New Developments in Precision LHC Theory: QED⊗QCD Resummation, Shower/ME Matching, IR-Improved DGLAP-CS Theory and Implications for UV Finite Quantum Gravity

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INTRODUCTION

Motivation

- FNAL/RHIC $t\bar{t}$, $b\bar{b}$, J/Ψ PRODUCTION; POLARIZED pp PROCESSES: SOFT n(G) EFFECTS ALREADY NEEDED $\Delta m_t = 1.8$ GeV with SOFT n(G) UNCERTAINTY $\sim$ 1 GeV, ..., ETC.
- FOR THE LHC/TESLA/LC, THE REQUIREMENTS WILL BE EVEN MORE DEMANDING AND OUR QCD SOFT n(G) MC RESUMMATION RESULTS WILL BE AN IMPORTANT PART OF THE NECESSARY THEORY – (YFS)RESUMMED $\mathcal{O}(\alpha_s^2)L^n, \mathcal{O}(\alpha_s\alpha)L^{n'}, \mathcal{O}(\alpha^2)L^{n''}, n = 0,1,2, n' = 0,1,2, n'' = 2,1$, IN THE PRESENCE OF SHOWERS, ON AN EVENT-BY-EVENT BASIS, WITHOUT DOUBLE COUNTING AND WITH EXACT PHASE SPACE.
- HOW RELEVANT ARE QED HIGHER ORDER CORRECTIONS WHEN QCD IS CONTROLLED AT $\sim 1\%$ PRECISION?
- CROSS CHECK OF QCD LITERATURE:

  1. PHASE SPACE – CATANI, CATANI-SEYMOUR, ALL INITIAL PARTONS MASSLESS
  2. RESUMMATION – STERMAN, CATANI ET AL., BERGER ET AL., ...
  3. NO-GO THEOREMS–Di'Lieto et al.,Doria et al.,Catani et al.,Catani
  4. IR QCD EFFECTS IN DGLAP-CS THEORY

INTRODUCTION

- CROSS CHECK OF QED LITERATURE:

  1. ESTIMATES BY SPIESBERGER, STIRLING, ROTH and WEINZIERL, BLUMLEIN and KAWAMURA – FEW PER MILLE EFFECTS FROM QED CORRECTIONS TO STR. FN. EVOLUTION.

  2. WELL-KNOWN POSSIBLE ENHANCEMENT OF QED CORRECTIONS AT THRESHOLD, ESPECIALLY IN RESONANCE PRODUCTION

  $\Rightarrow$ HOW BIG ARE THESE EFFECTS AT THE LHC?

- TREAT QED AND QCD SIMULTANEOUSLY IN THE (YFS) RESUMMATION TO ASSESS THE ROLE OF THE QED AND TO REALIZE AN APPROACH TO SHOWER/ME MATCHING.

- QUANTUM GENERAL RELATIVITY:STILL NO PHENOMENOLOGICALLY TESTED THEORY

- OUTSTANDING ISSUES: FINAL STATE OF HAWKING RADIATION, ... – FERTILE GROUND FOR RESUMMATION; SEE ALSO WORK BY REUTER ET AL., LITIM, DONOGHUE ET AL., CAVAGLIA, SOLA ET AL., ETC.

PRELIMINARIES

- WE USE THE GPS CONVENTIONS OF JWW FOR SPINORS; PHOTON-GLUON POLARIZATION VECTORS FOLLOW THEREFROM:

$$(\epsilon^{\mu}_{\sigma}(\beta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}u_{\sigma}(\beta)}{\sqrt{2}\ \bar{u}_{-\sigma}(k)u_{\sigma}(\beta)}, \quad (\epsilon^{\mu}_{\sigma}(\zeta))^{*} = \frac{\bar{u}_{\sigma}(k)\gamma^{\mu}u_{\sigma}(\zeta)}{\sqrt{2}\ \bar{u}_{-\sigma}(k)u_{\sigma}(\zeta)}, \tag{1}$$

- REPRESENTATIVE PROCESSES

$pp \rightarrow V + n(\gamma) + m(g) + X \rightarrow \bar{\ell}\ell' + n'(\gamma) + m(g) + X,$
where $V = W^{\pm}, Z$,and $\ell = e, \mu,\ \ell' = \nu_e, \nu_\mu (e, \mu)$
respectively for $V = W^{+}(Z)$, and $\ell = \nu_e, \nu_\mu,\ \ell' = e, \mu$
respectively for $V = W^{-}$.

Quantum Gravity Loop Corrections to Elementary Particle Propagators

Review of Exact Amplitude-Based Resummation for QCD

QED CASE – S. Jadach et al., YFS2, YFS3, BHLUMI, BHWIDE, KORALZ, KKMC, YFSWW3, YFSZZ, KoralW

For $e^+(p_1)e^-(q_1) \rightarrow \bar{f}(p_2)f(q_2) + n(\gamma)(k_1, \cdot, k_n)$**, renormalization group improved YFS theory (PRD36(1987)939) gives precision preidtcions which are in agreemnet with LEPI&II precision data.**

QCD CASE– In hep-ph/0210357(ICHEP02), Acta Phys.Polon.B33,1543-1558,2002, Phys.Rev.D52(1995)108;ibid. 66 (2002) 019903(E);PLB342 (1995) 239, we have extended the YFS theory to QCD:

$$d\hat{\sigma}_{\rm exp} = \sum_n d\hat{\sigma}^n$$
$$= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3k_j}{k_j} \int \frac{d^4y}{(2\pi)^4} e^{iy\cdot(P_1+P_2-Q_1-Q_2-\sum k_j)+D_{\rm QCD}}$$
$$* \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3P_2}{P_2^{\,0}} \frac{d^3Q_2}{Q_2^{\,0}} \qquad (2)$$

where the new hard gluon residuals $\bar{\tilde{\tilde{\beta}}}_n(k_1,\ldots,k_n)$ defined by

$$\bar{\tilde{\tilde{\beta}}}_n(k_1,\ldots,k_n)=\sum_{\ell=0}^{\infty}\bar{\tilde{\tilde{\beta}}}^{(\ell)}_n(k_1,\ldots,k_n)$$

are free of all infrared divergences to all orders in $\alpha_s(Q)$.

- We stress that the arguments in (DeLaney *et al.* PRD52(1995)108, PLB342(1995)239) are not really sufficient to derive eq.(2)– they did not expose the compensation between the left over genuine non-Abelian IR virtual and real singularities between $\int dPh\bar{\beta}_n$ and $\int dPh\bar{\beta}_{n+1}$ respectively that allows to isolate $\bar{\tilde{\tilde{\beta}}}_j$ and distinguishes QCD from QED, where no such compensation occurs.
- Our exponent corresponds to the $N=1$ term in the exponent in Gatheral's non-Abelian eikonal formula (PLB133(1983)90), wherein everything that does not eikonalize and exponentiate is dropped – our result (2) is exact.
- We can include all of Gatheral's formula in (2) as desired.

Extension to QED⊗QCD and Quantum Gravity

Simultaneous exponentiation of QED and QCD higher order effects,

hep-ph/0404087,

gives

$$
\begin{aligned}
B^{nls}_{QCD} &\rightarrow B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED},\\
\tilde{B}^{nls}_{QCD} &\rightarrow \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED},\\
\tilde{S}^{nls}_{QCD} &\rightarrow \tilde{S}^{nls}_{QCD} + \tilde{S}^{nls}_{QED} \equiv \tilde{S}^{nls}_{QCED}
\end{aligned} \tag{3}
$$

which leads to

$$
d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3p_2}{p_2^{\,0}}\frac{d^3q_2}{q_2^{\,0}}, \tag{4}
$$

where the new YFS residuals

$\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$, **with n hard gluons and m hard photons,** represent the successive application of the YFS expansion first for QCD and subsequently for QED.

The infrared functions are now

$$\begin{aligned} \mathrm{SUM_{IR}(QCED)} &= 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED} \\ D_{\mathrm{QCED}} &= \int \frac{dk}{k^0}\left(e^{-iky} - \theta(K_{max} - k^0)\right)\tilde{S}^{nls}_{QCED} \end{aligned} \quad (5)$$

where K_{max} is a dummy parameter – here the same for QCD and QED.

Infrared Algebra(QCED):

$x_{avg}(QED) \cong \gamma(QED)/(1+\gamma(QED))$

$x_{avg}(QCD) \cong \gamma(QCD)/(1+\gamma(QCD))$

$\gamma(A) = \frac{2\alpha_A \mathcal{C}_A}{\pi}(L_s - 1)$, $A = QED, QCD$

$\mathcal{C}_A = Q_f^2, C_F$, respectively, for $A = QED, QCD$

$\Rightarrow$ QCD dominant corrections happen an order of magnitude earlier than those for QED.

$\Rightarrow$ Leading $\tilde{\tilde{\beta}}^{(0,0)}_{0,0}$-level gives a good estimate of the size of the effects we study.

RESUMMED QUANTUM GRAVITY

APPLY (4) TO QUANTUM GENERAL RELATIVITY:

$\Rightarrow$

$$i\Delta'_F(k)|_{\text{resummed}} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)} \tag{6}$$

FOR

$$B''_g(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \tag{7}$$

THIS IS THE BASIC RESULT.

NOTE THE FOLLOWING:

- Σ'_s **STARTS IN** $\mathcal{O}(\kappa^2)$**, SO WE MAY DROP IT IN CALCULATING ONE-LOOP EFFECTS.**

- **EXPLICIT EVALUATION GIVES, FOR THE DEEP UV REGIME,**

$$B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k^2|}\right), \tag{8}$$

$\Rightarrow$ THE RESUMMED PROPAGATOR FALLS FASTER THAN ANY POWER OF $|k^2|$!

- **IF m VANISHES, USING THE USUAL $-\mu^2$ NORMALIZATION POINT WE GET $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2} \ln\left(\frac{\mu^2}{|k^2|}\right)$ WHICH AGAIN VANISHES FASTER THAN ANY POWER OF $|k^2|$!**

THIS MEANS THAT ONE-LOOP CORRECTIONS ARE FINITE!

INDEED, ALL QUANTUM GRAVITY LOOPS ARE UV FINITE(MPLA17(2002)2371)!

QED⊗QCD Threshold Corrections,

Shower/ME Matching & IRI-DGLAP-CS Theory at LHC

We shall apply the new simultaneous QED⊗QCD exponentiation calculus to the single Z production with leptonic decay at the LHC (and at FNAL) to focus on the ISR alone, for definiteness. See also the work of Baur *et al.*, **Dittmaier and Kramer, Zykunov for exact $\mathcal{O}(\alpha)$ results and Hamberg** *et al.*, **van Neerven and Matsuura and Anastasiou** *et al.* **for exact $\mathcal{O}(\alpha_s^2)$ results.**

For the basic formula

$$d\sigma_{exp}(pp \to V + X \to \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s), \tag{9}$$

we use the result in (4) here with semi-analytical methods and structure functions from Martin *et al.*.

A MC realization will appear elsewhere.

SHOWER/ME MATCHING

- Note the following: In (9) WE DO NOT ATTEMPT *HERE* TO REPLACE HERWIG and/or PYTHIA – WE INTEND *HERE* TO COMBINE OUR EXACT YFS CALCULUS, $d\hat{\sigma}_{exp}(x_i x_j s)$, WITH HERWIG and/or PYTHIA BY USING THEM/IT TO GENERATE A PARTON SHOWER STARTING FROM (x_1, x_2) AT FACTORIZATION SCALE μ AFTER THIS POINT IS PROVIDED BY $\{F_i\}$: THERE ARE TWO APPROACHES TO THE MATCHING UNDER STUDY, ONE BASED ON p_T-MATCHING AND ONE BASED ON SHOWER-SUBTRACTED RESIDUALS $\{\hat{\hat{\bar{\beta}}}_{n,m}(k_1, \ldots, k_n; k'_1, \ldots, k'_m)\}$, WHEREIN THE SHOWER FORMULA AND THE $QED \otimes QCD$ EXPONENTIATION FORMULA CAN BE EXPANDED IN PRODUCT AND REQUIRED TO MATCH THE GIVEN EXACT RESULT TO THE SPECIFIED ORDER – SEE hep-ph/0509003.

- THIS COMBINATION OF THEORETICAL CONSTRUCTS CAN BE SYSTEMATICALLY IMPROVED WITH EXACT RESULTS ORDER-BY-ORDER IN α_s, α, WITH EXACT PHASE SPACE.

- THE RECENT ALTERNATIVE PARTON EVOLUTION ALGORITHM BY JADACH and SKRZYPEK,Acta. Phys. Pol.B35, 745 (2004), CAN ALSO BE USED.

- LACK OF COLOR COHERENCE $\Rightarrow$ ISAJET NOT CONSIDERED HERE.

With this said, we compute , with and without QED, the ratio

$r_{exp} = \sigma_{exp}/\sigma_{Born}$

to get the results (We stress that we *do not* use the narrow resonance approximation here.)

$$r_{exp} = \begin{cases} 1.1901 & ,\ \text{QCED} \equiv \text{QCD+QED, LHC} \\ 1.1872 & ,\ \text{QCD, LHC} \\ 1.1911 & ,\ \text{QCED} \equiv \text{QCD+QED, Tevatron} \\ 1.1879 & ,\ \text{QCD, Tevatron} \end{cases} \tag{10}$$

$\Rightarrow$

$\ast$***QED IS AT .3% AT BOTH LHC and FNAL.***

$\ast$***THIS IS STABLE UNDER SCALE VARIATIONS.***

$\ast$***WE AGREE WITH BAUR ET AL., HAMBERG ET AL., van NEERVEN and ZIJLSTRA.***

$\ast$***QED EFFECT SIMILAR IN SIZE TO STR. FN. RESULTS.***

$\ast$***DGLAP-CS SYNTHESIZATION HAS NOT COMPROMISED THE NORMALIZATION.***

IR-Improved DGLAP-CS Theory

Exponentiation of QCD higher order effects: Where to apply?

hep-ph/0508140,

consider

$$\frac{dq^{NS}(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y} q^{NS}(y,t) P_{qq}(x/y) \tag{11}$$

where the well-known result for the kernel $P_{qq}(z)$ **is, for** $z < 1$**,**

$$P_{qq}(z) = C_F \frac{1+z^2}{1-z}, \tag{12}$$

$t = \ln \mu^2/\mu_0^2$ **for some reference scale** μ_0**.** $\Rightarrow$

Unintegrable singularity at $z = 1$**, usually regularized by**

$$\frac{1}{(1-z)} \rightarrow \frac{1}{(1-z)_+} \tag{13}$$

with $\frac{1}{(1-z)_+}$ **such that**

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)}. \tag{14}$$

$\Rightarrow$

$$\frac{1}{(1-z)_+} = \frac{1}{(1-z)}\theta(1-\epsilon-z) + \ln\epsilon\,\delta(1-z) \qquad (15)$$

with the understanding that $\epsilon \downarrow 0$.

Require

$$\int_0^1 dz P_{qq}(z) = 0, \qquad (16)$$

$\Rightarrow$ add virtual corrections to get

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right). \qquad (17)$$

Observations

- Smooth, divergent $1/(1-z)$ behavior as $z \to 1$ replaced with a mathematical artifact: the regime $1-\epsilon < z < 1$ now has no probability at all; at $z = 1$ we have a large negative integrable contribution $\Rightarrow$ a finite (zero) value for the total integral of $P_{qq}(z)$
- LEP1,2 experience: such mathematical artifacts, while correct, impair precision.

Why set $P_{qq}(z)$ to 0 for $1-\epsilon < z < 1$ where it actually has its largest values?

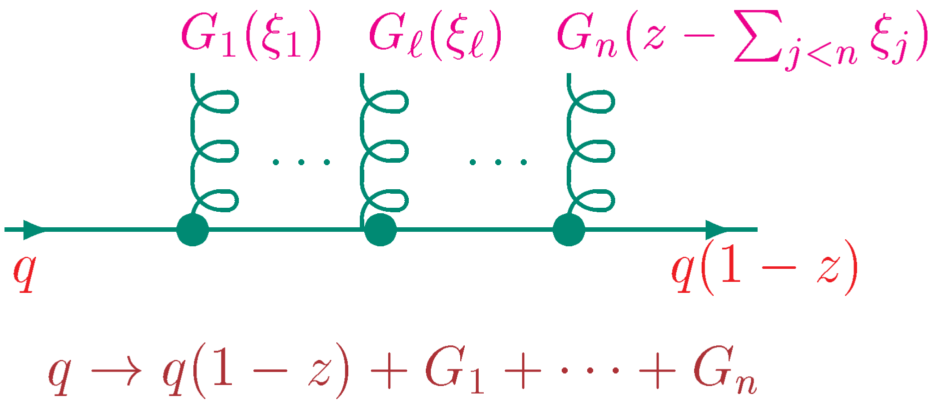
- **USE EXPERIENCE FROM LEP1,2: $\frac{1}{(1-z)_+}$ SHOULD BE EXPONENTIATED –SEE CERN YELLOW-BOOKS, CERN-89-08., YIELDING FROM (2) THE REPLACEMENT**

$$P_{BA} = \frac{1}{2}z(1-z)\overline{\sum_{spins}}\frac{|V_{A\to B+C}|^2}{p_\perp^2}$$

$$\Rightarrow \tag{18}$$

$$P_{BA} = \frac{1}{2}z(1-z)\overline{\sum_{spins}}\frac{|V_{A\to B+C}|^2}{p_\perp^2} z^{\gamma_q}F_{YFS}(\gamma_q)e^{\frac{1}{2}\delta_q}$$

WHERE $A=q$, $B=G$, $C=q$ AND $V_{A\to B+C}$ IS THE LOWEST ORDER AMPLITUDE FOR $q\to G(z)+q(1-z)$.



$\Rightarrow$

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+z^2}{1-z}(1-z)^{\gamma_q} \quad (19)$$

where

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0} \tag{20}$$

$$\delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}) \tag{21}$$

and

$$F_{YFS}(\gamma_q) = \frac{e^{-C_E \gamma_q}}{\Gamma(1+\gamma_q)}. \tag{22}$$

Note: $\int_{k_0} dz/z = C_0 - \ln k_0$

is experimentally distinguishable from

$\int_{k_0} dz/z^{1-\gamma} = C'_0 - k_0^{\gamma}/\gamma$.

- NORMALIZATION CONDITION (16) $\Rightarrow$:

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z)\right] \qquad (23)$$

where

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{\gamma_q+1} + \frac{1}{\gamma_q+2}. \qquad (24)$$

- THIS IS OUR IR-IMPROVED P_{qq} DGLAP-CS KERNEL.

$\Rightarrow$ STANDARD DGLAP-CS THEORY:

for $z < 1$, we have

$$P_{Gq}(z) = P_{qq}(1-z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}. \qquad (25)$$

$\Rightarrow$ TEST OF NEW THEORY – QUARK MOMENTUM SUM RULE:

$$\int_0^1 dz z \left(P_{Gq}(z) + P_{qq}(z)\right) = 0. \qquad (26)$$

$\Rightarrow$ **CHECK VANISHING OF**

$$I = \int_0^1 dzz\left(\frac{1+(1-z)^2}{z}z^{\gamma_q} + \frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z)\right). \quad (27)$$

NOTE,

$$\frac{z}{1-z} = \frac{z-1+1}{1-z} = -1 + \frac{1}{1-z}. \quad (28)$$

$\Rightarrow$

$$I = \int_0^1 dz\{(1+(1-z)^2)z^{\gamma_q} - (1+z^2)(1-z)^{\gamma_q} + \frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z)\} = 0$$

QUARK MOMENTUM SUM RULE IS SATISFIED.

- **For $P_{qG}(z),\ P_{GG}(z)$, we get, with the replacement $C_F \rightarrow C_G$ in the IR algebra, that the usual results**

$$P_{GG}(z) = 2C_G(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z))$$
$$P_{qG}(z) = \frac{1}{2}(z^2 + (1-z)^2) \quad (29)$$

become

$$P_{GG}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G} + \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z)\}, \quad (30)$$

$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \}, \quad (31)$$

where

$$\gamma_G = C_G \frac{\alpha_s}{\pi} t = \frac{4C_G}{\beta_0} \tag{32}$$

$$\delta_G = \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right), \tag{33}$$

$$f_G(\gamma_G) = \frac{n_f}{C_G} \frac{1}{(1+\gamma_G)(2+\gamma_G)(3+\gamma_G)} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} \tag{34}$$

$$+ \frac{1}{(1+\gamma_G)(2+\gamma_G)} + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} \tag{35}$$

$$+ \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)}. \tag{36}$$

THE GLUON MOMENTUM SUM RULE HAS BEEN USED.

- **THIS DEFINES THE NEW IR-IMPROVED DGLAP-CS THEORY.**

IR-IMPROVED DGLAP-CS KERNELS

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \left[\frac{1+z^2}{1-z}(1-z)^{\gamma_q} - f_q(\gamma_q)\delta(1-z) \right], \tag{37}$$

$$P_{Gq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1+(1-z)^2}{z} z^{\gamma_q}, \tag{38}$$

$$P_{GG}(z) = 2C_G F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \{ \frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G} + \frac{1}{2}(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G}) - f_G(\gamma_G)\delta(1-z) \}, \tag{39}$$

$$P_{qG}(z) = F_{YFS}(\gamma_G) e^{\frac{1}{2}\delta_G} \frac{1}{2} \{ z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G} \}. \tag{40}$$

Higher Order DGLAP-CS Kernels

Connection with the exact $\mathcal{O}(\alpha_s^2),\ \mathcal{O}(\alpha_s^3)$ kernel results of Curci, Furmanski and Petronzio, Floratos et al., Moch et al., etc., is immediate:

For example, non-singlet case, using standard notation,

$$P_{ns}^{+} = P_{qq}^{v} + P_{q\bar{q}}^{v} \equiv \sum_{n=0}^{\infty} (\frac{\alpha_s}{4\pi})^{n+1} P_{ns}^{(n)+} \qquad (41)$$

where at order $\mathcal{O}(\alpha_s)$ we have

$$P_{ns}^{(0)+}(z) = 2C_F\{\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\} \qquad (42)$$

$\Rightarrow P_{ns}^{(0)+}(z)$ agrees with the unexponentiated result for P_{qq} except for an overall factor of 2. Floratos et al., etc., have exact result for $P_{ns}^{(1)+}(z)$, and Moch et al. have

exact results for $P^{(2)+}_{ns}(z)$. Applying (2) to $q \rightarrow q + X$, $\bar{q} \rightarrow q + X'$, we get

$$P^{+,exp}_{ns}(z) = (\frac{\alpha_s}{4\pi})2P^{exp}_{qq}(z) + F_{YFS}(\gamma_q)e^{\frac{1}{2}\delta_q}\Big[(\frac{\alpha_s}{4\pi})^2\{(1-z)^{\gamma_q}\bar{P}^{(1)+}_{ns}(z) + \bar{B}_2\delta(1-z)\} + (\frac{\alpha_s}{4\pi})^3\{(1-z)^{\gamma_q}\bar{P}^{(2)+}_{ns}(z) + \bar{B}_3\delta(1-z)\}\Big] \tag{43}$$

where $P^{exp}_{qq}(z)$ is given above and the resummed residuals $\bar{P}^{(i)+}_{ns}$, $i = 1, 2$ are related to the exact results for $P^{(i)+}_{ns}$, $i = 1, 2$, as follows:

$$\bar{P}^{(i)+}_{ns}(z) = P^{(i)+}_{ns}(z) - B_{1+i}\delta(1-z) + \Delta^{(i)+}_{ns}(z) \tag{44}$$

where

$$\Delta^{(1)+}_{ns}(z) = -4C_F\pi\delta_1\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\}$$
$$\Delta^{(2)+}_{ns}(z) = -4C_F(\pi\delta_1)^2\{\frac{1+z^2}{1-z} - f_q\delta(1-z)\} - 2\pi\delta_1\bar{P}^{(1)+}_{ns}(z) \tag{45}$$

and

$$\bar{B}_2 = B_2 + 4C_F\pi\delta_1 f_q$$
$$\bar{B}_3 = B_3 + 4C_F(\pi\delta_1)^2 f_q - 2\pi\delta_1\bar{B}_2. \quad (46)$$

The constants $B_i,\ i = 2,3$ are given by

$$B_2 = 4C_GC_F(\frac{17}{24} + \frac{11}{3}\zeta_2 - 3\zeta_3) - 4C_Fn_f(\frac{1}{12} + \frac{2}{3}\zeta_2) + 4C_F^2(\frac{3}{8} - 3\zeta_2 + 6\zeta_3)$$
$$B_3 = 16C_GC_Fn_f(\frac{5}{4} - \frac{167}{54}\zeta_2 + \frac{1}{20}\zeta_2^2 + \frac{25}{18}\zeta_3)$$
$$+ 16C_GC_F^2(\frac{151}{64} + \zeta_2\zeta_3 - \frac{205}{24}\zeta_2 - \frac{247}{60}\zeta_2^2 + \frac{211}{12}\zeta_3 + \frac{15}{2}\zeta_5)$$
$$+ 16C_G^2C_F(-\frac{1657}{576} + \frac{281}{27}\zeta_2 - \frac{1}{8}\zeta_2^2 - \frac{97}{9}\zeta_3 + \frac{5}{2}\zeta_5)$$
$$+ 16C_Fn_F^2(-\frac{17}{144} + \frac{5}{27}\zeta_2 - \frac{1}{9}\zeta_3)$$
$$+ 16C_F^2n_F(-\frac{23}{16} + \frac{5}{12}\zeta_2 + \frac{29}{30}\zeta_2^2 - \frac{17}{6}\zeta_3)$$
$$+ 16C_F^3(\frac{29}{32} - 2\zeta_2\zeta_3 + \frac{9}{8}\zeta_2 + \frac{18}{5}\zeta_2^2 + \frac{17}{4}\zeta_3 - 15\zeta_5). \quad (47)$$

Contact with Wilson Expansion

N-th moment of the invariants $T_{i,\ell},\ i=L,2,3,\ \ell=q,G,$ **of the forward Compton amplitude in DIS:(Gorishni et al.)**

$$\mathcal{P}_N \equiv \left[\frac{q^{\{\mu_1}\cdots q^{\mu_N\}}}{N!}\frac{\partial^N}{\partial p^{\mu_1}\cdots\partial p^{\mu_N}}\right]\Big|_{p=0}, \tag{48}$$

$x_{Bj}=Q^2/(2qp)$ **in the standard DIS notation – Projects the coefficient of** $1/(2x_{Bj})^N$**. Terms which we resum here** $\Leftrightarrow$ **Formally** γ_q**-dependent anomalous dimensions associated with the respective coefficient, not in Wilson's expansion by usual definition.:**

LARGE λ NOT ALL ON TIP OF LIGHTCONE.

COMMENTS

$(*)$ **IRI-DGLAP-CS RESUMS IR SINGULAR ISR;BY FACTORIZATION THIS IS NOT CONTAINED IN ANY RESUMMATION OF HARD SHORT-DISTANCE COEFFICIENT FN CORRECTIONS AS IN THE STERMAN, CATANI-TRENTADUE, COLLINS ET AL. FORMULAS**

$(**)$ **WE DO NOT CHANGE THE PREDICTED HADRON CROSS SECTION:**

$$\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) \hat{\sigma}(x_1 x_2 s)$$
$$= \sum_{i,j} \int dx_1 dx_2 {F'}_i(x_1) {F'}_j(x_2) \hat{\sigma}'(x_1 x_2 s) \quad (49)$$

ORDER BY ORDER IN PERTURBATION THEORY.

$\{P^{exp}\}$ **factorize** $\hat{\sigma}_{\text{unfactorized}} \Rightarrow \hat{\sigma}'$

$\{P\}$ **factorize** $\hat{\sigma}_{\text{unfactorized}} \Rightarrow \hat{\sigma}$

$(***)$ **QUARK NUMBER CONSERVATION AND CANCELLATION OF IR SINGULARITIES IN XSECTS: Quaranteed by fundamental quantum field theoretic principles: Global Gauge Invariance, Unitarity – Everybody may use these principles.**

Effects on Parton Distributions

Moments of kernels ⇔ Logarithmic exponents for evolution

$$\frac{dM_n^{NS}(t)}{dt} = \frac{\alpha_s(t)}{2\pi} A_n^{NS} M_n^{NS}(t) \tag{50}$$

where

$$M_n^{NS}(t) = \int_0^1 dz z^{n-1} q^{NS}(z,t) \tag{51}$$

and the quantity A_n^{NS} is given by

$$\begin{aligned} A_n^{NS} &= \int_0^1 dz z^{n-1} P_{qq}(z), \\ &= C_F F_{YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)] \end{aligned} \tag{52}$$

where $B(x,y)$ is the beta function given by

$$B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$$

.

Compare the usual result

$$A_n^{NS^o} \equiv C_F\left[-\frac{1}{2}+\frac{1}{n(n+1)}-2\sum_{j=2}^{n}\frac{1}{j}\right]. \quad (53)$$

- **ASYMPTOTIC BEHAVIOR: IR-improved goes to a multiple of $-f_q$, consistent with** $\lim_{n\to\infty} z^{n-1} = 0$ **for** $0 \le z < 1$**;** **usual result diverges as** $-2C_F \ln n$**.**

- **Different for finite n as well: for** $n = 2$ **we get, for example, for** $\alpha_s \cong .118$**,**

$$A_2^{NS} = \begin{cases} C_F(-1.33) & , \text{ un-IR-improved} \\ C_F(-0.966) & , \text{ IR-improved} \end{cases} \quad (54)$$

- **For completeness we note**

$$M_n^{NS}(t) = M_n^{NS}(t_0) e^{\int_{t_0}^{t} dt' \frac{\alpha_s(t')}{2\pi} A_n^{NS}(t')}$$
$$= M_n^{NS}(t_0) e^{\bar{a}_n [Ei(\frac{1}{2}\delta_1 \alpha_s(t_0)) - Ei(\frac{1}{2}\delta_1 \alpha_s(t))]}$$
$$\underset{t,t_0 \text{ large with } t>>t_0}{\Longrightarrow} M_n^{NS}(t_0) \left(\frac{\alpha_s(t_0)}{\alpha_s(t)} \right)^{\bar{a'}_n} \tag{55}$$

where $Ei(x) = \int_{-\infty}^{x} dr e^r / r$ **is the exponential integral function,**

$$\bar{a}_n = \frac{2C_F}{\beta_0} F_{YFS}(\gamma_q) e^{\frac{\gamma_q}{4}} [B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q)]$$
$$\bar{a'}_n = \bar{a}_n \left(1 + \frac{\delta_1}{2} \frac{(\alpha_s(t_0) - \alpha_s(t))}{\ln(\alpha_s(t_0)/\alpha_s(t))} \right) \tag{56}$$

with

$\delta_1 = \frac{C_F}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right)$.

Compare with un-IR-improved result where last line in eq.(55) holds exactly with $\bar{a'}_n = 2A_n^{NS^o}/\beta_0$.

- For $n = 2$, taking $Q_0 = 2$GeV and evolving to $Q = 100$GeV, with $\Lambda_{QCD} \cong .2GeV$ and $n_f = 5$ for illustration, (55,56) $\Rightarrow$ a shift of evolved NS moment by $\sim\ 5\%$, of some interest in view of the expected HERA precision (see for example, T. Carli et al., Proc. HERA-LHC Wkshp, 2005).
- Introduction of IR-Improved DGLAP-CS kernels in HERWIG and PYTHIA in progress.

QUARK MASSES and RESUMMATION in PRECISION QCD THEORY

- **Di'Lieto et al.(NPB183(1981)223), Doria et al.(*ibid.*168(1980)93), Catani et al.(*ibid.*264(1986)588;Catani(ZPC37(1988)357): IN ISR, BLOCH-NORDSIECK CANCELLATION FAILS AT $\mathcal{O}(\alpha_s^2)$ for $m_q \neq 0$.**
- **FOR $q + q' \to q'' + q''' + V + X$, THEY GET**

$$\text{flux}\ \frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{(d-4)32\pi^2}\left(\frac{1-\beta}{\beta}\right)\left(\frac{1}{\beta}\ln(\frac{1+\beta}{1-\beta}) - 2\right) \tag{57}$$

- **HERE, $\bar{H}$ IS THE HARD PROCESS DRESSED AS**

$$F_1 = C_2(G) H_{ab}^{\alpha\beta} (T_i)^{\beta\alpha} (T_i)_b a \tag{58}$$

FOR

$$f_{ijk} f_{ijl} = C_2(G)\delta_{kl}$$
$$(T_i T_i)_{ab} = C_2(F) I_{ab}.$$

THEY EVALUATE THE GRAPHS IN FIG.1 USING MUELLER'S THM.

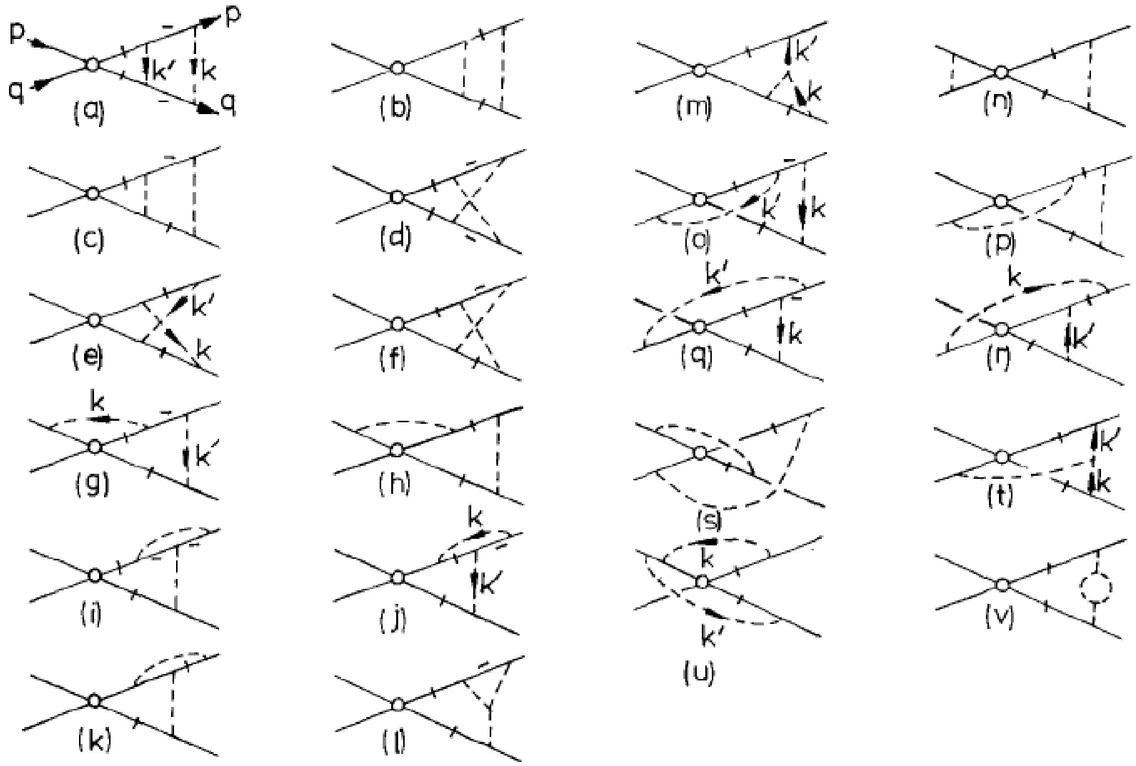


Figure 1. Graphs evaluated in Ref. [2] (see the first paper therein especially) in arriving at the result in (3) using Mueller's theorem for the respective cross section. The usual Landau-Bjorken-Cutkosky (LBC) [10] rules obtain so that a slash puts the line on-shell and a dash changes the iε-prescription; and, graphs that have cancelled or whose contributions are implied by those in the figure are not shown explicitly.

- SINCE BN VIOLATION VANISHES FOR $m_q \to 0$, MUST SET $m_q = 0$ IN ISR for $\mathcal{O}(\alpha_s^n),\ n \geq 2$: NOTE, $m_b \cong 5$GeV.

- SOURCE OF BN-VIOLATION: LOOK AT CONTRIBUTION OF DIAGRAMS (q-o) IN FIG.1:

$$A_{q-o} = \frac{1}{\beta^2}\int \frac{d^3k d^3k' 2k_z}{(k_z + k'_z + i\epsilon)(\beta^2 k_z^2 - \mathbf{k}^2)(\beta^2 k_z^2 - \mathbf{k'}^2 + i\epsilon)(k_z^2 + \epsilon^2)} \tag{59}$$

UV-REGULATED RESULT: USE THE REGULATOR $e^{-\mathbf{k}^2/\Lambda^2}$,

$\Rightarrow$

$$A_{q-o}|_{UV-reg} = \frac{4\pi^{n+1}(\Lambda^2)^{n-3}}{\beta^2}\left\{\frac{1}{(n-3)^2} + \frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right\}. \tag{60}$$

$\Rightarrow$

$$F_{nbn} = \frac{(1-\beta)(\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta)}{\ln\left(\frac{1+\beta}{1-\beta}\right)} \tag{61}$$

IS FRACTION OF SINGLE-POLE TERM UN-CANCELLED.

- LANDAU-BJORKEN-CUTKOSKY ANALYSIS: INTEGRATE OVER ${k'}_z$ IN (59)
$\Rightarrow$ TWO POLES BELOW REAL AXIS
${k'}_z = -k_z - i\epsilon$, ${k'}_z = -\sqrt{\beta^2 k_z^2 - {k'}_\perp^{\,2} + i\epsilon}$
WHERE ENERGY OF k'-gluon $= -\beta k_z$ BY LBC RULES.
ONLY THE LATTER POLE GIVES ON-SHELL k' RADIATION:
$\Re = \{0 \le {k'}_\perp^{\,2} \le \beta^2 k_z^2\}$ IS ON-SHELL k'-gluon REGIME.

- WE GET THERE

$$A_{q-o}|_{\Re} = \Re \frac{1}{\beta^2} \int d^3k \int_0^{\beta^2 k_z^2} \pi d({k'}_\perp^{\,2}) \frac{-2\pi i}{-(-2)\sqrt{\beta^2 k_z^2 - {k'}_\perp^{\,2}}} \frac{1}{k_z - \sqrt{\beta^2 k_z^2 - {k'}_\perp^{\,2}} + i\epsilon} \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2}. \quad (62)$$

WE NEED THE REAL PART:

$$
\begin{aligned}
A_{q-o}|_{\Re} &= \Re \frac{-\pi i}{\beta^2} \int d^3k \int_0^{\beta^2 k_z^2} \pi d({k'_\perp}^2) \frac{1}{\sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} \\
&\quad \frac{k_z + i\epsilon + \sqrt{\beta^2 k_z^2 - {k'_\perp}^2}}{(k_z + i\epsilon)^2 - (\beta^2 k_z^2 - {k'_\perp}^2)} \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
&= \Re \frac{-\pi i}{\beta^2} \int d^3k \int_0^{\beta^2 k_z^2} \pi d({k'_\perp}^2) \frac{1}{\sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} \\
&\quad \frac{k_z + i\epsilon}{(k_z + i\epsilon)^2 - (\beta^2 k_z^2 - {k'_\perp}^2)} \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
&= \Re \frac{-\pi i}{\beta^2} \int d^3k \int_0^{\beta^2 k_z^2} \pi d({k'_\perp}^2) \frac{1}{\sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} \\
&\quad \frac{1}{2} \left(\frac{1}{k_z + i\epsilon - \sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} + \frac{1}{k_z + i\epsilon - \sqrt{\beta^2 k_z^2 - {k'_\perp}^2}} \right) \\
&\quad \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2} \\
&= \Re \frac{-i\pi^2}{\beta^2} \int d^3k \left(-\ln(k_z + i\epsilon - \beta|k_z|) + \ln(k_z + i\epsilon + \beta|k_z|) \right) \\
&\quad \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2},
\end{aligned}
\tag{63}
$$

WHERE ON-SHELL REGIME ACTUALLY HAS $k_0' = -\beta k_z < 0$, REAL RADIATIVE CONTRIBUTION, BY THE STANDARD LBC METHODS, HAS $k_z > 0$.

- FOR INTEGRATION OVER $k_z > \sqrt{\epsilon}$, RHS OF THE LAST EQUATION HAS NO REAL PART AS $\epsilon \rightarrow 0$.

  THE REAL EMISSION PART OF (63) ARISES FROM $0 \leq k_z \leq \sqrt{\epsilon}$.

  BRANCH CUTS FOR THE LOGS:JOIN THEM BETWEEN $k_{z1} = -i\epsilon/(1-\beta)$ and $k_{z2} = -i\epsilon/(1+\beta)$;

  AND WE CLOSE THE CONTOUR BELOW THE REAL AXIS AS SHOWN IN Fig. 2

  $\Rightarrow$

$$\oint_C dk_z \left(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z)\right) \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \bar{\epsilon}^2} = 0, \tag{64}$$

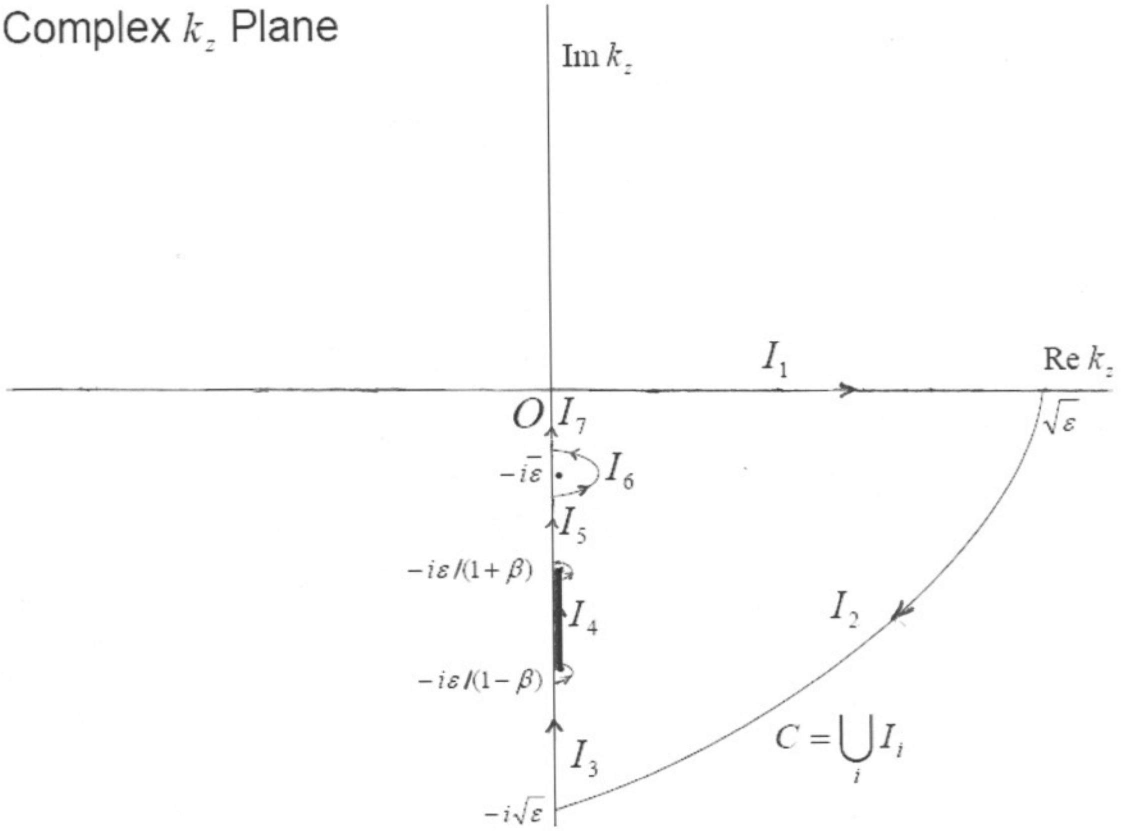


Figure 2. The contour C used in the complex k_z–plane to evaluate the real emission part of the contribution of diagrams (q-o) in Fig. 1 to the RHS of (3). See the text for further discussion.

WHERE WE USE THE INTRINSIC FREEDOM IN THE FEYNMAN $i\epsilon$-PRESCRIPTION TO TAKE EACH SUCH INFINITESIMAL PARAMETER INDEPENDENTLY TO 0 FROM ABOVE AND THE CURVE C IS GIVEN IN FIG. 2. WE TAKE HERE $k_\perp > \sqrt{\epsilon}$, $\bar{\epsilon} = \epsilon^{\frac{3}{2}}$

- BY CAUCHY'S THEOREM,

$$I_1 = \int_0^{\sqrt{\epsilon}} dk_z \left(-\ln(k_z + i\epsilon - \beta^2|k_z|) + \ln(k_z + i\epsilon + \beta|k_z|\right) \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \bar{\epsilon}^2} = -\sum_{i=2}^{7} I_i. \quad (65)$$

- **TREAT EACH INTEGRAL IN TURN:**

  FOR I_2, USE THE CHANGE OF VARIABLE $k_z = \sqrt{\epsilon} e^{i\theta}$,

  FOR$0 \geq \theta \geq -\frac{\pi}{2}$. THEN, WE GET

$$
\begin{aligned}
I_2 &= \int_0^{-\frac{\pi}{2}} id\theta k_z \left(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z)\right) \\
&\qquad \frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \bar{\epsilon}^2} \\
&= 2 \int_0^{-\frac{\pi}{2}} id\theta \left(-\ln(1-\beta) + \ln(1+\beta)\right) \frac{1}{-\mathbf{k}_\perp{}^2} \\
&= -i\pi \ln\left(\frac{1+\beta}{1-\beta}\right) \frac{1}{(-\mathbf{k}_\perp{}^2)}.
\end{aligned}
\tag{66}
$$

- **For I_3, USE THE CHANGE OF VARIABLE $k_z = -iy$: IT IS PURE REAL SO THAT IT WILL NOT CONTRIBUTE THE THE IMAGINARY PART OF I_1 VIA (64).**

- FOR I_4 WE SEE FROM PASSING AROUND THE LOWER BRANCH POINT IN FIG. 2 THAT THE RESPECTIVE IMAGINARY CONTRIBUTION IS

$$i\Im I_4 = \int_{\frac{-i\epsilon}{1-\beta}}^{\frac{-i\epsilon}{1+\beta}} dk_z\,(-\pi i)\,\frac{1}{-\mathbf{k}_\perp{}^2}\,\frac{2k_z}{k_z^2+\bar{\epsilon}^2}$$
$$= 2\pi i \ln\left(\frac{1+\beta}{1-\beta}\right)\frac{1}{(-\mathbf{k}_\perp{}^2)}. \tag{67}$$

- FOR I_5, WE SEE BY THE CHANGE OF VARIABLE $k_z = -iy$ THAT IT TOO IS PURE REAL AND DOES NOT CONTRIBUTE TO THE IMAGINARY PART OF I_1 VIA (64).

- For I_6, WE GET THE RESULT

$$I_6 = \pi i Res(-i\bar{\epsilon}) = 0 \tag{68}$$

SINCE $\bar{\epsilon}/\epsilon \rightarrow 0$ WHEN $\epsilon \rightarrow 0$.

- **FOR I_7 THE CHANGE OF VARIABLE $k_z = -iy$ SHOWS THAT IT TOO IS PURE REAL AND DOES NOT CONTRIBUTE TO THE IMAGINARY PART OF I_1 VIA (64).**
- **NET RESULT:**

$$\begin{aligned} i\Im I_1 &= -\{2\pi i - \pi i\}\frac{-1}{{\mathbf{k}_\perp}^2}\ln\left(\frac{1+\beta}{1-\beta}\right) \\ &= \frac{\pi i}{{\mathbf{k}_\perp}^2}\ln\left(\frac{1+\beta}{1-\beta}\right). \end{aligned} \tag{69}$$

$\Rightarrow$

$$A_{q-o}|_{\Re,\text{real rad.}} = \frac{2\pi^3}{\beta^2}\left(\frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)\right)\int\frac{d^2k_\perp}{{\mathbf{k}_\perp}^2}, \tag{70}$$

- **INTEGRAL OVER $\mathbf{k}_\perp$ in (70):**

$$\mathcal{I}_{\textsf{UV reg.}} = \int \frac{d^2k_\perp e^{-{\mathbf{k}_\perp}^2/\Lambda^2}}{{\mathbf{k}_\perp}^2}$$
$$= \int \frac{d^3k\delta(k_z)e^{-\mathbf{k}^2/\Lambda^2}}{\mathbf{k}^2}$$
$$= \int \frac{d^nk\delta(k_z)e^{-\mathbf{k}^2/\Lambda^2}}{\mathbf{k}^2} \quad (71)$$
$$= \int_0^\infty d\rho \int d^nk\delta(k_z)e^{-\mathbf{k}^2/\Lambda^2-\rho\mathbf{k}^2}$$
$$= \frac{2\pi^{\frac{(n-1)}{2}}}{n-3}(\Lambda^2)^{\frac{n-3}{2}}.$$

- $\Rightarrow$

$$A_{q-o}\big|_{\mathfrak{R},\textsf{real rad., UV reg.}} = \frac{4\pi^4(\Lambda^2)^{\frac{n-3}{2}}}{\beta^2}\left(\frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right), \quad (72)$$

$\Rightarrow$ **REAL EMISSION IN A_{q-o} SATURATES SINGLE IR POLE.**

- **THUS, WE WRITE**

$$\text{flux}\,\frac{d\sigma}{d^3Q} = \frac{-g^4\bar{H}}{64\pi^6}F_{nbn}A_{q-o}|_{\mathfrak{R},\textbf{real rad., IR pole part}}, \tag{73}$$

WHERE FROM (72) WE HAVE

$$A_{q-o}|_{\mathfrak{R},\textbf{real rad., IR pole part}} = \frac{4\pi^4}{\beta^2}\left(\frac{1}{2(n-3)}\ln\left(\frac{1+\beta}{1-\beta}\right)\right). \tag{74}$$

- **APPLY QCD RESUMMATION TO REAL EMISSION IN $A_{q-o}|_{\mathfrak{R}}$: APPLY IT TO THE FRACTION F_{nbn}; REMAINING $1-F_{nbn}$ CANCELLED BY VIRTUAL CORRECTIONS**

- **USING**

$$d\hat{\sigma}_{\rm exp} = e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-p_X-\sum k_j)}$$
$$* \, e^{D_{\rm QCD}} \tilde{\bar{\beta}}_n(k_1,\ldots,k_n) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} \frac{d^3 p_X}{p_X^0} \tag{75}$$

WE GET

$$F_{nbn} A_{q-o}\big|_{\Re,\text{real rad., resummed}} = F_{nbn} \Re \frac{-i\pi^2}{\beta^2} \int d^2 k_\perp \int_0^{\sqrt{\epsilon}} dk_z F_{YFS}(\bar{\gamma}_q) e^{\bar{\delta}_q/2}$$
$$(\beta k_z)^{\bar{\gamma}_q} \left(-\ln(k_z + i\epsilon - \beta k_z) + \ln(k_z + i\epsilon + \beta k_z)\right)$$
$$\frac{1}{\beta^2 k_z^2 - \mathbf{k}^2} \frac{2k_z}{k_z^2 + \epsilon^2}, \tag{76}$$

WHERE WE HAVE DEFINED

$$\bar{\gamma}_q = 2C_F \frac{\alpha_s(Q^2)}{\pi}(\ln(s/m^2) - 1) \tag{77}$$

$$\bar{\delta}_q = \frac{\bar{\gamma}_q}{2} + \frac{2\alpha_s C_F}{\pi}(\frac{\pi^2}{3} - \frac{1}{2}). \tag{78}$$

- USING THE SUBSTITUTION $k_z = \sqrt{\epsilon}\bar{k}_z$, WE HAVE

$$F_{nbn} A_{q-o}|_{\Re,\text{real rad., resummed}} = F_{nbn}\Re\frac{-i\pi^2\epsilon^{\frac{\bar{\gamma}_q}{2}}}{\beta^2}\int d^2k_\perp \int_0^1 d\bar{k}_z F_{YFS}(\bar{\gamma}_q)e^{\bar{\delta}_q/2}$$
$$(\beta\bar{k}_z)^{\gamma_q}\left(-\ln(\bar{k}_z + i\sqrt{\epsilon} - \beta\bar{k}_z) + \ln(\bar{k}_z + i\sqrt{\epsilon} + \beta\bar{k}_z)\right)$$
$$\frac{1}{-(1-\beta^2)\epsilon\bar{k}_z^2 - \mathbf{k}_\perp{}^2}\frac{2\bar{k}_z}{\bar{k}_z^2 + \epsilon}. \tag{79}$$

THE RHS OF THIS LAST EQUATION VANISHES AS $\epsilon \longrightarrow 0$, REMOVING THE VIOLATION OF BLOCH-NORDSIECK CANCELLATION IN (57).

CONCLUSION:

- RESUMMATION CURES LACK OF BN CANCELLATION IN MASSIVE QCD

FINAL STATE OF HAWKING RADIATION

CONSIDER THE GRAVITON PROPAGATOR IN THE THEORY OF GRAVITY COUPLED TO A MASSIVE SCALAR(HIGGS) FIELD(Feynman). WE HAVE THE GRAPHS

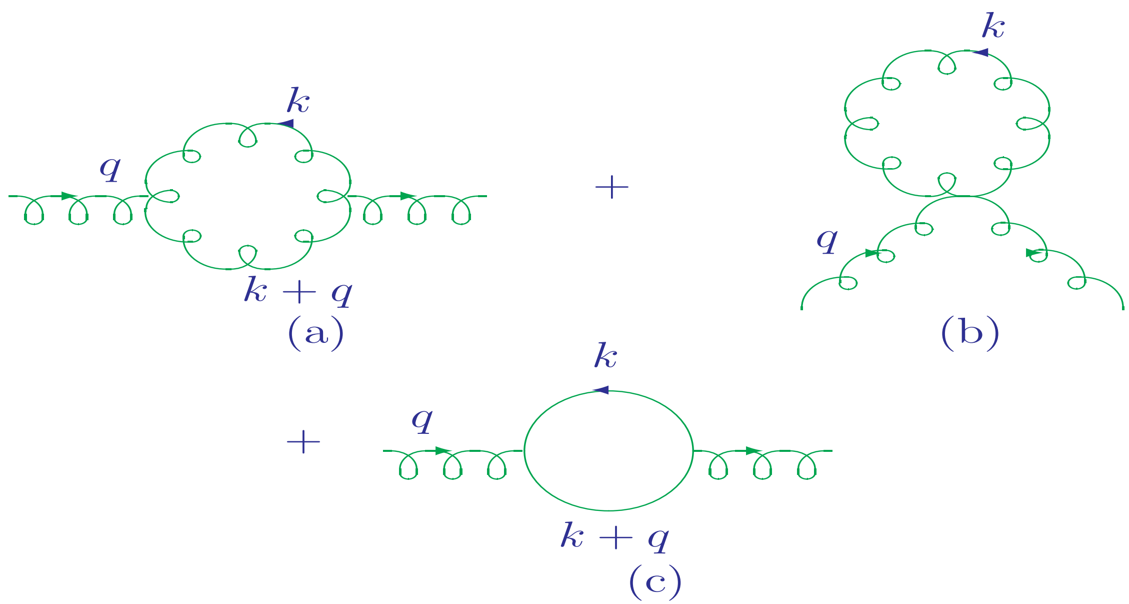


Figure 1: The graviton((a),(b)) and its ghost((c)) one-loop contributions to the graviton propagator. q is the 4-momentum of the graviton.

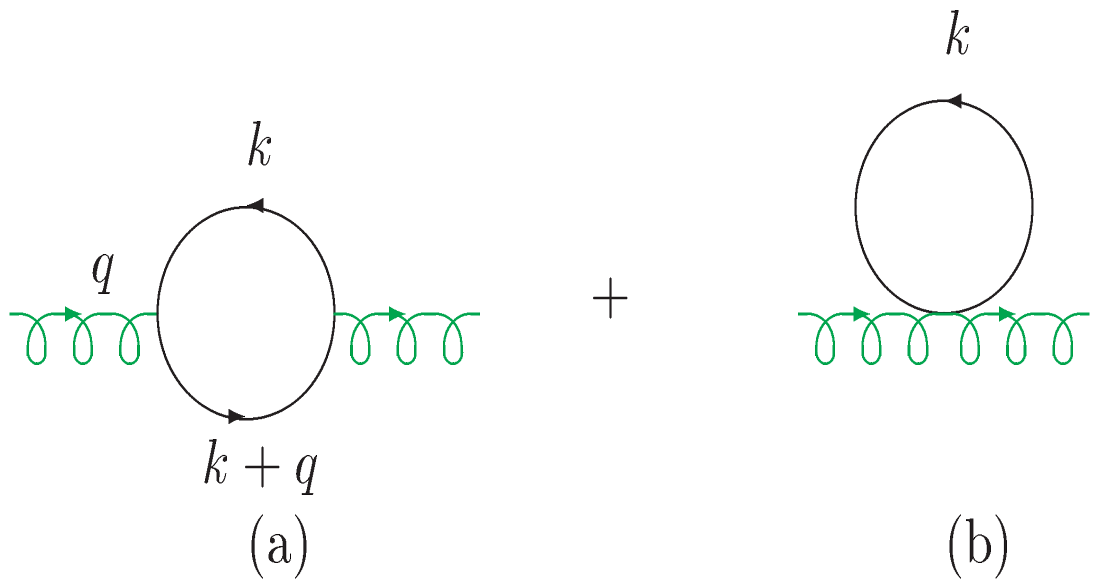


Figure 2: The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

.

USING THE RESUMMED THEORY, WE GET THAT THE NEWTON POTENTIAL BECOMES

$$\Phi_N(r) = -\frac{G_N M}{r}(1 - e^{-ar}), \tag{80}$$

FOR

$$a \cong 0.210 M_{Pl}. \tag{81}$$

CONTACT WITH AYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G(k) = G_N/(1 + \frac{k^2}{a^2})$$

$\Rightarrow$ FIXED POINT BEHAVIOR FOR $k^2 \rightarrow \infty$,

IN AGREEMENT WITH THE PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONNANNO & REUTER IN PRD62(2000) 043008.

- OUR RESULTS IMPLY THAT AN ELEMENTARY PARTICLE HAS

  NO HORIZON WHICH ALSO AGREES WITH BONNANNO'S & REUTER'S

  RESULT THAT A BLACK HOLE WITH A MASS LESS THAN

  $M_{cr} \sim M_{Pl}$

  HAS NO HORIZON.

  BASIC PHYSICS:

  $G(k)$ VANISHES FOR $k^2 \to \infty$.

- A FURTHER "AGREEMENT": FINAL STATE OF HAWKING RADIATION OF AN ORIGINALLY VERY MASSIVE BLACKHOLE

  BECAUSE OUR VALUE OF THE COEFFICIENT,

  $$\frac{1}{a^2},$$

  OF k^2 IN THE DENOMINATOR OF $G(k)$

  AGREES WITH THAT FOUND BY BONNANNO & REUTER(B-R),

  IF WE USE THEIR PRESCRIPTION FOR THE

  RELATIONSHIP BETWEEN k AND r

  IN THE REGIME WHERE THE LAPSE FUNCTION VANISHES,

  WE GET THE SAME HAWKING RADIATION PHENOMEMNOLOGY AS THEY DO:

  THE BLACK HOLE EVAPORATES IN THE B-R ANALYSIS UNTIL IT REACHES A MASS

  $$M_{cr} \sim M_{Pl}$$

  AT WHICH THE BEKENSTEIN-HAWKING TEMPERATURE VANISHES,

  LEAVING A PLANCK SCALE REMNANT.

- FATE OF REMNANT? IN hep-ph/0503189 $\Rightarrow$ OUR QUANTUM LOOP EFFECTS COMBINED WITH THE $G(r)$ OF B-R IMPLY HORIZON OF THE PLANCK SCALE REMNANT IS OBVIATED – CONSISTENT WITH RECENT RESULTS OF HAWKING.

TO WIT, IN THE METRIC CLASS

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2 \tag{82}$$

THE LAPSE FUNCTION IS, FROM B-R,

$$\begin{aligned} f(r) &= 1 - \frac{2G(r)M}{r} \\ &= \frac{B(x)}{B(x) + 2x^2}\Big|_{x=\frac{r}{G_N M}}, \end{aligned} \tag{83}$$

WHERE

$$B(x) = x^3 - 2x^2 + \Omega x + \gamma\Omega \tag{84}$$

FOR

$$\Omega = \frac{\tilde{\omega}}{G_N M^2} = \frac{\tilde{\omega} M_{Pl}^2}{M^2}. \tag{85}$$

AFTER H-RADIATING TO REGIME NEAR $M_{cr} \sim M_{Pl}$, QUANTUM LOOPS ALLOW US TO REPLACE $G(r)$ WITH $G_N(1-e^{-ar})$ IN THE LAPSE FUNCTION FOR $r < r_>$, THE OUTERMOST SOLUTION OF

$$G(r) = G_N(1-e^{-ar}). \tag{86}$$

IN THIS WAY, WE SEE THAT THE INNER HORIZON MOVES TO NEGATIVE r AND THE OUTER HORIZON MOVES TO $r = 0$ AT THE NEW CRITICAL MASS $\sim 2.38 M_{Pl}$.

NOTE: M. BOJOWALD *et al.*, gr-qc/0503041, – LOOP QG CONCURS WITH GENERAL CONCLUSION.

PREDICTION: ENERGETIC COSMIC RAYS AT $E \sim M_{Pl}$ DUE THE DECAY OF SUCH A REMNANT.

Conclusions

YFS-TYPE METHODS (EEX AND CEEX) EXTEND TO NON-ABELIAN GAUGE THEORY AND ALLOWS SIMULTANEOUS RESMN OF QED AND QCD WITH PROPER SHOWER/ME MATCHING BUILT-IN.

FOR QED$\otimes$QCD

- FULL MC EVENT GENERATOR REALIZATION OPEN.
- SEMI-ANALYTICAL RESULTS FOR QED (AND QCD) THRESHOLD EFFECTS AGREE WITH LITERATURE ON Z PRODUCTION; CHECK WITH W-PROD. IMMINENT
- AS QED IS AT THE .3% LEVEL, IT IS NEEDED FOR 1% LHC THEORY PREDICTIONS.

- A FIRM BASIS FOR THE COMPLETE $\mathcal{O}(\alpha_s^2, \alpha\alpha_s, \alpha^2)$ MC RESULTS NEEDED FOR THE FNAL/LHC/RHIC/TESLA/LC PHYSICS HAS BEEN DEMONSTRATED AND ALL THE LATTER IS IN PROGRESS, WITH M. Kalmykov, S. Majhi, S. Yost and S. Joseph.– SEE JHEP0702(2007)040,arxiv:0707.3654,0803 NEW RESULTS FOR HO F-Int's,etc. –no time to discuss here

THE THEORY ALLOWS A NEW APPROACH
TO QUANTUM GENERAL RELATIVITY:

- RESUMMED QG UV FINITE
- MANY CONSEQUENCES:
  BLACK HOLES EVAPORATE TO FINAL MASS $\sim M_{Pl}$
  WITH NO HORIZON
  $\Rightarrow E \sim M_{Pl}$ COSMIC RAYS, $\cdots$ – EXPT'S IN PROGRESS.