Is there something in between SUGRA and Strings?

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- Double Field Theory

  Siegel (1993), Hull, Zwiebach (2009), O.H., Hull, Zwiebach (2010 –),

  O.H., Siegel, Zwiebach (2013 –)

- Exceptional Field Theory

  de Wit, Nicolai (1986), O.H., Samtleben (2013 –)

- Bossard, Kleinschmidt (2015)

  Ashoke Sen (2016)

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Plan of the talk:

- Duality-covariant Geometry of DFT/ExFT
- Generalized Scherk-Schwarz Compactification & Consistency of Kaluza-Klein
- Higher-derivative α' Corrections
- Consistent theory beyond supergravity?

Duality-covariant Geometry of DFT/ExFT

(Hidden) duality groups of SUGRA/Strings for toroidal compactification:

$O(d,d), \quad E_{6(6)}, \quad E_{7(7)}, \quad E_{8(8)}$

DFT/ExFT: extended coordinates to make dualities manifest

Section constraint for doubled coordinates $X^M = (\tilde{x}_i, x^i)$

- strongly constrained:

$$\forall A, B: \qquad \partial^M \partial_M A = 2\tilde{\partial}^i \partial_i A = 0$$

$$\partial^M A \, \partial_M B = 0 \qquad\qquad \eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- solved by $\tilde{\partial}^i = 0$, up to $O(D,D)$, but explains hidden symmetry:

  $X^{\mathcal{M}} = (\, \cancel{\tilde{x}_\mu}\,,\ x^\mu\,, \cancel{Y^M}\,)\,, \quad M = 1,\ldots,2d \quad \Rightarrow$ unbroken $O(d,d)$!

- weakly constrained in full string (field) theory:

  level-matching $\partial^M \partial_M A = 0\,,$ non-trivial string product

  consistent theory for massless fields plus their KK/winding modes?

Generalized Geometry of DFT

Generalized Lie derivatives for Generalized metric $\mathcal{H}_{MN}(g,b) \in O(D,D)$

$$\widehat{\mathcal{L}}_\xi \mathcal{H}_{MN} \;=\; \xi^P \partial_P \mathcal{H}_{MN} + \left(\partial_M \xi^P - \partial^P \xi_M\right) \mathcal{H}_{PN} + \left(\partial_N \xi^P - \partial^P \xi_N\right) \mathcal{H}_{MP}$$

Gauge algebra C-bracket: $\left[\,\xi_1\,,\;\xi_2\,\right]_C \;=\; \widehat{\mathcal{L}}_{\xi_1}\xi_2 - \widehat{\mathcal{L}}_{\xi_2}\xi_1$

$\tilde{\partial}^i = 0$: $\delta g = \mathcal{L}_\xi g$, $\delta b = \mathrm{d}\tilde{\xi} + \mathcal{L}_\xi b$ $\rightarrow$ Courant bracket in Gen. Geom.

$$\left[\,\xi_1 + \tilde{\xi}_1, \xi_2 + \tilde{\xi}_2\,\right] \;=\; \left[\,\xi_1\,, \xi_2\,\right] \;+\; \mathcal{L}_{\xi_1}\tilde{\xi}_2 - \mathcal{L}_{\xi_2}\tilde{\xi}_1 - \tfrac{1}{2}\mathrm{d}\left(i_{\xi_1}\tilde{\xi}_2 - i_{\xi_2}\tilde{\xi}_1\right)$$

exact term by B-shifts: $\tilde{\xi}_i \rightarrow \tilde{\xi}_i + B_{ij}\xi^j$, $\quad h_B = \begin{pmatrix} 1 & B \\ 0 & 1 \end{pmatrix} \in O(D,D)$

C-bracket: $\xi^{M\prime} = h^M{}_N \xi^N \quad \forall h \in O(D,D)$

$\Rightarrow$ Any diff. & b-field gauge invariant theory compatible with Gen. Geom.

$\Rightarrow$ only DFT makes $O(d,d)$ manifest $\rightarrow$ constrains α' corrections!

Scherk-Schwarz Compactification & Consistency of Kaluza-Klein

Ansatz in terms of twist matrix $U(Y)$ in duality group & $\rho(Y)$

$$\begin{aligned}
\mathcal{H}_{MN}(x,Y) &= U_M{}^K(Y)\, U_N{}^L(Y)\, \mathcal{H}_{KL}(x) \\
A_\mu{}^M(x,Y) &= (U^{-1})_N{}^M(Y)\, A_\mu{}^N(x) \\
e^{-2\phi(x,Y)} &= \rho^{-(n-2)}(Y)\, e^{-2\phi(x)} \quad \ldots
\end{aligned}$$

Y-dependence factors out consistently $\rightarrow$ geometric constraints on U, ρ

$$\boxed{\widehat{\mathcal{L}}_{U_M^{-1}} U_N^{-1} = X_{MN}{}^K\, U_K^{-1}}$$

with constant $X_{MN}{}^K = \Theta_M{}^\alpha (t_\alpha)_N{}^K + \cdots$, Generalized Parallelizability

- Reduction fully consistent, including scalar potential, fermions, etc.
- U, ρ for spheres $\Rightarrow$ consistency of $AdS_4 \times S^7$ [de Wit & Nicolai (1986)], $AdS_7 \times S^4$ [Nastase, Vaman, van Nieuwenhuizen (1999)] and $AdS_5 \times S^5$
- Non-geometric compactifications upon relaxing section constraint

Higher-derivative α' corrections

- string theory: infinite number of higher-derivative α' corrections;
  $O(d, d; \mathbb{R})$ symmetry preserved [Sen (1991)]

- $O(d, d; \mathbb{R})$ α'-deformed [Meissner (1997), O.H. & Zwiebach (2011, 2015)]

$$\mathcal{H}_{MN} = \begin{pmatrix} \widehat{g}^{-1} & -\widehat{g}^{-1}\widehat{b} \\ \widehat{b}\,\widehat{g}^{-1} & \widehat{g} - \widehat{b}\,\widehat{g}^{-1}\,\widehat{b} \end{pmatrix}, \qquad \widehat{g} = g + \alpha'(\partial g)(\partial g) + \alpha'(\partial b)(\partial b) + \cdots$$

- Deformed gauge structure in DFT, $K_{MN} \equiv \partial_M \xi_N - \partial_N \xi_M$

$$\xi_{12}^M = \left[\xi_2, \xi_1\right]_C^M + \frac{\alpha'}{2}(\gamma^+ \bar{\mathcal{H}}^{KL} - \gamma^- \eta^{KL}) K_{[2K}{}^P \partial^M K_{1]LP}$$

- Uniquely determines $\mathcal{O}(\alpha')$ correction up to two parameters:
  [O.H. & Zwiebach (2014), Nunez & Marques (2015)]

$$\begin{cases} \gamma^+ = 1\,, \quad \gamma^- = 0 & \text{bosonic string} \\ \gamma^+ = \frac{1}{2}\,, \quad \gamma^- = \frac{1}{2} & \text{heterotic string} \end{cases}$$

- $\gamma^+ = 0\,,\ \ \gamma^- = 1$ (HSZ): exactly duality and gauge invariant
  $\Rightarrow$ infinite number of α' corrections!

Consistent theory beyond supergravity?

- $\partial^M \partial_M(\phi_1 \phi_2) \neq 0$ without strong constraint $\Rightarrow$ non-trivial product

$$\big(\phi_1 \star \phi_2\big)(X) \;=\; \sum_{P_1,P_2 \in \mathbb{Z}^{2d}} \delta(P_1 \cdot P_2)\, \phi^1_{P_1} \phi^2_{P_2}\, e^{i(P_1+P_2)\cdot X} \;,$$

- Non-associative, but 'associator' total derivative

$$(\phi_1 \star \phi_2) \star \phi_3 - \phi_1 \star (\phi_2 \star \phi_3) \;=\; \partial_M F^M(\phi_1, \phi_2, \phi_3)$$

$\Rightarrow$ cubic theory consistent, beyond that: L_∞ or A_∞ algebra?

[Zwiebach (1993), O.H., Hull & Zwiebach, unpublished]

- Theory for "SUGRA" fields plus their KK & winding modes?

  Yes: other massive modes can be integrated out [Sen (2016)]

  in some sense full string theory; amplitudes for mass. modes hidden

- One-Loop in ExFT with massive modes $\Rightarrow$ improved UV behavior

  [Bossard & Kleinschmidt (2015)]

Summary & Outlook

- DFT and ExFT make duality symmetries $O(d,d)$, $E_{d(d)}$ manifest
- strongly constrained theory fully background independent, reformulation of SUGRA
- very powerful formalism: consistency of KK truncations, higher-derivative α' corrections
- but at least on T^d extended coordinates physical and real; consistent theory of SUGRA fields plus KK/winding modes from strings

  Can it be given explicitly?
- *Consistent truncation* with KK and winding modes?

  $\rightarrow$ analogous to 5D SUGRA consistent truncation of IIB on $AdS_5 \times S^5$