Supersymmetric solutions of supergravities and the gauge/gravity duality

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Supergravity at 40

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Plan

A biased recollection of some of the achievements of supergravity so far...

1. The role of supergravity in the discovery of AdS/CFT
2. The role of Kaluza-Klein spectroscopy in AdS/CFT
3. Evolution of supersymmetric solutions relevant for holography
4. G-structures as a tool to analyse supersymmetric solutions
5. Consistent truncations
6. Rigid supersymmetry as framework for exact computations in QFT

Supergravity or string theory?

- Our favourite theory of quantum gravity is string theory
- String theory prefers 10 dimensions and likes supersymmetry
- It comes in a few versions: type IIA, IIB, I, Heterotic. But these are all related (via dualities)
- Key point: they all have a low energy limit, where only low-lying modes are kept, interacting consistently $\longrightarrow$ these are respectively 10 dimensional type IIA, IIB [Howe;Schwarz;West] (1983), I, Heterotic, supergravities
- Also an 11 dimensional supergravity exists [Cremmer,Julia,Scherk] (1978): this has been proposed to be the "down-to-earth" limit of M-theory

Supersymmetric solutions

- 11d supergravity is the "mother" of all supergravities, and it is relatively simple to describe

Fields: metric $\mathbf{g_{MN}}$, 3-form potential $\mathbf{C_{MNP}}$ (with $\mathbf{G = dC}$), gravitino $\psi_{\mathbf{M}}$

Action: $$\mathbf{S} = \int \left(\mathbf{R} * \mathbf{1} - \frac{\mathbf{1}}{\mathbf{2}} \mathbf{G} \wedge * \mathbf{G} - \frac{\mathbf{1}}{\mathbf{6}} \mathbf{C} \wedge \mathbf{G} \wedge \mathbf{G} \right) + \text{fermionic terms}$$

Supersymmetry: $$\delta\psi_{\mathbf{M}} = \nabla_{\mathbf{M}}\epsilon + \frac{\mathbf{1}}{\mathbf{288}} \left(\Gamma_{\mathbf{M}}{}^{\mathbf{NPQR}} - \mathbf{8}\delta^{\mathbf{N}}_{\mathbf{M}} \Gamma^{\mathbf{PQR}} \right) \mathbf{G_{NPQR}}\epsilon$$

Supersymmetric solutions are given by bosonic fields $\mathbf{g_{MN}}$, $\mathbf{C_{MNP}}$ obeying the equations of motion, plus a spinor ϵ, all obeying $\delta\psi_{\mathbf{M}} = \mathbf{0}$

The birth of the AdS/CFT correspondence

[Maldacena] (1997)

- Conjectured that (in a particular limit) "string theory in the background of $\mathrm{AdS_5} \times \mathbf{S^5}$ is dual to $\mathcal{N} = \mathbf{4}$ SYM", which is a SCFT $[\frac{\mathbf{R^2_{S^5}}}{\boldsymbol{\ell^2_s}} = \sqrt{\mathbf{4\pi g^2_{YM} N}}]$

- The supergravity solution comprises the "round" metric on $\mathrm{AdS_5} \times \mathbf{S^5}$ and 5-form RR flux $\mathbf{F_5} \propto \mathbf{N(1 + *)vol(S^5)}$

- Motivated by dual viewpoint on **N D3**-branes in type IIB string theory: 1) solitonic solutions of supergravity 2) describing SYM on world-volume

- Maldacena noticed that the conformal group of a CFT in $\mathbf{d}$ dimensions is $\mathbf{SO(2, d)}$, the same of the isometry of $\mathrm{AdS}_{\mathbf{d+1}}$. Moreover, for the specific case above, $\mathbf{SO(6)} \simeq \mathbf{SU(4)}$ is the **R**-symmetry group of $\mathcal{N} = \mathbf{4}$ SYM, the same of the isometry of $\mathbf{S^5}$

Maldacena's original conjecture(s)

In the same 1997 paper, Maldacena makes conjectures about a number of other cases involving AdS spaces of different dimensions

- Multiple M5 branes in 11d $\rightarrow \mathrm{AdS}_{\mathbf{7}} \times \mathbf{S}^{\mathbf{4}}$ with $\mathbf{N}$ units of flux $\mathbf{G}$ on $\mathbf{S}^{\mathbf{4}}$ is dual to the 6d "(0,2) conformal field theory"

- Multiple M2 branes in 11d $\rightarrow \mathrm{AdS}_{\mathbf{4}} \times \mathbf{S}^{\mathbf{7}}$ with $\mathbf{N}$ units of flux $*\mathbf{G}$ on $\mathbf{S}^{\mathbf{7}}$ is dual to the "SCFT on multiple M2 branes"

  So far these are all examples with maximal supersymmetry preserved

- Multiple D1/D5 intersection in type IIB $\rightarrow \mathrm{AdS}_{\mathbf{3}} \times \mathbf{S}^{\mathbf{3}} \times \mathbf{K3}$ is dual to "1+1 dimensional $\mathbf{(4,4)}$ SCFT describing the Higgs branch of the D1+D5"
- Soon after this, proposals for extensions in various directions start to flourish: less supersymmetry, non-conformal theories, thermal theories, ...

Dynamical content of holography

The computational power of holography is emphasised by two papers by [Gubser,Klebanov,Polyakov] and [Witten] (1998)

- Precise prescriptions for how to compute correlation functions of operators in the dual field theories in terms of calculations in the bulk of AdS

- Schematically, the "master formula" of the gauge/gravity duality is

$$\mathbf{e}^{-\mathbf{S}_{\text{supergravity}}[\mathbf{M}_{\mathbf{d+1}};\phi|_{\partial \mathbf{M}_{\mathbf{d+1}}}]} \simeq \mathbf{Z}_{\mathbf{QFT}}[\mathbf{M}_{\mathbf{d}} = \partial \mathbf{M}_{\mathbf{d+1}}; \mathbf{J}]$$

  In this formula the supergravity action depends on the asymptotic boundary values of the bulk fields in the background space $\mathbf{M}_{\mathbf{d+1}}$, which are identified with the sources in the QFT generating function: $\phi|_{\partial \mathbf{M}_{\mathbf{d+1}}} = \mathbf{J}$

Witten's paper contains much more: e.g. introduces the idea of study of phase transitions, holographic realizations of anomalies, comparison of KK spectra with conformal dimensions of operators...

Decreasing supersymmetry I

A first direction of generalization consists in considering SCFT's (hence AdS spaces), but decreasing the supersymmetry from maximal

- In the '80's the "Kaluza-Klein supergravity" literature had produced a list of such $AdS_p \times \mathbf{M_q}$ backgrounds, in particular for $\mathbf{(p, q) = (5, 5)}$ and $\mathbf{(p, q) = (4, 7)}$, and had conveniently studied their properties
- In type IIB [Romans] (1985): $AdS_5 \times \mathbf{Y_5}$, where $\mathbf{Y_5}$ is a Sasaki-Einstein manifold. A particular example given by Romans is $\mathbf{Y_5 = T^{1,1}}$, which is a coset manifold (for 30 years this was the only explicit example! In 2004 we constructed the $\mathbf{Y^{p,q}}$ manifolds, in 2005 the slightly more general $\mathbf{L^{a,b,c}}$ – there haven't been found new explicit metrics since then)
- All these solutions predicted a set of 4d and 3d SCFTs

Decreasing supersymmetry II

- In 11d supergravity: $\mathrm{AdS}_4 \times \mathbf{Y}_7$, where $\mathbf{Y}_7$ is a weak $\mathbf{G}_2$, Sasaki-Einstein, 3-Sasakian manifold. A summary of the "old" examples is given in the 1986 Physics Report by [Duff,Nilsson,Pope]

Table 6

Solution	G	$\mathscr{H}$	N	b_2	Stable?
Round S^7	$SO(8)$	1	8	0	Yes
Squashed S^7	$SO(5)\times SU(2)$	G_2	1	0	Yes
$S^5\times S^2 = M(1,0)$	$SU(4)\times SU(2)$	$SO(7)$	0	1	No
$S^4\times S^3$	$SO(5)\times SU(2)\times SU(2)$	$SO(7)$	0	0	No
$S^2\times S^2\times S^3 = Q(0,0,1)$	$[SU(2)]^4$	$SO(7)$	0	2	No
$S^2\times T_1S^3 = Q(0,1,1)$	$[SU(2)]^3\times U(1)$	$SO(7)$	0	2	No
Twisted $(S^2\times S^2)\times S^3$	$[SU(2)]^3\times U(1)$	$SO(7)$	0	2	No
$CP^2\times S^3 = M(0,1)$	$SU(3)\times SU(2)\times SU(2)$	$SO(7)$	0	1	No
$\dfrac{SU(3)}{SO(3)_{max}}\times S^2$	$SU(3)\times SU(2)$	$SO(7)$	0	1	No
$\dfrac{SO(5)}{SO(3)_{max}}$	$SO(5)$	G_2	1	0	Yes
$V_{5,2}$	$SO(5)\times U(1)$	$SU(3)$	2	0	Yes
$M(3,2)$	$SU(3)\times SU(2)\times U(1)$	$SU(3)$	2	1	Yes
$M(m,n)$	$SU(3)\times SU(2)\times U(1)$	$SO(7)$	0	1	See below
$Q(1,1,1)$	$[SU(2)]^3\times U(1)$	$SU(3)$	2	2	Yes
$Q(p,q,r)$	$[SU(2)]^3\times U(1)$	$SO(7)$	0	2	See below
$N(1,1)_{I}$	$SU(3)\times SU(2)$	$SU(2)$	3	1	Yes
$N(1,1)_{II}$	$SU(3)\times SU(2)$	G_2	1	1	Yes
$N(k,l)_{I,II}$	$SU(3)\times U(1)$	G_2	1	1	Yes
T^7	$[U(1)]^7$	1	8	21	Yes
$K3\times T^3$	$[U(1)]^3$	$SU(2)$	4	25	Yes

Kaluza-Klein spectroscopy I

- (Scalar) fields in $\mathbf{AdS_{d+1}}$ with mass $\mathbf{m}$ correspond to operators in the dual CFT with conformal dimension $\boldsymbol{\Delta = \frac{1}{2}(d + \sqrt{d^2 + 4m^2})}$
- Scalar, and other fields, in an $\mathbf{AdS_{d+1} \times M_q}$ space arise as Kaluza-Klein harmonics $\mathbf{Y_I(y)}$ on $\mathbf{M_q}$, schematically: $\boldsymbol{\phi(x, y) = \mathring{\phi}(y) + \sum_I \varphi_I(x) Y_I(y)}$
- Example [Witten]: chiral operators $\mathrm{Tr}\boldsymbol{[\Phi^{(z_1}\Phi^{z_2}\dots\Phi^{z_k)}]}$ where $\boldsymbol{\Phi^i}$ are the 3 adjoint scalars of $\boldsymbol{\mathcal{N} = 4}$ SYM, have conformal dimension $\boldsymbol{\Delta = k}$, thus they should arise from scalar KK harmonics with $\mathbf{m^2 = k(k-4)}$, $\mathbf{k = 2, 3, \dots}$
- Complete spectrum of type IIB on $\mathbf{S^5}$ computed in 1985 by [Kim,Romans,van Nieuwenhuizen]

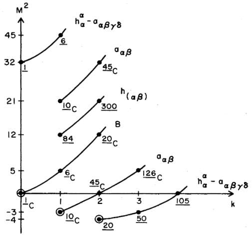

FIG. 2. Mass spectrum of scalars.

Kaluza-Klein spectroscopy II

- In 1998 [Klebanov,Witten] proposed an $\mathcal{N} = 1$ gauge theory dual to the $\mathrm{AdS}_5 \times \mathbf{T}^{1,1}$ solution

- It was a simple quiver theory, with gauge group $\mathbf{SU(N)} \times \mathbf{SU(N)}$ and bi-fundamental chiral fields interacting through a quartic superpotential

- They matched the flavour/baryonic/R-symmetry $\mathbf{SU(2)} \times \mathbf{SU(2)} \times \mathbf{U(1)_R}$ to the isometry of $\mathbf{T}^{1,1}$ plus $\mathbf{U(1)_B}$ from modes of $\mathbf{C}_4$ KK reduced on $\mathbf{S}^3 \subset \mathbf{T}^{1,1}$, argued that the theory flows to a SCFT in the IR, matched the central charge $\mathbf{c} = \mathbf{a}$ in the large $\mathbf{N}$ limit, and other things..

- A non-trivial test of this proposal was performed a year later by [Ceresole,Dall'Agata,D'Auria,Ferrara]: worked out complete KK spectrum on $\mathbf{T}^{1,1}$ and matched this to dimensions of operators constructed with the fields of the Klebanov-Witten model, transforming in various representations of the superconformal group $\mathbf{SU(2,2|1)}$

Kaluza-Klein spectroscopy III

- For $\mathrm{AdS}_4 \times \mathbf{Y_7}$ solutions, where $\mathbf{Y_7}$ is one of the three homogeneous Sasaki-Einstein manifolds, the KK spectra where worked out as follows
  - In 1985, [Castellani,D'Auria,Fré,Pilch,van Nieuwenhuizen] for $\mathbf{Y_7} = \mathbf{M^{3,2}}$
  - In 1999, a second [Ceresole,Dall'Agata,D'Auria,Ferrara] for $\mathbf{Y_7} = \mathbf{V_{5,2}}$
  - In 2000, [Merlatti] for $\mathbf{Y_7} = \mathbf{Q^{1,1,1}}$
- Based on these spectra, and "mimicking" [Klebanov,Witten], in 1999 [Fabbri,Fré,Gualtieri,Reina,Tomasiello,Zaffaroni,Zampa] proposed three-dimensional quiver guage theories dual to the $\mathbf{M^{3,2}}$, $\mathbf{Q^{1,1,1}}$ solutions
- However, the matching didn't quite work. The reason is that they missed a key ingredient, the Chern-Simons terms, that were introduced only a decade later, by ABJM

Matching Kaluza-Klein spectra post-ABJM

- In 2008 [Aharony,Bergman,Jafferis,Maldacena] – inspired by the work of [Bagger,Lambert] – proposed a three-dimensional quiver gauge theory as AdS/CFT dual to $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbb{Z}_k$
- Curiously, this field theory was nothing but the reduction of the Klebanov-Witten model, augmented with suitable Chern-Simons terms
- This immediataly (two months later!) prompted various groups [DM,Sparks],[Jafferis,Tomasiello],[Hanany,Zaffaroni] to put forward constructions of $\mathcal{N} \geq 2$ Chern-Simons-matter theories dual to $\mathrm{AdS}_4 \times \mathbf{Y}_7$ solutions, where $\mathbf{Y}_7$ is a Sasaki-Einstein manifold
- The field theory dual to $\mathbf{V}_{5,2}$ was constructed a year later in [DM,Sparks]

  Eventually the "old" Kaluza-Klein spectra were successfully compared with the dimensions of operators in these Chern-Simons-matter theories

Other types of supegravity solutions relevant for the gauge/gravity duality

- Holographic RG flows: "GPPZ", [Freedman,Gubser,Pilch,Warner], [Klebanov,Strassler], [Maldacena,Nunez], ...
- Warped $AdS_p \times \mathbf{M_q}$ $(\mathbf{p}+\mathbf{q}=\mathbf{10}$ or $\mathbf{11})$, with generic fluxes
- $\mathbf{M_p} \times \mathbf{M_q}$, where $\mathbf{M_p}$ are asymptotically locally AdS. E.g. black-holes,
- $\mathbf{M_p} \times \mathbf{M_q}$, with even more "exotic" $\mathbf{M_p}$, e.g. space-times with non-relativistic symmetries
- ....

The classification program of supersymmetric $\mathrm{AdS}_{\mathbf{p}}$ solutions: general idea

1. Pick a specific 10d/11d supergravity
2. Impose that the bosonic fields preserve a symmetry group containing $\mathbf{SO(p-1,2)}$ (i.e. isometry, plus all fluxes invariant)
3. Demand (minimal or a given fraction of) preserved supersymmetry

- Practically this leads to a general ansatz where the space-time takes the form of a warped $\mathrm{AdS}_{\mathbf{p}} \times \mathbf{M_q}$ and the fluxes are only "along" $\mathbf{M_q}$. The supersymmetry determines the (local) geometry of $\mathbf{M_q}$
- In the past $\sim \mathbf{12}$ years this search has been applied to all 10d/11d supergravities, with various values of $\mathbf{p}$, and demanding various fractions of supersymmetry

Classification of $\mathrm{AdS}_\mathbf{p}$ solutions: virtues and limitations

Example: all AdS_5 solutions in 11d sugra [Gauntlett,DM,Sparks,Waldram] (2004)

Pros

- All fluxes are determined algebraically and the equations of motion and Bianchi identities are automatic
- The quotient $\mathbf{M_6/U(1)}$ space involves some kind of Khäler geometry
- Supergravity "knows" about the dual SCFT: the $\mathbf{U(1)}$ R-symmetry emerges as a symmetry of all solutions
- It is possible to determine quite generally the central charge of the dual SCFT's in term of geometric data of $\mathbf{M_6}$

Cons

- The differential equations characterising $\mathbf{M_6}$ are in general a system of PDEs
- Finding explicit solutions of the general system is hard – presumably it hides many more solutions, with dual 4d SCFTs to be discovered

Classification program supersymmetric $\mathbf{AdS_p}$ solutions

Let me give some further representative references

- AdS_5 solutions in type IIB [Gauntlett,DM,Sparks,Waldram] (2005)
- $\mathcal{N} = 2$ AdS_4 solutions in 11d [Gabella,DM,Passias,Sparks] (2012)
- AdS_7 solutions in type II [Apruzzi,Fazzi,Rosa,Tomasiello] (2013)
- AdS_6 solutions of type II [Apruzzi,Fazzi,Passias,Rosa,Tomasiello] (2014)
- AdS_5 solutions in massive type IIA [Apruzzi,Fazzi,Passias,Tomasiello] (2015)
- More.... (e.g. partial AdS_3 classifications)

The lower dimensional point of view

- The study of supersymmetric solutions of lower (than 10) dimensional supergravities has been also pursued, leading to vast number of examples – many of these are relevant for holography
- For example, solutions that are asympotically (locally) AdS can be interpreted holographically (generically either as explicit deformations of SCFTs or as SSB)
- The holographic community is divided in two:
  1. Bottom-up: work in a (super)gravity model of your choice and apply the ideas of holography to compute quantities that should be relevant for some (they usually don't tell you which) CFT
  2. Top-down: consider only supergravities that can be embedded into 10d/11d $\rightarrow$ consistent truncations: solutions to the lower dimensional supergravities can be uplifted to solutions of 10d/11d supergravities

The latter is, in my opinion, a much more controlled set-up

Strategies for constructing supersymmetric solutions

1. G-structure analysis
2. Consistent truncations
3. Combine both

G-structure analysis I

Idea: derive a set of necessary and sufficient conditions for supersymmetry, expressed in terms of exterior differential equations on differential forms

A generalization of the perhaps more familiar idea of special holonomy manifolds

- Let's consider a "trivial" example: $AdS_5 \times \mathbf{Y_5}$, with only 5-form flux $\mathbf{F_5 = 4m(}\mathrm{vol}_{AdS_5} + \mathrm{vol}_\mathbf{5}\mathbf{)}$, with metric on AdS_5 obeying $\mathbf{R}_{\mu\nu} = -4\mathbf{m}^2\mathbf{g}_{\mu\nu}$
- Supersymmetry $\delta\psi_\mathbf{M} = \mathbf{0} \quad \Leftrightarrow \quad \nabla_\mathbf{m}\xi + \frac{\mathbf{im}}{\mathbf{2}}\gamma_\mathbf{m}\xi = \mathbf{0}$
- Solutions to this equations are well-known, but let's pretend we didn't know them, and proceed with the strategy of the **G**-structure analysis
- Define all possible bilinears: $\mathbf{K} \equiv \bar{\xi}\gamma_{(1)}\xi,\ \mathbf{J} \equiv -\mathbf{i}\bar{\xi}\gamma_{(2)}\xi,\ \Omega \equiv \bar{\xi}^\mathbf{c}\gamma_{(2)}\xi$

G-structure analysis II

1. $\nabla_{(i}K_{j)} = 0$: $\mathbf{K}$ is Killing $\Rightarrow$ $\quad ds^2(Y_5) = ds^2(X_4) + (d\psi + \rho)^2$
2. $dK = -2mJ \qquad \Rightarrow dJ = 0, \quad d\Omega = -3imK \wedge \Omega$
3. J, Ω define an $\mathbf{SU(2)}$ structure on $\mathbf{X_4}$: this is just an algebraic/group theoretic statement
4. The two differential conditions in point **2** above mean that $\mathbf{X_4}$ is (locally): i) Kähler and ii) Einstein
5. A (local) metric as above, where the 4d part is Kähler-Einstein is one of the definitions of Sasaki-Einstein metric
6. These are clearly necessary conditions; one can prove they are also sufficient for the existence of the Killing spinor ξ

Consistent truncations

- Given a "reference" solution, e.g. an internal sphere $\mathbf{S^n}$, recall that the Kaluza-Klein ansatz is schematically: $\phi(\mathbf{x},\mathbf{y}) = \mathring{\phi}(\mathbf{y}) + \sum_{\mathbf{I}} \varphi_{\mathbf{I}}(\mathbf{x})\mathbf{Y}_{\mathbf{I}}(\mathbf{y})$
- In KK spectroscopy one assumes that $\varphi_{\mathbf{I}}(\mathbf{x})$ are fluctuations, and hence the equations for the KK ansatz have to be solved only at linearized order
- One could ask whether a similar ansatz can be used to produce a set of consistent equations for the $\varphi_{\mathbf{I}}(\mathbf{x})$, not assuming that these are small, thus solving the full non-linear equations in higher dimension
- When this program is successful, the equations for the fields in lower dimension $\varphi_{\mathbf{I}}(\mathbf{x})$, can be repackaged into a (supersymmetric) Lagrangian, which is called a consistent reduction of the higher dimensional theory
- Intuitively, this program has a chance to work if one considers a finite set of massless modes, thus this is called a "consistent truncation"

Consistent truncations to maximal gauged supegravities

Some representative references:

- [de Wit,Nicolai] 1986, [Nicolai,Pilch] 2012: consistent truncation of 11d sugra on $\mathbf{S^7}$ to 4d $\mathcal{N} = \mathbf{8}$ gauged supergravity
- [Nastase,Vaman,van Nieuwenhuizen] 1999: consistent truncation of 11d sugra on $\mathbf{S^4}$ to 7d $\mathcal{N} = \mathbf{4}$ gauged supergravity
- [Gunaydin,Romans,Warner], [Pernici,Pilch,van Nieuwenhuizen] 1985; [Baguet,Hohm,Samtleben] 2015: consistent truncation of type IIB sugra on $\mathbf{S^5}$ to 5d $\mathcal{N} = \mathbf{8}$ gauged supergravity
- [Guarino,Varela] 2015: consistent truncation of massive IIA on $\mathbf{S^6}$ to 4d $\mathcal{N} = \mathbf{8}$ gauged supergravity
- Many more...

Massive consistent truncations

- In [Maldacena,DM,Tachikawa] 2008 we constructed the first examples of consistent truncations including massive KK modes (in type IIB)
- Idea: if the internal manifold preserves supersymmetry, then it has a natural set of **G**-structure forms, which can be used to construct the KK ansatz. For example, for any Sasaki-Einstein solution in type IIB: $\mathbf{B_2 = A \wedge K}$, etc.
- This gives rise to a massive consistent truncation in 5d (at least two KK scalars $\mathbf{u, v}$ have to be included for consistency), with action

$$\mathbf{S_{5d}} = \int \mathbf{d^5x\,[R - f(\phi, u, v)F_{\mu\nu}F^{\mu\nu} - g(\phi, u, v)A_\mu A^\mu + \text{scalar part}]}$$

- $\mathbf{g(0,0,0) = m_A^2 = 8}$ is a special mode in the universal KK spectrum of type IIB on Sasaki-Einstein manifolds (including $\mathbf{S^5}$ and $\mathbf{T^{1,1}}$)
- Supersymmetrized and extended by *four* groups in 2010 [Liu et al;Cassani et al;Gauntlett et al;Skenderis et al]
- [Gauntlett,Kim,Varela,Waldram] 2009 constructed a supersymmetric massive consistent truncation of 11d supergravity

Rigid supersymmetry on curved manifolds

- One can try to define supersymmetric field theories on Riemannian (or Lorentzian) curved manifolds: clearly $\partial_\mu \to \nabla_\mu$, but this is not sufficient. The supersymmetry transformations and Lagrangians must be modified
- Rigid supersymmetry in curved space (Euclidean or Lorentzian) addressed systematically only in the 2010's
- [Festuccia,Seiberg] (2011): take supergravity with some gauge and matter fields and appropriately throw away gravity $\to$ "rigid limit"
- Important: in the process of throwing away gravity, some extra fields of the supergravity multiplet remain, but are non-dynamical $\to$ background fields
- This procedure produces systematically supersymmetric Lagrangians of field theories in curved backgrounds $\to$ key role in the context of localization computations

Rigid new minimal supersymmetry

- Not any background is allowed – only those with vanishing gravitino variation
- For $\mathbf{d=4}$ field theories with an $\mathbf{R}$-symmetry, one can use (Euclidean) new minimal supergravity [Sohnius,West] (1981). Gravitino variation:

$$\delta\psi_\mu \sim (\nabla_\mu - i\mathbf{A}_\mu)\,\zeta + i\mathbf{V}_\mu\zeta + i\mathbf{V}^\nu\sigma_{\mu\nu}\zeta = 0$$

- $\mathbf{A}_\mu, \mathbf{V}_\mu$ are background fields and ζ is the supersymmetry parameter
- Existence of ζ solving this equation is equivalent to complex manifold [Klare,Tomasiello,Zaffaroni], [Dumitrescu,Festuccia,Seiberg] (2012)
- Lorentzian version addressed in [Cassani,Klare,DM,Tomasiello,Zaffaroni] (2012): existence of ζ solving the equation above is equivalent to existence of a null conformal Killing vector

So, "what next?"

Late '70s: 1st supergravity revolution – discovery, (simple) compactifications, and Kaluza-Klein spectroscopy

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Early '10s: 3rd supergravity revolution – rigid limit key for exact QFT computations

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- In 40 years supergravity evolved from candidate "theory of everything" to essential "tool" for exploring the gauge/gravity duality, charting string theory vacua, understanding black holes, devicing models for inflation, performing exact non-perturbative QFT calculations $\longrightarrow$ more to do...

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- One possible new direction is to try to compute exact path integrals in supergravity, using the ideas of localization — perhaps we’ll have to wait until 2027 for this program to be up and running...

But "what next?", really?

In the near future, can we detect *or* rule out supersymmetry from experiments or cosmological observations?

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Happy 40th birthday supergravity!!