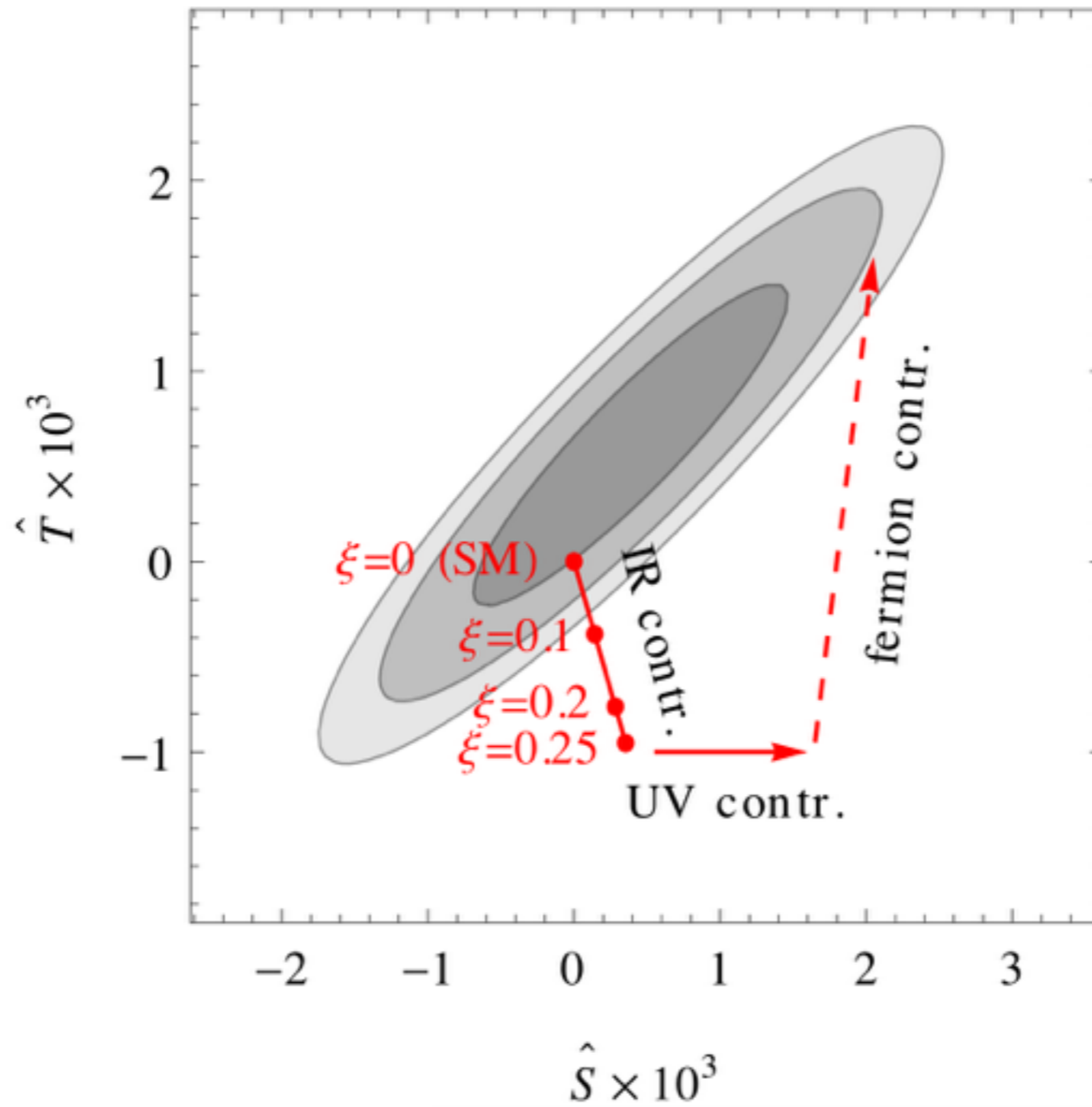
IR contribution due to Higgs coupling modifications

short distance effects

[Grojean, Matsedonskyi, Panico: 1306.4655]

- $\alpha$  and  $\beta$  constants of order 1
- define  $\chi^2(\xi, m_\rho, \alpha, \beta)$  and marginalise

# EWPT



# EWPT

- define  $\chi^2(\xi, m_\rho, \alpha, \beta)$  and marginalise
- to avoid unnatural cancellations

$$\delta_{\chi^2} = \frac{\chi^2(\xi, m_\rho, \alpha = 0, \beta = 0)}{\chi^2(\xi, m_\rho, \alpha, \beta)}$$

