NNLO QCD Analysis of the Virtual Photon Structure Functions

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1. Introduction and Motivation

Two-photon process
in e^+e^- collision

Photon structure functions

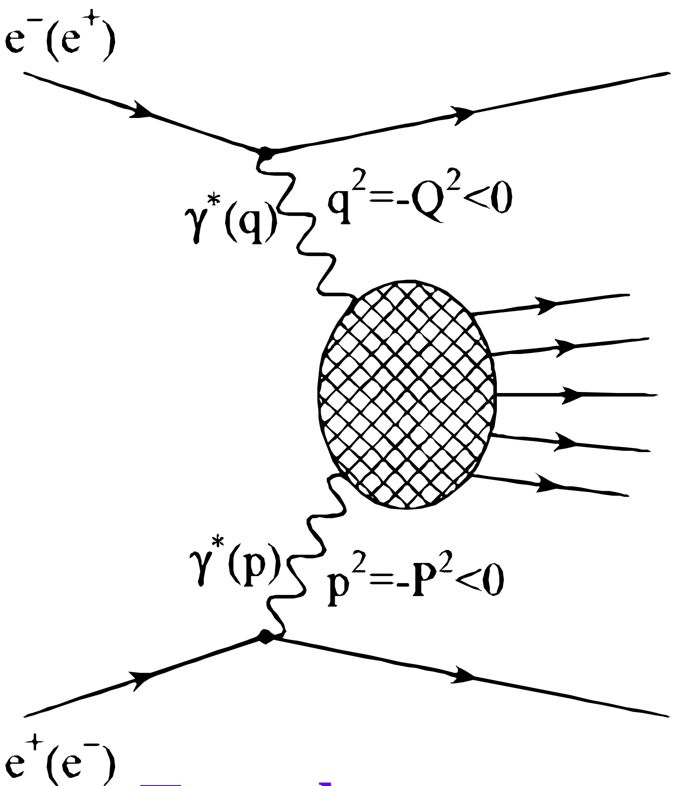
F_2^γ and F_L^γ

for the case of virtual
photon ($P^2 \neq 0$) target

as in the case of nucleon
structure functions

$F_2^{p(n)}$ and $F_L^{p(n)}$

in the future ⇨ linear collider ILC



Two-photon process

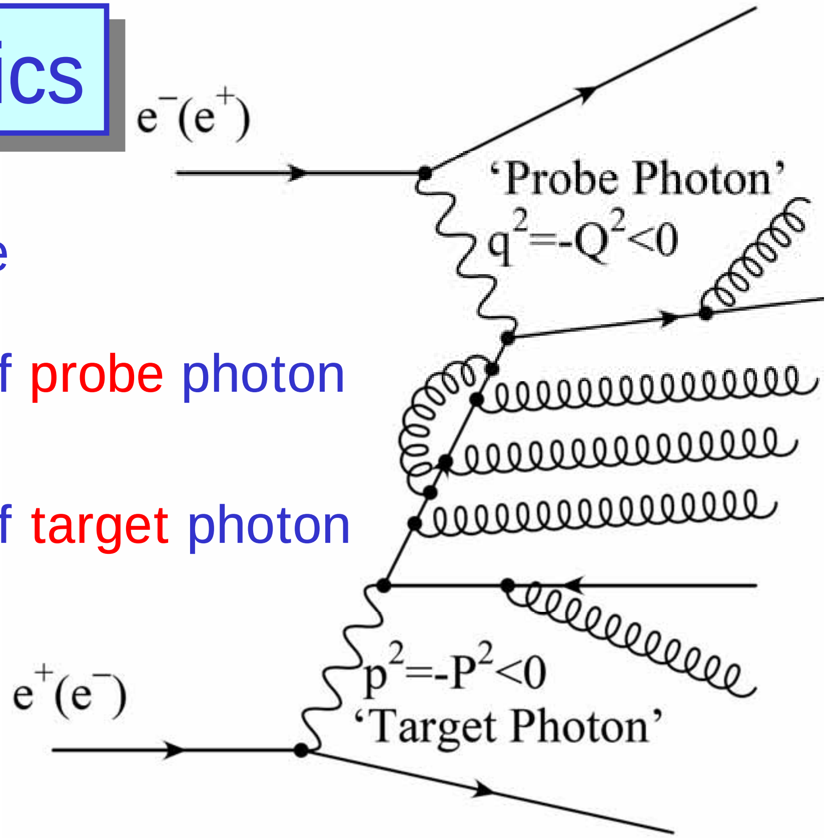
$$e^+e^- \to (e^+e^-\gamma\gamma) \to e^+e^-X$$

Virtual-Photon Kinematics

$$x = \frac{Q^2}{2p \cdot q}$$: Bjorken variable

$$Q^2 = -q^2 > 0$$: Mass squared of probe photon

$$P^2 = -p^2 > 0$$: Mass squared of target photon



In the kinematic region:

$$\Lambda^2 \ll P^2 \ll Q^2$$

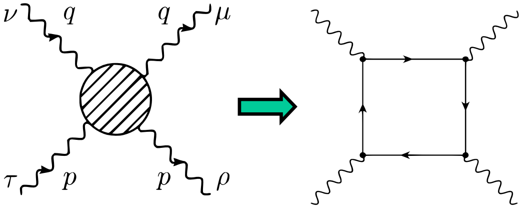
e^+e^-collision

& QCD interactions

structure fns. F_2^γ and F_L^γ
perturbatively calculable !

F_2^γ in Perturbative QCD

Simple parton model



- For real photon target $(P^2 \approx 0)$ $P^2 \ll Q^2$

$$F_2^\gamma(x, Q^2) = \alpha \left[\frac{1}{\alpha_s(Q^2)} A + B + B' + \mathcal{O}(\alpha_s) \right]$$

$\sim \ln \frac{Q^2}{\Lambda^2}$ (LO) (NLO) Hadronic piece

Point-like contribution dominates $\sim \ln Q^2$

Walsh-Zerwas (1973)

OPE+RGE Witten (1977)

lowest order in α Bardeen-Buras (1979)

QCD improved parton model

Gluck-Reya (1983),…

- For highly virtual photon target $(\Lambda^2 \ll P^2 \ll Q^2)$

$$F_2^\gamma(x, Q^2, P^2) = \alpha \left[\frac{1}{\alpha_s(Q^2)} \tilde{A} + \tilde{B} + \mathcal{O}(\alpha_s) \right]$$

Λ : QCD scale parameter

(LO) (NLO) Walsh-TU (1981,1982)

Hadronic piece can also be dealt with perturbatively

Definite prediction of F_2^γ , its shape and magnitude, is possible

Present Motivation

⇨ NNLO extension Moch-Vermaseren-Vogt (2002, 2006)

Motivated by the calculation of 3-loop anomalous dimensions Vogt-Moch-Vermaseren (2004,2006)

We extend the analysis of virtual photon structure functions to NNLO ($\alpha\alpha_s$)

K. Sasaki, T. Ueda and T.U., Phys. Rev.D75 (2007) 114009

Here we also consider target mass effects to NNLO

Y. Kitadono, K. Sasaki, T. Ueda and T.U., in preparation

For parton distributions inside virtual photon in the NNLO, see Ken Sasaki's next talk

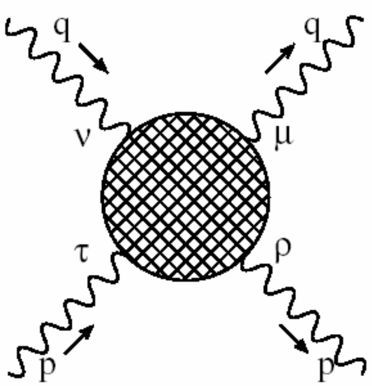
Structure Functions

$$W^{\gamma}_{\mu\nu}(p,q) = \frac{1}{2}\sum_{\lambda} \epsilon^{\rho *}_{\lambda}(p) W_{\mu\nu\rho\tau} \epsilon^{\tau}_{\lambda}(p)$$

Spin-averaged Structure Tensor

$$= \frac{1}{2\pi}\int d^4x e^{iq\cdot x} \langle \gamma(p)|J_{\mu}(x)J_{\nu}(0)|\gamma(p)\rangle_{\rm spin\ av.}$$

$W_{\mu\nu\rho\tau}$



Forward photon-photon scattering

$$W^{\gamma}_{\mu\nu} = e_{\mu\nu}\frac{1}{x}F^{\gamma}_{L}(x,Q^2,P^2) + d_{\mu\nu}\frac{1}{x}F^{\gamma}_{2}(x,Q^2,P^2)$$

$$e_{\mu\nu} \equiv g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \qquad d_{\mu\nu} \equiv -g_{\mu\nu} + \frac{p_{\mu}q_{\nu}+p_{\nu}q_{\mu}}{p\cdot q} - \frac{p_{\mu}p_{\nu}q^2}{(p\cdot q)^2}$$

cf. Structure tensor no spin averaged

$$W_{\mu\nu} = W^{S}_{\mu\nu} + iW^{A}_{\mu\nu}$$

Symmetric part ⇨ unpolarized photon structure functions F^{γ}_{2} and F^{γ}_{L}

Anti-symmetric part ⇨ polarized photon structure functions g^{γ}_{1} and g^{γ}_{2}

2. Theoretical framework : OPE and RGE

- $$W^{\gamma}_{\mu\nu}(p,q) = \frac{1}{2\pi}\int d^4z\, e^{iq\cdot z}\langle\gamma(p)|J_\mu(z)J_\nu(0)|\gamma(p)\rangle_{\text{spin ave.}}$$
  $$= e_{\mu\nu}\frac{1}{x}F^{\gamma}_{L}(x,Q^2,P^2) + d_{\mu\nu}\frac{1}{x}F^{\gamma}_{2}(x,Q^2,P^2)$$

- The OPE near the light-cone $Q^2 \to \infty$ ⟺ light-cone

  $$J(z)J(0) \sim \sum_i C_i(z)O_i(0)$$

  i : over relevant ops.

  ⇒ $$\langle\gamma(p)|J(z)J(0)|\gamma(p)\rangle \sim \sum_i C_i(z)\langle\gamma(p)|O_i|\gamma(p)\rangle$$

  (hadronic ops. + photon op.)

- Spin-n twist-2 operators

  (n-th moment) (dominant) $\tau = d_O - n$

  quark : $O^{\mu_1\cdots\mu_n}_{\psi}$ (flavor singlet) gluon : $O^{\mu_1\cdots\mu_n}_{G}$

  $O^{\mu_1\cdots\mu_n}_{NS}$ (flavor non-singlet) photon : $O^{\mu_1\cdots\mu_n}_{\gamma}$

  QCD with massless quarks with n_f flavors

- Moment sum rule of F_2^γ

$$M_2^\gamma(n,Q^2,P^2)=\int_0^1 dx\, x^{n-2}F_2^\gamma(x,Q^2,P^2)$$

we take $\mu^2=-p^2=P^2$

$$=\sum_{i=\psi,G,NS,\gamma} C_{2,n}^i\left(\frac{Q^2}{P^2},\bar g(P^2),\alpha\right)A_n^i\left(\bar g(P^2),\alpha\right)$$

for even n

RG improved coefficients

Photon matrix element $\langle\gamma(p)|O_i^n|\gamma(p)\rangle_{\rm spin\ ave.}$

Perturbatively calculable when $\Lambda^2\ll P^2$

$$C_{2,n}^i\left(\frac{Q^2}{P^2},\bar g(P^2),\alpha\right)=\left(T\exp\left[\int_{\bar g(Q^2)}^{\bar g(P^2)}dg\frac{\gamma_n(g,\alpha)}{\beta(g)}\right]\right)_{ij}C_{2,n}^j\left(1,\bar g(Q^2),\alpha\right)$$

Solution of RG eq. for coefficient functions

$\beta(g)$: QCD beta function

γ_n : anomalous dimensions

Expand $M_2^\gamma(n,Q^2,P^2)$ up to NNLO ($\alpha\alpha_s$)

- Inverse Mellin transformation: n-space ⇨ x-space

$$F_2^\gamma(x,Q^2,P^2)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dn\, x^{1-n}M_2^\gamma(n,Q^2,P^2)$$

numerically inverted

Anomalous dimension matrix

$$\gamma_n(g,\alpha) = \left(\begin{array}{c|c} \hat{\gamma}_n(g) & \mathbf{0} \\ \hline \boldsymbol{K}_n(g,\alpha) & 0 \end{array}\right)$$

4 x 4 matrix

Anomalous dimensions of hadronic operators

1-loop anom. dim.

$$\widehat{\gamma}_n(g) = \begin{pmatrix} \gamma^n_{\psi\psi}(g) & \gamma^n_{G\psi}(g) & 0 \\ \gamma^n_{\psi G}(g) & \gamma^n_{GG}(g) & 0 \\ 0 & 0 & \gamma^n_{NS}(g) \end{pmatrix}$$

$$\widehat{\gamma}^{(0)}_n = \sum_{i=+,-,NS} \lambda^n_i \; P^n_i$$

$$\sum_i P_i = 1 \qquad P_i P_j = \delta_{ij} P_i$$

Mixing anomalous dimensions of photon-hadron ops.

$$\boldsymbol{K}_n(g,\alpha) = \left(K^n_\psi(g,\alpha) \;\; K^n_G(g,\alpha) \;\; K^n_{NS}(g,\alpha) \right)$$

Moment Sum Rule for F_2^γ

- Summarized as

$$d_i^n = \lambda_i^n / 2\beta_0 \quad i = +, -, NS$$

LO ($\alpha\alpha_s^{-1}$)

for even n

$$\int_0^1 dx\, x^{n-2} F_2^\gamma(x, Q^2, P^2)$$

$$= \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left\{ \frac{4\pi}{\alpha_s(Q^2)} \sum_i \mathcal{L}_i^n \left[1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right] \right.$$

NLO (α)

$$+ \sum_i \mathcal{A}_i^n \left[1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right] + \sum_i \mathcal{B}_i^n \left[1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right] + \mathcal{C}^n$$

$$+ \frac{\alpha_s(Q^2)}{4\pi}\left(\sum_i \mathcal{D}_i^n \left[1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n-1}\right] + \sum_i \mathcal{E}_i^n \left[1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n}\right] \right.$$

$$\left.\left. + \sum_i \mathcal{F}_i^n \left[1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_i^n+1}\right] + \mathcal{G}^n \right) + \mathcal{O}(\alpha_s^2) \right\}$$

NNLO ($\alpha\alpha_s$)

Renomalization Group parameters

$\overline{\mathrm{MS}}$

- QCD β -function

Tarasov-Vladimirov-Zarkov (1980)
Larin-Vermaseren (1993)

$$\beta(g) = -\frac{g^3}{16\pi^2}\beta_0 - \frac{g^5}{(16\pi^2)^2}\beta_1 - \frac{g^7}{(16\pi^2)^3}\beta_2 + \mathcal{O}(g^9)$$

1-loop 2-loop 3-loop

- Anomalous dimensions of hadronic operators

Moch-Vermaseren-Vogt (2004)

$$\hat{\gamma}^n(g) = \frac{g^2}{16\pi^2}\hat{\gamma}^{(0),n} + \frac{g^4}{(16\pi^2)^2}\hat{\gamma}^{(1),n} + \frac{g^6}{(16\pi^2)^3}\hat{\gamma}^{(2),n} + \mathcal{O}(g^8)$$

1-loop 2-loop 3-loop

- Hadronic coefficient functions van Neerven-Zijlstra (1991,1992)

$$\boldsymbol{C}_2^n(g) = \boldsymbol{C}_2^{(0),n} + \frac{g^2}{16\pi^2}\boldsymbol{C}_2^{(1),n} + \frac{g^4}{(16\pi^2)^2}\boldsymbol{C}_2^{(2),n} + \mathcal{O}(g^6)$$

tree 1-loop 2-loop

Coefficients of the moment sum rule

		$C_2^n(g)$	$\hat{\gamma}^n(g)$	$K^n(g,\alpha)$	$A^n(g,\alpha)$
LO :	$\mathcal{L}_i^n$	tree	1-loop	1-loop	tree
NLO :	$\mathcal{A}_i^n$, $\mathcal{B}_i^n$, $\mathcal{C}^n$	1-loop	2-loop	2-loop	1-loop
NNLO :	$\mathcal{D}_i^n$, $\mathcal{E}_i^n$, $\mathcal{F}_i^n$, $\mathcal{G}^n$	2-loop	3-loop	3-loop	2-loop

[box] New evaluation needed

Mixing anomalous dimensions (→ NNLO $K^n(g,\alpha)$: 3-loop)

known (→ NNLO $\hat{\gamma}^n(g)$: 3-loop)

Photon coefficient function (→ NNLO $C_2^n(g)$: 2-loop)

Photon matrix elements (→ NNLO $A^n(g,\alpha)$: 2-loop)

- Mixing anomalous dimensions $\overline{\mathrm{MS}}$

$$\boldsymbol{K}^n(g,\alpha) = -\frac{\alpha}{4\pi}\left[\boldsymbol{K}^{(0),n} + \frac{g^2}{16\pi^2}\boldsymbol{K}^{(1),n} + \frac{g^4}{(16\pi^2)^2}\boldsymbol{K}^{(2),n} + \mathcal{O}(g^6)\right]$$

1-loop 2-loop 3-loop

published

No exact expressions for 3-loop $\boldsymbol{K}_{\psi}^{(2),n},\ \boldsymbol{K}_{NS}^{(2),n},\ \boldsymbol{K}_{G}^{(2),n}$ yet

But the compact parametrization of 3-loop photon-parton splitting functions $\boldsymbol{P}_{ns\gamma}^{(2),\mathrm{approx}}(x),\ \boldsymbol{P}_{G\gamma}^{(2),\mathrm{approx}}(x)$

(and exact $\boldsymbol{P}_{ps\gamma}^{(2)}(x)$) available

(the deviation < 0.1 %)

A.Vogt, S.Moch and J.Vermaseren
Acta Phys.Polon. B37,683(2006)

We adopt this parametrization

- Photon coefficient function $\overline{\mathrm{MS}}$

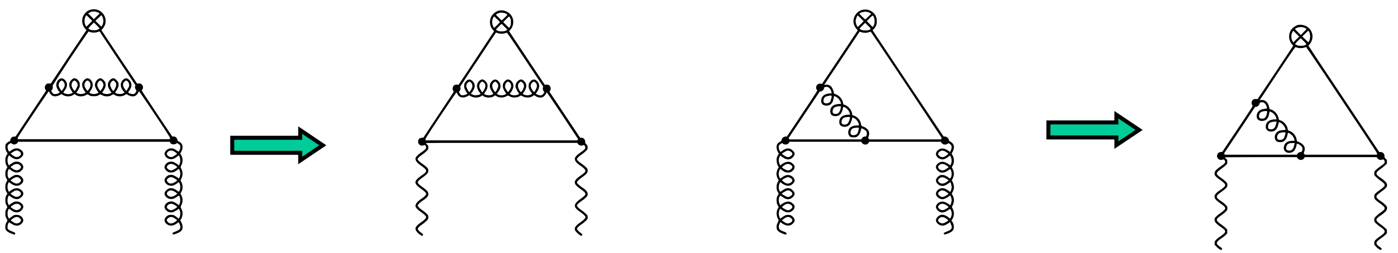
$$C_{2,\gamma}^{n}(g,\alpha) = \frac{\alpha}{4\pi}\left[C_{2,\gamma}^{(1),n} + \frac{g^2}{16\pi^2}C_{2,\gamma}^{(2),n} + \mathcal{O}(g^4)\right]$$

1-loop 2-loop

$\boldsymbol{K}^{(1),n}$, $C_{2,\gamma}^{(2),n}$ are obtained by the replacement of group factors

[example]

$\gamma_{\psi G}^{(1),n} \Rightarrow K_{\psi}^{(1),n}$



$n_f T_F C_F$ $\quad -3\langle e^2\rangle n_f C_F$ $\quad n_f T_F\left(C_F - \frac{C_A}{2}\right)$ $\quad -3\langle e^2\rangle n_f C_F$

$n_f T_F \rightarrow -3\langle e^2\rangle n_f$ $\quad C_A \rightarrow 0$

• Photon matrix elements $\overline{\text{MS}}$

Perturbatively calculable when $\Lambda^2 \ll P^2$

$$A^n(g,\alpha) = \frac{\alpha}{4\pi}\left[A^{(1),n} + \frac{g^2}{16\pi^2}A^{(2),n} + \mathcal{O}(g^4)\right]$$

1-loop 2-loop

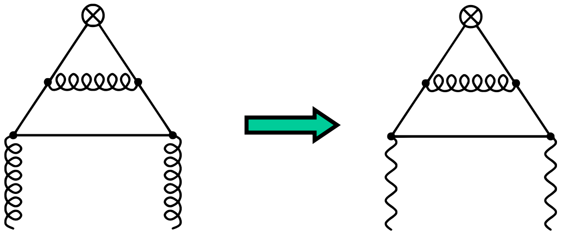
– Unrenormalized gluon matrix elements of hadronic operators were calculated in x -space

Matiounine-Smith-van Neerven (1998)

– We obtain $A_\psi^{(2),n}$, $A_{NS}^{(2),n}$, $A_G^{(2),n}$, after taking moments,

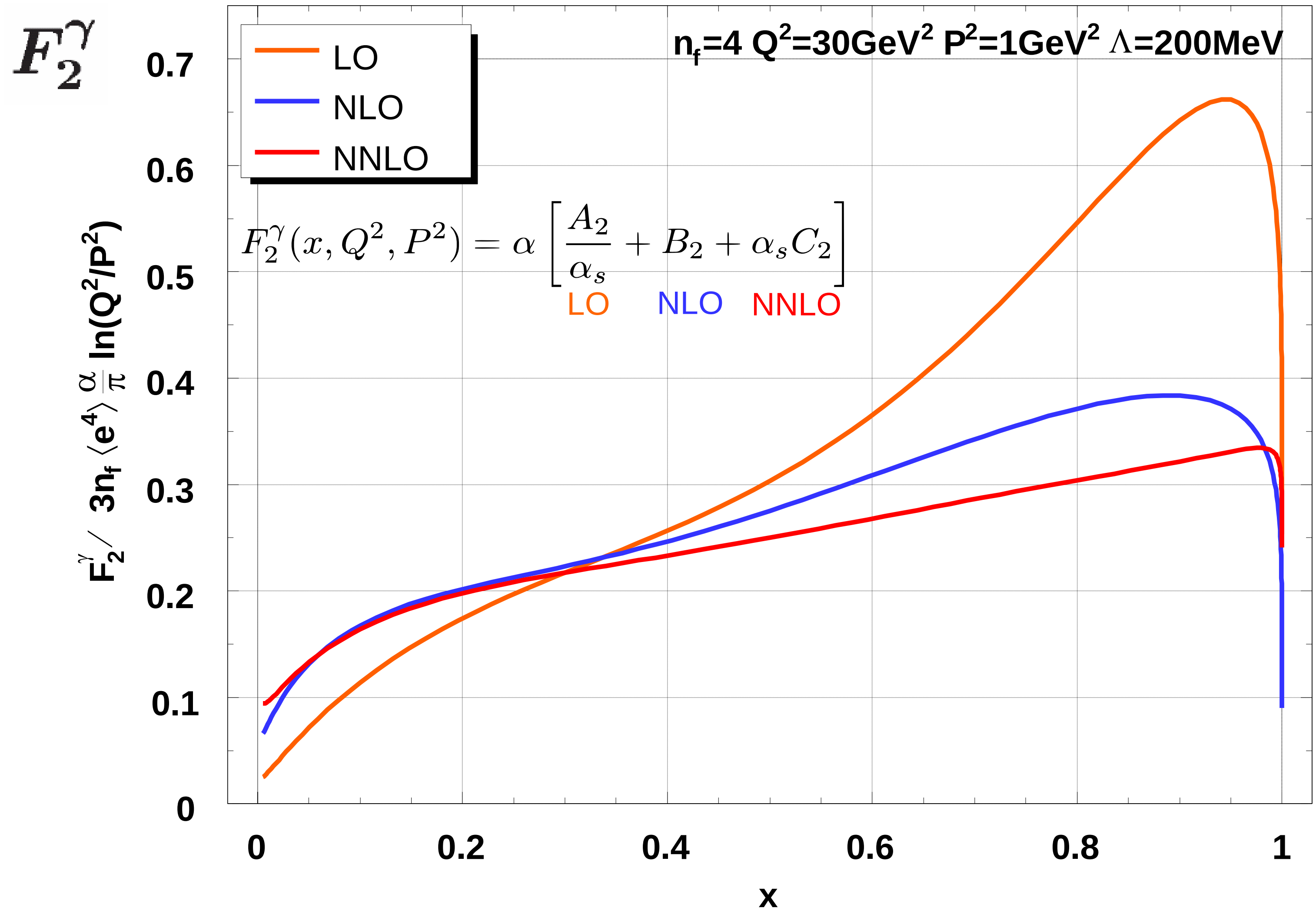
$$\hat{A}^k_{qg}\left(n, \frac{-p^2}{\mu^2}, \frac{1}{\epsilon}\right) = \int_0^1 dx\, x^{n-1}\hat{A}^k_{qg}\left(x, \frac{-p^2}{\mu^2}, \frac{1}{\epsilon}\right)$$

renormalization and the replacement of group factors

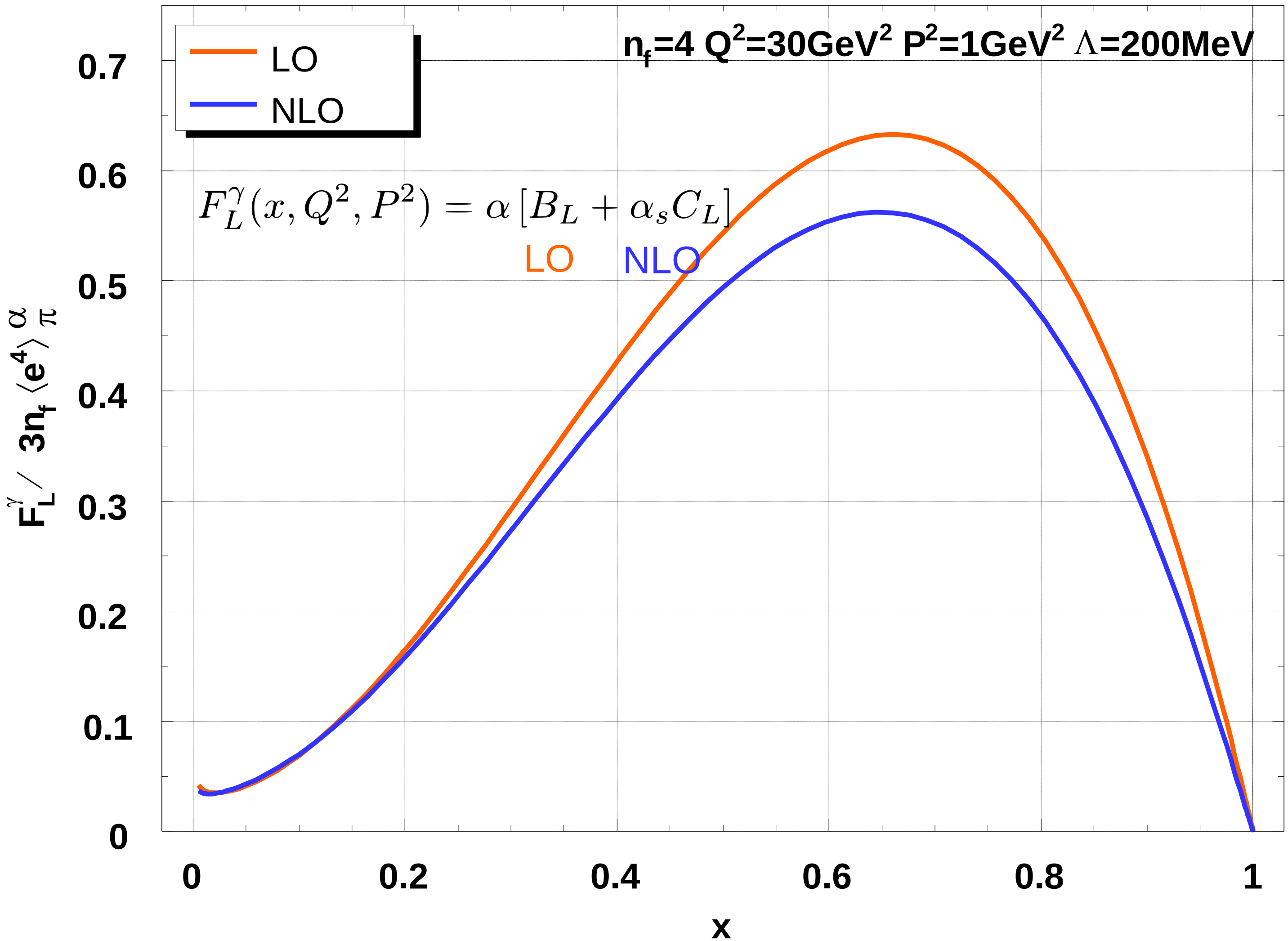


$n_f T_F C_F$ $\qquad$ $-3\langle e^2\rangle n_f C_F$

3. Numerical Analysis



Numerical Plot F_L^γ



Sum rule : first moment

By taking the limit $n \to 2$ we get a sum rule

$$\int_0^1 dx\, F_2^\gamma(x, Q^2, P^2) = \frac{\alpha}{4\pi}\frac{1}{2\beta_0}\left\{\frac{4\pi}{\alpha_s(Q^2)}c_{\rm LO} + c_{\rm NLO} + \frac{\alpha_s(Q^2)}{4\pi}c_{\rm NNLO} + \mathcal{O}(\alpha_s^2)\right\}$$

Corrections $\sim$ 8-10 %

e.g.

$$\mathrm{NNLO/(LO+NLO)} \simeq -0.08 \quad n_f = 4,\ Q^2 = 100\ \mathrm{GeV}^2\ P^2 = 3\ \mathrm{GeV}^2$$

Note: $\mathcal{A}_-^n$ and $\mathcal{E}_-^n$ develop singularities at $n=2$

which are cancelled by a factor $1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)}\right)^{d_-^n}$

$d_-^n = \lambda_-^n/2\beta_0 \quad d_-^{n=2} = 0$ vanishing of anom. dim of energy-mom tensor

4. Target mass effects for virtual photon structures

Y.Kitadono, K.Sasaki, T.Ueda and T.U.

- For the virtual photon target,

  the maximal value of the Bjorken variable

  ⇨ $x_{\max} = \dfrac{1}{1 + (P^2/Q^2)} < 1$ $\quad \xi(x_{\max}) = 1$

  $\xi = 2x/(1 + \sqrt{1 - 4x^2 P^2/Q^2})$

  due to the constraint $(p + q)^2 \geq 0$

  In the nucleon case $x_{\max} = 1$ $\quad \xi(x_{\max}) < 1$

- We study the Target Mass Effects (TME) based on Operator Product Expansion (OPE) taking into account the trace terms of the matrix elements of the operators.
- This amounts to consider Nachtmann moments.

  By using the orthogonality in the Gegenbauer polynomial

  we project out the definite spin contributions.

Nachtmann moments for $F_2^\gamma(x,Q^2,P^2)$ and $F_L^\gamma(x,Q^2,P^2)$

$$\mu_{2,n}^\gamma(Q^2,P^2) \equiv \int_0^{x_{\max}} dx \frac{1}{x^3}\xi^{n+1}\left[\frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)}\right] F_2^\gamma(x,Q^2,P^2)$$

$$\begin{aligned}\mu_{L,n}^\gamma(Q^2,P^2) &\equiv \int_0^{x_{\max}} dx \frac{1}{x^3}\xi^{n+1}\Big[F_L^\gamma(x,Q^2,P^2) \\ &+ \frac{4P^2x^2}{Q^2}\frac{(n+3)-(n+1)\xi^2P^2/Q^2}{(n+2)(n+3)}F_2^\gamma(x,Q^2,P^2)\Big]\end{aligned}$$

where $x_{\max}=\dfrac{1}{1+(P^2/Q^2)}, \quad r=\sqrt{1-\dfrac{4P^2x^2}{Q^2}}, \quad \xi=\dfrac{2x}{1+r}$

Inverting the moments

$$F_2^\gamma(x,Q^2,P^2)=\frac{x^2}{r^3}F(\xi)-6\kappa\frac{x^3}{r^4}H(\xi)+12\kappa^2\frac{x^4}{r^5}G(\xi)$$

$$F_L^\gamma(x,Q^2,P^2)=\frac{x^2}{r}F_L(\xi)-4\kappa\frac{x^3}{r^2}H(\xi)+8\kappa^2\frac{x^4}{r^3}G(\xi)$$

where $\kappa=\dfrac{P^2}{Q^2}$

where the four functions F, G, H and FL are defined as Mellin-inverted moments :

$$F(\xi) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dn\ \xi^{-n-1} M_2^\gamma(n)$$

$$H(\xi) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dn\ \xi^{-n} \frac{M_2^\gamma(n)}{n}$$

$$G(\xi) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dn\ \xi^{-n+1} \frac{M_2^\gamma(n)}{n(n-1)}$$

$$F_L(\xi) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} dn \xi^{-n-1} M_L^\gamma(n)$$

with $M_2^\gamma(n)$ and $M_L^\gamma(n)$ defined as ordinary n-th moment given as

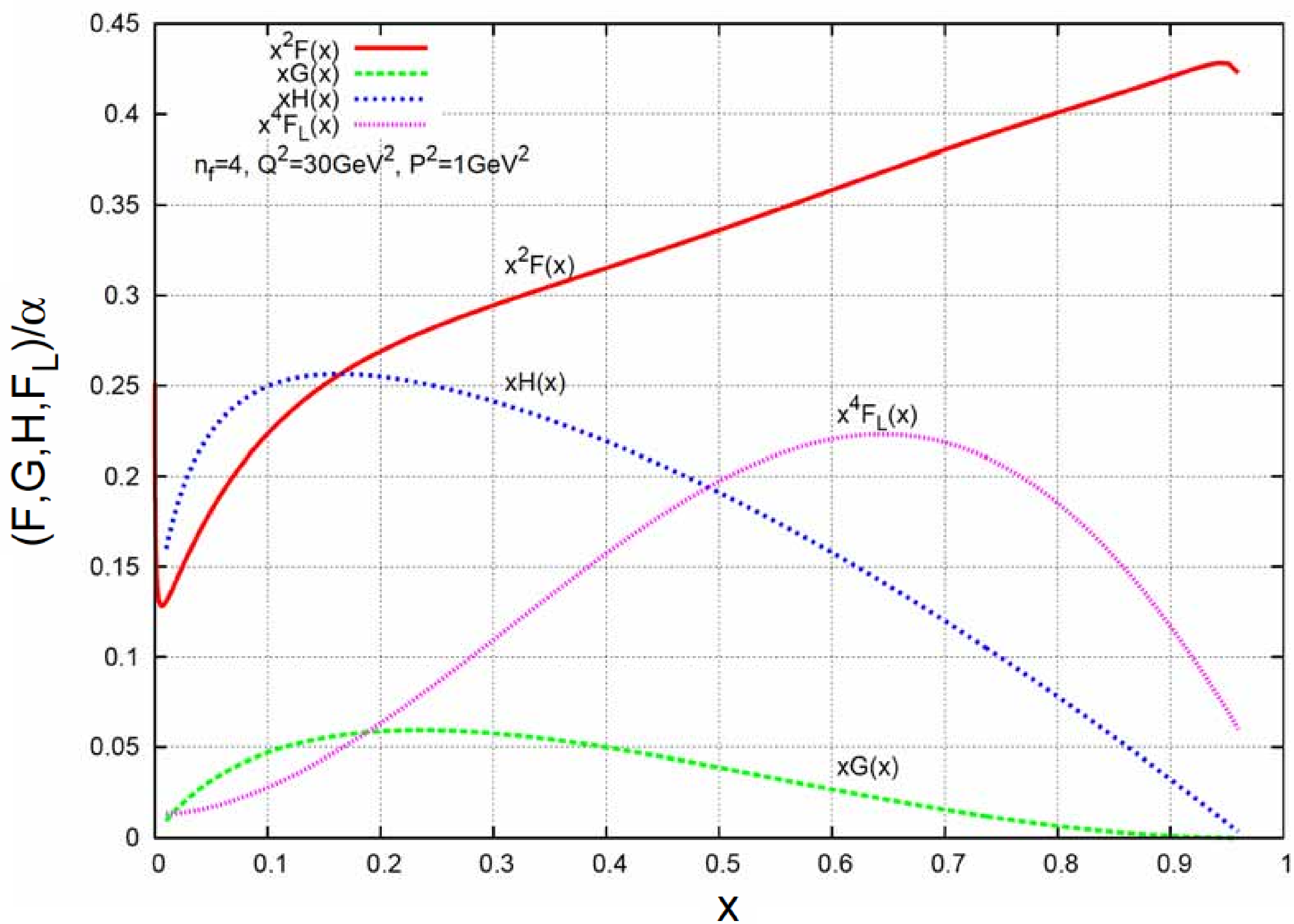
$$M_2^\gamma(n) = \int^1 dx x^{n-2} F_2^\gamma(x, Q^2, P^2)$$

$$M_L^\gamma(n) = \int_0^1 dx x^{n-2} F_L^\gamma(x, Q^2, P^2)$$

} obtained to NNLO

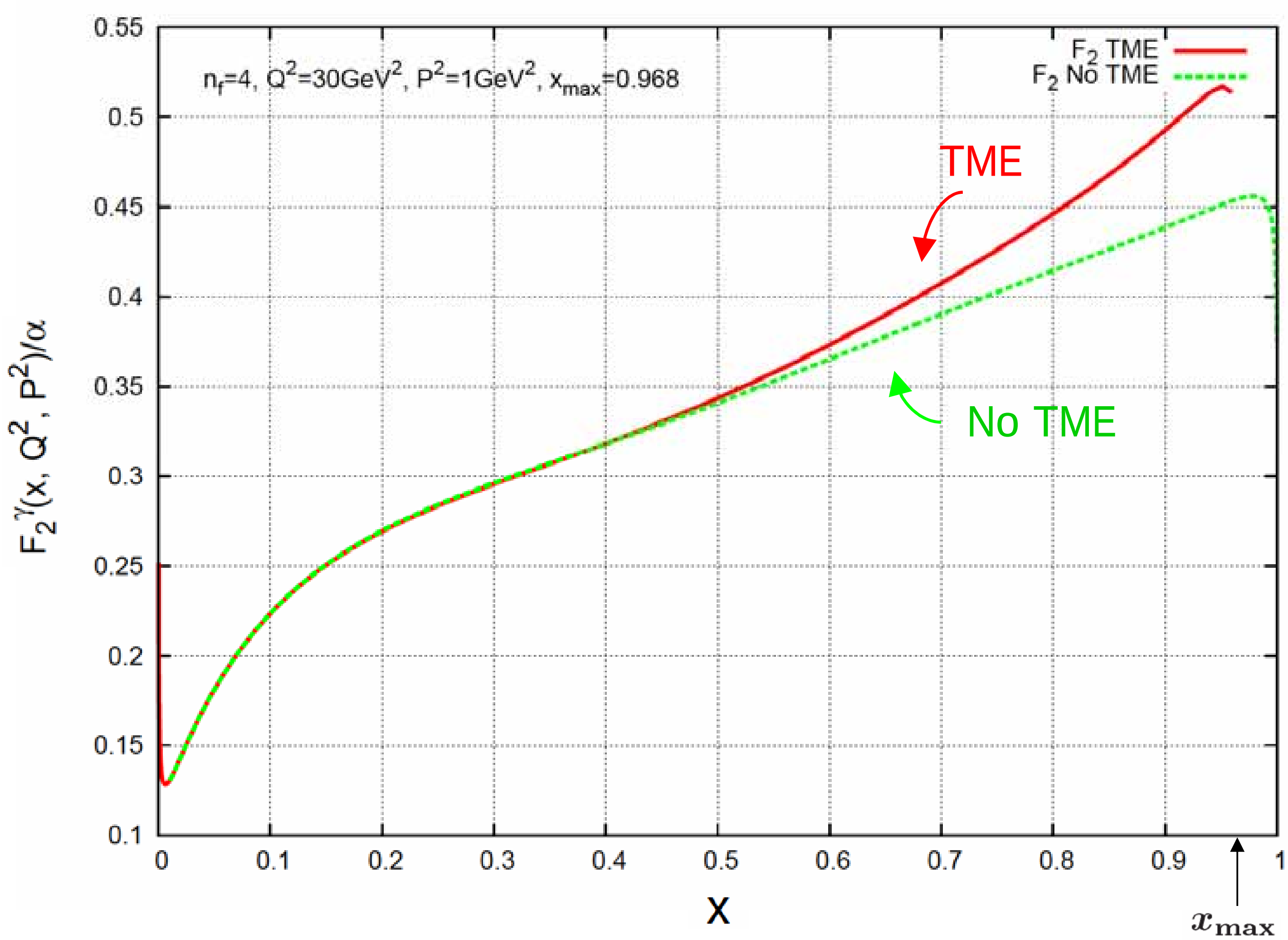
Numerical Results

(1) F, G, H, F_L



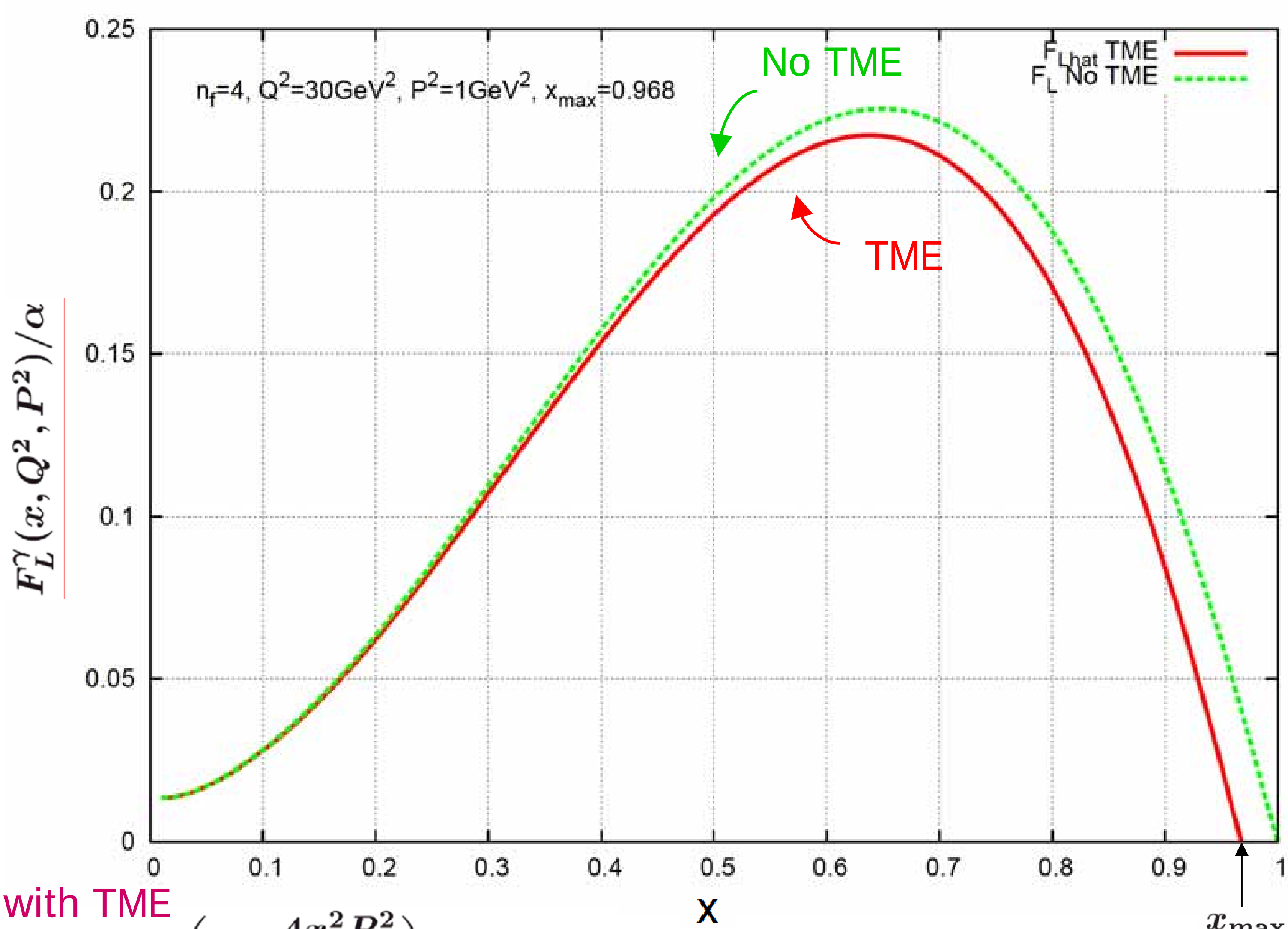
(2) $F_2^\gamma(x, Q^2, P^2)$

F_2 TME > F_2 no TME at larger x



(3) $F_L^\gamma(x, Q^2, P^2)$

F_L TME < F_L no TME
at larger x

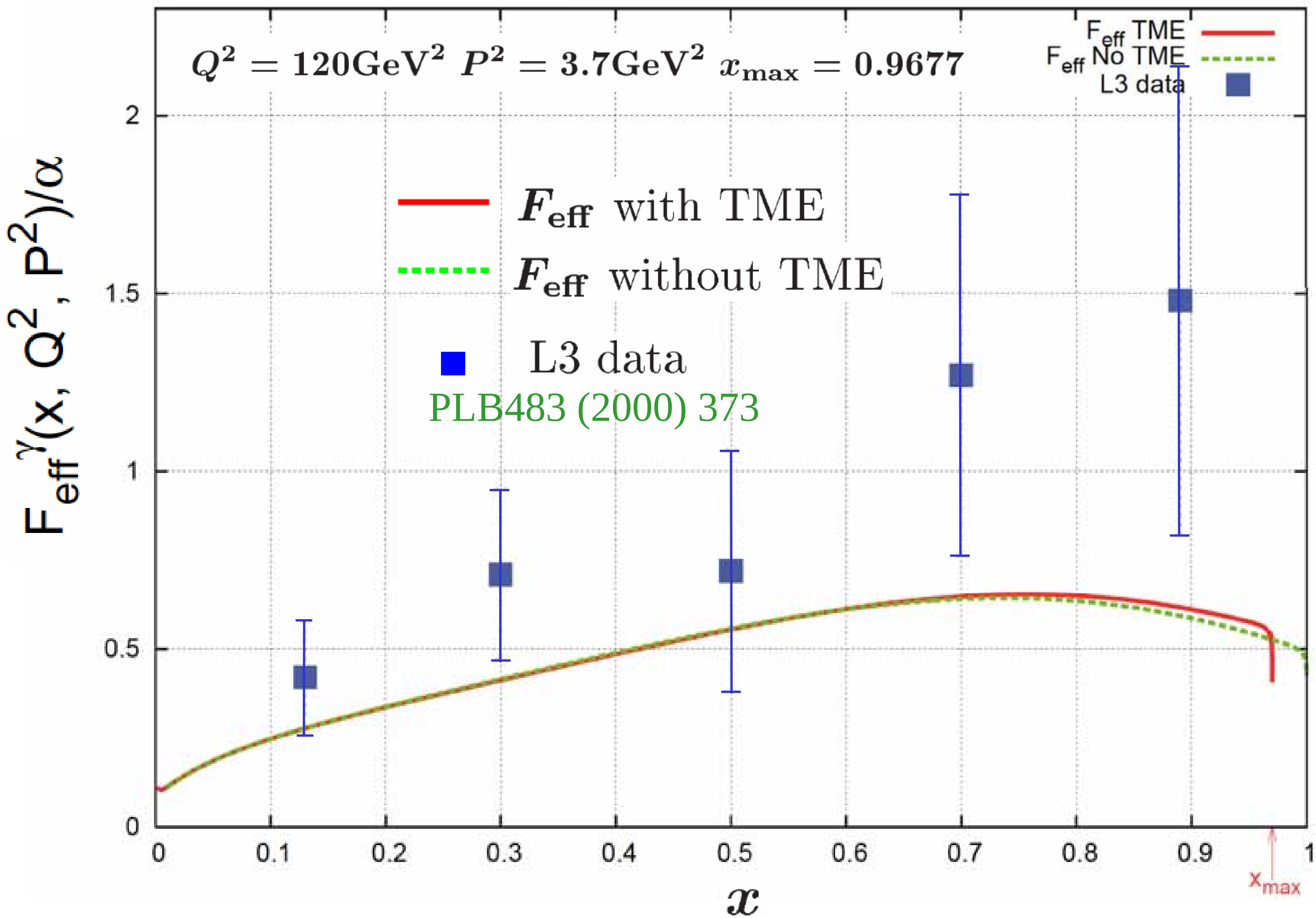


Note with TME

$$F_L^\gamma = \left(1 - \frac{4x^2P^2}{Q^2}\right) F_2^\gamma - xF_1^\gamma$$

Comparison with existing experimental data

$$F_{\rm eff}^{\gamma} = F_2^{\gamma} + \frac{3}{2} F_L^{\gamma}$$: Effective photon structure fn.



5. Summary

- The virtual photon structure function $F_2^\gamma(x, Q^2, P^2)$ investigated in the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$
- Definite predictions made up to NNLO ($\alpha\alpha_s$)
- NNLO corrections appear sizable at large x
- We also analyzed the virtual photon structure function $F_L^\gamma(x, Q^2, P^2)$ to NLO ($\alpha\alpha_s$)
- The target mass effects (TME) investigated
- TME becomes sizable at larger x. TME reduces F_L^γ and enlarges F_2^γ at larger x.

Future subjects

- Power corrections of the form $(P^2/Q^2)^k \quad (k = 1, 2, \cdots)$ due to higher-twist effects should also be studied
- Here we have treated active flavors as massless quarks

  Off course, surely we have to take into account mass effects of heavy quark flavors
- So far we have studied the NNLO QCD correction to the 1st moment of $g_1^\gamma(x, Q^2, P^2)$

  We should also interested in full NNLO analysis of $g_1^\gamma(x, Q^2, P^2)$ and $g_2^\gamma(x, Q^2, P^2)$
- Experimental confrontation in the future is anticipated