

Turbulence on APE: towards <u>channel@apeNEXT</u>

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The past, the present and the future

- APE100
 - RB, Channel flow
- APEmille
 - RB, Channel flow, Fast Fourier Transform
- apeNEXT
 - RB, Channel flow, Lagrangian Turbulence,
 - Microfluidic ?

Introduction to turbulence

- What is fluid dynamics turbulence??
 - Deterministic, non-linear & chaotic system.
 - Characterized by an infinite number of active degrees of freedom, in the infinite Re number limit (field theory)
- Navier-Stokes is an open problem for math, physics, engineering (more later).



Statistical observables in turbulence

Structure functions:

$$S^{(p)}(\boldsymbol{r}) = \langle \left[(\boldsymbol{v}\left(\boldsymbol{r}\right) - \boldsymbol{v}\left(0\right) \right) \cdot \boldsymbol{\hat{r}} \right]^{p} \rangle$$

Well known, structure function behaviour, in the inertial range, for homogeneous and isotropic turbulence:

 $\eta \ll |\boldsymbol{r}| \ll L_0$ $S^{(p)}(|\boldsymbol{r}|) \sim r^{\zeta_p}$

With famous Kolmogorov's prediction 1941: $\zeta_p = p/3 \cdots + {
m intermittency}$

Exact result for homogeneous/isotropic turbulence: $\zeta_3=1$

Turbulence, a challenge for:

- Mathematics
 - Existence of NS solutions.
- Physics
 - How to compute anomalous scaling exponents ?
 (exponents = quantification of intemittency)
 - Universality issue !
- Engineering
 - Ability to simulate or reproduce realistic systems.
- Computer science
 - Efficient computational methods.

The method of choice on APE: LBE

Stream and collide Particularly tailored to APE topology





Boundary conditions for LBE code

BC: good for APE easy, local, overlap communications & comp. BC: good for physics LBE scheme allow a big flexibility in bc !!!



Illustration of population injection from the "buffer" layer

What have we done with this?

RB cell & plumes







Motivation

- Hot topics (still open today!):
- •Scaling of Nu vs. Ra and Pr

- Bolgiano scaling
 - and in particular on the statistics of velocity, temperature fields

What we studied over the years...

- Studied several variants of convective cell (also periodic case !!)
- Always cubic geometry
- With different boundary conditions !!
- Modest resolutions i.e. 160³ and 240³
- Very high statistics
 (i.e. hundreds of eddy turnover times)

Bolgiano scaling

Boussinesq equations: $\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla) \boldsymbol{v} = -\nabla p + \nu \nabla^2 \boldsymbol{v} + \alpha g T \hat{\boldsymbol{z}}$ $\partial_t T + (\boldsymbol{v} \cdot \nabla) T = \kappa \nabla^2 T$

Temperature difference ΔT

Cell's heigth $\, H \,$

$$L_B(z) = \frac{\varepsilon(z)^{5/4}}{N(z)^{3/4} (\alpha g)^{3/2}}$$

 $Ra = \alpha q \Delta T H^3 / (\nu \chi)$

 $Pr = \frac{\nu}{\chi}$

Kolmogorov scaling
$$r \ll L_B$$
 $\delta v(r) \sim arepsilon^{1/3} r^{1/3}$ $\delta T(r) \sim N^{1/2} arepsilon^{-1/6} r^{1/3}$

Bolgiano scaling

 $r \gg L_B$

The standard RB cell and $L_B(z)$



From measuring this quantity one can understand how

strongly non homogeneous a convective cell is

R. Benzi, F. Toschi, R. Tripiccione

On the Heat Transfer in Rayleigh-Bénard systems

Journal of Statistical Physics 93 3 (1998)

The homogeneous Rayleigh-Bénard cell

Where do the eqns. of HRB comes from?

Here some more details...

From Boussinesq $\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla) \, \boldsymbol{v} = -\nabla p + \nu \nabla^2 \boldsymbol{v} + \alpha g T \hat{\boldsymbol{z}}$ approximation: $\partial_t T + (\boldsymbol{v} \cdot \nabla) \, T = \kappa \nabla^2 T$

Supposing temperature is the sum of a linear profile, plus fluctuating part: $T(x, y, z; t) = T_{lin}(z) + T'(x, y, z; t)$ with $T_{lin}(z) = +\frac{\Delta T}{2} \cdot \left(1 - \frac{2z}{H}\right)$

one ends up with:

$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla) \, \boldsymbol{v} = -\nabla p + \nu \nabla^2 \boldsymbol{v} + \alpha g T' \, \hat{\boldsymbol{z}}$$

$$\partial_t T' + (\boldsymbol{v} \cdot \nabla) \, T' = \kappa \nabla^2 T' - \frac{\Delta T}{H} v_z$$

Supplemented with periodic boundary conditions in all directions

Inside the HRB cell...

- The system auto-mantains itself: no external forcing!
- No boundary layers! (see Lohse & Toschi PRL 2003)
- The system is fully homogeneous BUT not isotropic
- L_B too big to see Bolgiano scaling



Istituto per le Applicazioni del Calcolo Mauvo Picone

Thermal plume

is fully periodic

Results from the standard cell

Bolgiano scaling maybe close to walls

From the behaviour of L_B one learns that to see Bolgiano scaling one has to move close to the top/bottom isothermal walls (z) q-Is this enough? Is it so $simple?^{\circ}$ What happens near to the walls (inside a boundary layers)?

The boundary layer problem

Structure functions from a boundary layer experiment



Phys. Rev. Lett., 85 (2000) 1436-1439

The boundary layer problem



Structure functions of order 3 and 6 at y⁺=102 from a boundary layer experiment

How to get the scaling exponents



Exponents from the standard cell

Scaling exponents for velocity $\zeta_3(z)$ and temperature $\chi_3(z)$



Consistent with Bolgiano scaling !!

Results from the homogeneous cell

Ultimate regime for RB

...idea. Use the homogenous cell to

check the Kraichnan regime: R. H. Kraichnan, Phys. Fluids 5, 1374 (1962)

Prediction:

 $Nu \sim Ra^{1/2}$

results from our DNS...



Channel flow: non homogeneous turb.

Turbulent cascade: $L_0 \rightarrow \eta$ $\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \nu \Delta \boldsymbol{v} + \boldsymbol{f}$ $\partial \cdot \boldsymbol{v} = 0$ Energy flux $Re = \frac{L_0 v}{1}$ LO FORCING $\eta \ll r \ll L_0$ ENERGY FLUX DISSIPATION Inertial range DISSIPATIVE SCALE 0000 0000 0000 0000 0000000

Idealized turbulence: H/I

Exact relation for homogeneous and isotropic turbulence

$$\left< \delta v^3 \right> = -\frac{4}{5} \varepsilon \cdot r + 6 \nu \frac{d}{dr} \left< \delta v^2 \right>$$

Exact result for homogeneous/isotropic turbulence: $\zeta_3=1$. What about fluctuations ?

Fluctuations intermittent very complicated but the following remarkable relation holds for any inertial distance, r, Refined Kolmogorov Similarity Hypothesis RKSH $\delta v(r)^3 \sim \varepsilon_r \cdot r$

Eddy viscosity is a crazy idea

- Large eddy simulation: resolve only scales larger than Δ
- Eddy viscosity: $\varepsilon_{\Delta} = \nu(\Delta) \cdot \left(\frac{\delta v(\Delta)}{\Delta}\right)^2$ model the subgrid scales in terms of a cutoff dependent effective viscosity (unresolved scales act on resolved ones through a renormalized "eddy" viscosity)

Eddy viscosity

If such an eddy viscosity exist it must be able to "eat" the energy flux

DICCI

Definition of eddy viscosity

$$\varepsilon_{\Delta} = \nu(\Delta) \cdot \left(\frac{\delta v(\Delta)}{\Delta}\right)^{2} \qquad \varepsilon_{\Delta} \sim \frac{\delta v(\Delta)^{3}}{\Delta}$$
$$\nu(\Delta) \sim \Delta \cdot \delta v(\Delta) \sim \Delta^{2} \cdot \sqrt{\left(\frac{\delta v(\Delta)}{\Delta}\right)^{2}} \sim \Delta^{2} \cdot S_{\Delta}$$

RKSH -> Smagorinsky

Boundary layer Turbulence

Surprise!

First flow where violations to RKSH has been reported

Non ideal turbulence: boundary layers





F. Toschi, E. Levêque, G.-R. Chavarria, Shear effects in nonhomogeneous turbulence, Phys. Rev. Lett., **85** (2000) 1436-1439

Mapping non ideal on ideal turbulence



$$\delta v(r)^3 \sim r \cdot \varepsilon_r \longrightarrow \delta v(r)^2 \sim \varepsilon_r / S$$

Results from experimental boundary layer



Compensated structure functions for several orders

Eddy viscosity in presence of shear In general, in presence of shear:

$$\begin{array}{l} \text{Definition of eddy viscosity} \\ \varepsilon_{\Delta} = \nu(\Delta) \cdot \left(\frac{\delta v(\Delta)}{\Delta}\right)^2 & \qquad \text{RKSH} \quad \frac{\delta v(\Delta)^3}{\Delta} \\ \end{array}$$

$$\frac{\delta v(\Delta)^3}{\Delta} + \alpha \left\langle \mathcal{S} \right\rangle \cdot \delta v(\Delta)^2 = \varepsilon_\Delta = \nu(\Delta) \left(\frac{\delta v(\Delta)}{\Delta}\right)^2$$

$$\nu(\Delta) = C_S^2 \Delta^2 \left(S_\Delta + \alpha \left\langle S \right\rangle \right) \quad \alpha = -1$$

Generalized RKSH -> SISM

Shear Improved Smagorinsky Model (SISM)

$$egin{aligned} & au_{ij} - rac{1}{3} \delta_{ij} au_{kk} = -2
u_T \overline{S}_{ij} \ &|\overline{S}| \equiv (2 \overline{S}_{ij} \overline{S}_{ij})^{1/2} \end{aligned}$$

SISM model:

$$egin{aligned}
u_T &= ig(C_s \Deltaig)^2 ig(ig|\overline{S}ig| - ig|ig\langle\overline{S}ig|ig) \ &ig(\overline{S}ig) ig) \ &igg(\mathbf{x},t) &= \int \phi(\mathbf{x}',t) G_\Delta(\mathbf{x}-\mathbf{x}') d\mathbf{x}' &igc(C_s \sim 0.17) \ &igc(\overline{S}_i)(\mathbf{x},t) &= rac{1}{2} igg(rac{\partial \overline{u}_i}{\partial x_j}(\mathbf{x},t) + rac{\partial \overline{u}_j}{\partial x_i}(\mathbf{x},t)igg) & rac{\partial \overline{u}_i}{\partial t} + \overline{u}_j rac{\partial \overline{u}_i}{\partial x_j} + rac{\partial \overline{p}}{\partial x_i} = -rac{\partial \overline{p}}{\partial x_i} +
u rac{\partial^2 \overline{u}_i}{\partial x_k \partial x_k} & ext{with} \quad rac{\partial \overline{u}_i}{\partial x_i} = 0 \end{aligned}$$

Test of the SISM

1) Spectral channel flow

2) Finite difference backward facing step



FIGURE 2. Left: (•) mean-velocity profile (in wall units) at $Re_{\tau} = 395$. The computational domain (in outer units) is $4\pi H \times 2H \times 2\pi H$ with $64 \times 65 \times 64$ grid points. In comparison with (-) the DNS data obtained by Moser *et al.* (1999) in the domain $2\pi H \times 2H \times \pi H$ with $256 \times 193 \times 192$ grid points, and (\triangle) a computation of the dynamic Smagorinsky model carried out by Piomelli (private communication) in the domain $5\pi H/2 \times 2H \times \pi H/2$ with $48 \times 49 \times 48$ grid points (using a pseudo-spectral solver). Right: (•) mean-velocity profile at $Re_{\tau} = 590$ with $96 \times 97 \times 96$ grid points. In comparisons with (-) the DNS data with $384 \times 257 \times 384$ grid points.

Reynolds stress



FIGURE 6. Left: Reynolds stress at $Re_{\tau} = 395$ (computed from the resolved velocity). Right: The Reynolds stress at $Re_{\tau} = 590$. The insets focus on the near-wall behavior.

Lagrangian turbulence

Roadmap

Any **realistic** approach to Lagrangian turbulence requires going through (at least) the following steps:

- Neutrally byoyant case
 - Smaller that the dissipative scale of turbulence and with same density of advecting field
- · Heavy partice case
 - Smaller that the dissipative scale of turbulence but with density much higher that advecting field
 - One way coupling
 - Two way coupling
- Generic density contrast case
 - One way coupling
 - Two way and four way coupling (collision
- Non idealized particles
 - Finite particle size, non spherical geometry case,
- Thermal effects (both stable and unstable conditions)
- Intrinsic dynamics (i.e.droplest in clouds)
 - Radii growth
 - Coalescence, etc...

We are here ...

Will present two cases:

Lagrangian tracers

(i.e. pointwise, neutrally buoyant particles)

Heavy particles

(i.e. particle density much larger than fluid density)

Equation of motion for Lagrangian Tracers

The simplest case of Lagrangian turbulence is the evolution of small (infinitesimal) fluid elements. This is equivalent to the evolution of very small particles with density matched with that of the advecting turbulent field.



Equation of motion for "real" particles

$$m_p \ddot{\boldsymbol{x}} = m_f \frac{D\boldsymbol{u}(\boldsymbol{x},t)}{Dt} - 6\pi a\mu \left[\dot{\boldsymbol{x}} - \boldsymbol{u}(\boldsymbol{x},t) \right] - \frac{m_f}{2} \left[\ddot{\boldsymbol{x}} - \frac{d}{dt} \boldsymbol{u}(\boldsymbol{x},t) \right] - \frac{6\pi a^2 \mu}{\sqrt{\pi\nu}} \int_0^t \frac{ds}{\sqrt{t-s}} \frac{d}{ds} \left[\dot{\boldsymbol{x}} - \boldsymbol{u}(\boldsymbol{x},s) \right]$$

Maxey, M. & Riley, J. 1983 Equation of motion of a small rigid sphere in a nonuniform flow. Phys. Fluids 26, 883-889.

$$\frac{d\boldsymbol{v}(t)}{dt} = \beta \frac{D\boldsymbol{u}(\boldsymbol{x},t)}{Dt} - \frac{1}{\tau} \left[\boldsymbol{v}(t) - \boldsymbol{u}(\boldsymbol{x}(t),t) \right]$$

Stokes number
$$St = \frac{\tau}{\tau_{\eta}} \qquad \beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

$$\frac{d\boldsymbol{v}(t)}{dt} = -\frac{1}{\tau} \left[\boldsymbol{v}(t) - \boldsymbol{u}(\boldsymbol{x}(t), t) \right]$$

$$m_p \gg m_f$$

Experimental state of the art

Experimental Lagrangian measurements are intrinsically difficult: one has to follow (many) Lagrangian trajectories for long time at high Reynolds (i.e. high sampling frequency)



Ott and Mann experiment at Risø conventional 3D PTV - Re_{λ} =100 (now $Re_{\lambda} \approx$ 300)



Bodenschatz experiment at Cornell fast silicon strip detectors (now fast CCD cameras) $Re_{\lambda} \approx 1000-1500$



Pinton experiment at ENSL ultrasonic Doppler tracking - Re_{λ} =740 (single particle tracking)

Lagrangian Tracers integration

Ν	$\operatorname{Re}_{\lambda}$	η	L	T _L	τ _η	Т	δx	N _p
512	183	0.01	3.14	2.1	0.048	5	0.012	0.96 106
1024	284	0.005	3.14	1.8	0.033	4.4	0.006	1.92 106

Pseudo spectral code - dealiased 2/3 rule - normal viscosity -2 millions of passive tracers- code fully parallelized with MPI+FFTW - Platform IBM SP4 (sust. Performance 150Mflops/proc) - duration of the run: 40 days

Energy spectrum 10⁻² Spectral flux k^{-5/3} E(K) 1 10⁻⁴ II(K) 10⁻⁶ 0 100 10 1 10 100 1 k

Lagrangian database (x(†),v(†), $a(†)=-\nabla p+v\Delta u$) at high resolution

Heavy particles - Lagrangian integration

L ³	256 ³	512 ³		
Total particles	32 Mparticles	120 Mparticles		
Stokes/ LyapStokes	16/32	16/32		
Slow dumps 10	2.000.000	7.500.000		
Fast dumps 0.1	250.000	500.000		
dt	8 10 ⁻⁴	4 10 ⁻⁴		
Time step ch0+ch1	756 + 1744	900 + 2100		
$ au_\eta$	0.0746	0.0466		
Τ	0.0, 0.0120, 0.0200, 0.0280, 0.0360, 0.0440, 0.0520, 0.0600, 0.0680, 0.0760, 0.0840, 0.1000, 0.1200, 0.152, 0.200, 0.248	0.0, 0.00753454, 0.0125576, 0.0175806, 0.0226036, 0.0276266, 0.0326497, 0.0376727, 0.0426957, 0.0477187, 0.0527418, 0.0627878, 0.0753454, 0.0954375, 0.125576, 0.155714		
Disk space used	400 GByte	1 TByte		

What happens to Lagrangian tracers?



Typical evolution of tracers: Large scale view

And to particles with inertia ...



Acceleration statistics for tracers and heavy particles

L. Biferale, G. Boffetta , A. Celani, B. J. Devenish, A. Lanotte and F. Toschi Multifractal Statistics of Lagrangian Velocity and Acceleration in Turbulence PHYSICAL REVIEW LETTERS **93**, 6 (2004)

J. Bec, L. Biferale, G. Boffetta, A. Celani, M. Cencini, A. Lanotte, S. Musacchio and F. Toschi, **Acceleration statistics of heavy particles in turbulence** Journal of Fluid Mechanics, **550** (2006) 349-358 10.1017/S002211200500844X

Acceleration p.d.f., DNS results



Kaplan-Yorke dimension

Balance dimension between expansion and contraction



Acceleration: pdf(a) vs. St



Small scale bottleneck and vortex filaments



-600

 t/τ

FIG. 1: Trajectory and time series. Left panel: 3D trajectory of a trapping event in vortex filament. Acceleration and velocity fluctuations here reach about 30 and 2 r.m.s. values, respectively (right panels).

For the future:

- Neutrally buoyant case
 - Smaller that the dissipative scale of turbulence and with same density of advecting field
- Heavy particle case
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Conclusions

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• We have NOW the expertise and the appropriate understanding of the phenomenology of different aspects of turbulence:

- Thermal convection
- Wall bounded flow turbulence
- Lagrangian transport of passive particles

apeNEXT

- IS the candidate platform to put all this physics together
- Stably and unstably stratified channel flow seeded with passive tracers
 - Study particle dispersion
 - Drag reduction
- This system interests several physics and engineering groups in Italy

Preliminary performance test



