Massive Gravitinos
In Curved Space time

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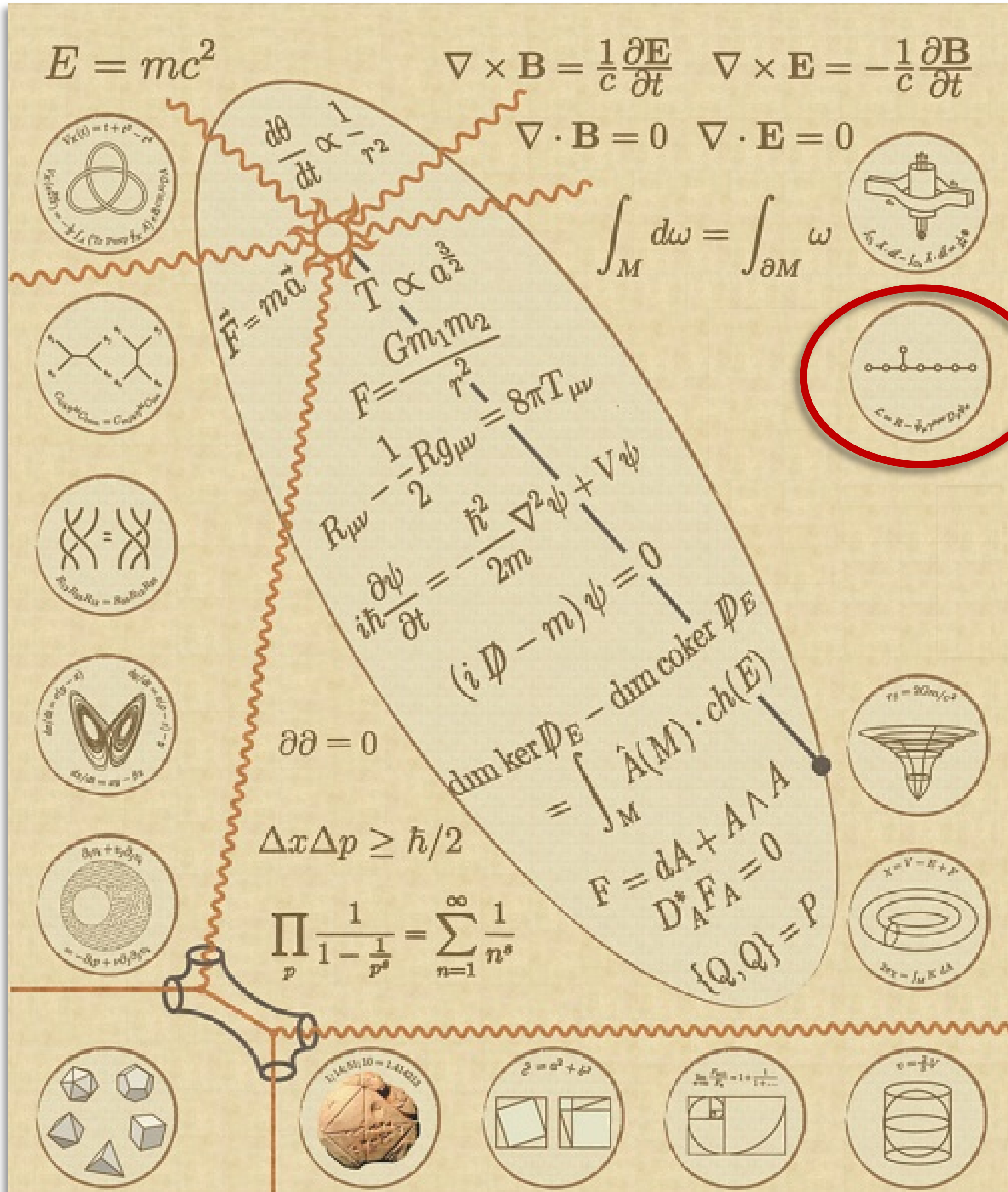

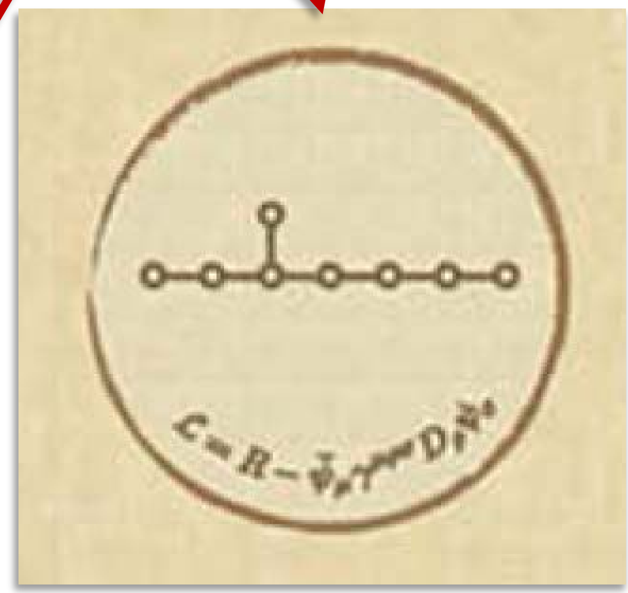

The Iconic Wall
Simons Center for Geometry and Physics
Stony Brook, New York

I am glad to have the opportunity to speak at the GGI final Conference of the Supergravity Workshop, which is devoted to the 40^{th} anniversary of Supergravity.

As a member of the Organizing Committee I should not speak here, but I think the exception was made by the Organizers, especially Toine, since with Dan and Peter I participated in the first construction of a supergravity theory (1976).

On top of the recent developments, which can be widely covered by the speakers at this final Conference, I would like to mention other developments that followed that year, especially those where I was involved, which were also relevant for what is considered today a consistent UV completion of Supergravity, namely Superstring Theory.

Role of Supergravity in the three String Revolutions

1) Green-Schwarz anomaly cancellation in chiral N=1,D=10 Supergravity coupled to 10D super-Yang-Mills;

2) Witten's embedding of 11D Supergravity in M-theory;

3) Maldacena's AdS/CFT correspondence and duality between type-IIB supergravity on AdS_5xS_5 and N=4 Yang-Mills on the 4D boundary of AdS

Applications of Supergravity to Physics beyond the Standard Model

1) Grand-unification, Proton decay, dark matter

No-scale Supergravity → flux compactifications

Gauge Supergravity → flux compactifications, geometrical and non-geometrical fluxes

Extended Supergravity → String and M-theory reductions

Examples: N=1,2,4,8 Supergravity in D=4

Effective theories in D=4

N=1 → Heterotic strings on a Calabi-Yau threefold

N=2 → Type-II strings on Calabi-Yau threefolds, moduli spaces,

Special Geometry, c-map, mirror symmetry

N=4 → Heterotic strings on T_6 (coupled to matter)

N=8 → type-II strings on T_6, M-theory on T_7

N=1 → M-theory on G_2

BPS states, in particular N=2 black holes in D=4, extremality and "Attractor Mechanism"

String developments: microscopic state counting, split attractor flow, Bekenstein-Hawking entropy-area formula

Cosmology: Starobinsky and Higgs inflation, application to Cosmology of non-linear Supersymmetry coupled to Supergravity, nilpotent superfields coupled to N=1 Supergravity

Spontaneously Broken SUSY in curved space-time

Super BEH mechanism:

The Goldstino is eaten by the Gravitino, which becomes «massive»

$$m_{\frac{3}{2}}\ \overline{\psi}_{\mu}\,\gamma^{\mu\nu}\,\psi_{\nu}$$

BUT: the Lagrangian mass term is an «apparent» mass, not an «effective» mass.

Spontaneously Broken SUSY in curved space-time

The effective gravitino mass in curved space-time it is not the Lagrangian mass

$$m_{\frac{3}{2}} \;=\; W\, e^{\frac{K}{2}}$$

But rather

$$\left|m_{\frac{3}{2}}^{(eff)}\right|^2 \;=\; \left|m_{\frac{3}{2}}\right|^2 \;+\; \frac{V}{3}$$

This quantity is in fact non vanishing when Supersymmetry is broken and vanishes when it is unbroken.

$$m_{\frac{3}{2}}^{(eff)} \;=\; 0 \longrightarrow \begin{cases} \text{Minkowski}: \quad m_{\frac{3}{2}} = 0\;, \quad V = 0 \\ \text{Anti de Sitter}: \quad 3\left|m_{\frac{3}{2}}\right|^2 \;=\; -\,V \end{cases}$$

Mass Sum Rules in curved space-time

$(V \neq 0\,, \quad V_{,i} \neq 0)$

(SF, Van Proeyen, 2016)

These formulas are particularly relevant in applications of Supergravity to Cosmology (inflaton potential, slow-roll inflation, stability problems).

In Particle Physics these formulas may describe soft-breaking terms of Supergravity-mediated breaking, and lie at the heart of mass splittings not captured by rigid Supersymmetry.

Early 1980's

(Cremmer, SF, Girardello, Van Proeyen, 1983)

Mass formulae in flat background (and for flat Kahler manifold)

$$\frac{1}{2}\, Str M^2 \;=\; \frac{1}{2}\sum_{J=0}^{\frac{3}{2}}(-1)^{2J}(2J+1)m_J^2 \;=\; (N-1)\left|m_{\frac{3}{2}}\right|^2 \;+\; \mathrm{D-term}$$

In Particle Physics these expressions may describe soft-breaking terms of Supergravity-mediated breaking, and lie at the heart of mass splittings not captured by rigid Supersymmetry.

$$\left|m_{\frac{3}{2}}\right| \;=\; |W|\, e^{\frac{K}{2}} \;=\; e^{\frac{G}{2}} \qquad (G \;=\; K \;+\; \log W\,\overline{W})$$

W : (holomorhic) superpotential evaluated at $\mathrm{V}_{,\mathrm{i}} = 0$ (with $\mathrm{V} = 0$)

(still vanishing curvature of the Kahler manifold)

Early 1980's

The factor (N-1) reflects the fact that there is one preferred multiplet, the «sgoldstino» multiplet, which is responsible for the BEH mechanism (*i.e.* Polony model) (N=1)

$$\frac{1}{2}\left(m_A^2 + m_B^2\right) = 2\, m_{\frac{3}{2}}^2 \quad \Leftrightarrow \quad \mathrm{SuperTrace}\, M^2 = 0$$

Generalization to (non-flat) Kahler manifolds was soon obtained, taking into account also generic D-terms (still at $V = 0, V_i = 0$)

(Grisaru, Rocek, Karhede, 1983)

Flat space-time

(Grisaru, Rocek, Karlhede, 1983)

Mass formulae in flat background (and for non-flat Kahler manifold)

$$\frac{1}{2}\,StrM^2 \;=\; \frac{1}{2}\sum_{J=0}^{\frac{3}{2}}(-1)^{2J}(2J+1)m_J^2 \;=\; (N-1)m_{\frac{3}{2}}^2 \;+\; e^G\,G^i\,G^{\bar{j}}\,R_{i\bar{j}} \;+\; \mathrm{D-term}$$

$$(\ \text{Ricci curvature}\ :\quad R_{i\bar{j}} \;=\; -\,\partial_i\,\partial_{\bar{j}}\ \log\,\det G_{i\bar{j}}\ )$$

Note that in rigid Supersymmetry the analogous formula reads

$$\frac{1}{2}\,StrM^2 \;=\; \frac{1}{2}\sum_{J=0}^{\frac{1}{2}}(-1)^{2J}(2J+1)m_J^2 \;=\; \frac{\partial W}{\partial z^i}\,\frac{\partial \overline{W}}{\partial \bar{z}^{\bar{j}}}\,K^{i\bar{l}}\,K^{l\bar{i}}\,R_{l\bar{l}} \;+\; \mathrm{D-term}$$

This formula can be also used to explore non-linear limits for spontaneous Supersymmetry breaking. *(Volkov, Akulov, 1973)*

(Casalbuoni, De Curtis, Dominici, Feruglio, Gatto, 1989; Komargodsky and Seiberg, 2009)

Recent developments

(SF, Roest, 2016)

The generalization of the mass formulae for a single (sgoldstino) multiplet is important in applications to Cosmology.

In rigid Supersymmetry it reads:

$$\frac{1}{2}\left(m_A^2 \;+\; m_B^2\right) \;=\; V\,R \;+\; \frac{K^{z\overline{z}}\,V_z\,V_{\overline{z}}}{V}$$

(and for $V \neq 0, V_i = 0$ reduces to the previous one).

In local Supersymmetry:

$$\frac{1}{2}\;Str M^2 \;=\; K^{z\overline{z}}\,G_z\,G_{\overline{z}}\,R \;+\; \frac{2\,V_z\,G_{\overline{z}} \;+\; 2\,V_{\overline{z}}\,G_z \;+\; e^{-G}\,V_z\,V_{\overline{z}}}{G_z\,G_{\overline{z}}}$$

Generalization to an arbitrary potential

$$(V \neq 0, V_i \neq 0)$$

$$\frac{1}{2}\, StrM^2 \;=\; (N-1)\left(V \;+\; |m_{\frac{3}{2}}|^2\right) \;+\; e^G\, G^i\, G^{\bar{j}}\, R_{i\bar{j}} \;+\; \frac{e^G}{V \;+\; 3\, e^G}\,\left(V_i\, G^i \;+\; V_{\bar{i}}\, G^{\bar{i}}\right)$$

For $(V = 0, V_i = 0)$ it gives back the previous formulae, and in particular at $(V \neq 0, V_i = 0)$ the curved space-time correction comes from the V term that was absent in the previous derivation.

This formula covers all cases with broken or unbroken Supersymmetry, and the latter is non-trivial in an AdS background when $m_{\frac{3}{2}} \neq 0$.

Unbroken Supersymmetry and AdS curvature effects give rise to a splitting of the Lagrangian masses of the chiral multiplet bosons an fermions.

Unbroken anti-de Sitter $(G_i = 0)$ *(SF, Kehagias, Porrati, 2013)*

$$\frac{1}{2}\, Str M^2 \;=\; -\,2\,N\,e^G$$

$$(m_A^2 = E_0(E_0-3)\ ,\ \ m_B^2 = (E_0+1)(E_0-2)\ ,\ \ m_\psi^2 = (E_0-1)^2 \text{ in } Sp(4) \sim SO(3,2) \text{ language})$$

Effective gravitino mass term

The supertrace formula has a smooth limit in the case of broken or unbroken Supersymmetry if instead of the Lagrangian mass we use the effective mass for the gravitino $(V \neq 0)$.

Using the fact that $V + \left|m_{\frac{3}{2}}\right|^2 = \frac{2}{3} V + \left|m_{\frac{3}{2}}^{eff}\right|^2$

and inserting $m_{\frac{3}{2}}^{eff}$ in the supertrace formula gives

$$\frac{1}{2} StrM^2 + \frac{4}{3} V = (N-1) \left|m_{\frac{3}{2}}^{(eff)}\right|^2 + \frac{2}{3} N V$$

Which covers all cases, including the unbroken phases.

Pure de Sitter Vacua

Now: $m_{\frac{3}{2}} = 0 \,,\; V > 0 \,,\; V_{,\alpha} = 0 \longrightarrow W = 0 \text{ at } \nabla_\alpha W \neq 0$

$$\left(m^{(eff)}\right)^2 = \frac{V}{3}$$

The supertrace - $\left(m^{(eff)}\right)^2$ formula becomes

$$\frac{1}{2}\, Str\left(M^{(eff)}\right)^2 = (N-1)\,V - \frac{2}{3}\,V + e^K\,\overline{\nabla}^{\alpha}\,\overline{W}\,\nabla^{\overline{\beta}}W\,R_{\alpha\overline{\beta}}$$

where $V = e^K\,\nabla_\alpha W\,\overline{\nabla}_{\overline{\alpha}}\overline{W}\,K^{\alpha\overline{\alpha}}$, and the extramality condition reads

$$V_{,\alpha} = e^K\,\nabla_\alpha\nabla_\beta W\,\overline{\nabla}^{\overline{\beta}}\,\overline{W} = 0$$

and implies that the spin-1/2 mass matrix has a vanishing eigenvalue.