



ENHANCING CONSTRAINTS
ON MODIFIED GRAVITY AND INFLATION
WITH MULTI-TRACER COSMOLOGICAL SURVEYS

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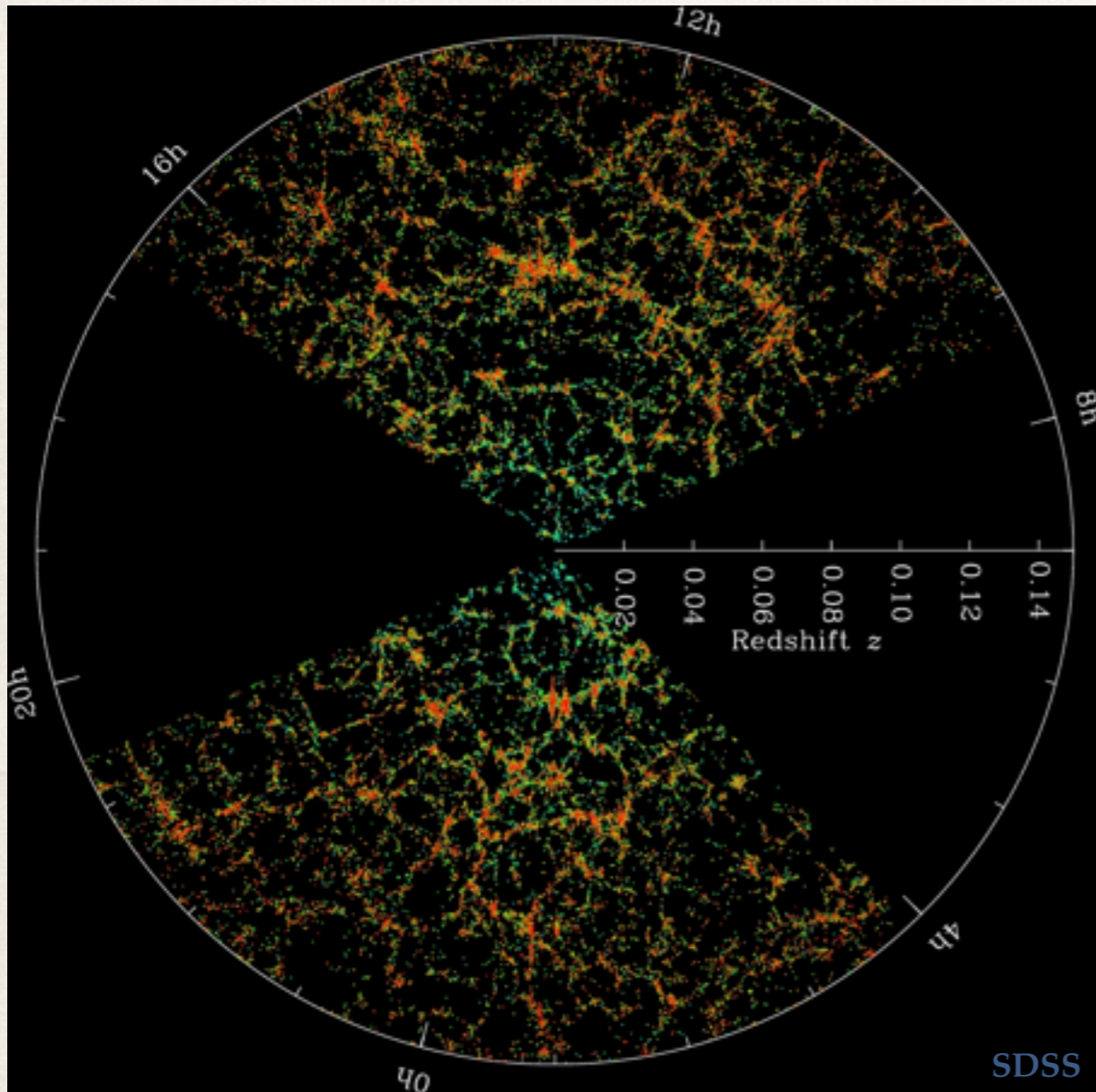


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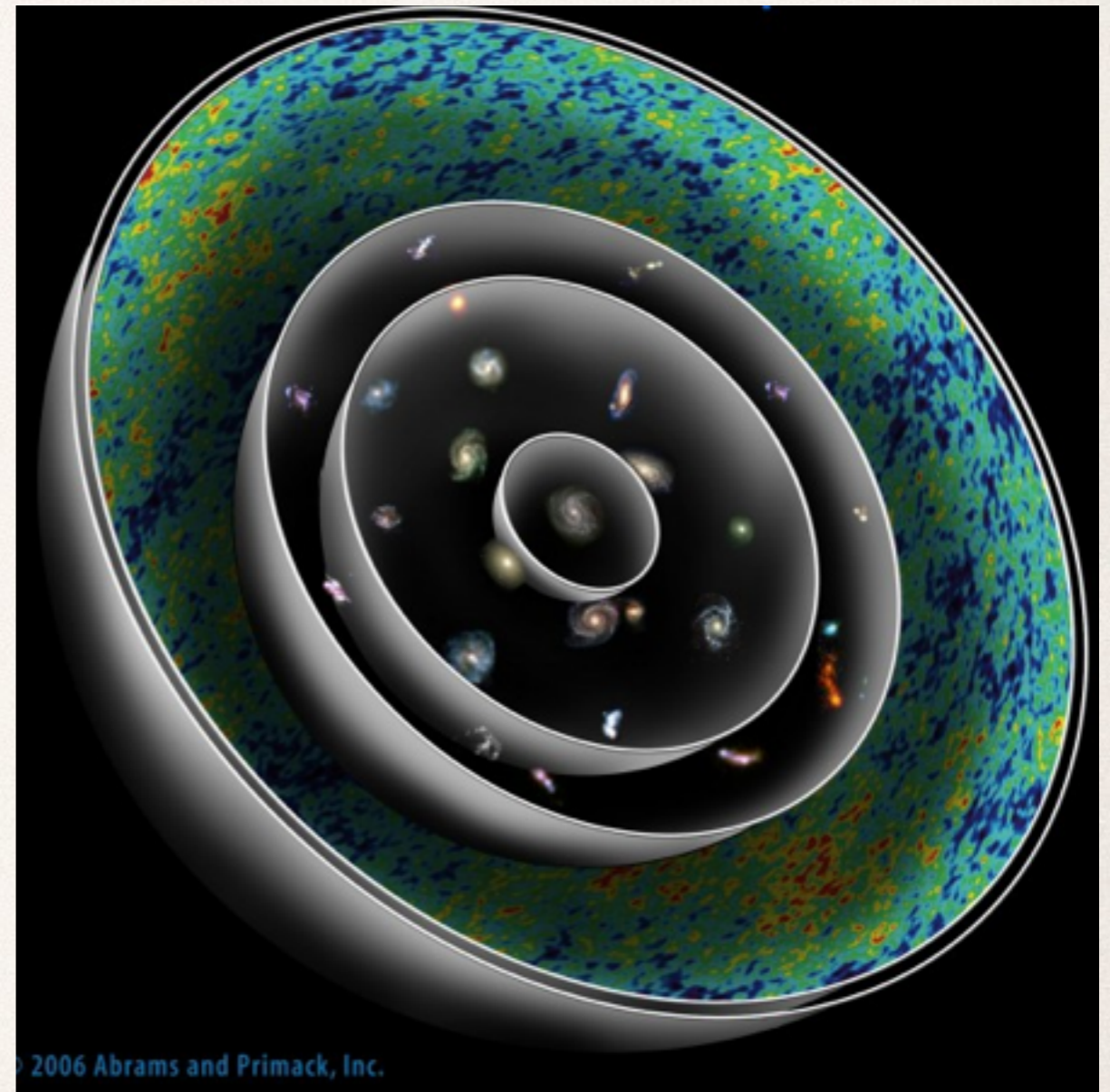
Galaxy surveys are evolving

We used to live in an era of shot (“counts”) noise



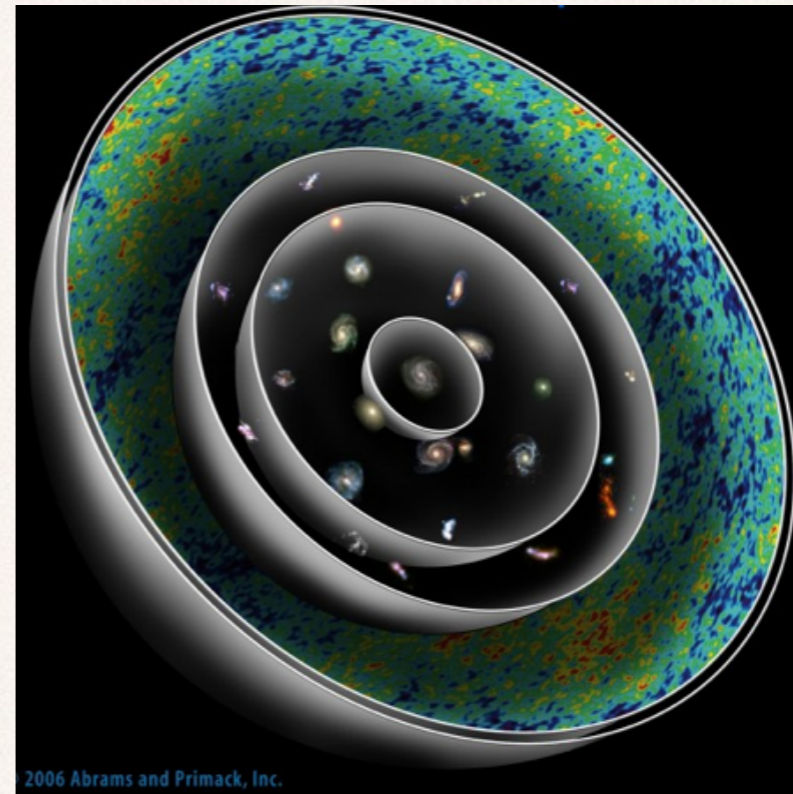
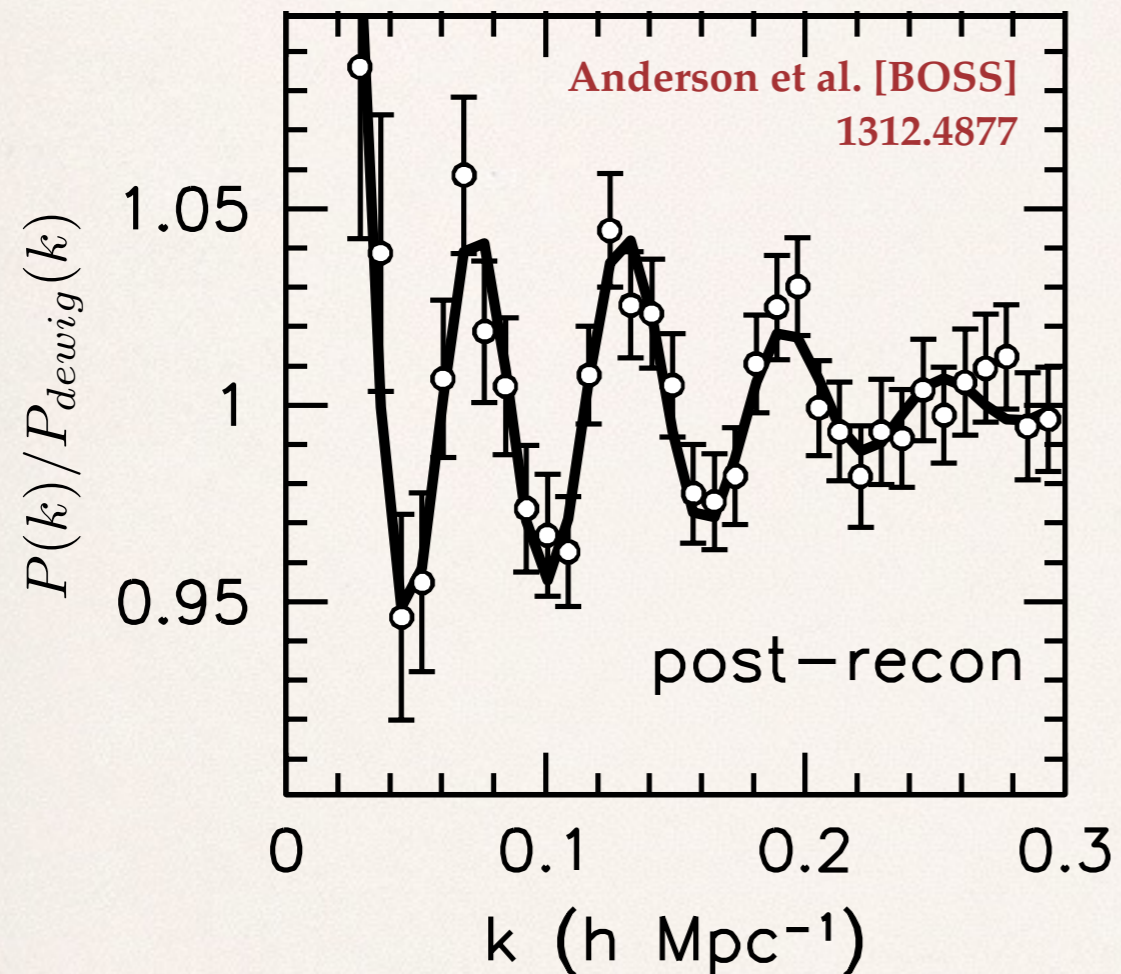
[Finding galaxies was the limiting factor]

We are now in the age of cosmic variance and systematics



[Volume and control are the limiting factors]

Surveys of large-scale structure are now limited by **cosmic variance** and **systematics**



However, up to any given redshift there is only a finite volume.
Moreover, we are reaching closer to the limits of the observable Universe!

Shot noise:

finite number of **counts** of the tracers
of the underlying density field
(Poisson statistics)

Cosmic variance:

finite **volume** inside which we can
estimate the **amplitudes and phases** of
the (Gaussian) random **modes** of the
density field

$$P_g(\vec{k}) \simeq b_g^2 P_m(\vec{k}) + \frac{1}{\bar{n}_g}$$

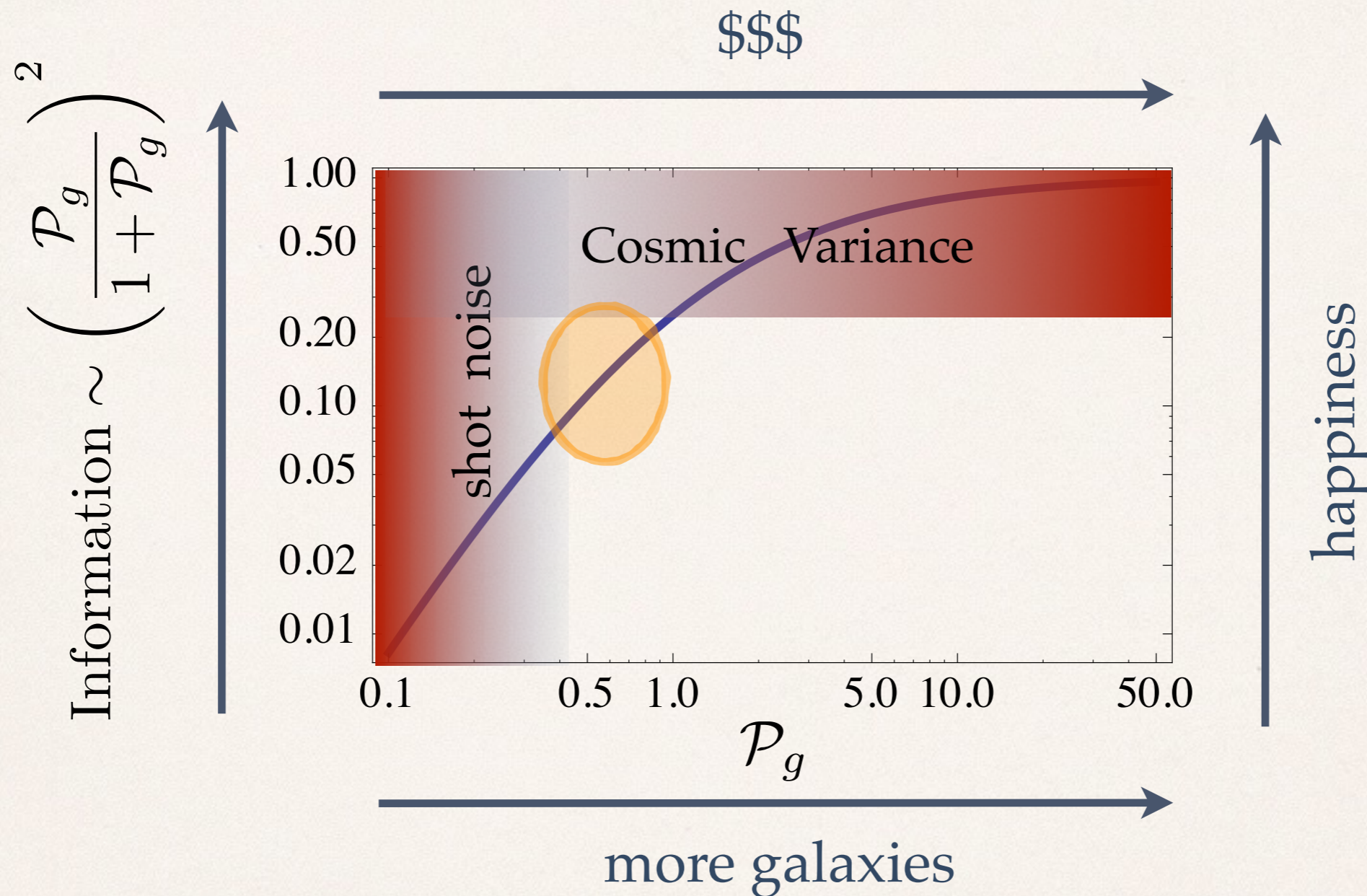
Clustering in units of shot noise

$$\bar{n}_g P_g(\vec{k}) \simeq \bar{n}_g b_g^2 P_m(\vec{k}) + 1$$

SNR

noise

Fisher information of galaxy surveys



Signal/Noise:
Clustering strength
of galaxy type "g" in
redshift space

$$\mathcal{P}_g(\vec{x}, \vec{k}) \equiv \bar{n}_g(\vec{x}) \left[b_g(z, k) + f(z) \mu_k^2 \right]^2 G^2(z) P_m(k)$$

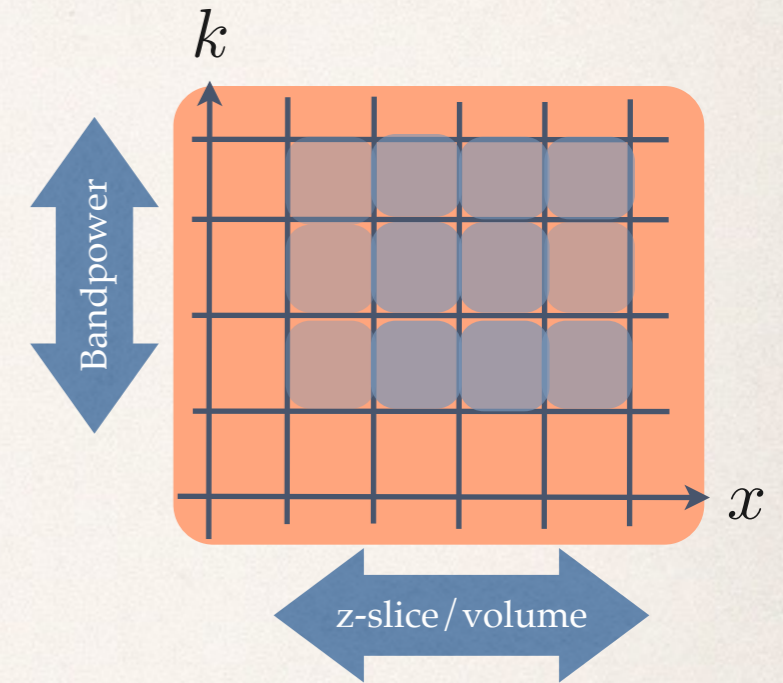
Fisher information in phase space

On each **unit volume** of **phase space** there is a certain **amount of information** about the clustering strength:

$$F[\log \mathcal{P}_g] \times \frac{\Delta V_x \Delta V_k}{(2\pi)^3} = \frac{1}{2} \left(\frac{\mathcal{P}_g}{1 + \mathcal{P}_g} \right)^2 \times \frac{\Delta V_x \Delta V_k}{(2\pi)^3}$$

Fisher information/
(phase space volume)

phase space
volume = $\Delta \mathcal{V}$.

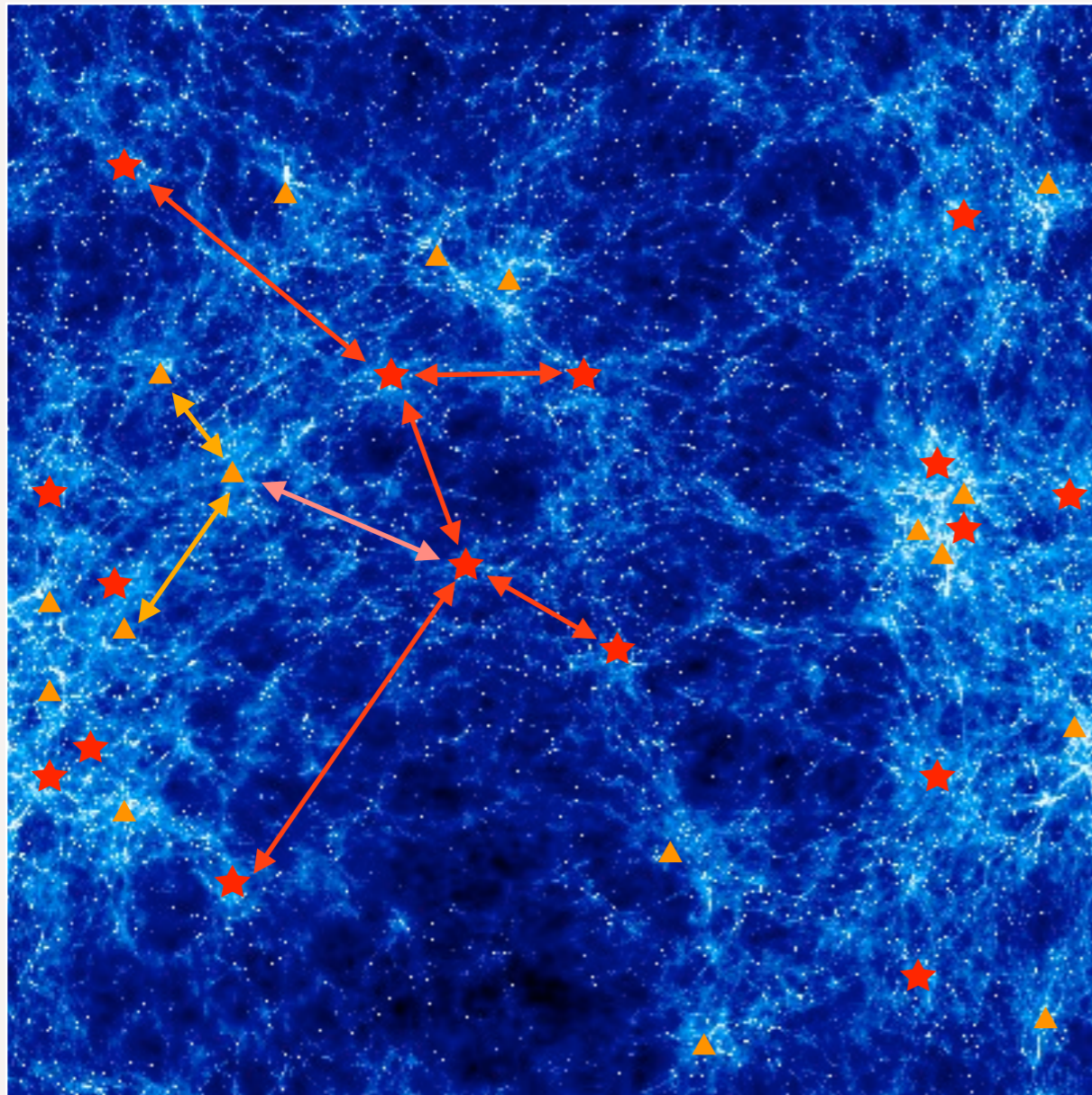


The **precision** (SNR) with which we can **estimate the clustering strength** is:

$$\frac{\mathcal{P}_g^2}{\sigma^2(\mathcal{P}_g)} = F[\log \mathcal{P}_g] \times \Delta \mathcal{V} = \frac{1}{2} \left(\frac{\mathcal{P}_g}{1 + \mathcal{P}_g} \right)^2 \Delta \mathcal{V}$$

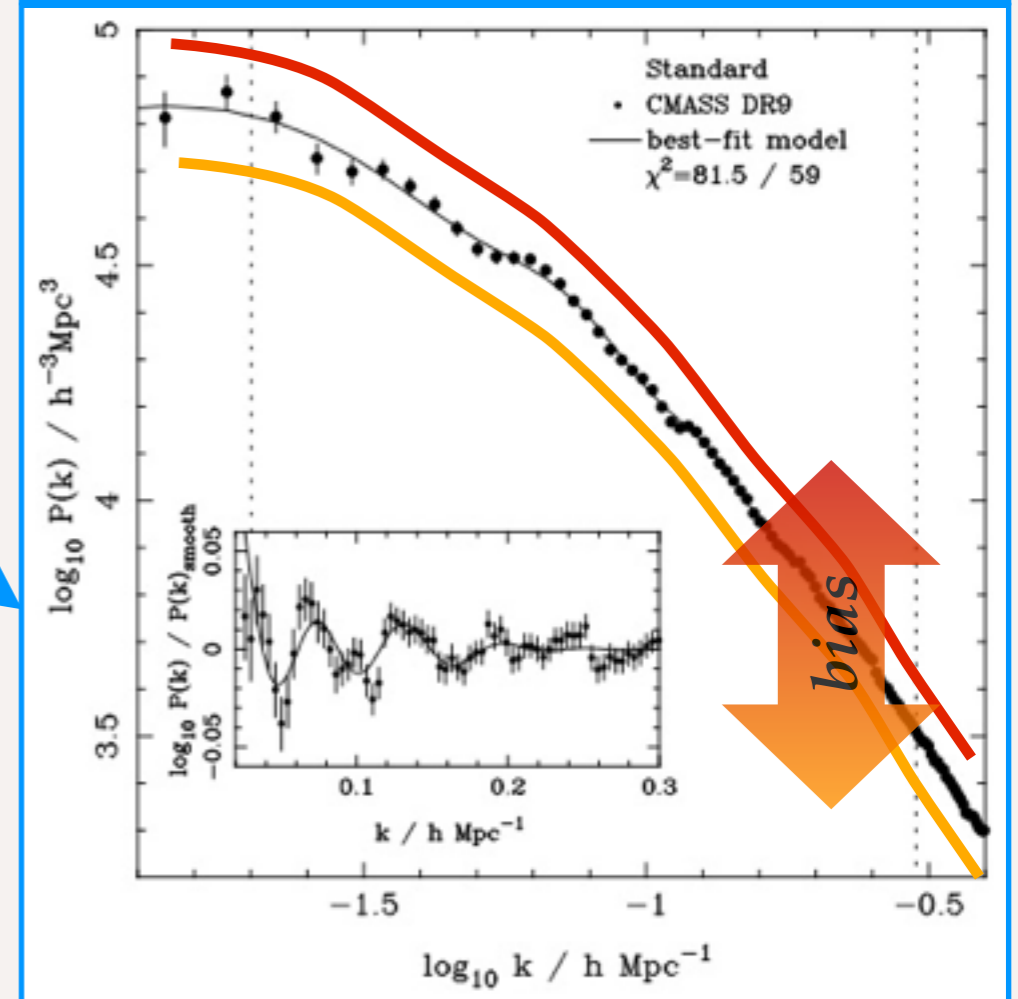
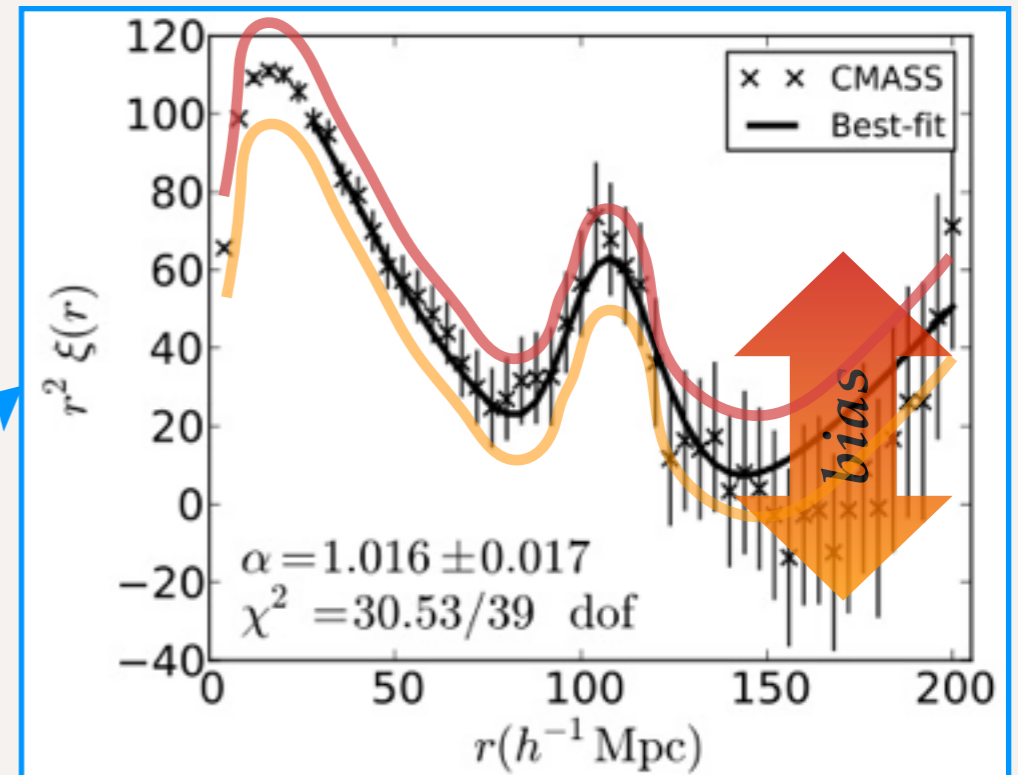
$$F = \frac{1}{2} \left(\frac{\text{signal}}{\text{signal} + \text{noise}} \right)^2 \leq \frac{1}{2}$$

The Universe has many different types of galaxies, halos, etc...



Clustering in position space

Clustering in Fourier space



Multi-tracer Fisher information matrix

R.A. 2012

R.A. & Katie Leonard 2013

Let's say we have **several** ($\alpha = 1, 2, \dots, N$) **different types of tracers** of large-scale structure. E.g. : $\alpha=1$ =LRGs , $\alpha=2$ =ELGs , $\alpha=3$ =quasars , etc.

$$\mathcal{P}_\alpha(k, \mu_k; z) = n_\alpha(z) [b_\alpha(z) + f(z) \mu_k^2]^2 P(k; z) \quad \mu_k = \frac{k_{||}}{k}$$

The **Fisher matrix** for the N clustering strengths (power spectra) is:

$$F_{\alpha\beta} = F(\log \mathcal{P}_\alpha, \log \mathcal{P}_\beta) = \frac{1}{4} \left[\delta_{\alpha\beta} \frac{\mathcal{P}_\alpha \mathcal{P}}{1 + \mathcal{P}} + \frac{\mathcal{P}_\alpha \mathcal{P}_\beta (1 - \mathcal{P})}{(1 + \mathcal{P})^2} \right] \quad , \quad \mathcal{P} = \sum_{\alpha} \mathcal{P}_\alpha$$

Multi-tracer Fisher information

Multi-tracer technique:

Seljak 2008

McDonald & Seljak 2008

Gil-Marín et al. 2011

Hamaus et al. 2011,2012

Cai & Bernstein 2011

Fisher matrix:

R.A. 2012

R.A. & K. Leonard 2013

$$F[\log \mathcal{P}_\alpha, \log \mathcal{P}_\beta] = \frac{1}{4} \left[\frac{\mathcal{P}_\alpha \mathcal{P}_\beta (1 - \mathcal{P})}{(1 + \mathcal{P})^2} + \delta_{\alpha\beta} \frac{\mathcal{P}_\alpha \mathcal{P}}{1 + \mathcal{P}} \right]$$

1 tracer $F = \frac{1}{2} \left(\frac{\mathcal{P}}{1 + \mathcal{P}} \right)^2$

2 tracers $\Rightarrow \frac{1}{4} \begin{pmatrix} \frac{\mathcal{P}_1^2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} + \frac{\mathcal{P}_1 \mathcal{P}}{1 + \mathcal{P}} & \frac{\mathcal{P}_1 \mathcal{P}_2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} \\ \frac{\mathcal{P}_1 \mathcal{P}_2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} & \frac{\mathcal{P}_2^2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} + \frac{\mathcal{P}_2 \mathcal{P}}{1 + \mathcal{P}} \end{pmatrix}$

3 tracers $\Rightarrow \frac{1}{4} \begin{pmatrix} \frac{\mathcal{P}_1^2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} + \frac{\mathcal{P}_1 \mathcal{P}}{1 + \mathcal{P}} & \frac{\mathcal{P}_1 \mathcal{P}_2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} & \frac{\mathcal{P}_1 \mathcal{P}_3 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} \\ \frac{\mathcal{P}_1 \mathcal{P}_2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} & \frac{\mathcal{P}_2^2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} + \frac{\mathcal{P}_2 \mathcal{P}}{1 + \mathcal{P}} & \frac{\mathcal{P}_2 \mathcal{P}_3 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} \\ \frac{\mathcal{P}_1 \mathcal{P}_3 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} & \frac{\mathcal{P}_2 \mathcal{P}_3 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} & \frac{\mathcal{P}_3^2 (1 - \mathcal{P})}{(1 + \mathcal{P})^2} + \frac{\mathcal{P}_3 \mathcal{P}}{1 + \mathcal{P}} \end{pmatrix}$

OK... but are we in fact **gaining** any information by **splitting galaxies into types**, or are we just “shuffling around” the information?

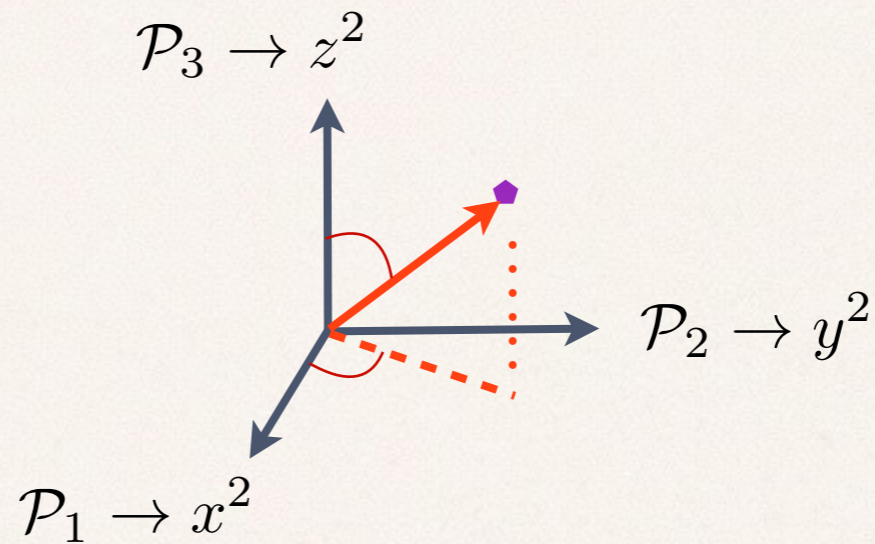
Multi-tracer Fisher information

Yes, we **gain** information

> In fact, with **multiple tracers** the Fisher information is **unbounded!**

We can **diagonalize** the multi-tracer Fisher matrix by changing variables:

⇒ (hyper) spherical coordinates!



$$\begin{array}{c} \left\{ \begin{array}{c} \mathcal{P}_1 \\ \mathcal{P}_2 \\ \mathcal{P}_3 \end{array} \right\} = \left\{ \begin{array}{c} x^2 \\ y^2 \\ z^2 \end{array} \right\} \iff \left\{ \begin{array}{c} r^2 \\ \tan^2 \theta \\ \tan^2 \phi \end{array} \right\} = \left\{ \begin{array}{c} \mathcal{P} \\ \frac{\mathcal{P}_3}{\mathcal{P}_1 + \mathcal{P}_2} \\ \frac{\mathcal{P}_2}{\mathcal{P}_1} \end{array} \right\} \end{array}$$

Total
clustering strength

Relative
clustering strengths

Multi-tracer Fisher information matrix

In “spherical” coordinates (i.e., using the total clustering strength and the relative clustering strengths) the **Fisher matrix becomes diagonal!**

E.g.: three species of tracers

Total clustering:
 $< 1/2$

$$F_{Sph} = \begin{pmatrix} \frac{1}{2} \left(\frac{\mathcal{P}}{1+\mathcal{P}} \right)^2 & 0 & 0 \\ 0 & \frac{1}{4} \frac{\mathcal{P}^2}{1+\mathcal{P}} \sin^2 \theta \cos^2 \theta & 0 \\ 0 & 0 & \frac{1}{4} \frac{\mathcal{P}^2}{1+\mathcal{P}} \sin^2 \theta \sin^2 \phi \cos^2 \phi \end{pmatrix}$$

Relative clusterings: information $\sim \mathcal{P} = \sum_{\alpha} \bar{n}_{\alpha} b_{\alpha}^2 P_m$

☛ unbounded

☛ extra information!

Why?

Cosmic variance is only inherited through the spectrum

By **comparing** the clustering between **different tracers** of large-scale structure (e.g.: LRGs, ELGs, etc.), we can **measure with arbitrary accuracy*** the physical parameters that distinguish the different **clustering strengths**:

$$\mathcal{P}_1 = n_1 (b_1 + f \mu_k^2)^2 P(k; z)$$

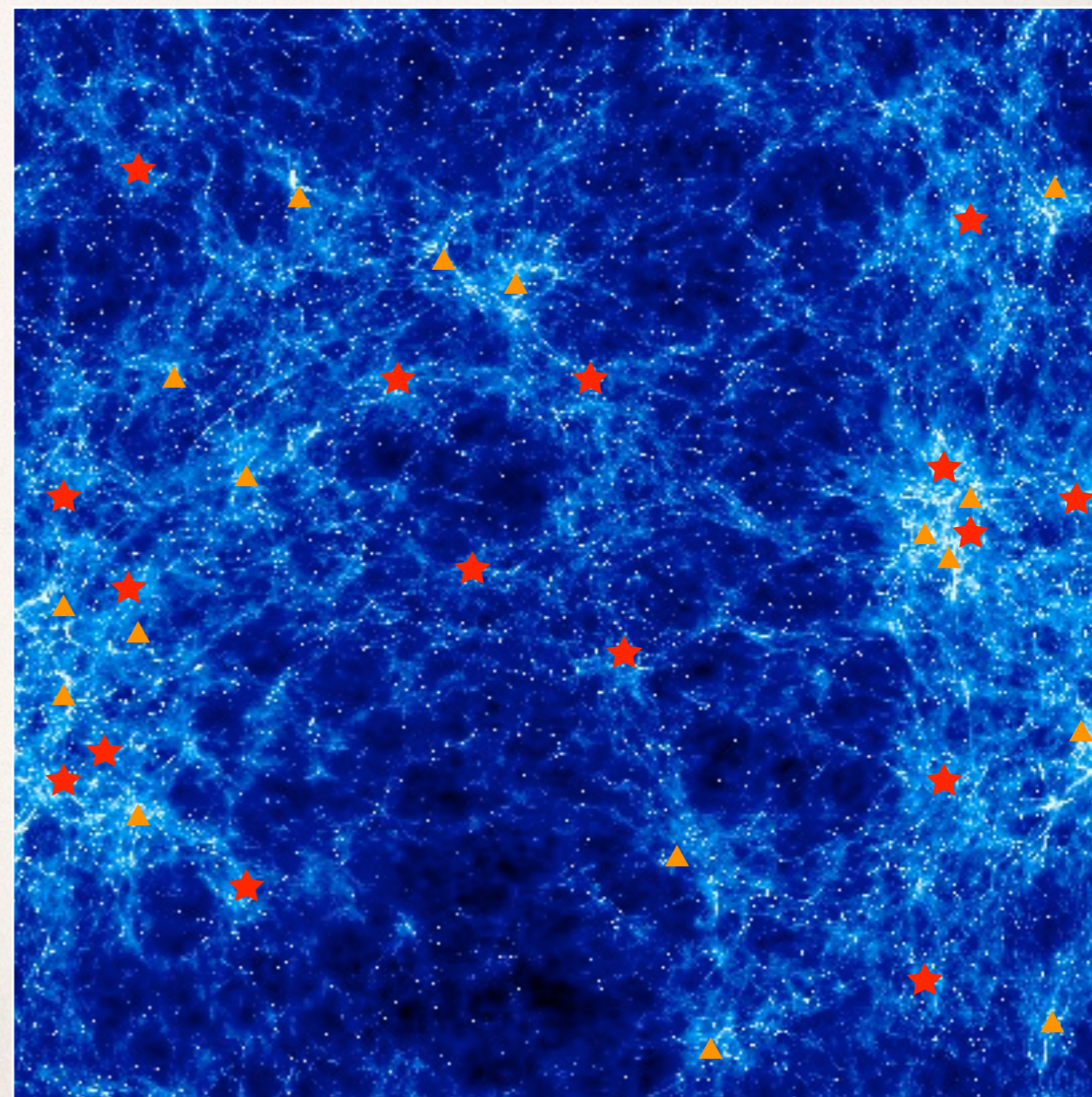
$$\mathcal{P}_2 = n_2 (b_2 + f \mu_k^2)^2 P(k; z)$$

$$\frac{\mathcal{P}_1}{\mathcal{P}_2} = \frac{n_1 (b_1 + f \mu_k^2)^2}{n_2 (b_2 + f \mu_k^2)^2}$$

Cosmic variance does **not** apply:

- *bias
- *RSDs
- *PNGs
- *HOD

The key: **high number densities** of distinct types of tracers (red galaxies, blue galaxies, emission-line galaxies, quasars, etc.)



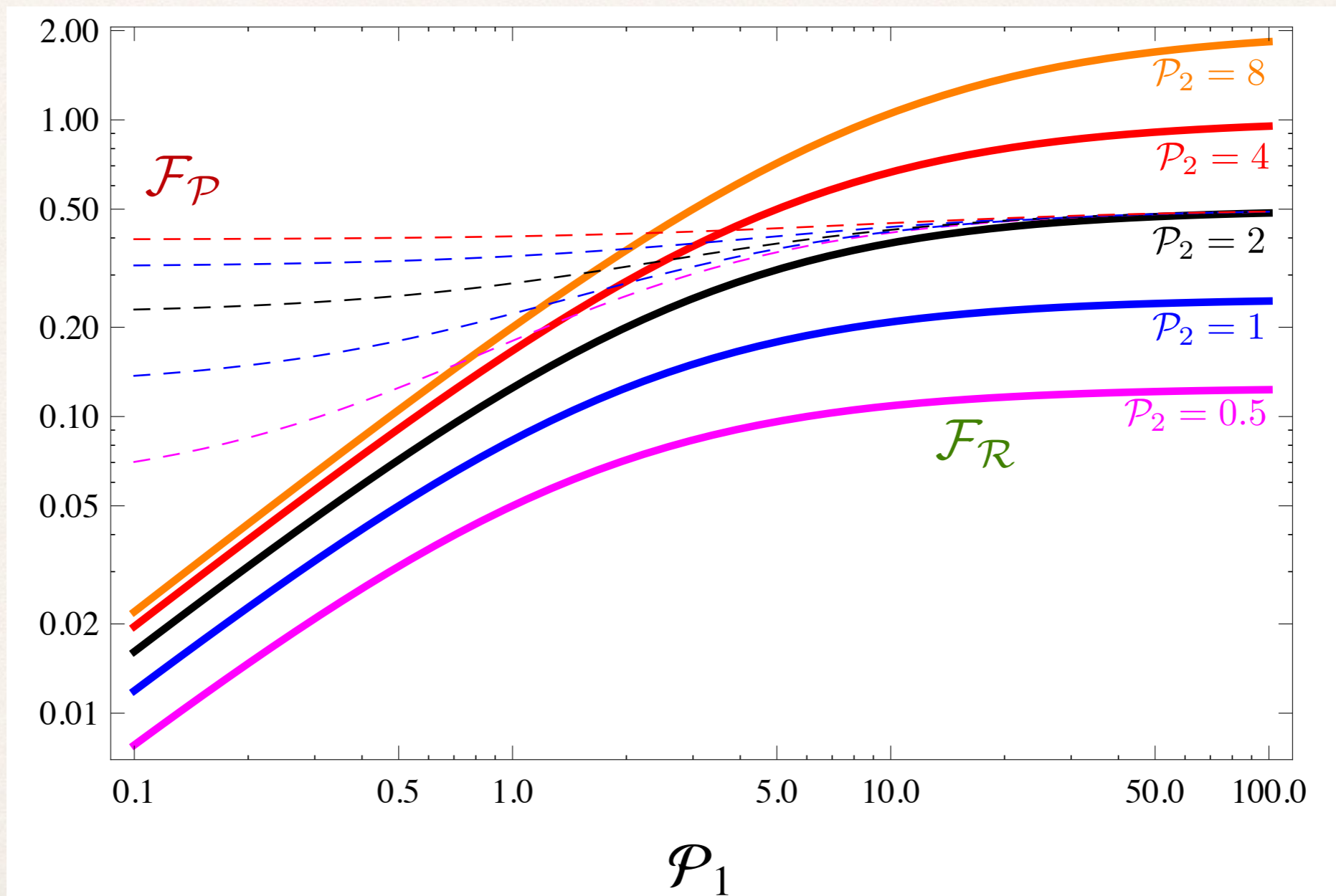
Simplest example: two types of tracers of large-scale structure

Total clustering

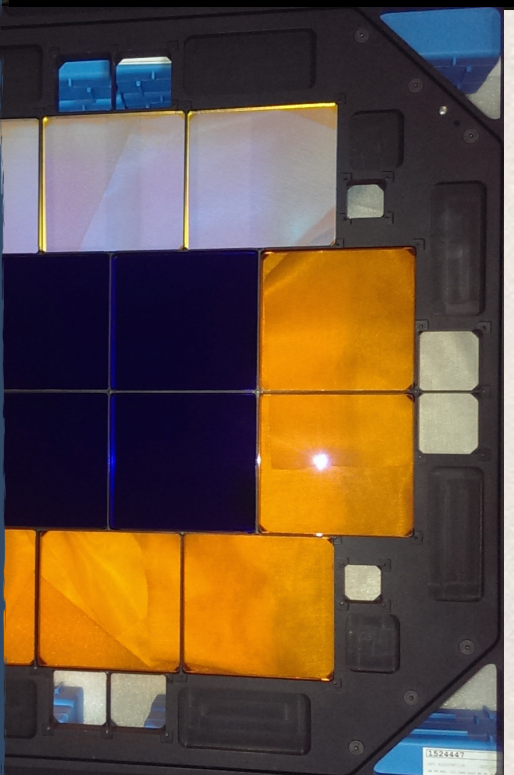
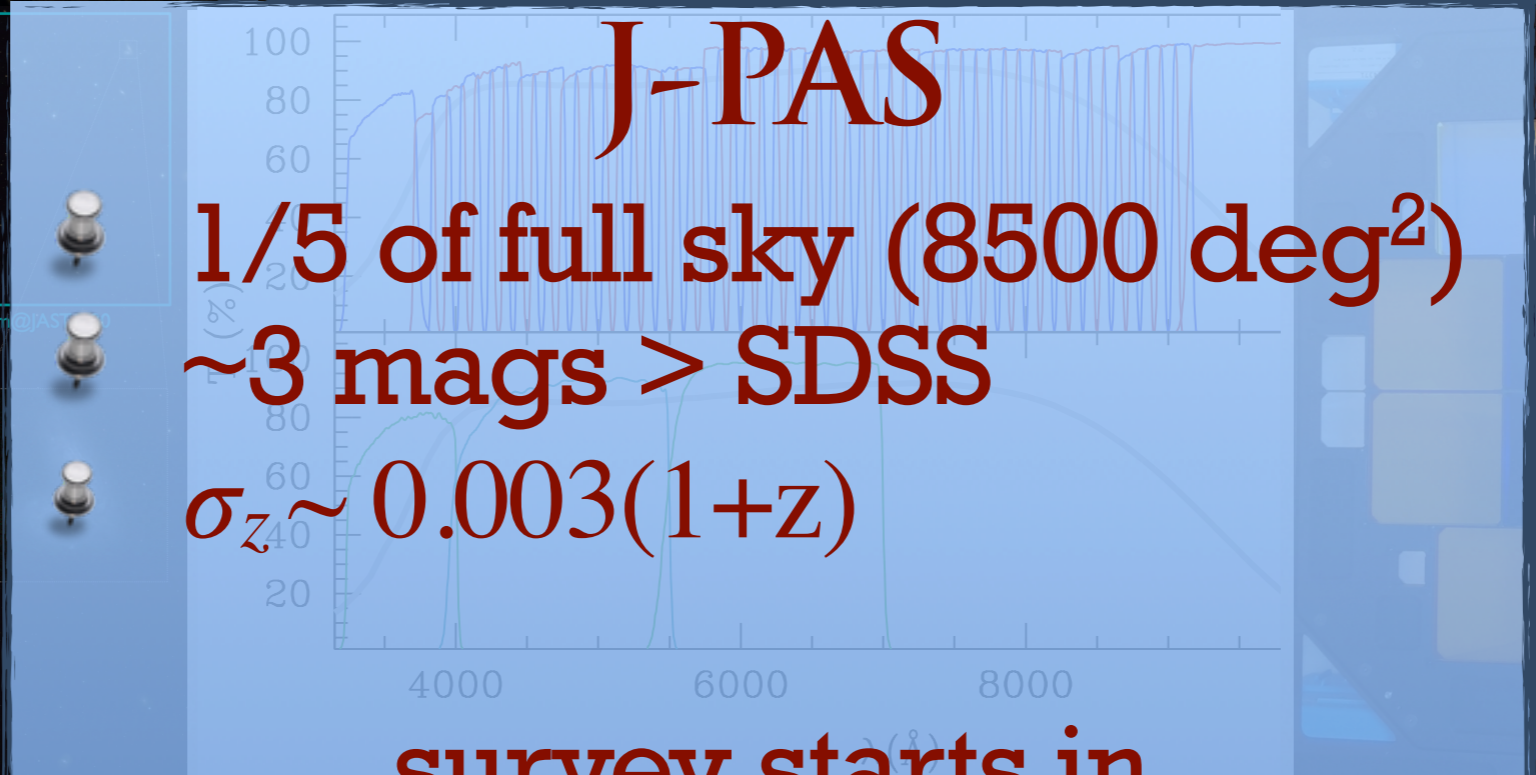
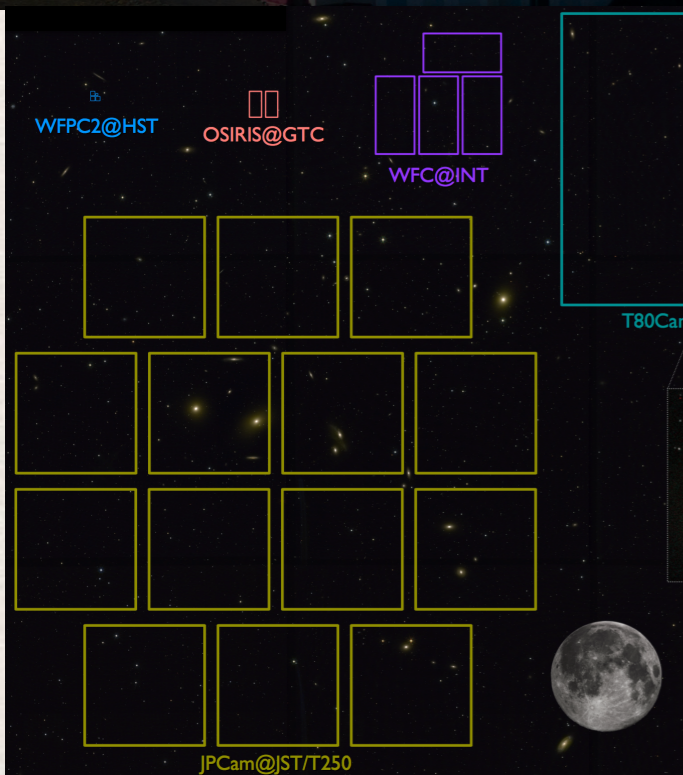
$$\mathcal{F}_{\mathcal{P}} = \frac{\mathcal{P}^2}{\sigma^2(\mathcal{P})} = \frac{1}{2} \frac{\mathcal{P}^2}{(1 + \mathcal{P})^2}$$

Relative clustering

$$\mathcal{F}_{\mathcal{R}} = \frac{(\mathcal{P}_2/\mathcal{P}_1)^2}{\sigma^2(\mathcal{P}_2/\mathcal{P}_1)} = \frac{1}{4} \frac{\mathcal{P}_1 \mathcal{P}_2}{1 + \mathcal{P}}$$



**Where the hell are we going to get
all those galaxies — with decent redshifts??**



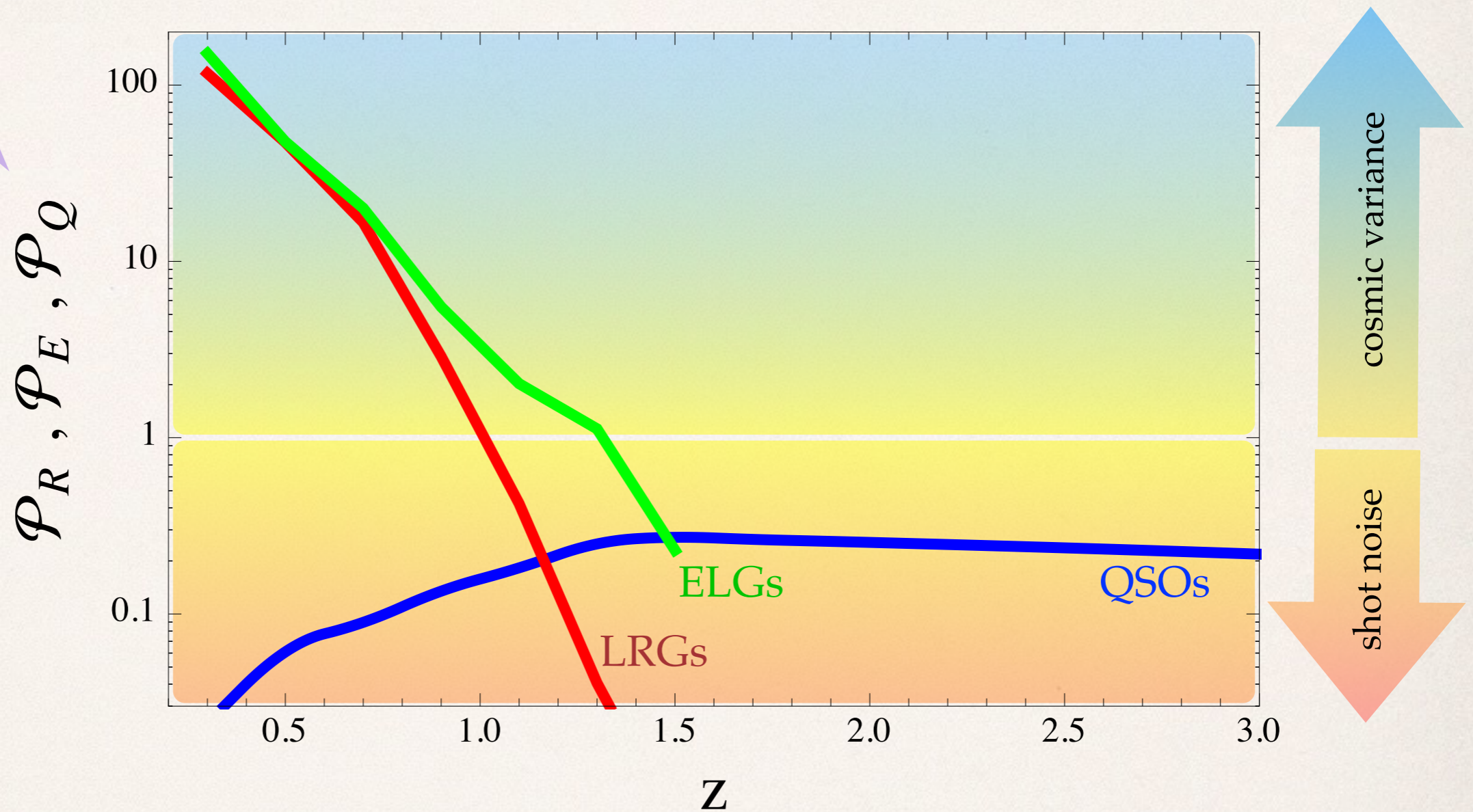
J-PAS
1/5 of full sky (8500 deg²)
~3 mags > SDSS
 $\sigma_z \sim 0.003(1+z)$
survey starts in
Q1 2017 !
 Benítez et al., 1403.5237
 Benítez et al. 2016 (to appear)



2017	2018	2019	2020	2021	2022	2023
J-PAS	J-PAS	J-PAS	J-PAS ¹	J-PAS	J-PAS ²	
		DESI (?)	DESI	DESI	DESI	
			Euclid	Euclid	Euclid	Euclid

Massive & deep multi-tracer survey with J-PAS

@ $k=0.1 \text{ h/Mpc}$



* GAMA - Blake et al., MNRAS 2013 : $\mathcal{P}_1 > 10$ for $z < 0.25$

* Radio galaxies & SKA - Ferramacho et al. 2014, Camera et al. 2015, ...

* 21cm intensity mapping - Bull, Ferreira, Patel & Santos 2015

* SKA + optical surveys - Fonseca, Camera, Santos & Maartens 2015

* DESI, Euclid...

Application: RSDs in J-PAS

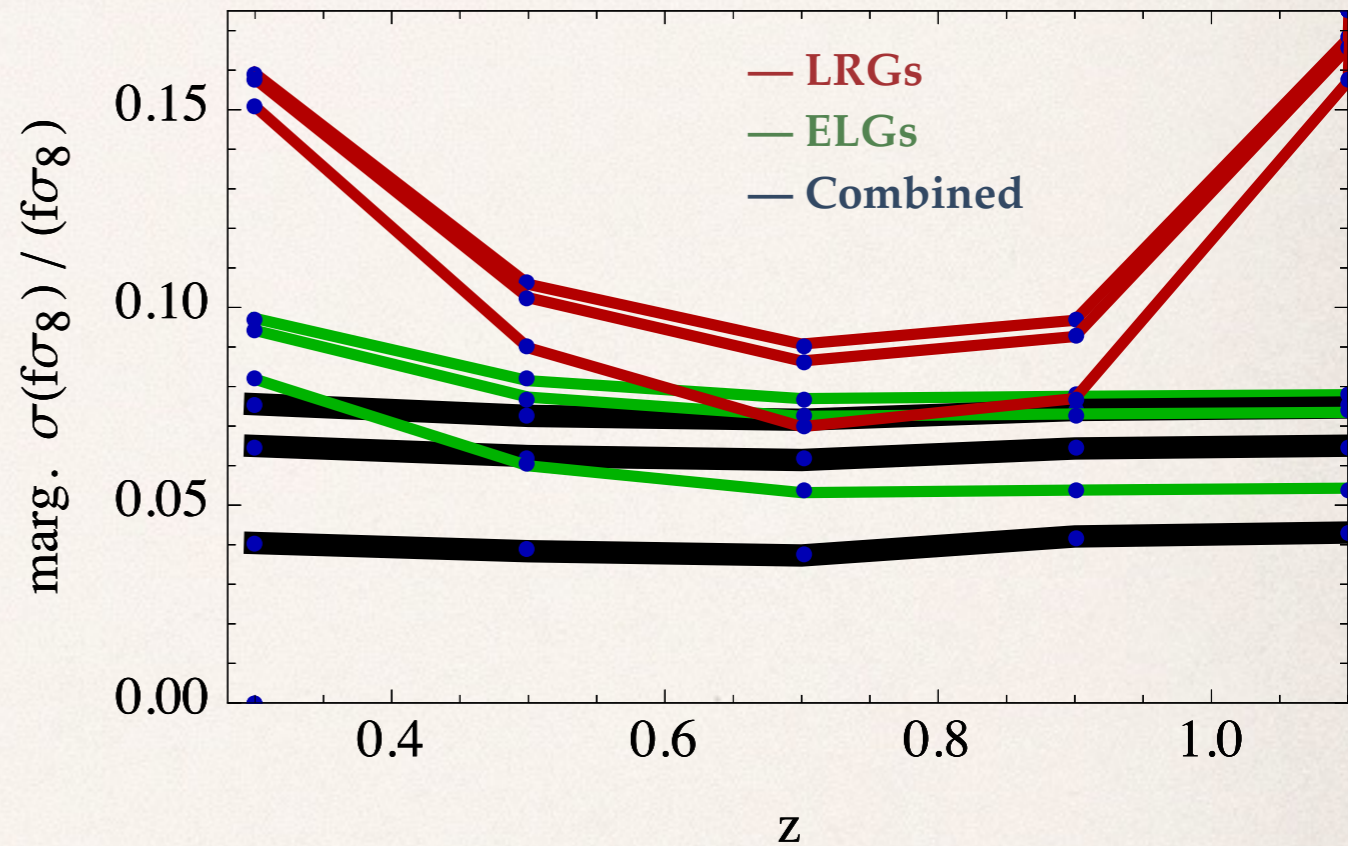
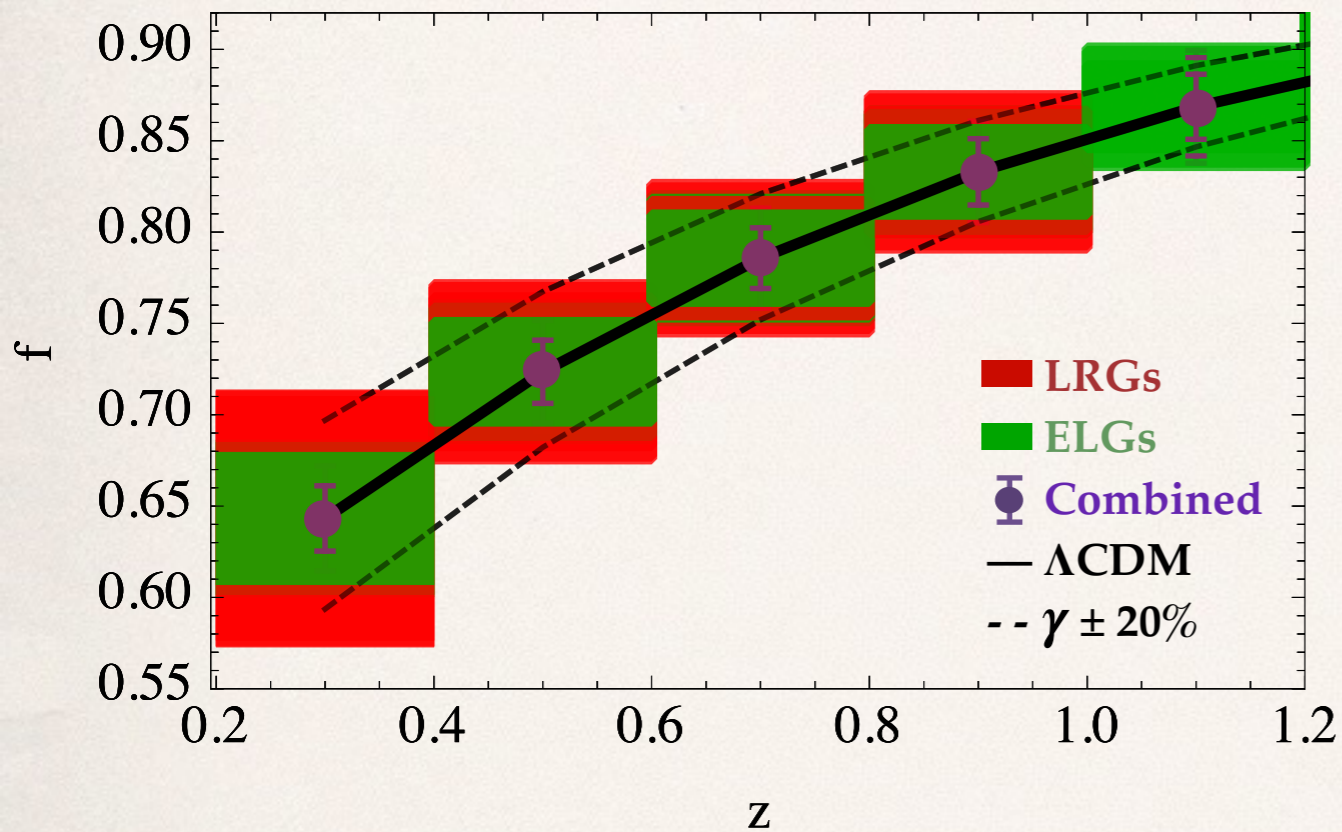
Marginalized* errors on matter growth rate



$$\mathcal{P}_g = n_g (b_g + f \mu_k^2)^2 P(k; z)$$

$$f = \frac{d \ln G}{d \ln a} \simeq \Omega_m^\gamma$$

~modified gravity
($\gamma_{GR} \approx 0.55$)



J-PAS forecast for constraint on γ :

$$\sigma(\gamma) = 0.025$$

No prior on bias
Weak (~20%) prior on bias
Strong (~5%) prior on bias

* Marginalized 7 "global" cosmological parameters (Ω_m, h , etc.) + 5 parameters on *each* redshift slice

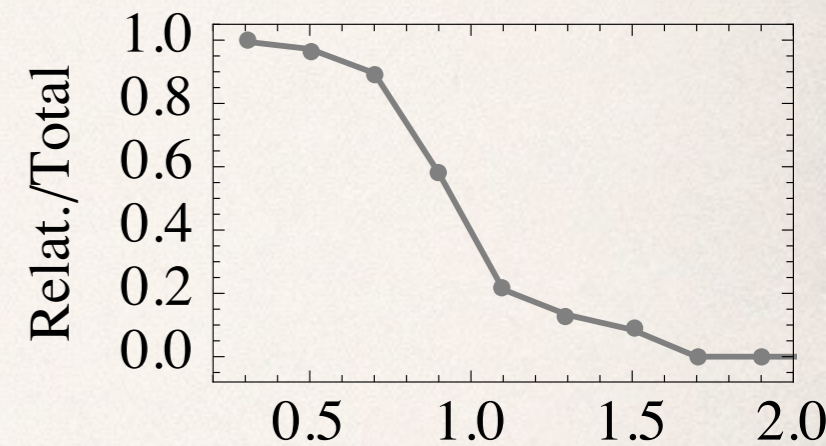
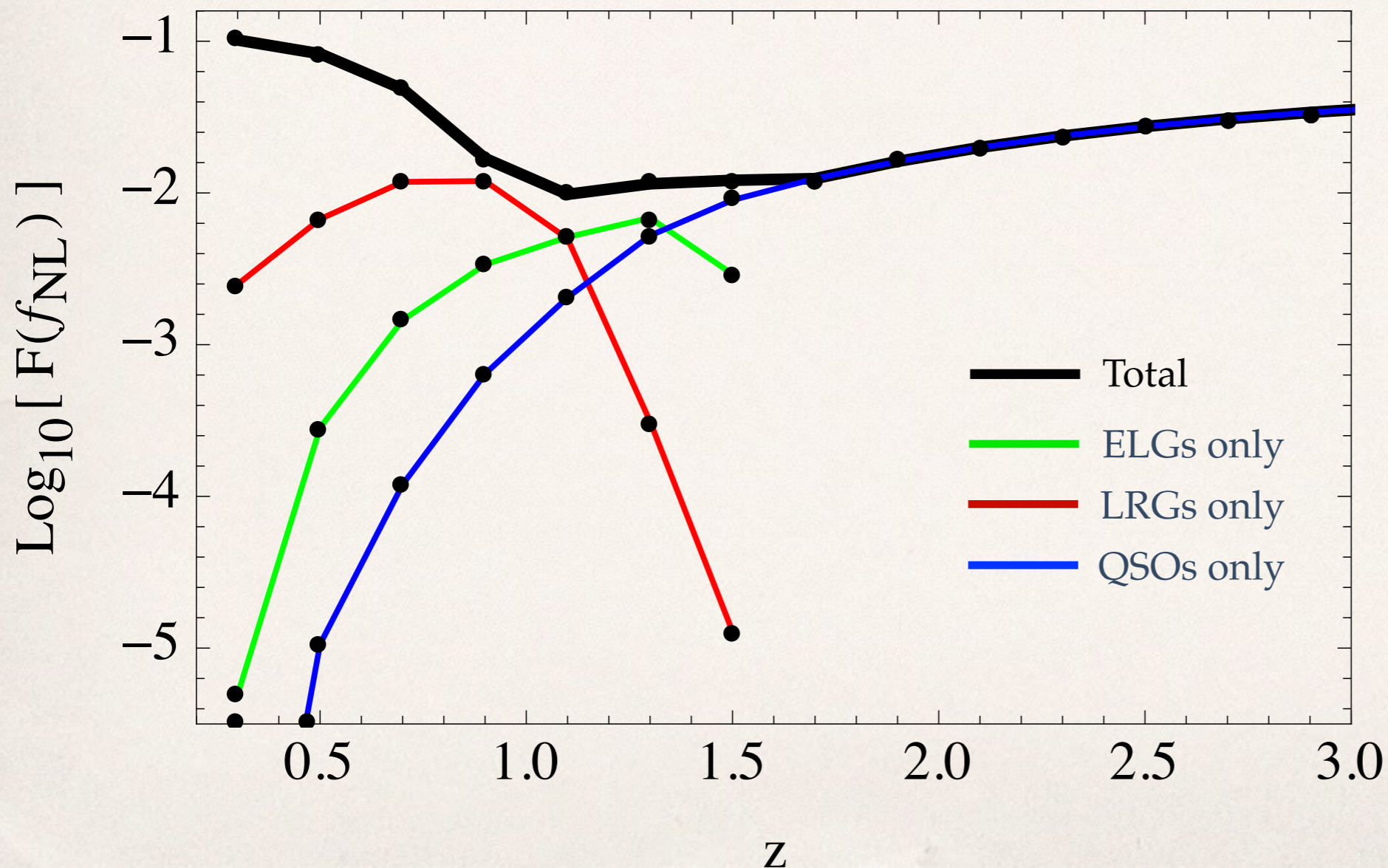
J-PAS constraints on local non-Gaussianity parameter f_{NL}



$$\mathcal{P}_g = n_g (b_g + f \mu_k^2)^2 P(k; z)$$

$$b_g \rightarrow b_g + \Delta b_g(f_{NL}, k)$$

$$F(\theta) = \frac{1}{\sigma_c^2(\theta)}$$

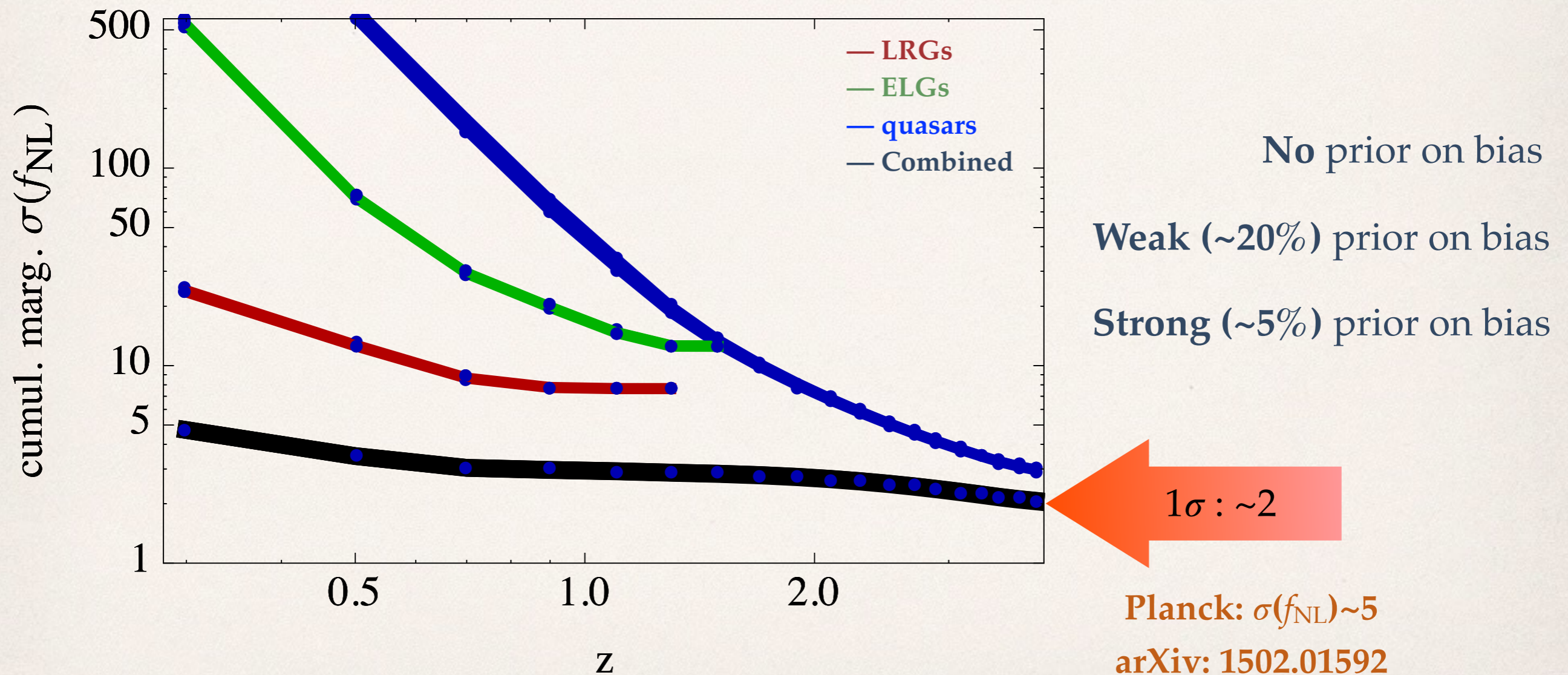


Information from
relative clustering can
improve constraints
on f_{NL} by ~ 5 at low- z !



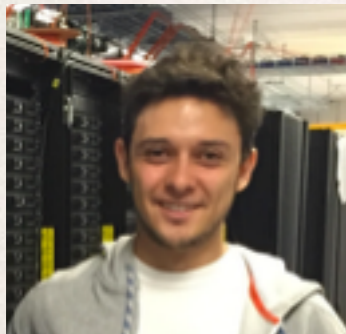
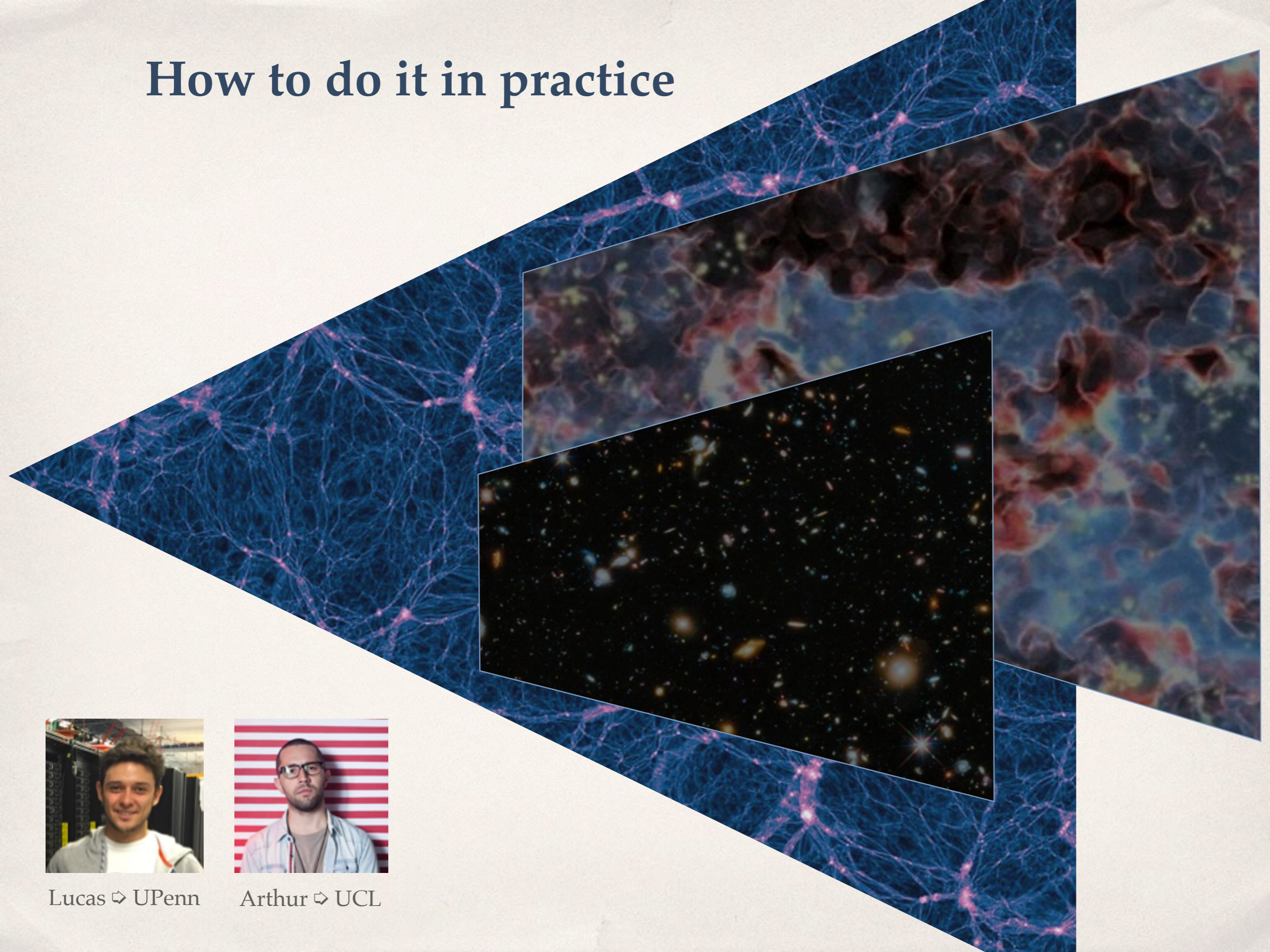
f_{NL} is almost unaffected by marginalization w.r.t. bias
The k -dependence of $\Delta b_{\text{NL}} \sim f_{\text{NL}} \times k^{-2}$ helps break the degeneracy

Cumulative uncertainty on f_{NL} when the redshift slices are combined



WARNING: this is *Fisherology* — not robust w.r.t. *systematics*

How to do it in practice



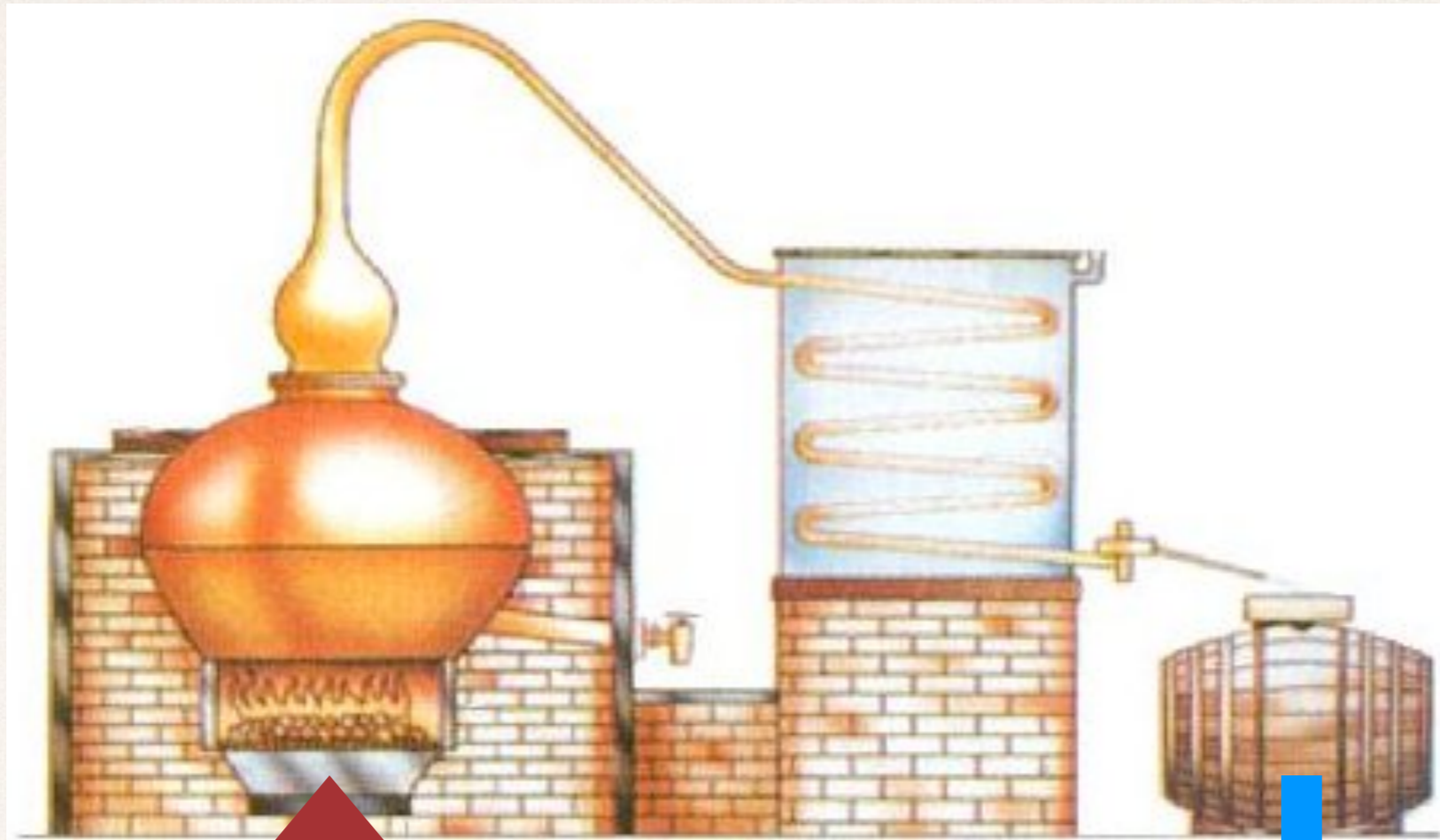
Lucas ⇨ UPenn



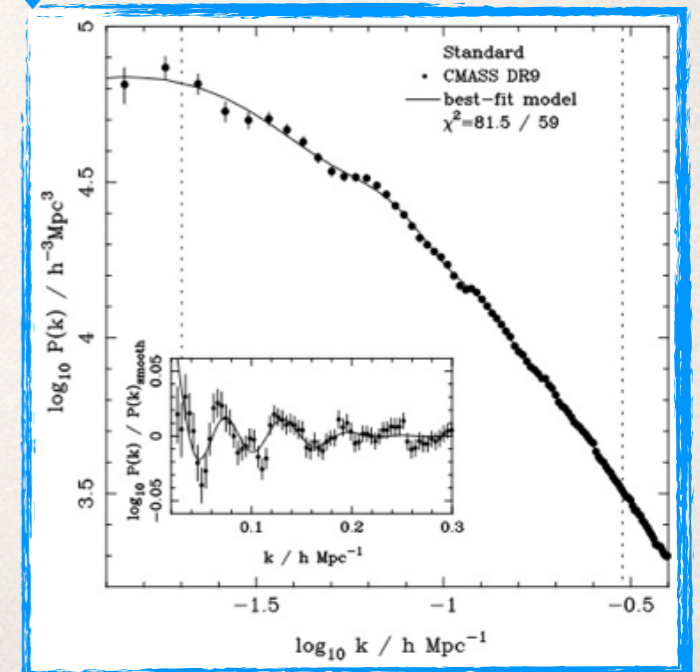
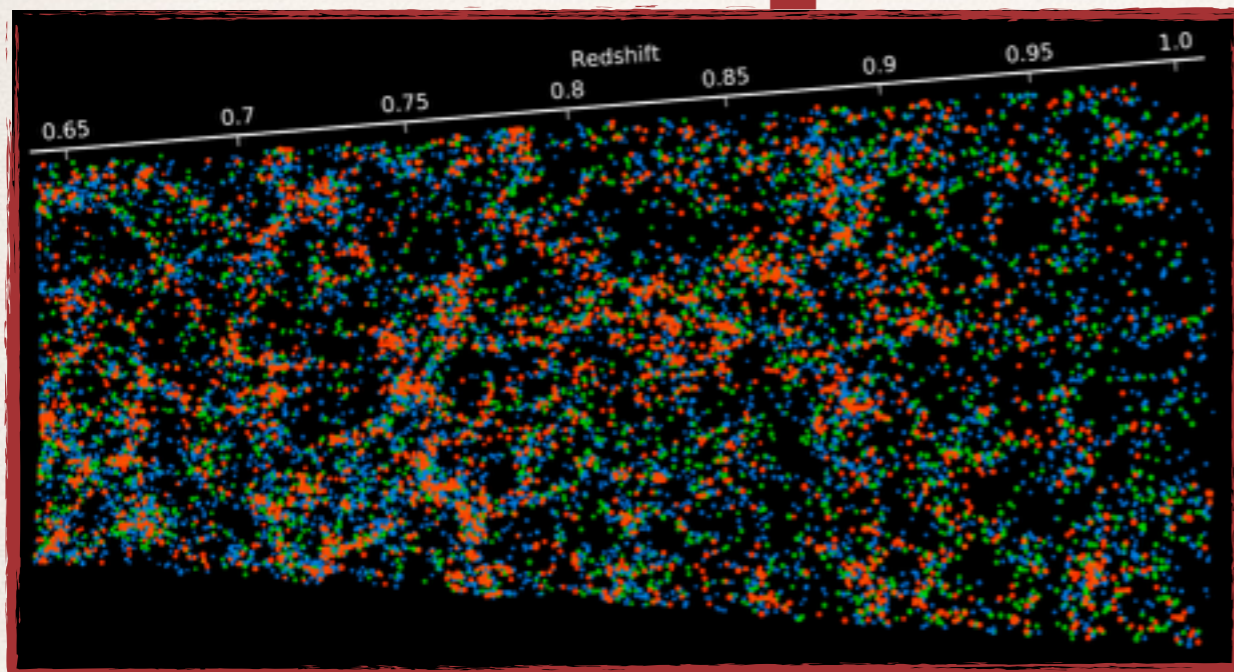
Arthur ⇨ UCL

From catalogs to the power spectrum

Types and positions of galaxies



$P(k)$, BAOs
RSDs
NGs, etc.



Fourier analysis of galaxy surveys

Given a **galaxy catalog** $n_g(\mathbf{x})$, the **optimal estimator** for the spectrum (FKP) is:

$$\frac{\delta n_g(\mathbf{x})}{\bar{n}_g} = \delta_g(\mathbf{x}) \longrightarrow f_g(\mathbf{x}) = w_{FKP}(\mathbf{x}) \delta_g(\mathbf{x})$$

The **FKP weights** express the best **compromise** between **cosmic variance** and **shot noise**:

$$w_{FKP} = \frac{1}{1 + \mathcal{P}_g} \bar{n}_g B_g \qquad B_g = b_g(z) + f(z) \mu_k^2 + \dots$$

The **estimated spectrum** for a Fourier bin k_i (the *bandpower*) is then:

$$\hat{P}_g(k_i) = \frac{1}{N} \langle |\tilde{f}_g|^2 \rangle_{k_i} \qquad \tilde{f}_g(\mathbf{k}) = \text{FFt}[f_g(\mathbf{x})]$$

The FKP estimator is **optimal** — it is **unbiased**, and it saturates the **Cramér-Rao bound**:

$$\text{Cov}[\hat{P}_g(k_i), \hat{P}_g(k_j)] \rightarrow [\text{Fisher}]^{-1}$$

Fourier analysis of multi-tracer surveys

Given any number of galaxy catalogs $n_\mu(x)$, the weighted fields are:

$$\delta_\mu(x) \longrightarrow f_\mu(x) = \sum_\nu w_{\mu\nu} \delta_\nu(x)$$

The multi-tracer **weights** are:

$$w_{\mu\nu} = \left[\delta_{\mu\nu} - \frac{\mathcal{P}_\mu}{1 + \mathcal{P}} \right] \bar{n}_\nu B_\nu$$

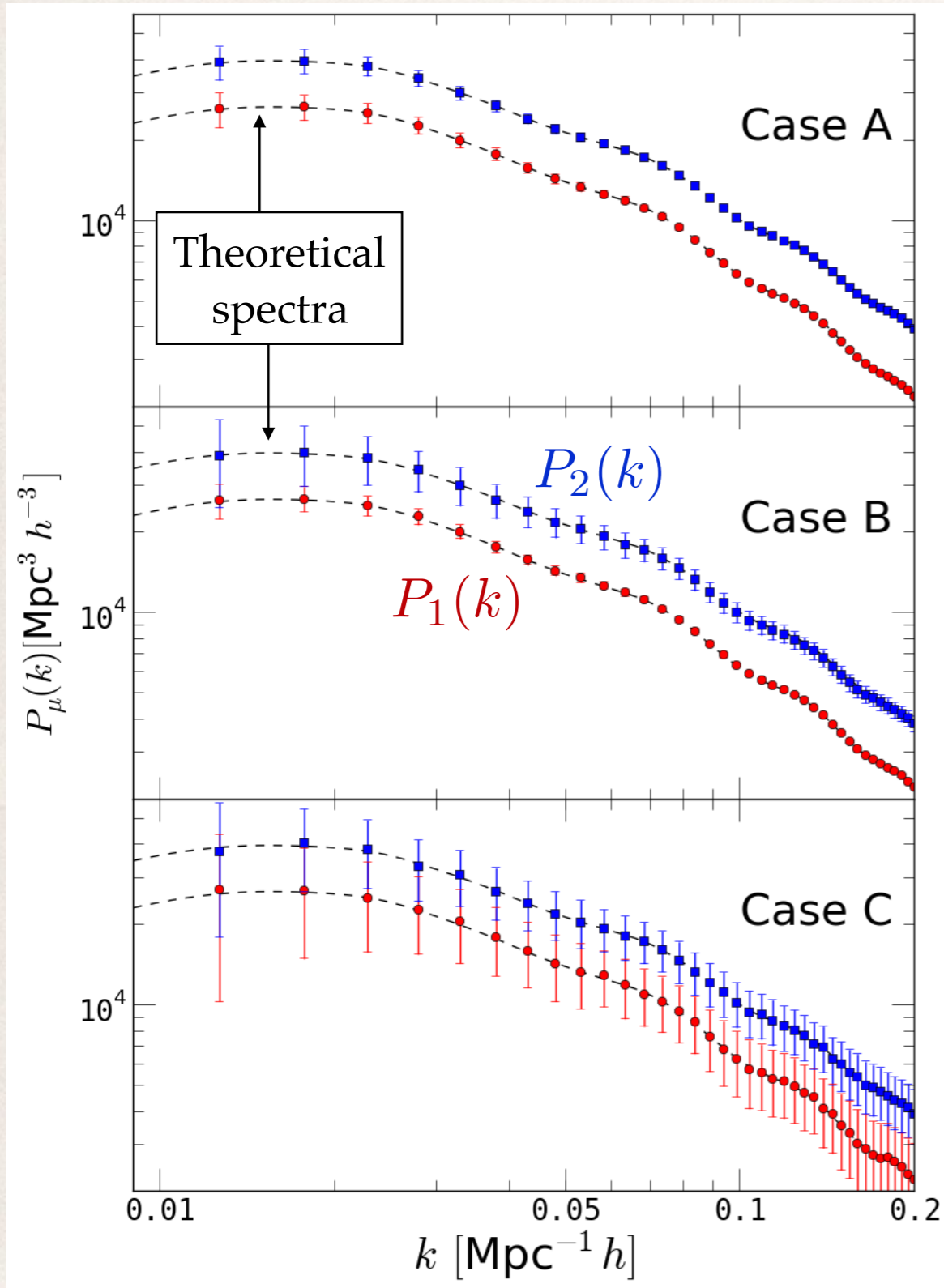
The **estimated auto-spectra** are:

$$\hat{P}_\mu(k_i) = \sum_\nu N_{\mu\nu}^{-1} \langle |\tilde{f}_\nu|^2 \rangle_{k_i}$$

These estimators are **optimal**: their **covariance** is the **inverse of the Fisher matrix**!

$$Cov[\hat{P}_\mu, \hat{P}_\nu] = [F_{\mu\nu}]^{-1}$$

Testing and validating the multi-tracer estimators



Case	$\bar{n}_1 (h^3 \text{ Mpc}^{-3})$	b_1	$\bar{n}_2 (h^3 \text{ Mpc}^{-3})$	b_2
A	$1 \cdot 10^{-2}$	1.0	$1 \cdot 10^{-2}$	1.2
B	$1 \cdot 10^{-2}$	1.0	$1 \cdot 10^{-5}$	1.2
C	$1 \cdot 10^{-5}$	1.0	$1 \cdot 10^{-5}$	1.2

* Volume = $(1280 h^{-1} \text{ Mpc})^3 = (128 \times 10 h^{-1} \text{ Mpc})^3$

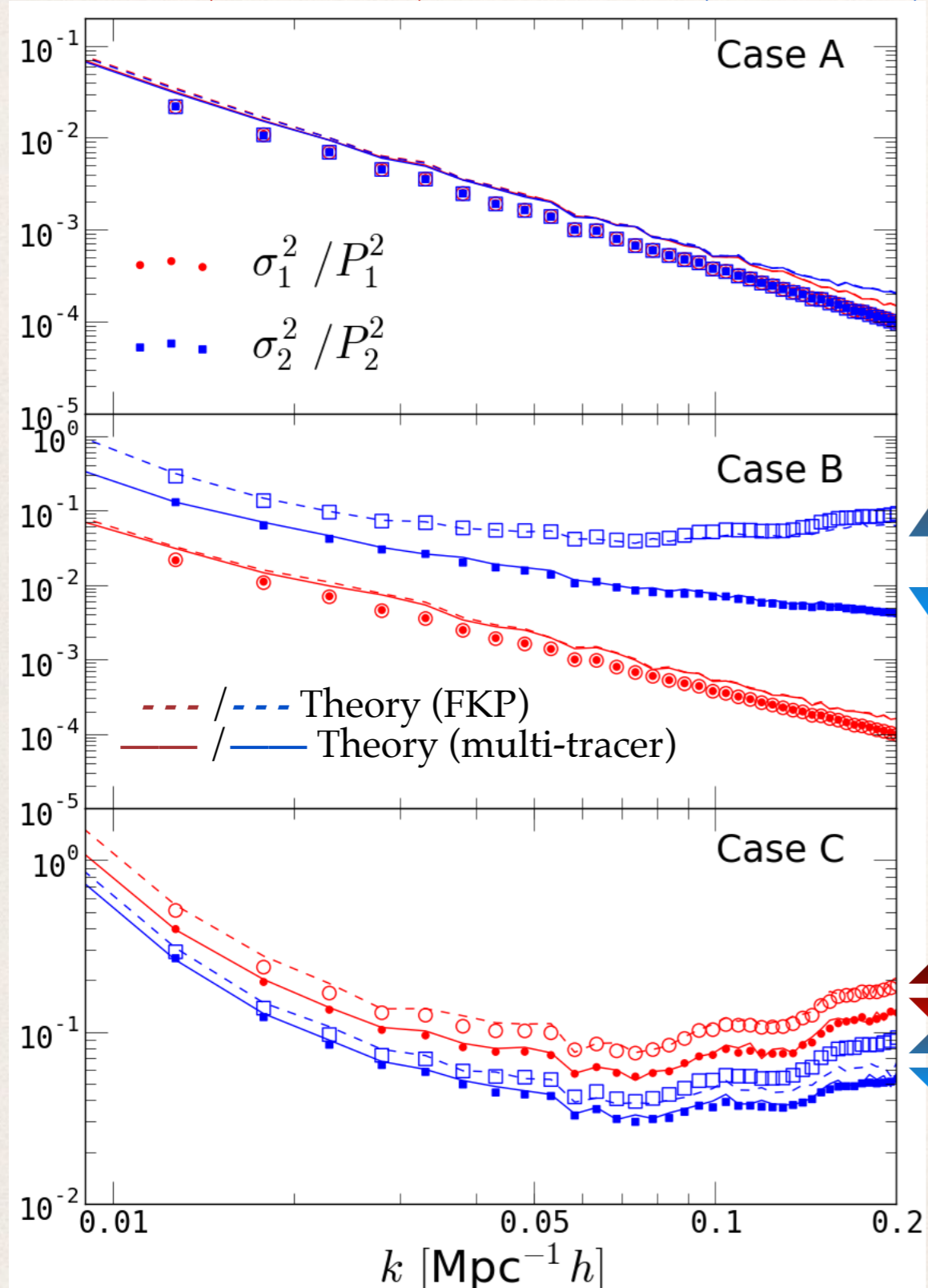
* 10^3 lognormal realizations

* Planck-vanilla fiducial parameters

✓ Unbiased

Testing and validating the multi-tracer estimators

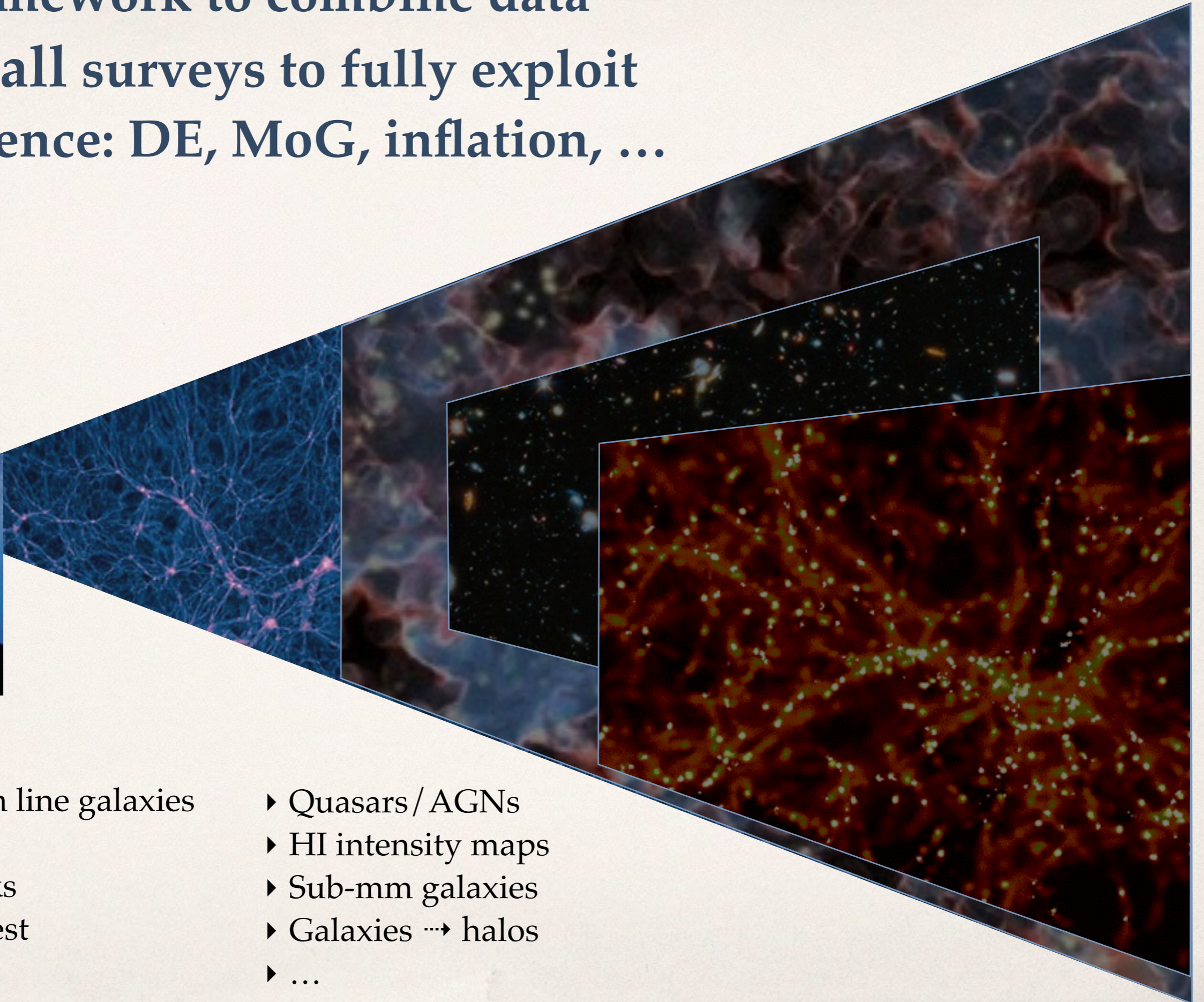
- tracer 1 (FKP) □ tracer 2 (FKP)
- tracer 1 (multi-tracer) ■ tracer 2 (multi-tracer)



Case	$\bar{n}_1 (h^3 \text{ Mpc}^{-3})$	b_1	$\bar{n}_2 (h^3 \text{ Mpc}^{-3})$	b_2
A	1.10^{-2}	1.0	1.10^{-2}	1.2
B	1.10^{-2}	1.0	1.10^{-5}	1.2
C	1.10^{-5}	1.0	1.10^{-5}	1.2

- ✓ Unbiased
- ✓ Optimal

Framework to combine data from all surveys to fully exploit the science: DE, MoG, inflation, ...



- ▶ Emission line galaxies
- ▶ LRGs
- ▶ Ly-breaks
- ▶ Ly- α forest

- ▶ Quasars / AGNs
- ▶ HI intensity maps
- ▶ Sub-mm galaxies
- ▶ Galaxies \rightsquigarrow halos
- ▶ ...

The End