Theoretical issues for luminosity determination at the ILC

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Outline

- luminosity & theory
- the LEP & flavour factories era
- the event generator BabaYaga
  1. theoretical ingredients: matching QED Parton Shower with exact $\mathcal{O}(\alpha)$ corrections
  2. theoretical accuracy
     - comparison with 2-loop Bhabha calculations
     - other sources of uncertainties
  3. the process $e^+e^- \to \gamma\gamma$
- examples of BabaYaga at ILC energies
- conclusions

Luminosity & Theory

- luminosity is a machine (and process independent) parameter entering every experimental cross-section

$$\frac{N_{obs}}{\mathcal{L}} = \sigma$$

- precise experiments require a precise knowledge of $\mathcal{L}$, not achievable via machine's parameters
- the relation can be inverted and exploited if a well predictable process X is chosen

$$\mathcal{L} = \frac{N^X_{obs}}{\sigma^X_{theory}}$$

$$\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta N^X_{obs}}{N^X_{obs}} \oplus \frac{\delta\sigma^X_{theory}}{\sigma^X_{theory}}$$

Luminosity & Theory

- in order to minimize $\delta\mathcal{L}$, the process X has to
  1. have large statistics (δN small)
  2. be well calculable theoretically ($\delta\sigma$ small)
  3. be cleanly detectable (small sistematics)
- at e^+e^- machines, the best choice are QED processes, in particular $e^+e^- \to e^+e^-$ Bhabha scattering
  - ⋆ at small angles, at LEP & SLC
    - huge statistics
    - by far dominated by photon t-channel contribution (QED, no "Z contamination")
  - ⋆ at large angles at flavour factories
    - no need of dedicated detectors
    - also here dominated by t-channel photon exchange
- the theoretical error on σ has to be as small as possible, by including in the calculation all the relevant radiative corrections (RC) to achieve the aimed accuracy

Beamstralhung and RC at ILC

- at ILC, the *beamstrahlung* effect (beam loss of energy due to beam-beam interaction) has to be accounted for

$$\sigma(s) = \int dz_1 \int dz_2 \mathcal{D}_{bs}(z_1)\mathcal{D}_{bs}(z_2) \int d\sigma(z_1 z_2 s)\Theta(cuts)$$

- $\mathcal{L}$ has a continuum spectrum as a function of z_1 and z_2
- large part of RC in Bhabha is due to photonic corrections, driven at $\mathcal{O}(\alpha)$ by the collinear log $L = \log \frac{st}{um_e^2} - 1$
  - ⋆ $L \simeq \log(s/m_e^2) - 1 \simeq$ 14 at flavour factories (large angle)
  - ⋆ $L \simeq \log(-t/m_e^2) - 1 \simeq$ 16 at LEP1 (small angle)
  - ⋆ $L \simeq \log(-t/m_e^2) - 1 \simeq$ 18 at LEP2
  - ⋆ $L \simeq \log(-t/m_e^2) - 1 \simeq$ 20 at 500 GeV ILC
  - ⋆ $L \simeq \log(-t/m_e^2) - 1 \simeq$ 21 at 1 TeV ILC
- we naively and roughly expect that the impact of RC increases by $\sim$ 30% from LEP1 to an ILC at 1 TeV

The LEP era

- expected order of magnitude of the photonic corrections at LEP

		$\theta_{min} = 30$ mrad		$\theta_{min} = 60$ mrad	
		LEP1	LEP2	LEP1	LEP2
$\mathcal{O}(\alpha L)$	$\frac{\alpha}{\pi} 4L$	137×10^{-3}	152×10^{-3}	150×10^{-3}	165×10^{-3}
$\mathcal{O}(\alpha)$	$2\frac{1}{2}\frac{\alpha}{\pi}$	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}	2.3×10^{-3}
$\mathcal{O}(\alpha^2 L^2)$	$\frac{1}{2}\left(\frac{\alpha}{\pi}4L\right)^2$	9.4×10^{-3}	11×10^{-3}	11×10^{-3}	14×10^{-3}
$\mathcal{O}(\alpha^2 L)$	$\frac{\alpha}{\pi}\left(\frac{\alpha}{\pi}4L\right)$	0.31×10^{-3}	0.35×10^{-3}	0.35×10^{-3}	0.38×10^{-3}
$\mathcal{O}(\alpha^3 L^3)$	$\frac{1}{3!}\left(\frac{\alpha}{\pi}4L\right)^3$	0.42×10^{-3}	0.58×10^{-3}	0.57×10^{-3}	0.74×10^{-3}

Table: From CERN Yellow Report “Physics at LEP2”

Theoretical error at LEP1 on Bhabha process

My personal update of LEP1 theoretical error, Febr. 2003 (red/magenta)

Type of correction/error	Ref.[1]	Ref. [2]	Ref. [3]	My update
Technical precision	—	(0.030%)	(0.030%)	0.030%
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.10%	0.027%	0.027%	0.027%
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.015%	0.015%	0.015%
Vacuum polarization	0.04%	0.04%	0.040%	0.025%
Light pairs	0.03%	0.03%	0.010%	0.010%
Z-exchange	0.015%	0.015%	0.015%	0.015%
Total	0.11%	0.061% (0.068)	0.054% (0.061)	0.53%

[1] A. Arbuzov *et al. LEP Working Group 1996*, Phys. Lett. B **383** (1996) 238

[2] B. F. Ward, S. Jadach, M. Melles and S. A. Yost, Proc. of ICHEP 98, Vancouver arXiv:hep-ph/9811245 and Phys. Lett. B **450** (1999) 262

[3] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini and F. Piccinini, Phys. Lett. B **459** (1999) 649

- the impressive theoretical accuracy has been achieved with a hard work of many groups and comparing independent calculations/codes

The original `BabaYaga` (v.3.5)

- similar accuracy requirements ($\mathcal{O}(0.1\%)$) are needed also at low energy flavour factories
- `BabaYaga` is a MCEG for $e^+e^- \to e^+e^-, \gamma\gamma, \mu^+\mu^-, \pi^+\pi^-$ at flavour factories

C.M.C.C. et al., **NPB** 584 (2000)

C.M.C.C., **PLB** 520 (2001)

- the QED RC corrections were included with an (original) QED Parton Shower (PS), allowing for
  1. fully exclusive multi-photon generation (up to ∞ photons)
  2. natural inclusion of $\mathcal{O}(\alpha)$ and higher order QED photonic corrections in leading-log (LL) approximation
- theoretical error due to missing $\mathcal{O}(\alpha)$ non-log terms, not naturally reproduced by the PS.
  Estimated accuracies:
  - 0.5% for Bhabha
  - $\simeq \mathcal{O}(1\%)$ for $\gamma\gamma$ and $\mu^+\mu^-$

PS and exact $\mathcal{O}(\alpha)$ matrix elements (BabaYaga@NLO)

G. Balossini et al., Nucl. Phys. **B** 758 (2006)

PS and exact $\mathcal{O}(\alpha)$ (NLO) matrix elements must be combined and matched to reach the aimed accuracy. How?

- $d\sigma^{\infty}_{LL} = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \, |\mathcal{M}_{n,LL}|^2 \, d\Phi_n$
- $d\sigma^{\alpha}_{LL} = [1 + C_{\alpha,LL}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_{1,LL}|^2 d\Phi_1 \equiv d\sigma_{SV}(\varepsilon) + d\sigma_H(\varepsilon)$
- $d\sigma^{\alpha}_{exact} = [1 + C_{\alpha}] \, |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1$
- $F_{SV} = 1 + (C_{\alpha} - C_{\alpha,LL})$ $\qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$
- $d\sigma^{\alpha}_{exact} \overset{\text{at } \mathcal{O}(\alpha)}{=} F_{SV}(1 + C_{\alpha,LL})|\mathcal{M}_0|^2 d\Phi_0 + F_H |\mathcal{M}_{1,LL}|^2 d\Phi_1$

$$d\sigma^{\infty}_{matched} = F_{SV} \, \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} \, \left(\prod_{i=0}^{n} F_{H,i}\right) \, |\mathcal{M}_{n,LL}|^2 \, d\Phi_n$$

Contents of the *matched* formula

- F_{SV} and $F_{H,i}$ are infrared safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of LL
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{exact}$
- resummation of higher orders LL contributions preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
- as a by-product, the α^2 structure is richer than pure LL. E.g., part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV \mid H,i} \times LL$

G. Montagna et al., **PLB** 385 (1996)

- the error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m^2} \sim 5 \times 10^{-4}$$

Vacuum Polarization

- $\alpha \to \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\qquad \Delta\alpha = \Delta\alpha_{e,\mu,\tau,\text{top}} + \Delta\alpha^{(5)}_{\text{had}}$
- $\Delta\alpha^{(5)}_{\text{had}}$ is a non-perturbative contribution. Evaluated with HADR5N by F. Jegerlehner.

S. Eidelman and F. Jegerlehner, **Z. Phys. C** 67 (1995)

F. Jegerlehner, **NPB Proc. Supp.** 131 (2004)

- VP included both in lowest order and (at best) in one-loop diagrams $\Rightarrow$ part of the 2 loop factorizable corrections are included
- Z exchange included at lowest order.
  Its effect is $\mathcal{O}(0.1\%)$ @ 10 GeV

stolen to G. Montagna, talk at "Matter To The Deepest", Ustron, 5-11 September, 2007

Large–angle Bhabha: size of radiative corrections

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227

Selection criteria – ϕ and B factories

- a $\sqrt{s} = 1.02\,\mathrm{GeV},\ E_{\text{min}}^{\pm} = 0.408\,\mathrm{GeV},\ \vartheta_{\mp} = 20^\circ \div 160^\circ,\ \xi_{\text{max}} = 10^\circ$
- b $\sqrt{s} = 1.02\,\mathrm{GeV},\ E_{\text{min}}^{\pm} = 0.408\,\mathrm{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\text{max}} = 10^\circ$
- c $\sqrt{s} = 10\,\mathrm{GeV},\ E_{\text{min}}^{\pm} = 4\,\mathrm{GeV},\ \vartheta_{\mp} = 20^\circ \div 160^\circ,\ \xi_{\text{max}} = 10^\circ$
- d $\sqrt{s} = 10\,\mathrm{GeV},\ E_{\text{min}}^{\pm} = 4\,\mathrm{GeV},\ \vartheta_{\mp} = 55^\circ \div 125^\circ,\ \xi_{\text{max}} = 10^\circ$

Relative corrections (in %)

set up	a.	b.	c.	d.
δ_{VP}	1.73	2.43	4.59	6.03
δ_α	-13.06	-17.16	-19.10	-24.35
δ_{HO}	0.43	0.93	0.87	1.76
$\delta_\alpha^{\text{non-log}}$	-0.39	-0.66	-0.41	-0.70
$\delta_{\alpha^2 L}$	0.04	0.09	0.06	0.11

★ Both exact $\mathcal{O}(\alpha)$ and higher–order corrections (including vacuum polarization) necessary for 0.1% theoretical precision ★

- $\Delta\alpha_{\text{had}}^{(5)}$ contribution to vacuum polarization included through `HADR5N` routine, returning a data–driven error estimate

F. Jegerlehner, Nucl. Phys. Proc. Suppl. **131** (2004) 213

Estimate of the theoretical accuracy

- switching off VP, tuned comparisons with independent calculations/approaches (`Labspv`, `Bhwide`)
  - ⋆ $\Delta\sigma/\sigma < 0.03\%$ on cross sections
  - ⋆ up-to-0.5% differences between `BabaYaga` and `Bhwide` in distribution tails
- comparison with existing perturbative 2-loop calculations
  - ⋆ we compared to
    1. Penin: complete virtual 2-loop photonic corrections (for $Q^2 \gg m_e^2$) plus real radiation in the soft limit
    2. Bonciani et al.: virtual $N_F = 1$ [only electron in the loops] fermionic contributions plus real radiation in the soft limit
  - ⋆ the photonic and $N_F = 1$ $\mathcal{O}(\alpha^2)$ content of the S+V part in the `BabaYaga` matched formula can be easily extracted. The terms to be directely compared to 1. and 2. can be read out!
  - ⋆ the impact of the missing $\mathcal{O}(\alpha^2)$ S+V corrections can be quantified within realistic setup

Large–angle Bhabha: tuned comparisons at Φ and B factories

Without vacuum polarization, to compare consistently

At the Φ–factories (cross sections in nb)

set up	BabaYaga@NLO	BHWIDE	LABSPV	$\delta_{BBH}(\%)$	$\delta_{BL}(\%)$
a.	$6086.6(1)$	$6086.3(2)$	$6088.5(3)$	0.005	0.030
b.	$455.85(1)$	$455.73(1)$	$456.19(1)$	0.030	0.080

★ Agreement within 0.1%! ★

- Now at KLOE: $\frac{\delta\mathcal{L}}{\mathcal{L}} = \frac{\delta\mathcal{L}_{\rm exp}}{\mathcal{L}_{\rm exp}} \oplus \frac{\delta\sigma_{\rm th}}{\sigma_{\rm th}} = 0.3\%\,({\rm exp}) \oplus 0.1\%\,({\rm th}) = 0.3\%$

F. Ambrosino *et al.*, [KLOE Coll.], arXiv:0707.4078 [hep-ex]

At BABAR (cross sections in nb)

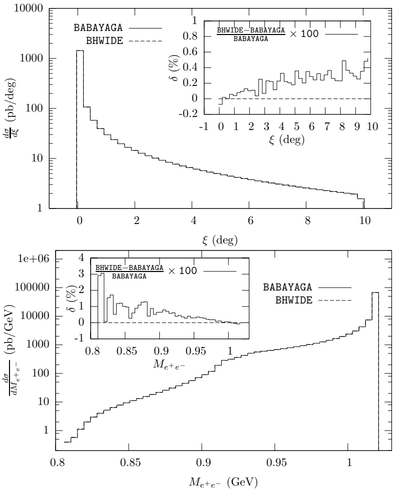
From talks by A. Denig and A. Hafner @LNF

angular range (c.m.s.)	BabaYaga@NLO	BHWIDE	$\delta_{BBH}(\%)$
$15^\circ \div 165^\circ$	$119.5(1)$	$119.53(8)$	0.025
$40^\circ \div 140^\circ$	$11.67(3)$	$11.660(8)$	0.086
$60^\circ \div 120^\circ$	$3.554(6)$	$3.549(3)$	0.141

★ Agreement at $\sim$ 0.1% level! ★

BabaYaga@NLO vs BHWIDE at DAΦNE

G. Balossini *et al.*, Nucl. Phys. **B758** (2006) 227



- Agreement within a few 0.1%, a few % only in the hard tails

NNLO QED calculations: large-angle Bhabha

- Massless two–loop virtual corrections

  Z. Bern, L. Dixon and A. Chingulov, Phys. Rev. **D63** (2001) 053007

- Exact coefficient of next–to–leading second order $\mathcal{O}(\alpha^2 L)$ corrections, w/o and with two–loop box contributions, plus soft bremsstrahlung

  A.B. Arbuzov, E.A. Kuraev and B.G. Shaikhatdenov, Mod. Phys. Lett. **A13** (1998) 2305
  E.W. Glover, J.B. Tausk and J.J. van der Bij, Phys. Lett. **B516** (2001) 33

- Complete virtual two–loop photonic corrections (in the limit $Q^2 \gg m_e^2$) plus real soft–photon radiation, up to non–logarithmic accuracy

  A. Penin, Phys. Rev. Lett. **95** (2005) 010408
  A. Penin, Nucl. Phys. **B734** (2006) 185

- Two–loop $N_F = 1$ [only electron loops] fermionic corrections, with finite mass terms, plus soft bremsstrahlung and real pair corrections

  R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121
  R. Bonciani *et al.*, Nucl. Phys. **B716** (2005) 280
  R. Bonciani and A. Ferroglia, Phys. Rev. **D72** (2005) 056004

- Two–loop $N_F = 2$ [electron and muon loops] fermionic corrections, with finite mass terms, plus soft bremsstrahlung

  T. Becher and K. Melnikov, `arXiv:0704.3582 [hep-ph]`

- Two–loop heavy fermion [$e, \mu, \tau, \mathrm{top}$] corrections combined with all available non–fermionic contributions

  M. Czakon, J. Glusza and T. Riemann, Nucl. Phys. **B751** (2006) 1
  S. Actis, M. Czakon, J. Glusza and T. Riemann, `arXiv:0704.2400 [hep-ph]`

NLLO content

By expanding the matched PS formula up to $\mathcal{O}(\alpha^2)$, the approximate 2nd order `BabaYaga` cross section can be cast in the form

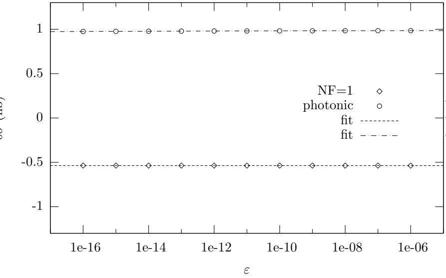
$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{SV} + \sigma^{\alpha^2}_{SV,H} + \sigma^{\alpha^2}_{HH}$$

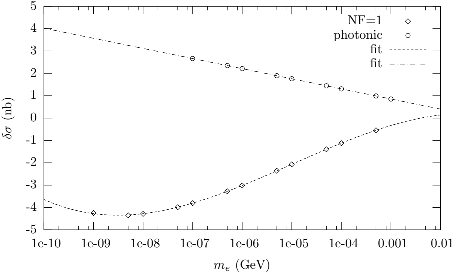
where

1. $\sigma^{\alpha^2}_{SV}$ represents soft+virtual RC at $\mathcal{O}(\alpha^2) \to$ to be compared with available NNLO exact corrections
2. $\sigma^{\alpha^2}_{SV,H}$ represents soft+virtual corrections to one real photon emission $\to$ error estimated relying on existing (partial) results
3. $\sigma^{\alpha^2}_{HH}$ represents the two real photons emission $\to$ compared to the exact $e^+e^{\cdot}- \to e^+e^-\gamma\gamma$ matrix elements. Differences on cross sections are negligible (at the level of 0.001%)

Differences from Penin & Bonciani et al.

- diff. between Penin and Bonciani et al. and the corresponding `BabaYaga` content, as $f(\varepsilon)$ and $g(\log(m_e))$. E.g. LABS at 1 GeV





- ⋆ differences are infrared safe
- ⋆ $\delta\sigma(phot.)/\sigma_0 \propto \alpha^2 L$ $\qquad$ $\delta\sigma(N_F = 1)/\sigma_0 \propto \alpha^2 L^2$
- ⋆ Numerically, in LABS and VLABS,

$$\delta\sigma(phot.) + \delta\sigma(N_F = 1) < 0.015\% \times \sigma_0$$

Summary of theoretical errors

- for Bhabha, within realistic setups for luminometry at flavour factories, the theoretical errors of `BabaYaga` are summarized

$\|\delta^{err}\|$ (%)	(a)	(b)	(c)	(d)
$\|\delta^{err}_{VP}\|$	0.01	0.00	0.02	0.04
$\|\delta^{err}_{pairs}\|$	0.02	0.03	0.03	0.04
$\|\delta^{err}_{H,H}\|$	0.00	0.00	0.00	0.00
$\|\delta^{err}_{phot+N_f=1}\|$	0.01	0.01	0.00	0.01
$\|\delta^{err}_{SV,H}\|$	0.05	0.05	0.05	0.05
$\|\delta^{err}_{total}\|$	0.09	0.09	0.10	0.14

Table: LABS (a) (c), VLABS (b) (d), 1.02 GeV (a) (b), 10 GeV (c) (d)

- missing (virt. & real) pair corrections estimated in the soft limit
  [Jadach et al. ('97), Kniehl ('90), Burgers ('85), Barbieri et al. ('72); Arbuzov et al. ('97)]
- Vacuum polarization uncertainty as returned by `HADR5N`

Resummation beyond α^2

- with a complete 2-loop generator at hand, (leading-log) resummation beyond α^2 can be neglected?

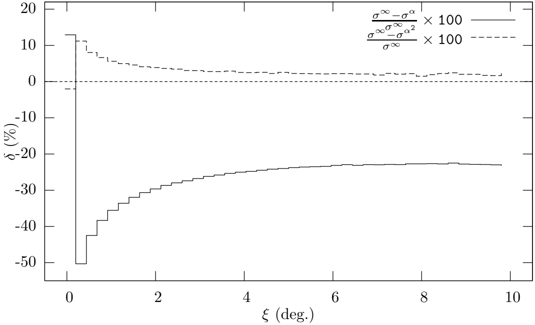


Figure: Impact of α^2 (solid line) and resummation of higher order ($\geq \alpha^3$) (dotted) corrections on the acollinearity distribution

- resummation beyond α^2 still important

$e^+e^- \to \gamma\gamma$

- $e^+e^- \to \gamma\gamma$ can be used to cross-check independently $\mathcal{L}$ measurements

⋆ the matching is now applied also to $\gamma\gamma$, relying on the 1-loop formulae in Berends and Kleiss **NPB** 186 (1981) and Berends et al. **NPB** 202 (1981)

- e.g., E_{cms} = 1 GeV, at least 2 photons with 20º $< \vartheta_\gamma <$ 160º, $E_\gamma >$ 0.3 GeV and varying the acollinearity cut

$\zeta_{\gamma\gamma}$ (º)	σ_0 (nb)	$O(\alpha)_{LL}$	$O(\infty)_{LL}$	$O(\alpha)_{ex}$	$O(\infty)_{matched}$
5	329.8	302.5	304.0	304.4	305.6
10	329.8	314.3	314.8	316.3	316.6
15	329.8	320.2	320.4	322.2	322.2
20	329.8	323.6	323.6	325.6	325.4

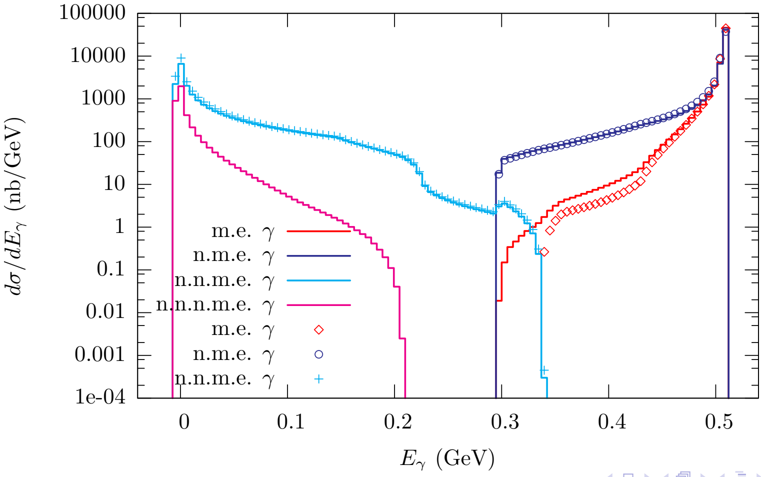
- $\mathcal{O}(\alpha)$ non-log $\simeq 0.7\%$, now included

⋆ estimated theoretical error $\leq 0.1\%$ (VP error is not present here)

$e^+e^- \to \gamma\gamma$ distributions

- photon energies

  markers = $\mathcal{O}(\alpha)$, hist. = $\mathcal{O}(\infty)$



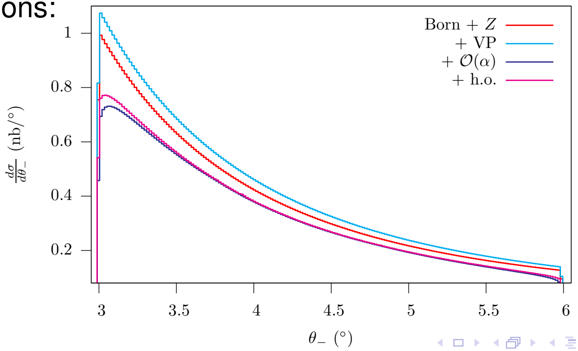
BabaYaga for Bhabha at ILC

- the MC can run also at ILC, in the small angle regime (QED)
- *e.g.*, $E_{cms} = 500$ GeV, $3^\circ < \theta_- < 6^\circ$, $174^\circ < \theta_+ < 177^\circ$, $E_\pm > 200$ GeV

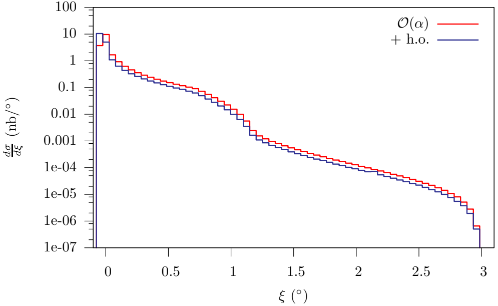
	Born	$+Z$	+VP	$+\mathcal{O}(\alpha)$	+ h.o.
σ (nb)	1.13762	1.13757	1.23816	0.97689	0.99550
%	-	-0.004	+8.84	-12.98	+1.91

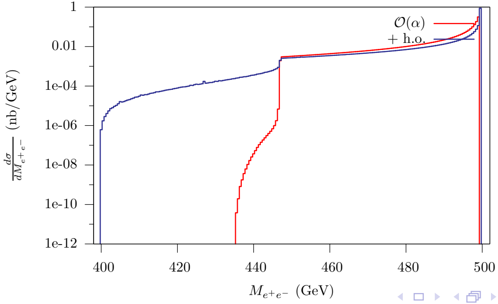
at higher energies, t-channel Z exchange is larger

- distributions:



BabaYaga at ILC





Conclusions

- precise luminosity determination at ILC can greatly benefit from past experience, LEP and flavour factories
- tools to reach a theoretical accuracy at a few 0.1%, with small angle Bhabhas, are already on the market
- room for improvements exists
  - new data for hadronic contribution to VP
  - huge efforts to calculate complete 2-loop corrections
  - inclusion in MCs of exact $\mathcal{O}(\alpha)$ EW corrections to Z s and t channel exchange diagrams
- exponentiation still needed with a full 2-loop calculation at hand
- due to increasing code complexity, careful comparisons and cross-checks among independent calculations and codes (MCs) are mandatory, as usual