Four-fermion production near the W-pair production threshold

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In collaboration with:

M. Beneke, C. Schwinn, A. Signer, G. Zanderighi

Overview

- Introduction
- Effective Field Theory Formalism
- Born-level results
- Radiative corrections
- Uncertainties on W-mass determination
- Conclusion

Motivation

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The masses of the top quark, the W boson and yet undiscovered particles like supersymmetric partners could be accurately measured using threshold scan at a future $e^- e^+$ linear collider

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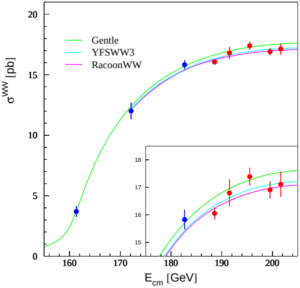

Measurements of the four-fermion production cross section near the W-pair production threshold could reduce δM_W to $\approx 6\,\mathrm{MeV}$ (*G. Wilson, 2nd ECFA/DESY Study, 1498-1505, Desy LC note LC-PHSM-2001-009*)

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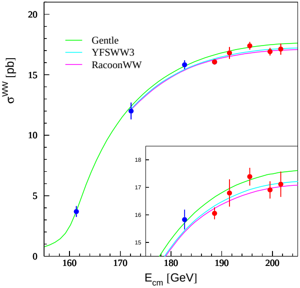

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Theoretical uncertainties must be reduced to $\sim 0.1\%$!

$\Downarrow$

Accurate theoretical predictions for $e^-e^+ \to 4f$ in the energy range $\sqrt{s} \approx 155 - 170\,\mathrm{GeV}$ strongly motivated by future phenomenological applications

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2. Effective Field Theory Approach

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   - Computationally simple + final analytic expressions
   - At the moment only for inclusive observables

Effective Field Theory Approach

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The process is characterized by two well-separated scales: $\Lambda^2 \equiv M_W^2 \gg M_W \Gamma_W \equiv \lambda^2$
$\rightarrow$ Effective Field Theory (EFT) techniques are used to integrate out the large scale M_W^2

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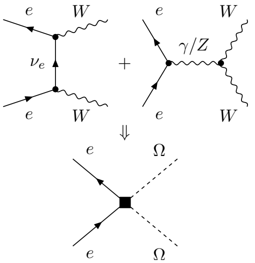

- Effective Lagrangian describing long-distance degrees of freedom
  $(k^2 - m_p^2 \lesssim M_W \Gamma_W)$
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  - potential $(k^2 \sim M_W \Gamma_W)$ and soft $(k^2 \sim \Gamma_W^2)$ photons
  - high-energetic external fermions $(k^2 = 0)$
- Matching coefficients determined by short-distance physics
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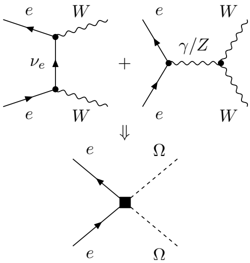

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$$\mathcal{L}_{\text{EFT}} = \sum_{\mp} \Omega_{\mp}^{i*} \left(iD^0 + \frac{\vec{D}^2}{2M_W} + i\frac{\Gamma_W^{(0)}}{2} - \frac{(\vec{D}^2 - iM_W\Gamma_W^{(0)})^2}{8M_W^3} + i\frac{\Gamma_W^{(1)}}{2} + \ldots \right) \Omega_{\mp}^{i} +$$

$$+ \frac{g^2 C_p}{2M_W^2} (\bar{e}_L \gamma^{[i} i n^{j]} e_L)(\Omega_-^{i*} \Omega_+^{j*}) + \frac{K_{4e}}{2M_W^2} (\bar{e}_L \gamma^\mu e_L)(\bar{e}_L \gamma_\mu e_L) + \ldots$$

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EFT calculation organized as a simultaneous expansion of the matrix elements in powers of α, α_s, the ratios Γ_W/M_W and the non-relativistic energy of the Ws $E/M_W \equiv (\sqrt{s} - 2M_W)/M_W$

$$\alpha_s^2 \sim \alpha_{ew} \sim \frac{\Gamma_W}{M_W} \sim \frac{E}{M_W}$$

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For counting purposes the expansion parameters are collectively indicated as δ!

$$\sigma(s) = \sum_{n \geq 0} \sigma^{(n/2)}(s) \quad \text{where} \quad \frac{\sigma^{(n/2)}(s)}{\sigma^{(0)}(s)} \sim \delta^{n/2}$$

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EFT formalism applied to the calculation of total cross-section for $e^+e^- \to \mu^-\overline{\nu}_\mu u\bar{d}X$ up to NLO in $\alpha_s^2 \sim \alpha_{ew} \sim \Gamma_W/M_W \sim E/M_W$

(*M. Beneke, P. Falgari, C. Schwinn, A. Signer, G. Zanderighi, ArXiv:0707.0773[hep-ph]*)

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The cross section is extracted from appropriate cuts of the forward-scattering amplitude!

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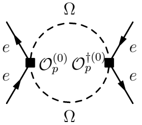
Leading-order forward-scattering amplitude obtained from the matrix element of lowest-order production operators:

$$i\mathcal{A}^{(0)}_{\text{Born}} = \int d^4x \langle e^- e^+ | T[i\mathcal{O}_p^{(0)\dagger}(0) i\mathcal{O}_p^{(0)}(x)] | e^- e^+ \rangle =$$



where $\mathcal{O}_p^{(0)} = i\frac{g^2}{2M_W^2}\bar{e}_L(\gamma^i n^j + \gamma^j n^i)e_L\Omega_-^{i*}\Omega_+^{j*}$ (with n^i the direction of the incoming electron).

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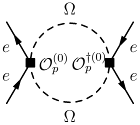

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The flavor-specific final state is selected by multiplying the imaginary part of $\mathcal{A}$ with the leading-order branching ratios, $\mathrm{Br}^{(0)}(W^- \to \mu^- \bar{\nu}_\mu) \mathrm{Br}^{(0)}(W^+ \to u\bar{d}) = 1/27$:

$$\sigma^{(0)}_{\rm Born} \quad = \quad \frac{1}{27s} \mathrm{Im} \mathcal{A}^{(0)}_{\rm Born} = -\frac{\pi \alpha^2}{27 s_w^4 s} \mathrm{Im} \left[\sqrt{-\frac{(E + i\Gamma_W^{(0)})}{M_W}} \right]$$

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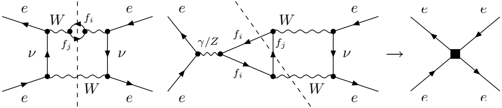
$\sqrt{\text{N}}$LO

From singly-resonant kinematical configurations



$$\Rightarrow \sigma^{(1/2)}_{\text{Born}} = \frac{\alpha^3}{27 s_w^6 s} \sum_h K_h C^h(s)$$

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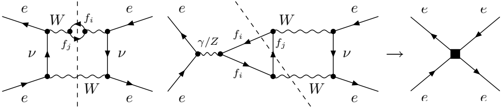

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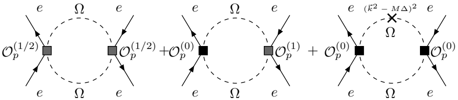

$$\Rightarrow \sigma^{(1)}_{\text{Born}} = \frac{\pi \alpha^2}{27 s_w^4 s} \Bigg\{ \mathcal{F}(s) \text{Im} \left[\left(-\frac{E + i\Gamma_W^{(0)}}{M_W} \right)^{3/2} \right]$$

$$+ \text{Im} \left[\left(\frac{3E}{8M_W} + \frac{17 i \Gamma_W^{(0)}}{8M_W} \right) \sqrt{-\frac{E + i\Gamma_W^{(0)}}{M_W}} \right.$$

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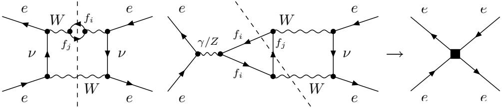

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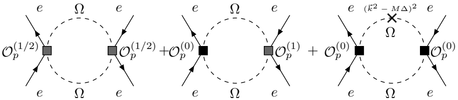

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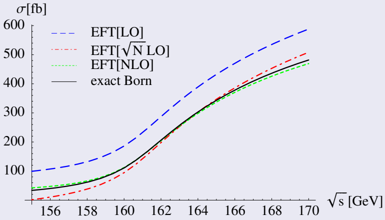
Comparison with the exact cross section

Numerical result from Whizard/CompHep: *W. Kilian; E. Boos et al., Nucl. Instrum. Meth.A534(2004); A. Pukhov et al., hep-ph/9908288*



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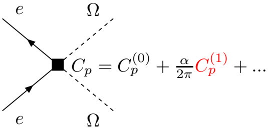

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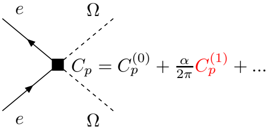

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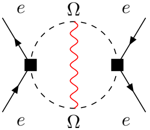

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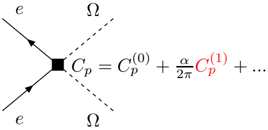

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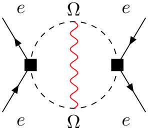
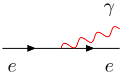


- Universal corrections from Initial State Radiation (ISR)



NLO matching coefficients

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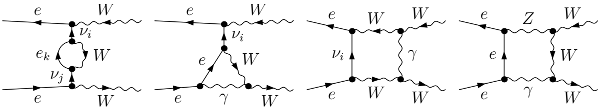
- $O(\alpha)$ production vertex: $\frac{g^2 \alpha C_p^{(1)}}{4\pi M_W^2} (\bar{e}_L \gamma^{[i} in^{j]} e_L)(\Omega_-^{i*} \Omega_+^{j*})$

  Extracted from the one-loop corrections to the on-shell process $e^+ e^- \to W^+ W^-$
  At lowest order set $s = 4M_W^2 \to$ only corrections to t-channel diagram survive!



$$\Delta\sigma^{(1)}_{\text{production}} = \frac{\alpha}{\pi} \text{Re} \left[\left(-\frac{1}{\varepsilon^2} - \frac{3}{2\varepsilon} \right) \left(-\frac{4M_W^2}{\mu^2} \right)^{-\varepsilon} + c_p^{(1,\text{fin})} \right] \sigma^{(0)}_{\text{Born}}$$

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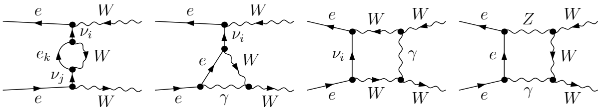

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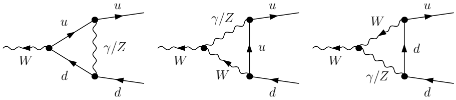
- $O(\alpha)$ decay vertices

  Extracted from EW virtual and real corrections to the decays $W^- \to \mu^- \bar{\nu}_\mu$ and $W^+ \to u\bar{d}$

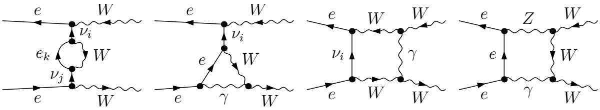


$$\Delta\sigma^{(1)}_{\text{decay}} = \frac{\alpha}{\pi} \left[\mathrm{Re}\left[c^{(1,\text{fin})}_{\mu\bar{\nu}} + c^{(1,\text{fin})}_{u\bar{d}} \right] + \frac{101}{12} - \frac{7\pi^2}{12} + \left(\frac{19}{4} - \frac{\pi^2}{12} \right) Q_u Q_d \right] \sigma^{(0)}_{\text{Born}}$$

NLO matching coefficients

- $O(\alpha)$ production vertex: $\frac{g^2 \alpha C_p^{(1)}}{4\pi M_W^2} (\bar{e}_L \gamma^{[i} i n^{j]} e_L)(\Omega_-^{i*} \Omega_+^{j*})$

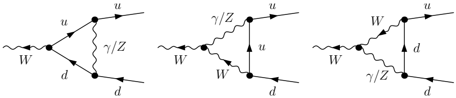
  Extracted from the one-loop corrections to the on-shell process $e^+ e^- \to W^+ W^-$
  At lowest order set $s = 4M_W^2 \to$ only corrections to t-channel diagram survive!



$$\Delta\sigma^{(1)}_{\text{production}} = \frac{\alpha}{\pi} \text{Re} \left[\left(-\frac{1}{\varepsilon^2} - \frac{3}{2\varepsilon} \right) \left(-\frac{4M_W^2}{\mu^2} \right)^{-\varepsilon} + c_p^{(1,\text{fin})} \right] \sigma^{(0)}_{\text{Born}}$$

- $O(\alpha)$ decay vertices

  Extracted from EW virtual and real corrections to the decays $W^- \to \mu^- \bar{\nu}_\mu$ and $W^+ \to u\bar{d}$



$$\Delta\sigma^{(1)}_{\text{decay}} = \frac{\alpha}{\pi} \left[\text{Re} \left[c^{(1,\text{fin})}_{\mu\bar{\nu}} + c^{(1,\text{fin})}_{u\bar{d}} \right] + \frac{101}{12} - \frac{7\pi^2}{12} + \left(\frac{19}{4} - \frac{\pi^2}{12} \right) Q_u Q_d \right] \sigma^{(0)}_{\text{Born}}$$

QCD corrections are taken into account by multiplying the cross sections with the universal factor for massless quarks $\delta_{\text{QCD}} = 1 + \alpha_s/\pi + 1.409\alpha_s^2/\pi^2$

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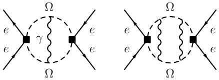
- Coulomb corrections

  Arise from exchange of potential photons ($q^2 \sim M_W \Gamma_W$) between the Ws: n^{th} Coulomb correction scales as $\alpha^n (M_W/\Gamma_W)^{n/2} \sim \alpha^{n/2} \longrightarrow$ first and second correction must be included!



$$\rightarrow \Delta\sigma^{(1)}_{\text{Coulomb}} = \frac{\pi\alpha^2}{27 s_w^4 s} \,\text{Im}\left[-\frac{\alpha}{2} \ln\left(-\frac{E + i\Gamma_W^{(0)}}{M_W} \right) + \frac{\alpha^2\pi^2}{12} \sqrt{ -\frac{M_W}{E + i\Gamma_W^{(0)}} } \right]$$

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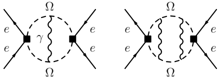

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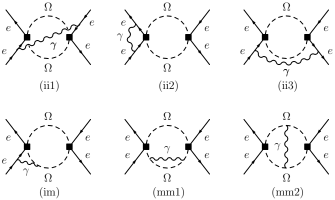
- **Soft-photon corrections**

  Arise from soft photons ($q^2 \sim \Gamma_W^2$) exchange between different subprocesses
  Large cancellations due to residual gauge-invariance of the EFT Lagrangian!



$$\rightarrow \Delta\sigma^{(1)}_{\text{soft}} = \frac{\pi\alpha^2}{27 s_w^4 s}\frac{\alpha}{\pi}\left(\frac{M_W}{2\mu}\right)^{-\varepsilon}\left(\frac{1}{\varepsilon^2}+\frac{5}{\varepsilon}+30+\frac{7\pi^2}{3}\right)$$
$$\times \text{Im}\left[-\sqrt{-\frac{E+i\Gamma_W^{(0)}}{M_W}}\left(-\frac{8(E+i\Gamma_W^{(0)})}{\mu}\right)^{-3\varepsilon}\right]$$

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Leading logs ($\sim \alpha^n \ln^n \left(\frac{2M_W}{m_e}\right)$) can be resummed to all orders!

$$\sigma^{\mathrm{NLO}}(s) = \int_0^1 dx_1 \int_0^1 dx_2 \Gamma_{ee}^{\mathrm{LL}}(x_1) \Gamma_{ee}^{\mathrm{LL}}(x_2) \hat{\sigma}(x_1 x_2 s)$$

where $\Gamma_{ee}^{\mathrm{LL}}$ is the electron structure function in Leading Log (LL) approximation and

$$\hat{\sigma}(s) = \sigma_{\mathrm{Born}}(s) + \hat{\sigma}^{(1)}(s) = \sigma_{\mathrm{Born}}(s) + \sigma^{(1)}(s) - 2\int_0^1 dx \Gamma_{ee}^{\mathrm{LL},(1)}(x) \sigma_{\mathrm{Born}}^{(0)}(xs)$$

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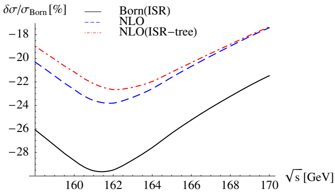
	$\sigma(e^- e^+ \rightarrow \mu^- \bar{\nu}_\mu u \bar{d} X)$(fb)			
$\sqrt{s}$ [GeV]	Born	Born(ISR)	NLO	NLO(ISR-tree)
158	61.67(2)	45.64(2) [-26.0%]	49.19(2) [-20.2%]	50.02(2) [-18.9%]
161	154.19(6)	108.60(4) [-29.6%]	117.81(5) [-23.6%]	120.00(5) [-22.2%]
164	303.0(1)	219.7(1) [-27.5%]	234.9(1) [-22.5%]	236.8(1) [-21.8%]
167	408.8(2)	310.2(1) [-24.1%]	328.2(1) [-19.7%]	329.1(1) [-19.5%]
170	481.7(2)	378.4(2) [-21.4%]	398.0(2) [-17.4%]	398.3(2) [-17.3%]



Comparison with the full four-fermion calculation

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Comparison with the full $e^+e^- \to 4f$ result

(*Denner, Dittmaier, Roth, Wieders, Phys. Lett. B612: 223-232,2005*)

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- Strict NLO electroweak corrections

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$\sqrt{s}$ [GeV]	Born	NLO(EFT)	ee4f	DPA
161	150.05(6)	104.97(6)	105.71(7)	103.15(7)
170	481.2(2)	373.74(2)	377.1(2)	376.9(2)

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- QCD corrections and higher-order ISR

	$\sigma(e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}\,X)$(fb)			
$\sqrt{s}$ [GeV]	Born(ISR)	NLO(EFT)	ee4f	DPA
161	107.06(4)	117.38(4)	118.12(8)	115.48(7)
170	381.0(2)	399.9(2)	401.8(2)	402.1(2)

Difference between EFT and full four-fermion result $\sim 0.6\%$ in the range $160 - 170$ GeV!

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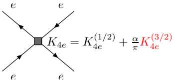
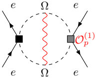
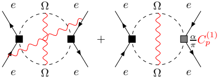
  - $O(\alpha)$-improved four-electron operators from radiative corrections to singly-resonant diagrams. Included in the full four-fermion calculation



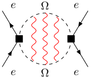
  - Interference of Coulomb correction with higher-dimensional production operators. Included in the full four-fermion calculation



  - Interference of Coulomb correction with soft corrections or $O(\alpha)$ matching coefficients



  - Third Coulomb correction (Known but negligible)



Uncertainties on W mass determination

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Assume six experimental points O_i at $\sqrt{s_i} = 160, 161, 162, 163, 164, 170\,\text{GeV}$
$(O_i = \sigma^{\text{NLO}}_{\text{EFT}}(s_i; M_W = 80.377\,\text{GeV}))$

Determine the uncertainty δM_W on the W mass for different theoretical prediction E_i minimizing the function

$$\chi^2(\delta M_W) = \sum_{i=1}^{6} \frac{(O_i - E_i(\delta M_W))^2}{2\rho O_i}$$

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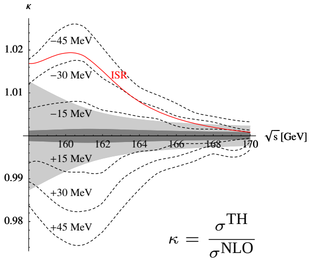

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- With further inputs from the full four-fermion calculation and higher-order corrections in the EFT framework the theoretical error on the W mass could be reduced to $\lesssim$ 5MeV