Lecture 1

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# Classical scattering

$dw$ - rate $\quad J_i$ - incident current

$$d\sigma = \frac{dw}{J_i} \qquad \text{units} \quad dw \sim P/s \qquad J_i = \frac{P}{\text{area} \cdot s}$$

$d\sigma$ - area

In classical scattering the problem is "determenistic", fixed trajectory

$$b(\theta) \Rightarrow \quad b \rightarrow \theta$$

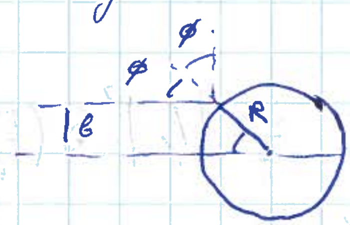

Under symmetry $\quad \frac{n}{t} = j \cdot b\, db\, d\varphi = dw$

$$d\sigma = b\, db\, d\varphi = b\left|\frac{db}{d\theta}\right| d\theta\, d\varphi \qquad d\Omega = \sin\theta\, d\theta\, d\varphi$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

Count does not depend on sign

**Example:** Sphere




$$\sin\varphi = \frac{b}{R} \qquad \theta = \pi - 2\varphi$$

$$\varphi = \frac{\pi}{2} - \frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = \frac{b}{R} \qquad \frac{d\sigma}{d\Omega} = \frac{R\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}\sin\frac{\theta}{2}} \frac{1}{2}\sin\frac{\theta}{2} R = \frac{R^2}{4}$$

Scattering C.S is constant, does not depend on angle

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \pi R^2$$

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Cross section may have several $\theta \leftarrow b_1, b_2$

Deflection angle & scattering angle

$$\frac{d\sigma}{d\Omega} = \frac{1}{\sin\theta} \sum_i b_i \left|\frac{db}{d\theta}\right|_i$$

Focusing: Rainbow effect

$$\frac{d\theta}{db} = 0$$

## Quantum scattering

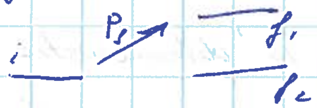
$\Psi_i \rightarrow \Psi_f$ (evolution of the w.f.)

$\Psi(t) = U(t, t_0)\Psi(t_0) \qquad S = U(+\infty, -\infty)$

Generally we have $H_0$ $\quad H_0 \Psi_i = E_i \Psi_i$

$$S_{fi} = \delta_{fi} - 2\pi i\, \delta(E_f - E_i)\, T_{fi}$$

Probability



$P_f = |S_{fi}|^2$ Sum of all probabilities is one

$$1 = \sum_f |S_{fi}|^2 = \sum S_{fi} S^*_{f f i} = \sum S_{fi} S^+_{if}$$

Unitarity $\quad S^+ S = I$

$$P_{\substack{fi \\ f\neq i}} = |2\pi T_{fi}|^2 \Big(\delta(E_f - E_i)\Big)^2 = |2\pi T_{fi}|^2 \delta(E_f - E_i) \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} e^{(E_f - E_i)t/\hbar}$$

$$\frac{dP_{fi}}{dt} = dw = \frac{2\pi}{\hbar}|T_{fi}|^2 \delta(E_f - E_i) \qquad \text{Fermi - Golden rule}$$

Rate of decay is determined by $|T|^2$ and density of states

$$\nu(E) = \sum_f \delta(E_f - E) \qquad w = \frac{2\pi}{\hbar}|T_{fi}|\, \nu(E)$$

<u>Example:</u> one-body decay

$$\nu(E) = V\int \frac{d^3 p}{(2\pi\hbar)^3} = V\int \frac{p^2 dp}{(2\pi\hbar)^3}\, \delta\left(E - \frac{p^2}{2m}\right) = \frac{V}{(2\pi\hbar)^3}\, p^2 \frac{dp}{dE} \cdot 4\pi$$

non-relativistic $\quad \dfrac{dE}{dp} = \dfrac{p}{m} \qquad \nu = \dfrac{V}{(2\pi\hbar)^3}\, p \cdot m \cdot 4\pi$

$$\frac{w}{V} = \frac{2\pi}{\hbar}|T|\frac{1}{(2\pi\hbar)^3}\, p\cdot m \sim \sqrt{E}$$

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Reaction cross-section

$$J_i = \frac{N}{V} \cdot v_i \qquad \omega = \frac{2\pi}{\hbar} |T|^2 \nu \cdot N$$

$$d\sigma = \frac{2\pi}{\hbar v_i} |T|^2 d\nu \cdot V \qquad d\sigma = V^2 \frac{2\pi}{\hbar v_i} \cdot \frac{p \cdot m \, d\Omega}{(2\pi\hbar)^3} |T|^2$$

How do we evaluate T-matrix

$H_0 + V$ - hamiltonian

$$i\hbar \frac{d\psi}{dt} = H\psi \qquad \psi = e^{-iH_0 t/\hbar} \psi'$$

$$H_0 e^{-iH_0 t/\hbar} \psi' + i\hbar e^{-iH_0 t} \frac{d\psi'}{dt} = H_0 e^{-iH_0 t/\hbar} \psi' + V e^{-iH_0 t/\hbar} \psi'$$

$$i\hbar \frac{d\psi'}{dt} = e^{iH_0 t} V e^{-iH_0 t} \psi'$$

interaction representation

$$i\hbar \frac{dU(t,t_0)}{dt} = e^{iH_0 t} V e^{-iH_0 t} U(t,t_0)$$

$$U(t,t_0) = U(t_0,t_0) + \frac{1}{i\hbar} \int^{t} e^{iH_0 t'} V e^{-iH_0 t'} U(t',t_0) \, dt'$$

$$S = 1 - \frac{i}{\hbar} V \int_{-\infty}^{\infty} e^{i(E_f - E_i) t/\hbar} dt = 1 - 2\pi i \, V_{fi} \, \delta(E_f - E_i)$$

Lowest Born approximation $\quad T_{fi} = V_{fi} = \langle \psi_f | V | \psi_i \rangle$

How to normalize $\quad \langle \psi_f | \psi_i \rangle = \delta_{fi}$

$$\psi_p = \frac{1}{\sqrt{V}} e^{ik \cdot p}$$

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{\hbar v_i} \frac{p \cdot m}{(2\pi\hbar)^3} |\langle \psi_p | V | \psi_p \rangle|^2$$

Momentum must be conserved & angular momentum is conserved

## Partial wave expansion

$$J = \frac{\hbar}{2mi}\left(\psi^* \nabla\psi - \psi\nabla\psi^*\right) \text{ - current} \qquad \rho = |\psi|^2$$

This is easy to show $\quad i\hbar \frac{d}{dt}\psi = (K+V)\psi$

Incident beam

$$e^{i\bar{k}\cdot\bar{r}} \qquad J = \frac{\hbar}{2mi}\left(e^{-ikr}\, ik\, e^{ikr} - (e^{ikr}(-ik)e^{-ikr}\right)$$

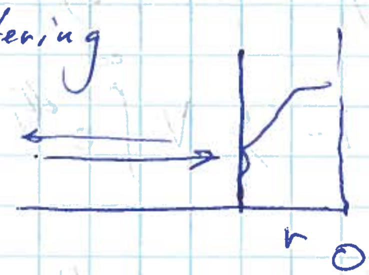
$$= \frac{\hbar k}{m} = v$$

Spherical coordinates $\quad \frac{j_\ell(kr)}{r} Y_{\ell m} \qquad \frac{d\rho}{dt} + \nabla J = 0$

Scattered beam

$$\frac{f(\theta)}{r} e^{ikr}$$

Example: One dimensional scattering



$$\psi = e^{ikx} - Ae^{-ikx}$$

$$\psi(R) = 0 \qquad e^{ik(R)} = Ae^{-ikR}$$

$$A = e^{2ikR} \quad \text{Missing phase } \delta = kR$$

$$dw = \int_{r\to\infty} d\Omega\, r^2 \quad \frac{\hbar}{m} \mathrm{Im}\left(\frac{f^*}{r} e^{-ikr}\, ik f \frac{e^{ikr}}{r}\right) = |f|^2 v$$

$$\frac{d\sigma}{d\Omega} = |f|^2$$

$$e^{i\bar{k}\cdot\bar{r}} \approx \frac{1}{2ikr}\sum_l (2l+1) P_l \left[e^{ikr} - (-1)^l e^{-ikr}\right]$$

$$e^{i\bar{k}\cdot\bar{r}} = \sum (2l+1)\, i^l j_l(kr) P_l$$

$$\psi \approx e^{ik\cdot r} + \frac{1}{r} f e^{ikr}$$

$$f(\theta) = \sum_l (2l+1) P_l f_l$$

$$\psi = \frac{1}{2ikr}\sum_l (2l+1) P_l \left[(1+2ikf_l)e^{ikr} - (-1)^l e^{-ikr}\right]$$

$$S_l = 1 + 2ikf_l \equiv e^{2i\delta_l}$$

$$f(\theta) = \sum_l (2l+1) P_l \frac{S-1}{2ik}$$ Total cross section

$$|f|^2 = \sum_{ll'} (2l+1)(2l'+1) f_l^* f_{l'} \int P_l P_{l'}\, d\Omega =$$

$$= 4\pi \sum_l (2l+1)|f_l|^2 = \frac{\pi}{k^2}\sum_l (2l+1)|S-1|^2$$

Everything is absorbed $S=0$ - initial flux

$$\cancel{\sigma_{in} = \frac{\pi}{k^2}\sum_l (2l+1)\left(1-|S-1|^2\right)}$$

$$\cancel{1-(S-1)(S^*-1) = 1+(S+S^*) - 1 - |S|^2}$$

$$\sigma_{abs} = \frac{\pi}{k^2}\sum_l (2l+1)\left(1-|S|^2\right)$$

Notes on possibilities Scattering must follow absorption

Classically $\frac{\pi}{k^2}(2l+1) = \pi b_{l+1}^2 - \pi b_l^2$ Max 4 $S=-1=e^{2i\delta}$

$\delta = \frac{\pi}{2}$ Resonance condition

$$\psi \approx \frac{1}{2ikr} \sum_l (2l+1) P_l \left[ (1+2ikf_l) e^{ikr} - (-1)^l e^{-ikr} \right]$$

$$e^{-ikr} - S e^{ikr} \to e^{-ikR} - S e^{ikR} = 0$$

$$S = e^{-2ikR}$$

Here $m=0$ $\quad \psi_l \simeq Y_{l0} \left[ e^{ikr+2i\delta} - e^{i\pi l} e^{-ikr} \right]$

$$\sim e^{i\delta \mp \frac{i\pi}{2}} \left[ e^{ikr+i\delta \mp i\pi/2} - e^{-ikr+i\pi/2-i\delta} \right] \sim \frac{1}{r}\sin(kr - \frac{\pi l}{2}$$

Cross section $\quad |1-S| \quad f_l = -\frac{1-S}{2ik} = \frac{e^{i\delta}}{k}\sin\delta$

$$\sigma = \frac{1}{k^2}\sum_l (2l+1)\sin^2\delta$$

How to compute cross-sections

$$-\frac{\hbar^2}{2m} u'' + \frac{\hbar^2 l(l+1)}{2mr^2} u + Vu = Eu$$

$$-u'' + \frac{l(l+1)}{r^2} u + \frac{2m}{\hbar^2} V u = k^2 u$$

Free solutions $F, G$

$$-F''G + G''F = 0 \qquad FG' - GF' = W[F,G]$$

$$F'G - G'F = \text{const}$$

$l=0$ these solutions are $\sin$ & $\cos$

$$u \simeq A(F\cos\delta + G\sin\delta) \qquad \begin{aligned} \sin\delta &= AW[F\,u] \\ \cos\delta &= -AW[G\,u] \end{aligned}$$

$$\tan\delta = -\frac{W[F\,u]}{W[G,u]}$$

Ph~~ras~~

# Phase shift at very low energies

$$\mathcal{E}\ uF'' - Fu'' = -\frac{V}{E} uF$$

$$\frac{d}{dx}\left[W[uF]\right] = -\frac{V}{E} uF$$

$$\sin\delta = \int_0^\infty \frac{V}{E} uF \sim \frac{2m}{\hbar k^2}\int_0^\infty V(r)\left[\frac{(kr)^{l+1}}{(2l+1)!!}\right]^2 dr$$

$$\sin\delta_l \approx \delta_l \sim k^{2l+1} \qquad \delta_0 \sim k \qquad \delta_0 = -k a_s$$

Scattering length $\quad e^{2i\delta} = 1 + 2ikf \qquad f = \dfrac{e^{2i\delta}-1}{2ik}$

$$= \frac{e^{i\delta}}{k}\sin\delta = \frac{\sin\delta}{k(\cos\delta - i\sin\delta)} = \frac{1}{k\cot\delta - ik}$$

$$(1+2ikf)(1-2ikf^*) = 1 \qquad 2ik(f-f^*) + 4k^2|f|^2 = 0$$

$$-4k\,\mathrm{Im} f + 4k^2|f|^2 = 0$$

$$k = \frac{\mathrm{Im} f}{|f|^2} = -\frac{\mathrm{Im}(f)^*}{ff^*} = -\mathrm{Im}\frac{1}{f}$$

$k\cot\delta$ is most interesting $\qquad f = -\dfrac{1}{\frac{1}{a} + ik}$

$$\sigma = 4\pi \frac{1}{\frac{1}{a^2}+k^2} = \frac{4\pi a^2}{1+k^2a^2} = 4\pi \frac{\frac{\hbar^3}{2m}}{\frac{\hbar^2}{2ma^2}+E}$$

$$\sigma = \frac{4\pi\hbar^3}{2m}\frac{1}{\varepsilon + E}$$

$\uparrow$ virtual state

## Coulomb problem

$$\eta = \frac{\alpha}{\beta} Z_1 Z_2 \qquad F_\ell \simeq \sin\left(x - \frac{\ell\pi}{2} - \eta\,\ln(2x) + \delta\right)$$

$$\delta_\ell^c = \arg\Gamma(1+\ell+i\eta) \qquad \delta_0 \sim e^{-2\pi\eta} \quad \text{Gamow fa}$$

## Analytic properties of S-matrix

$$u(k,r) = a(k)\,H^- - b(k)\,H^+$$

$$S = \cancel{\tfrac{\alpha}{\beta}}\;\frac{b(k)}{a(k)} \qquad \text{What happens if we flip } k$$

$$u(-k,r) = a(-k)H^+ - b(-k)H^-$$

$$S(+k) = \frac{a(-k)}{b(-k)} = \frac{1}{S(-k)}$$

$$S(k) = \frac{1}{S^*(k^*)}$$

Complex conjugated solution

$$u^* = a^*(k)H^+ - b^*(k)\,H^-$$

$$S(k) = \frac{a^*(k)}{b^*(k)} = \frac{1}{S^*(k)}$$

$$u = H^- - SH^+ \qquad e^{-ikr} \to e^{-i(i\varkappa)r}$$

$$\text{If } S(-i\varkappa) = 0 \qquad e^{-i(-i\varkappa)r} = e^{-\varkappa r} - \text{zero}$$

Bound state

$$S_\ell(k) = \frac{A_\ell}{k - i\varkappa}$$

$$\cancel{u_\ell = C\, i(-1)^{\ell+1}|C_\ell|} \qquad u = C_\ell\, e^{-\varkappa r}$$

$$\boxed{A_\ell = i(-1)^{\ell+1}|C_\ell|^2}$$

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Consider general S-matrix

$$S \sim \frac{1}{k-k_p}$$

$$S(-k) = \frac{1}{S(k)} \qquad S \sim \frac{k+k_p}{k-k_p}$$

$$S(k^*) = \frac{1}{S^*(k)} \qquad S \sim \frac{(k+k_p)(k-k_p^*)}{(k-k_p)(k+k_p^*)}$$

$k_p = k_0 + i\varkappa$

$$S \sim \frac{(k+k_0-i\varkappa)(k-k_0-i\varkappa)}{(k+k_0+i\varkappa)(k-k_0+i\varkappa)} = \frac{k^2-k_0^2-\varkappa^2-2i\varkappa k}{k^2-k_0^2-\varkappa^2-2i\varkappa k}$$

$$S = e^{2i\eta}\,\frac{E-E_p-\frac{i}{2}\Gamma}{E-E_p+\frac{i}{2}\Gamma} \qquad Re$$

$$\delta = \frac{1}{2i}\ln S = \eta - \arctan\frac{\Gamma/2}{E-E_p} \qquad \delta = \frac{1}{2i}\ln S$$

$$\frac{d}{dE}\delta = \frac{1}{2i}\,\frac{\frac{d}{dE}S}{S}$$

Cross-section near resonance

$$\sigma = \frac{\pi}{k^2}|S-1|^2 = \frac{\pi}{k^2}\left|\frac{i\Gamma}{E-E_p+\frac{i}{2}\Gamma}\right|^2 = \frac{\pi}{4k^2}\,\frac{\Gamma^2/4}{(E-E_p)^2+(\Gamma/2)^2}$$

$$\frac{d\delta}{dE} = \qquad S = 1 - \frac{i\Gamma}{E-E_p+\frac{i}{2}\Gamma}$$

$$\frac{dS}{dE} = \frac{i\Gamma}{(E-E_p+\frac{i}{2}\Gamma)^2} \qquad \frac{1}{S}\frac{d}{dS} = \frac{i\Gamma}{(E-E_p)^2+\frac{1}{4}\Gamma^3}$$

$$\frac{d\delta}{dE} = \frac{\Gamma/2}{(E-E_p)^2+\Gamma^2/4} \qquad \cancel{\frac{d\delta}{dE}} = \frac{\Gamma/2}{(E-E_p)^2} \quad \text{at}$$

resonance $\qquad \frac{d\delta}{dE} = \frac{2}{\Gamma}$ defines the width

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Linear Response theory

$$F(t) = \int_{-\infty}^{t} U(t-t')\, I(t')\, dt'$$

$$\Psi(t) = \Psi(E)\, e^{-iEt}$$

$$F(t) = \int F(E)\, e^{-iEt}\, dE$$

$$U(t) = \int U(E)\, e^{-iEt}\, dE$$

$$F(E) = \frac{1}{2\pi}\int E(t)\, e^{iEt}\, dt$$

$$F(t) = \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} U(E)\, e^{-iEt}\, e^{iEt'}\, I(t')\, dE$$

$$F(t) = \int_{-\infty}^{\infty} I(t')\, e^{iEt'}\, dt' \int U(E)\, e^{-iEt}\, dE$$

$$\cancel{F(E) = \int} \qquad F(t) = \int I(E)\, U(E)\, e^{-iEt}\, dE$$

$$F(E) = U(E)\, I(E)$$

$$\cancel{U(E) \Rightarrow} \quad U(\tau) = \int U(E)\, e^{-iE\tau}\, dE$$

$$U(\tau) = 0 \quad \text{if } \tau < 0$$

$$E_p = E - \frac{i}{2}\Gamma$$

$$e^{-i(E - \frac{i}{2}\Gamma)\tau} \to e^{-iE\tau}\, e^{-\frac{\Gamma}{2}\tau}$$

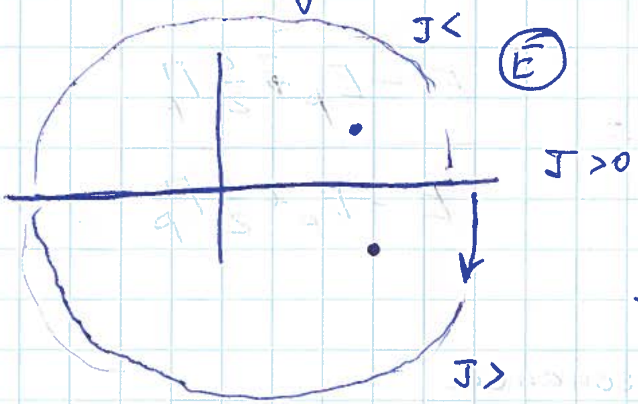
Poles related to causality

$$U(J) = \int U(E)\, e^{-iEJ}\, dE$$

$$\downarrow$$

$$S(E)$$

$U(J) = 0$ if $J<0$




$$E = E_0 \mp \frac{i}{2}\Gamma$$

if $J<0$

$$-i\left(-\frac{i}{2}\Gamma\right) J = -\frac{\Gamma}{2} J$$

if $\Gamma > 0$

if $J>0$

$$U(J) = \frac{1}{2\pi i}\sum_p R_p\, e^{-iE_p J} = \frac{1}{2\pi i}\sum_p R_p\, e^{-iE_0 J} + e^{i\frac{\Gamma}{2}J}$$

$$\psi_f(t) = \int U(E)\, \psi_i(E)\, e^{-iEt}$$

Breit wigner picture

$$S(k) = \frac{1}{k-k_p}$$

$$S(k) = \frac{k+k_p}{k-k_p} \qquad S(-k) = \frac{1}{S(k)}$$

$$S(k) = \frac{(k-k_p^*)(k+k_p)}{(k+k_p^*)(k-k_p)} \qquad S(k) = \frac{1}{S^*(k^*)}$$

This leads to

$$\cancel{(k-k_0+i\varkappa)(k}$$

$$S = \frac{(k+k_0 \mp i\varkappa)(k-k_0-i\varkappa)}{c\,c} = \frac{k^2-k_0^2-\varkappa^2-i\varkappa(k+k_0+k-k_0}{c\,c}$$

$$= \frac{k^2-(k_0+\varkappa^2)-2ik\varkappa}{c\,c} = \frac{E-E_p \mp \frac{i}{2}\Gamma_p}{E-E_p+\frac{i}{2}\Gamma_p}$$

$$\mathcal{E} = E_p - \frac{i}{2}\Gamma_p \quad \text{resonance}$$

Cross section at resonance

$$\sigma = \frac{\pi}{k^2}|S-1|^2 = \frac{\pi}{k^2}\frac{\Gamma^2}{(E-E_p)^2+\Gamma^2/4}$$

$$S-1 = \frac{E-E_p-\frac{i}{2}\Gamma}{E-E_p+\frac{i}{2}\Gamma} - 1 = \frac{-i\Gamma}{E-E_p+\frac{i}{2}\Gamma}$$

Phase shift $\quad \delta = \arg\left(E-E_p-\frac{i}{2}\Gamma_p\right) = -\arctan\frac{\Gamma/2}{E-E_r}$

Derivative of phase shift

$$S = 1 - \frac{i\Gamma}{E-E_p+\frac{i}{2}\Gamma}$$

$$\delta = \frac{1}{2i}\ln S. \qquad \frac{d\delta}{dE} = \frac{1}{2\cancel{i}}\,\frac{\cancel{E-E_p+\frac{i}{2}\Gamma}}{E-E_p-\frac{i}{2}\Gamma}\cdot\frac{\cancel{i}\Gamma}{\left(E-E_p+\frac{i}{2}\Gamma\right)^2}$$

$$\frac{d\delta}{dE} = \frac{1}{2}\,\frac{\Gamma}{(E-E_p)^2+\Gamma^2/4} \longrightarrow \frac{2}{\Gamma}$$

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# Fesbach projection

$|1\rangle$ - bound state $\quad \langle 1|2\rangle = \delta_{12}$

$|c; E\rangle$ - continuum state $\quad \langle c, E | c', E'\rangle = \delta_{cc'}\,\delta(E-E')$

$H|\alpha, E\rangle = E|\alpha, E\rangle$ - eigenstate

$$|\alpha, E\rangle = \underbrace{\sum_1 \alpha_1 |1\rangle}_{P} + \underbrace{\sum_c \int dE'\, \alpha_c(E, E')\,|c, E'\rangle}_{Q}$$

$$\begin{pmatrix} P, & Q\to P \\ P\to Q & Q \end{pmatrix} \qquad H = H_{PP} + H_{QQ} + H_{PQ} + H_{QP}$$

$$P^2 = P \qquad Q^2 = Q \qquad P + Q = 1$$

$$P|\alpha, E\rangle = \sum_1 \alpha_1(E)\,|1\rangle$$

$$H_{PP}|\psi\rangle + H_{PQ}|\psi\rangle = E\,P|\psi\rangle$$

$$H_{QQ}|\psi\rangle + H_{QP}|\psi\rangle = E\,Q|\psi\rangle$$

$$H_{QP}|\psi\rangle = \cancel{H_{QQ}}\,(E - H_{QQ})\,Q|\psi\rangle$$

$$Q|\psi\rangle = \frac{1}{E - H_{QQ}}\,H_{QP}|\psi\rangle$$

$$H_{PQ}\,\frac{1}{E - H_{QQ}}\,H_{QP}|\psi\rangle = (E - H_{PP})\,\cancel{Q}\,P|\psi\rangle$$

$$\left(H_{PP} + H_{PQ}\,\frac{1}{E - H_{QQ}}\,H_{QP}\right)|\psi\rangle = E\,P|\psi\rangle$$

In matrix form

$$\langle 1|\mathcal{H}|2\rangle = \langle 1|H|2\rangle +$$

$$+ \sum_{c'c''} \int dE' dE'' \langle 1|H|c'E'\rangle \langle c'E'| \frac{1}{E - H_{QQ}} |c''E''\rangle$$

$$\langle c''E''|H|2\rangle \qquad A_1^c = \langle 1|H|cE\rangle$$

Typical assumptions $\quad \langle q|cE\rangle = 0$ if not

$$A_1^c = \langle 1|H - E|cE\rangle$$

$$\langle 1|\mathcal{H}|2\rangle = \langle 1|H|2\rangle + \sum_c \int dE' \frac{A_1^c(E) A_2^{*c}(E)}{E - E' + i\varepsilon}$$

$$\langle 1|\mathcal{H}|2\rangle = \langle 1|H|2\rangle + \sum_c P_v \int$$

$$-\pi \sum_{c\,op.} |A_1^c|^2 \qquad W_{12} = 2\pi \sum_c |A^c|^2$$

# Nucleon - nucleon scattering problem

## Coulomb problem

$$-\frac{d^2 v}{dx^2} + \left[\frac{\ell(\ell+1)}{x^2} + \frac{e^2 Z_1 Z_2}{r}\frac{2m}{\hbar^2 k^2}\right] v = v$$

$$\frac{e^2 Z_1 Z_2}{x} \rightsquigarrow \frac{e^2 Z_1 Z_2\, m}{\hbar^2 k} = \eta \quad \text{Sommerfeld parameter}$$

$$\alpha = \frac{e^2}{\hbar c} = \frac{1}{137} \qquad \frac{e^2}{\hbar c}\,\frac{cm}{\hbar k} Z_1 Z_2 = \alpha \cdot \frac{c}{v} Z_1 Z_2$$

$k$ - small ?

$$E = \frac{\hbar^2 k^2}{2m} \sim \frac{(\hbar c)^2 k^2}{2mc^2} \approx \frac{(197\,\text{MeV}\cdot\text{fm})^2}{2\cdot 1000\,\text{MeV}}$$

$$\approx \frac{40000}{2} \approx 20\,\text{MeV}$$

$$F_\ell = \sin\left\{x - \frac{\ell\pi}{2} - \eta \ln(2x) + \delta_\ell\right\}$$

$$G_\ell \simeq \cos\left\{ \qquad \right\}$$

$$\delta_\ell \simeq \arg\Gamma(1+\ell+i\eta)$$

$$f = \frac{1}{2ik}\sum_\ell (2\ell+1) P_\ell (S_\ell - 1)$$

$$f = f^C(\theta) + \frac{1}{2ik}\sum_\ell (2\ell+1) P_\ell \left(e^{2i\delta_\ell^+} - 1\right) e^{2i\delta_\ell}$$

$$S_0 = e^{-2\pi\eta}$$ - cross section is exponentially suppressed. Gamow factor