High Multiplicity Influence on the Pion-Pion Correlations in Relativistic Nuclear Collisions

Dmitry Anchishkin

Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine

- Introduction: Two-particle correlations with accounting for final state interactions.
- Coulomb plus strong final state interaction.

- Final state interactions at high secondary multiplicities:

- Post-freeze-out phase density;
- Two-particle potential;
- Evaluations of the correlation function.
- Summary and conclusions.

Two-particle correlations with accounting for final state interactions

$$C(\mathbf{q}, \mathbf{K}) = \frac{P_2(\mathbf{p}_a, \mathbf{p}_b)}{P_1(\mathbf{p}_a)\, P_1(\mathbf{p}_b)}\,, \tag{1}$$

where

$$P_2(\mathbf{p}_a, \mathbf{p}_b) = E_a E_b \frac{d^6 N}{d^3 p_a d^3 p_b}, \qquad P_1(\mathbf{p}_{a,b}) = E_{a,b} \frac{d^3 N}{d^3 p_{a,b}}\,,$$

$$\mathbf{K} = \frac{1}{2}(\mathbf{p}_a + \mathbf{p}_b)\,, \qquad \mathbf{q} = \mathbf{p}_a - \mathbf{p}_b\,.$$

We are looking for two-particle probability to registrate particles with certain momenta $\mathbf{p}_a$ and $\mathbf{p}_b$: $\quad P_2(\mathbf{p}_a, \mathbf{p}_b) = ?$

D. Anchishkin, U. Heinz, and P. Renk,
Final state interactions in two-particle interferometry,
Phys. Rev. C **57** *(1998), No. 3, pp.1428-1439.*

$$P_2(\mathbf{p}_a, \mathbf{p}_b) = 2 \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \, \Phi\left(\tfrac{\mathbf{q}}{2}, \mathbf{k}'\right) \, \Phi^*\left(\tfrac{\mathbf{q}}{2}, \mathbf{k}\right)$$

$$\times \int d^4X \, d^4Y \, \theta(X^0 - Y^0) e^{i(k-k')\cdot(X-Y)} \, S\left(X, K + \tfrac{k+k'}{2}\right) \, S\left(Y, K - \tfrac{k+k'}{2}\right) ,$$

where

$$k^0 = K^0 - \omega(\mathbf{K} - \mathbf{k}) , \quad k^{0\prime} = K^0 - \omega(\mathbf{K} - \mathbf{k}') , \quad \omega(\mathbf{k}) \equiv \sqrt{m^2 + \mathbf{k}^2}$$

if the free time evolution between emission points is included, while $k^0 = k^{0\prime} = K^0$, if it is neglected.

$$S(X, K) = \int d^4x \, e^{iK\cdot x} \sum_{\gamma\gamma'} \rho_{\gamma\gamma'} \, \psi_\gamma\left(X + \tfrac{x}{2}\right) \, \psi^*_{\gamma'}\left(X - \tfrac{x}{2}\right)$$

Stationary Schrödinger equation

$$\widehat{H}(\mathbf{r})\,\phi_{\mathbf{q}/2}(\mathbf{r}) = E_{\text{rel}}\;\phi_{\mathbf{q}/2}(\mathbf{r})$$

with

$$\widehat{H}(\mathbf{r}) = -\frac{1}{2\mu}\nabla_r^2 + V(r)\,, \quad \lim_{|\mathbf{r}|\to\infty}\phi_{\mathbf{q}/2}(\mathbf{r}) = e^{\frac{i}{2}\mathbf{q}\cdot\mathbf{r}}\,, \quad E_{\text{rel}} = \frac{\mathbf{q}^2}{2\mu}\,, \quad \mu = \frac{m}{2}.$$

$$\tilde{\phi}\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) = \int d^3r\, e^{-i\mathbf{k}\cdot\mathbf{r}}\,\phi_{\mathbf{q}/2}(\mathbf{r})\,, \quad \text{where} \quad \phi_{-\mathbf{q}/2}(\mathbf{r}) = \phi_{\mathbf{q}/2}(-\mathbf{r})\,.$$

The symmetrized FSI distorted wave:

$$\Phi\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) = \frac{1}{\sqrt{2}}\left[\tilde{\phi}\left(\tfrac{\mathbf{q}}{2},\mathbf{k}\right) \pm \tilde{\phi}\left(-\tfrac{\mathbf{q}}{2},\mathbf{k}\right)\right]\,.$$

Smoothness approximation:

for massive particles the FSI cause mostly a change of the spatial momentum while the corresponding energy transfer is very small

$$\left[\omega(\mathbf{K}-\mathbf{k})-\omega(\mathbf{K}-\mathbf{k}')\right] \approx -\frac{1}{E_K}(\mathbf{k}-\mathbf{k}')\cdot\left(\mathbf{K}-\frac{\mathbf{k}+\mathbf{k}'}{2}\right).$$

$$P_2(\mathbf{p}_a,\mathbf{p}_b) = \int d^4x\,d^4y\,S\left(x+\tfrac{y}{2},p_a\right)\,S\left(x-\tfrac{y}{2},p_b\right)$$

$$\times\left[\theta(y^0)\left|\phi_{\mathbf{q}/2}(\mathbf{y}-\mathbf{v}_b y^0)\right|^2+\theta(-y^0)\left|\phi_{\mathbf{q}/2}(\mathbf{y}-\mathbf{v}_a y^0)\right|^2\right]\pm$$

$$\pm\int d^4x\,d^4y\,S\left(x+\tfrac{y}{2},K\right)\,S\left(x-\tfrac{y}{2},K\right)\phi^*_{-\mathbf{q}/2}(\mathbf{y}-\mathbf{v}y^0)\,\phi_{\mathbf{q}/2}(\mathbf{y}-\mathbf{v}y^0)\,,$$

$$\mathbf{v}=\frac{\mathbf{K}}{E_K},\quad \mathbf{v}_a=\frac{\mathbf{p}_a}{E_K},\quad \mathbf{v}_b=\frac{\mathbf{p}_b}{E_K},\quad E_K=\sqrt{m^2+\mathbf{K}^2}.$$

Further approximations:

$$\mathbf{v}_a \approx \mathbf{v}\,, \quad \mathbf{v}_b \approx \mathbf{v}.$$

In pair center off mass system $\mathbf{v} = 0$,

$$P_2(\mathbf{q}) = \int d^4x_a\, d^4x_b\, S\,(x_a, p_a)\; S\,(x_b, p_b)\, \left|\phi_{\mathbf{q}/2}(\mathbf{x}_a - \mathbf{x}_b)\right|^2 \pm$$

$$\pm \int d^4x_a\, d^4x_b\, S\,(x_a, K)\; S\,(x_b, K)\; \phi^*_{\mathbf{q}/2}(\mathbf{x}_b - \mathbf{x}_a)\, \phi_{\mathbf{q}/2}(\mathbf{x}_a - \mathbf{x}_b)\,.$$

Physical meaning: *Two single-particle probabilities to find particles in the time-space points* x_a *and* x_b *with certain momenta* $\mathbf{p}_a$ *and* $\mathbf{p}_b$*, which are expressed by* $S(x,p)$*, is weighted by the probability* $\left|\phi_{\mathbf{q}/2}(\mathbf{x}_a - \mathbf{x}_b)\right|^2$ *to find these particles with relative distance* $\mathbf{x}_a - \mathbf{x}_b$ *and relative momentum* $\mathbf{q}$*.*

The correlation function:

$$C(\mathbf{q}) = \frac{P_2(\mathbf{q})}{\int d^4x_a\, S\,(x_a, p_a)\, \int d^4x_b\, S\,(x_b, p_b)}\,,$$

where 4-vectors $p_a = (\mathbf{q}^2/4m,\, \mathbf{q}/2)$ and $p_b = (\mathbf{q}^2/4m,\, -\mathbf{q}/2)$.

In the noninteracting limit $\phi_{\mathbf{q}/2}(\mathbf{x}) \to \exp\,(i\mathbf{q}\cdot\mathbf{x}/2)$

$$C(\mathbf{q},\mathbf{K}) = 1 \pm \frac{\left|\int d^4x\, e^{i\mathbf{q}\cdot\mathbf{x}} S\,(x, K)\right|^2}{\int d^4x_a\, S\,(x_a, p_a)\, \int d^4x_b\, S\,(x_b, p_b)}\,.$$

Coulomb plus strong final state interaction

Source function:

$$S(x,p) = \frac{e^{-\omega(\mathbf{p})/T_{\mathsf{f}}}}{4\pi m^2 T K_1(m/T_{\mathsf{f}})} \frac{e^{-x_0^2/2\tau^2}}{(2\pi)^{1/2}\tau} \frac{e^{-\mathbf{x}^2/2R_0^2}}{(2\pi)^{3/2}R_0^3} .$$

In the pair c.m.s ($\mathbf{K}=0$) the two-particle probability reduces to the form

$$\begin{aligned} P_2(\mathbf{q}) \quad = \quad & \frac{e^{-2K^0/T_{\mathsf{f}}}}{[4\pi m^2 T K_1(m/T_{\mathsf{f}})]^2 \, \pi^{3/2} \, (2R_0)^3} \\ & \times \int d^3r \, e^{-r^2/4R_0^2} \left[\left| \phi_{\mathbf{q}/2}(\mathbf{r}) \right|^2 \pm \phi^*_{\mathbf{q}/2}(-\mathbf{r}) \, \phi_{\mathbf{q}/2}(\mathbf{r}) \right] , \end{aligned}$$

where time dependence is integrated out.

The single-particle probability reduces to the pure Boltzmann exponent

$$P_1(\mathbf{k}) = \frac{e^{-\omega(\mathbf{k})/T_\mathsf{f}}}{4\pi m^2 T K_1(m/T_\mathsf{f})}.$$

The correlation function:

$$\boxed{C(\mathbf{q}) \; = \; \frac{1}{\pi^{3/2}\,(2R_0)^3} \int d^3 r \, e^{-r^2/4R_0^2} \left[\left| \phi_{\mathbf{q}/2}(\mathbf{r}) \right|^2 \pm \phi^*_{\mathbf{q}/2}(-\mathbf{r}) \, \phi_{\mathbf{q}/2}(\mathbf{r}) \right]}$$

The numerical evaluations of the distorted wave function $\phi_{\mathbf{q}/2}(\mathbf{r})$ is provided by solution of the Schrödinger equation with the relevant potential which reflects two-pion interaction.

Potential energy:

$$V_{\mathrm{Coul}}(r) = \frac{\alpha}{r}, \qquad V_{eff}(r) = V_{\mathrm{Coul}}(r) + V_{\mathrm{str}}(r)\,,$$

where (*S. Pratt et al., PRC* ***42****, 2646 (1990)*)

$$V_{\mathrm{str}}(r) = V_0 \frac{e^{-m_\rho r}}{m_\rho r}, \qquad V_0 = 2.6\ \mathrm{GeV}, \quad m_\rho = 770\ \mathrm{MeV}\,.$$

Potential of the strong repulsion was chosen to match the behavior of the pion-pion scattering phase shifts. In any case the use of this strong potential can be considered as a model of the short-range repulsion which is possesed by pions.

Correlation function without FSI ($\mathbf{K} = 0$):

$$C(\mathbf{q}) = 1 + e^{-\mathbf{q}^2 R_0^2}$$

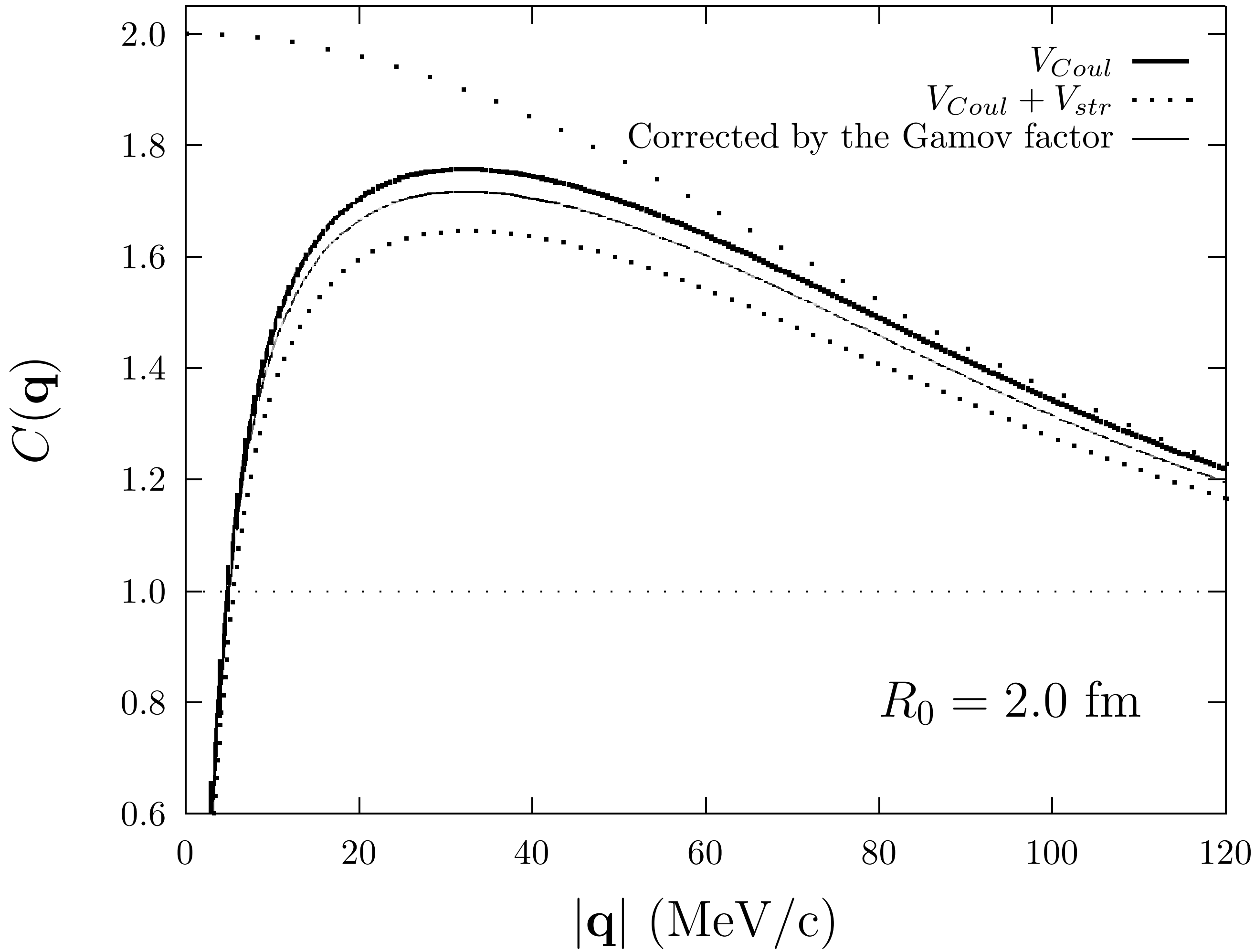
Correlation function, corrected by the Gamov factor $G(|\mathbf{q}|)$,

$$C(\mathbf{q}) \; = \; G(|\mathbf{q}|) \left(1 \, + \, e^{-\mathbf{q}^2 R_0^2} \right) \, ,$$

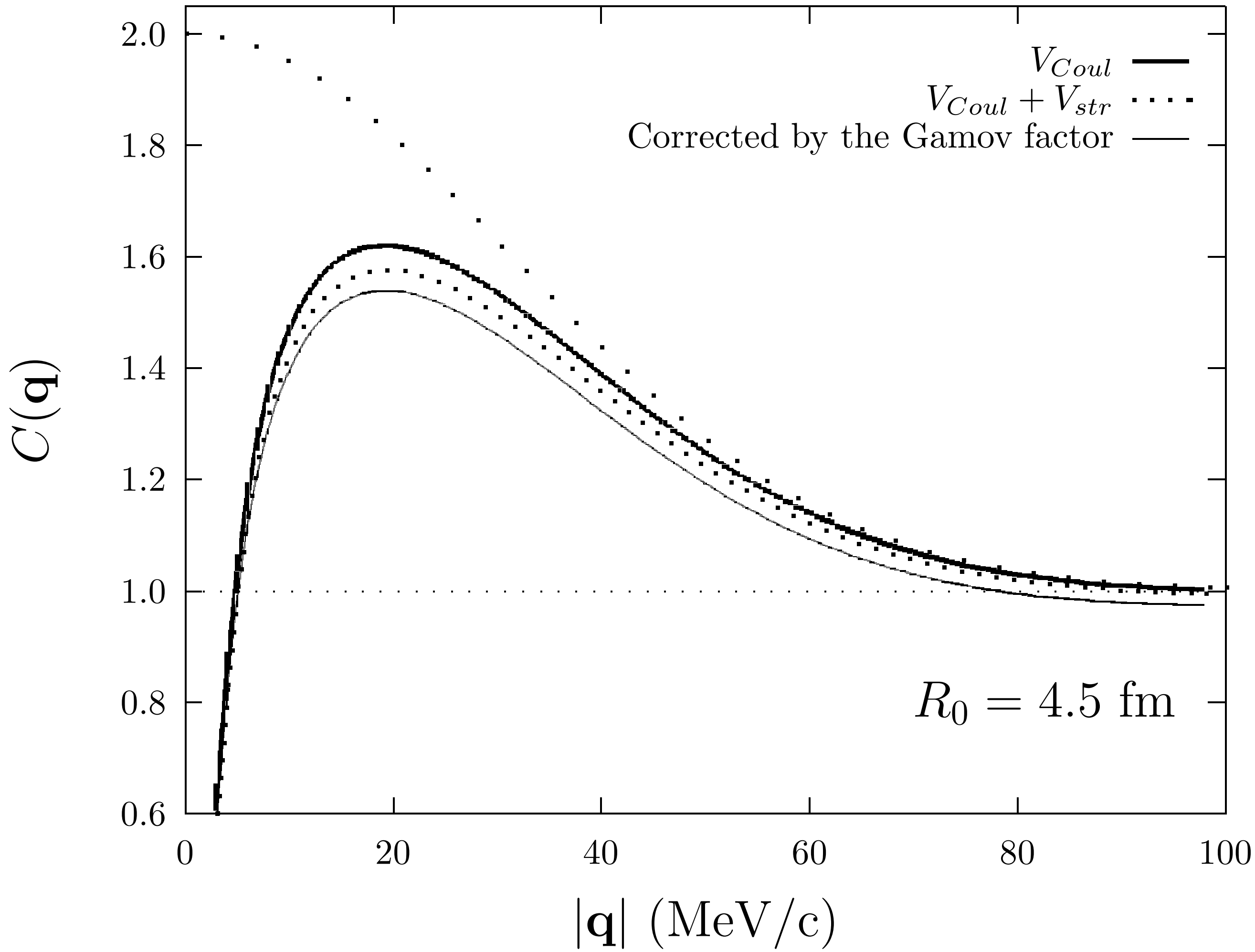
where

$$G(|\mathbf{q}|) = \left| \phi_{\mathbf{q}/2}(\mathbf{r} = 0) \right|^2 .$$

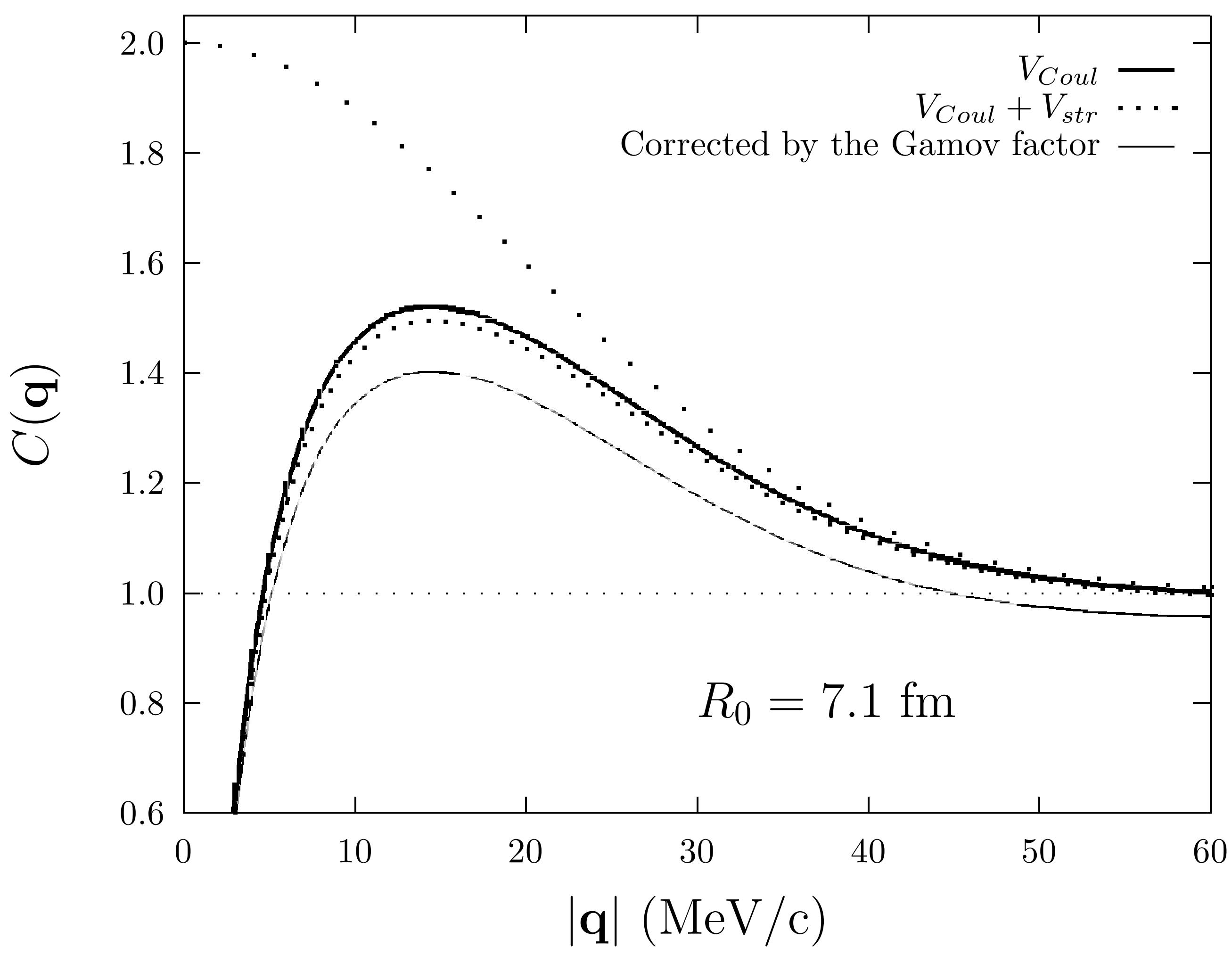
Pure Coulomb: $G(|\mathbf{q}|) = \frac{2\pi\eta}{e^{2\pi\eta} - 1}$ with $\eta = \alpha m_\pi / |\mathbf{q}|$.



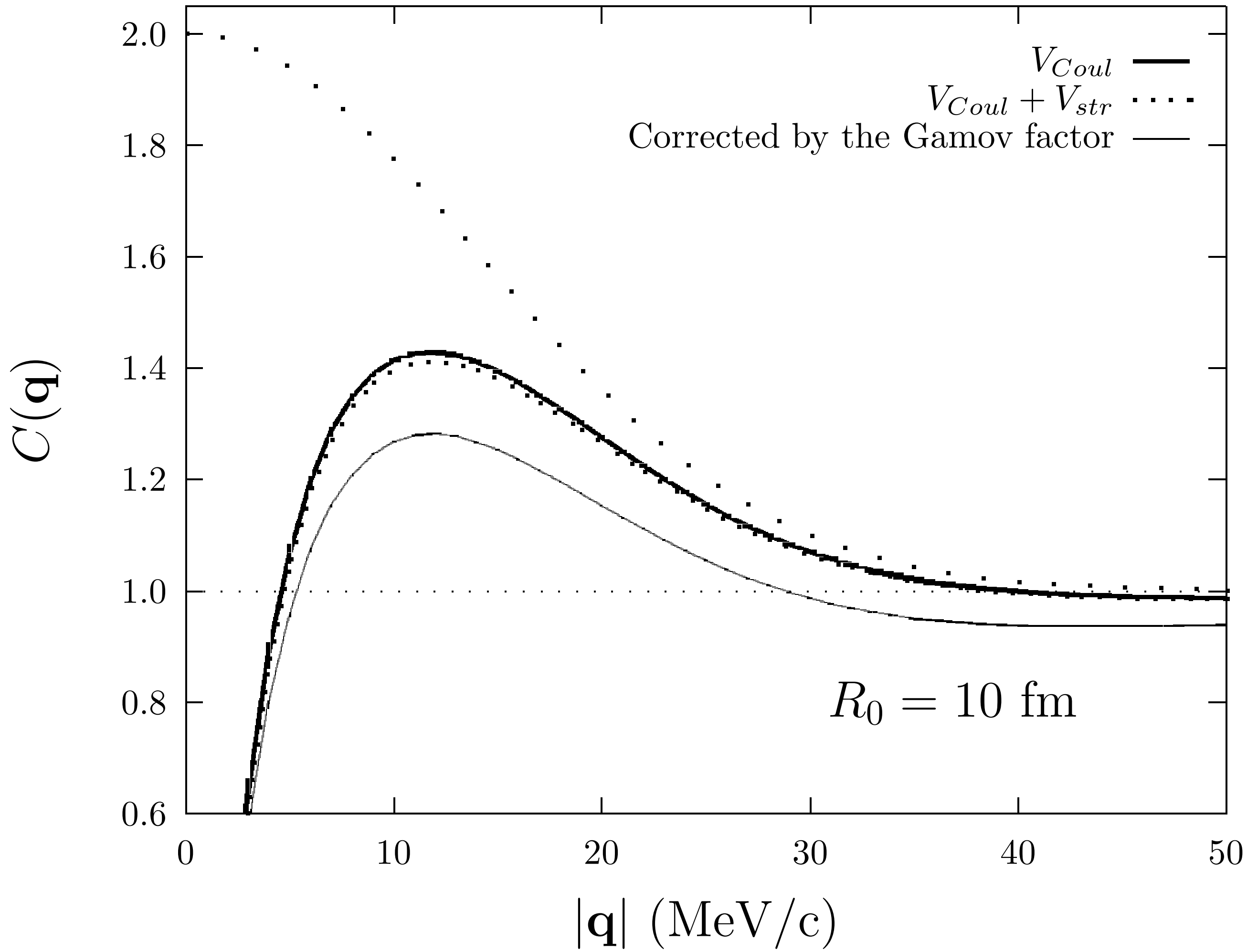
V_{Coul}
$V_{Coul} + V_{str}$
Corrected by the Gamov factor
$R_0 = 2.0$ fm
$C(\mathbf{q})$
$|\mathbf{q}|$ (MeV/c)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0
20
40
60
80
100
120



V_{Coul}
$V_{Coul} + V_{str}$
Corrected by the Gamov factor
$R_0 = 4.5$ fm
$C(\mathbf{q})$
$|\mathbf{q}|$ (MeV/c)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0
20
40
60
80
100



V_{Coul}
$V_{Coul} + V_{str}$
Corrected by the Gamov factor
$R_0 = 7.1$ fm
$C(\mathbf{q})$
$|\mathbf{q}|$ (MeV/c)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0
10
20
30
40
50
60



V_{Coul}
$V_{Coul} + V_{str}$
Corrected by the Gamov factor
$R_0 = 10$ fm
$C(\mathbf{q})$
$|\mathbf{q}|$ (MeV/c)
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0
10
20
30
40
50

As it seen in Figs. 1, 2 additional repulsive potential (hadron hard core) essentially supressed the correlation function at small relative momentum. Actually, the physical reasons for this are transparent: two strongly interacting particles emitted from the volume of $R_0 \approx 2--3$ fm, when their "own mean radius", reflected by repulsive strong potential V_{str}, is about $\langle r \rangle \approx 0.66$ fm, should sufficiently "feel" one another through mutual repulsion. On the other hand, when emision zone is much larger than the particle mean radius $\langle r \rangle$, for instance fireball radius is $R_0 = 10$ fm, the contribution of the short-range repulsion is negligible as it seen in Fig. 4. So, the strong FSI effectively extends the size of the region from which the particles are allowed for correlations. If we unite two cases where the radius of homogeneity is small $R_{homo} \leq 4$ fm and large $R_{homo} \geq 7$ fm (hence, it is comparable with the size of the fireball) then we come to the conclusion that the radius which is extracted from the correlation function is approximately the radius of the fireball.

Final state interactions at high secondary multiplicities

Post-freeze-out phase density

In order to take into account post-freeze-out expansion of the pion system let us consider a pion phase-space distribution

$$\frac{\partial f(x,p)}{\partial x^0} + \mathbf{v}\cdot\nabla f(x,p) = 0 \ , \qquad \mathbf{v} = \frac{\mathbf{p}}{\omega(\mathbf{p})}$$

where $\omega(\mathbf{p}) \equiv \sqrt{m_\pi^2 + \mathbf{p}^2}$.

Asymptotic condition:

$$\lim_{t\to\infty} f(t,\mathbf{x}=0;\mathbf{p}) = 0\,.$$

Solution:

$$\boxed{f(t,\mathbf{R},\mathbf{p}) = f_0(\mathbf{R}-\mathbf{v}t,\mathbf{p}) \ ,}$$

where $f_0(\mathbf{R},\mathbf{p})$ is the initial distribution at $t=0$.

We assume the initial $(t=0)$ distribution function:

$$f_0(\mathbf{R},\mathbf{p}) = n_0(\mathbf{R})g_0(\mathbf{p}) \ ,$$

where

$$g_0(\mathbf{p}) = \frac{2\pi^2}{m_\pi^2 T_\mathrm{f} K_2\left(\frac{m}{T_\mathrm{f}}\right)} e^{-\omega(\mathbf{p})/T_\mathrm{f}}\,, \qquad n_0(\mathbf{R}) = N_\pi \frac{e^{-R^2/2R_\mathrm{f}^2}}{(2\pi)^{3/2}R_\mathrm{f}^3}\,,$$

with

$$\int \frac{d^3p}{(2\pi)^3}\, g_0(\mathbf{p}) = 1\,, \qquad \int d^3R\, n_0(\mathbf{R}) = N_\pi\,.$$

N_π is the total number of pions, T_f and R_f are the temperature and the mean radius of the system at time $t=0$ (freeze-out).

The spatial distribution of the particles at time t:

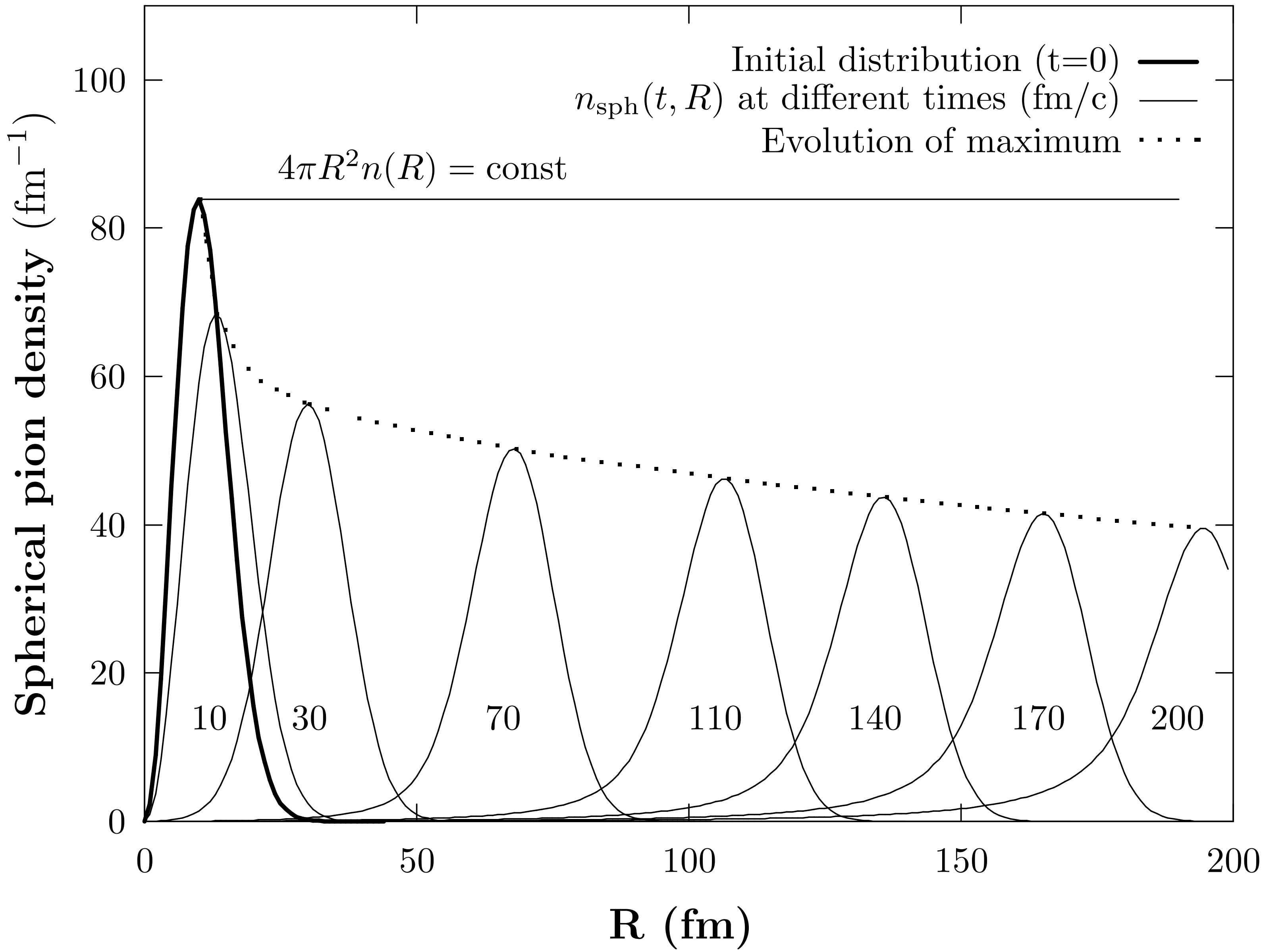
$$n(t,\mathbf{R}) = \int \frac{d^3p}{(2\pi)^3}\, n_0\left(\mathbf{R} - \frac{\mathbf{p}}{\omega(\mathbf{p})}t\right) g_0(\mathbf{p}) \ .$$

The "spherical density":

$$n_{\mathsf{sph}}(t,R) \equiv 4\pi R^2 n(t,R)\,, \quad \text{where} \quad \int_0^\infty dR n_{\mathsf{sph}}(t,R) = N_\pi\,.$$

This quantity may be treated as the number of pions in the shell with unit thickness at time t and at a distance $R = |\mathbf{R}|$ from the fireball center. Hence, $n_{\mathsf{sph}}(t,R)$ is a one-dimensional spatial distribution function and, evidently, the area under this curve at any time is equal to the particle number N_π, because of its normalization. We evaluate this function for the following freeze-out conditions:

$n_{\mathsf{f}} = 0.03\,\mathsf{fm}^{-1/3}$, $R_{\mathsf{f}} = 7.1$ fm, $T_{\mathsf{f}} = 180$ MeV.


Initial distribution (t=0)
$n_{\mathrm{sph}}(t,R)$ at different times (fm/c)
Evolution of maximum
$4\pi R^2 n(R) = \mathrm{const}$
10
30
70
110
140
170
200
0
20
40
60
80
100
0
50
100
150
200
Spherical pion density (fm^{-1})
R (fm)

The spherical distribution is always almost Gaussian-like and the velocity of the distribution maximum is very close to the velocity of light. The horizontal line denotes a constant spherical density

$$4\pi R^2 n(R) = \text{const} \quad \Rightarrow \quad n(R) = \frac{\text{const}'}{R^2}\,.$$

(for taken freeze-out conditions $\text{const}' \approx 85/4\pi$).

Obviously, at any time t:

$$n(t,R) \leq n_{\mathsf{f}} \frac{R_{\mathsf{f}}^2}{R^2}\ ,$$

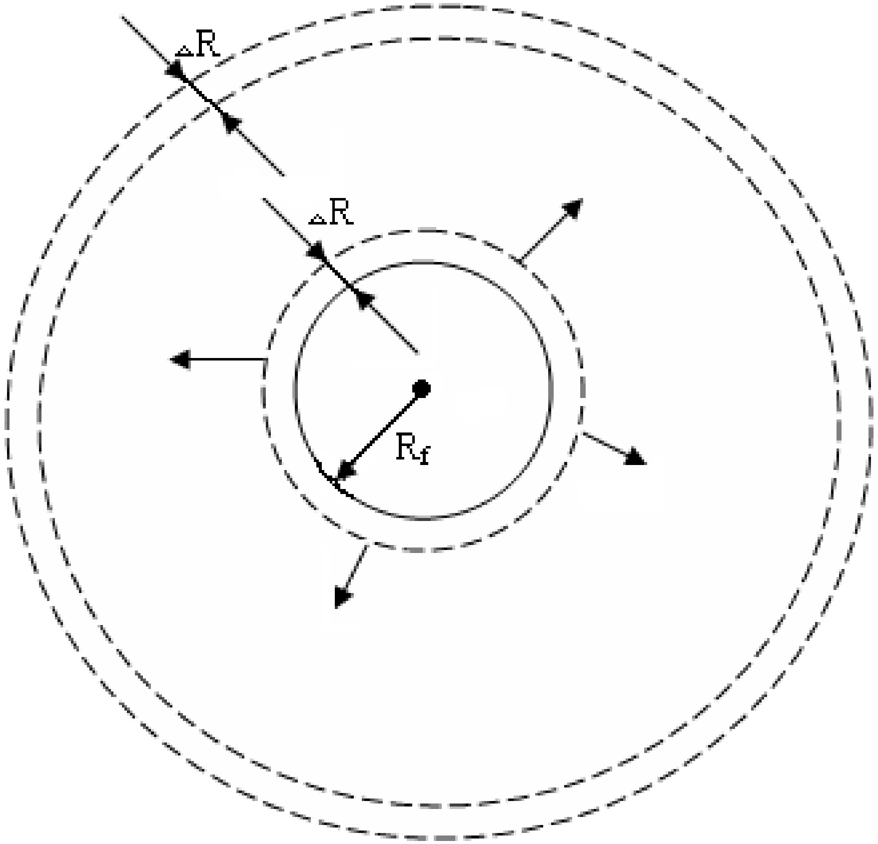
where $n_{\mathsf{f}} = \max\{n(t=0,R)\}$ and the equality is reached for an expanding system where the radial particle velocities are equal. Indeed, the spatial volume of the expanding pion system in the solid angle Ω is $\Delta V = \Omega \cdot R^2 \cdot \Delta R$, where R is the distance from the centre of the fireball and ΔR is the thickness of the layer, which we keep constant. If one keeps the radial velocities of the particles equal then, the number of particles ΔN in this volume is constant and the density reads:

$$n(R) = \frac{\Delta N}{\Omega \cdot R^2 \cdot \Delta R} = \frac{\mathrm{const}}{R^2} \, .$$

We parametrise the pion density at post-freeze-out expansion of the pion system as

$$\boxed{n(R) = n_{\mathrm{f}} \frac{R_{\mathrm{f}}^2}{R^2} \, ,}$$

where n_{f} is the freeze-out pion density and R_{f} is the freeze-out radius.



ΔR
ΔR
Rf

Two-particle potential

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi(t,\mathbf{r}) = 4\pi e\left(n^{(+)} - n^{(-)}\right) .$$

In the mean field approximation the density of charged pions $n^{(\pm)}$ is related to that of neutral pions $n^{(0)}$ via a Boltzmann factor:

$$n^{(\pm)} = n^{(0)} \ \mathrm{exp}\left(\mp\frac{e\phi}{T_{\mathsf{f}}}\right) .$$

The limit $e\phi \ll T_{\mathsf{f}}$ results in:

$$e\phi \ll T_{\mathsf{f}} \quad \Rightarrow \quad n^{(\pm)} = n^{(0)}\left(1 \mp \frac{e\phi}{T_{\mathsf{f}}}\right) .$$

This requires that the pions are not closer than $\sim 10^{-2}$ fm to one another at $T_{\mathsf{f}} \approx 180\,\mathrm{MeV}$.

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi(t,\mathbf{r}) = -\frac{8\pi\alpha}{3\,T_\mathrm{f}} n_\pi \phi(t,\mathbf{r})\,.$$

Constant density

$$U_{\pi\pi}(r) = \alpha \frac{e^{-r/R_{scr}}}{r}\,, \qquad \frac{1}{R_{scr}} = \sqrt{\frac{8\pi}{3}\alpha} \cdot \sqrt{\frac{n_\pi}{T_f}}$$

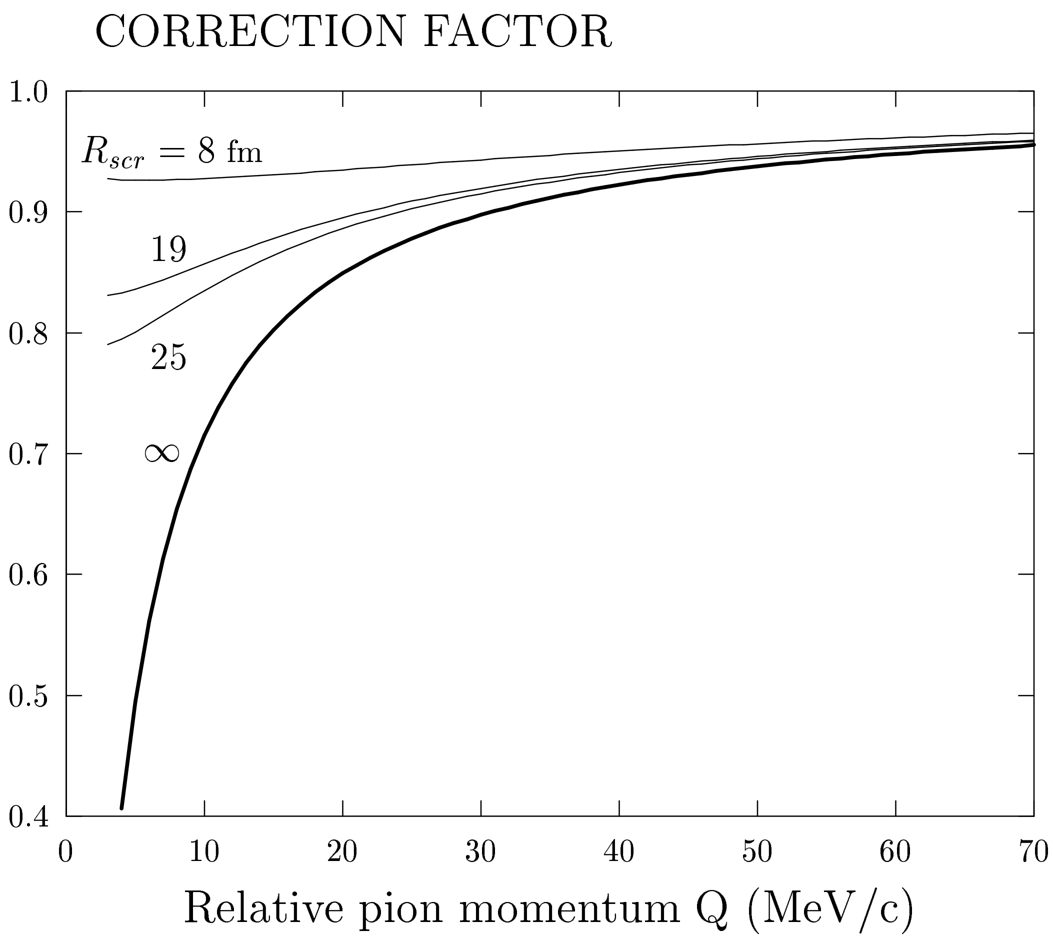
$$G_\mathrm{cor}(Q) = \mid \psi(\mathbf{r}=0) \mid^2$$

1) RHIC (LHC): $N_\pi = 8000$, $T_f = 180$ MeV, $R_f = 7.1$ fm, $\Rightarrow$ $R_{scr} = 7.9$ fm,

2) SPS-1: $T_f = 187\ MeV$, $\tau_f = R_L \approx 6.0$ fm, $R_T = 6$ fm, $\Delta y = 3$, $dN/dy = 40$,

$\Rightarrow$ $R_{scr} = 19.3$ fm,

3) SPS-2: $N_\pi = 800$, $T_f = 190$ MeV, $R_f = 7$ fm, $\Rightarrow$ $R_{scr} = 25$ fm.

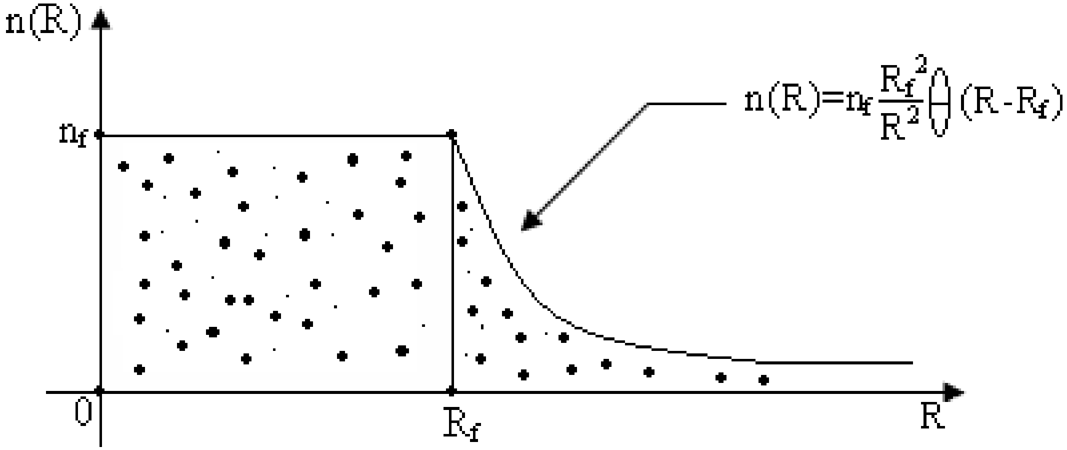


D.V. Anchishkin, W.A. Zajc, G.M. Zinovjev, Ukrainian J. Phys., **47** 451 (2002) .

Spatial dependence of the post freeze-out pion density

In the spherically expanding system the density of the environment depends on time t and distance from the fireball R.

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi(t,\mathbf{r}) = -\frac{8\pi\alpha}{3\,T_{\mathsf{f}}} n_\pi(R)\,\phi(t,\mathbf{r}) \ .$$



The presence of the potential $\phi(t, \mathbf{r})$ on the r.h.s. of this equation means that we assume polarization of the medium after freeze-out (polarization is accumulated by and its expansion).

It is really can be exploited in the frame of the adopted model of the post-freeze-out density, i.e. for the density profile.. Indeed, in the frame of this model Debye screen radius scales with R as $R_D = \left(\frac{3T}{8\pi\alpha n_\pi}\right)^{1/2} \frac{R}{R_\mathsf{f}}$, and mean distance between particles scales as $a = 1/n^{1/3}(R) = \frac{1}{n_\pi^{1/3}} \left(\frac{R}{R_\mathsf{f}}\right)^{2/3}$. Hence, if the plasma criterium was fulfilled at freeze-out times, i.e. $R_D = R^f_\mathsf{scr} \gg a = n_\pi^{-1/3}$, it would be fulfilled at the stage of expansion (we just follow the rules of a game which are dictated by the model). It means that our model is self-consistent in the sense of plasma criterium.

$$R \approx R_{\mathsf{f}} + v_{\mathsf{cm}} \cdot t \ , \qquad r \approx v_{\mathsf{rel}} \cdot t \ , \quad \Rightarrow \quad R = R_{\mathsf{f}} + \frac{v_{\mathsf{cm}}}{v_{\mathsf{rel}}} \cdot r \ .$$

$$n\big(R(r)\big) = \left(\frac{v_{\mathsf{rel}}}{v_{\mathsf{cm}}}\right)^2 \frac{n_{\mathsf{f}} R_{\mathsf{f}}^2}{(r+\overline{r})^2} \quad \text{where} \quad \overline{r} \equiv R_{\mathsf{f}} \frac{v_{\mathsf{rel}}}{v_{\mathsf{cm}}} \ .$$

$$c(q) = \frac{R_{\mathsf{f}}}{R^f_{\mathsf{scr}}} \frac{v_{\mathsf{rel}}(q)}{v_{\mathsf{cm}}} = \frac{\overline{r}}{R^f_{\mathsf{scr}}} , \qquad R^f_{\mathsf{scr}} = \left(\frac{3\, T_{\mathsf{f}}}{8\pi\alpha n_{\mathsf{f}}}\right)^{1/2} \ .$$

$$\boxed{\left(1 - v_{\mathsf{rel}}^2\right) \frac{d^2\phi(r)}{dr^2} + \frac{2}{r}\frac{d\phi(r)}{dr} - \frac{c^2(q)}{(r+\overline{r})^2}\phi(r) = 0 \ .}$$

$$\phi(r) = \frac{e}{r}\left(\frac{\overline{r}}{r+\overline{r}}\right)^b \ ,$$

where $\phi(r) \to e/r$ when $r \to 0$.

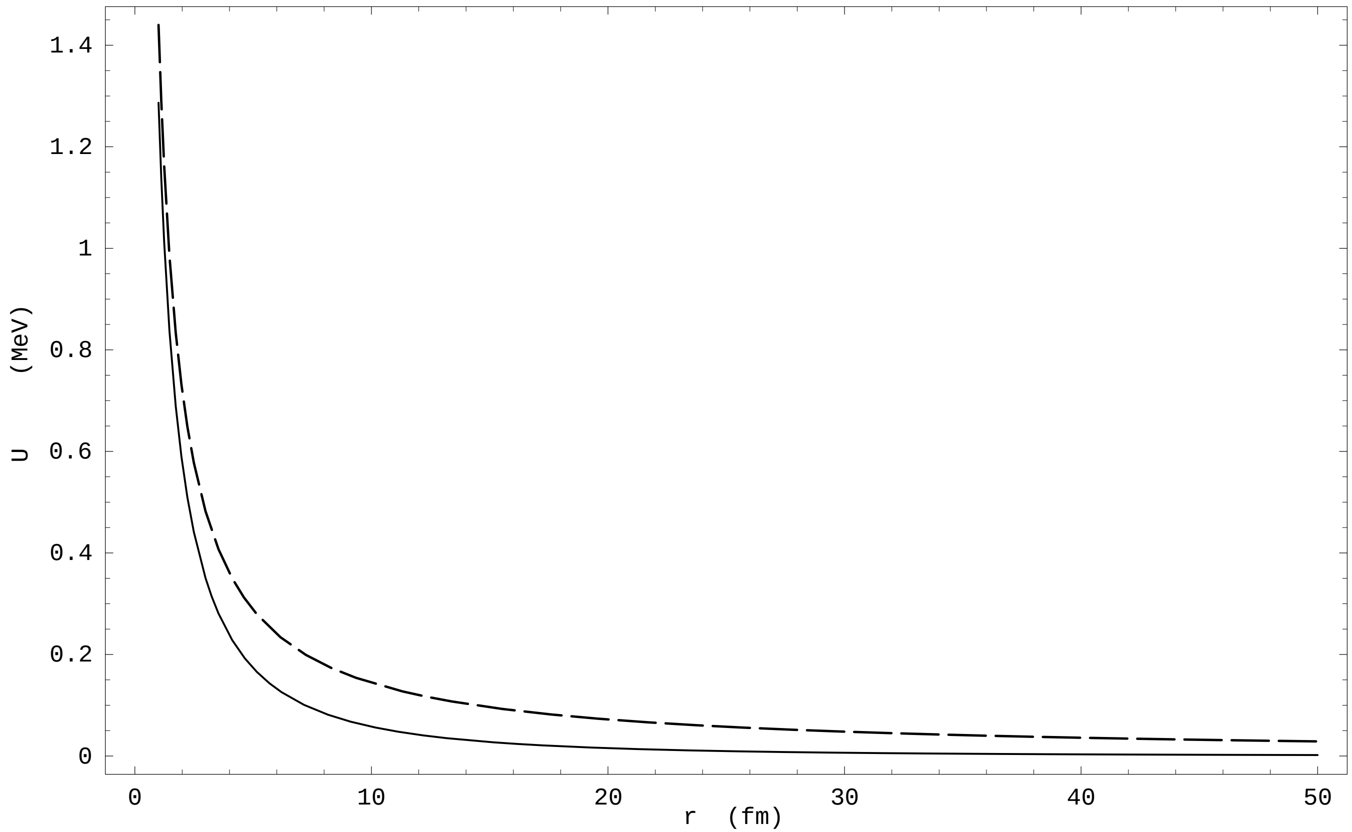
The exponent b is a solution of the quadratic equation:

$$b(q) = -\frac{1}{2} + \frac{1}{2}\sqrt{1+4c^2(q)}\,.$$

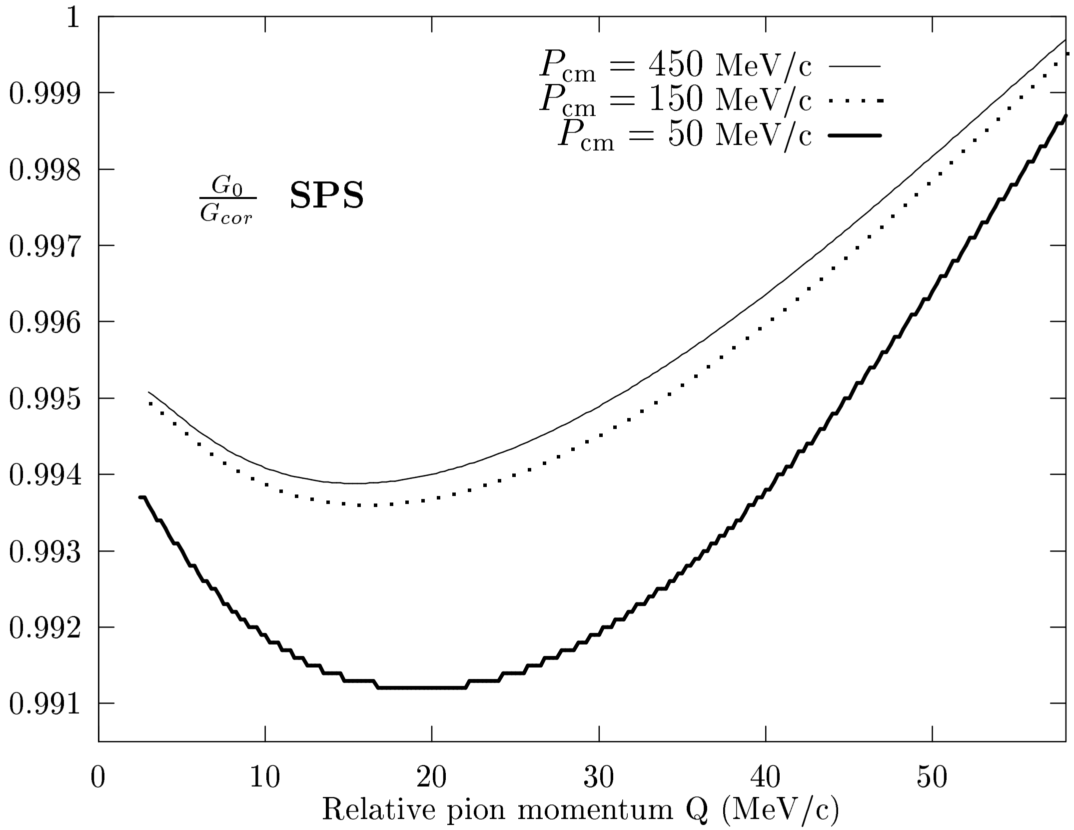
In the limit $n_{\mathsf{f}} \to 0$ one has $b \to 0$. This means that potential $\phi(r)$ smoothly transforms into the Coulomb potential when we go to the empty space.

Modifications to the Coulomb potential are of power-law:

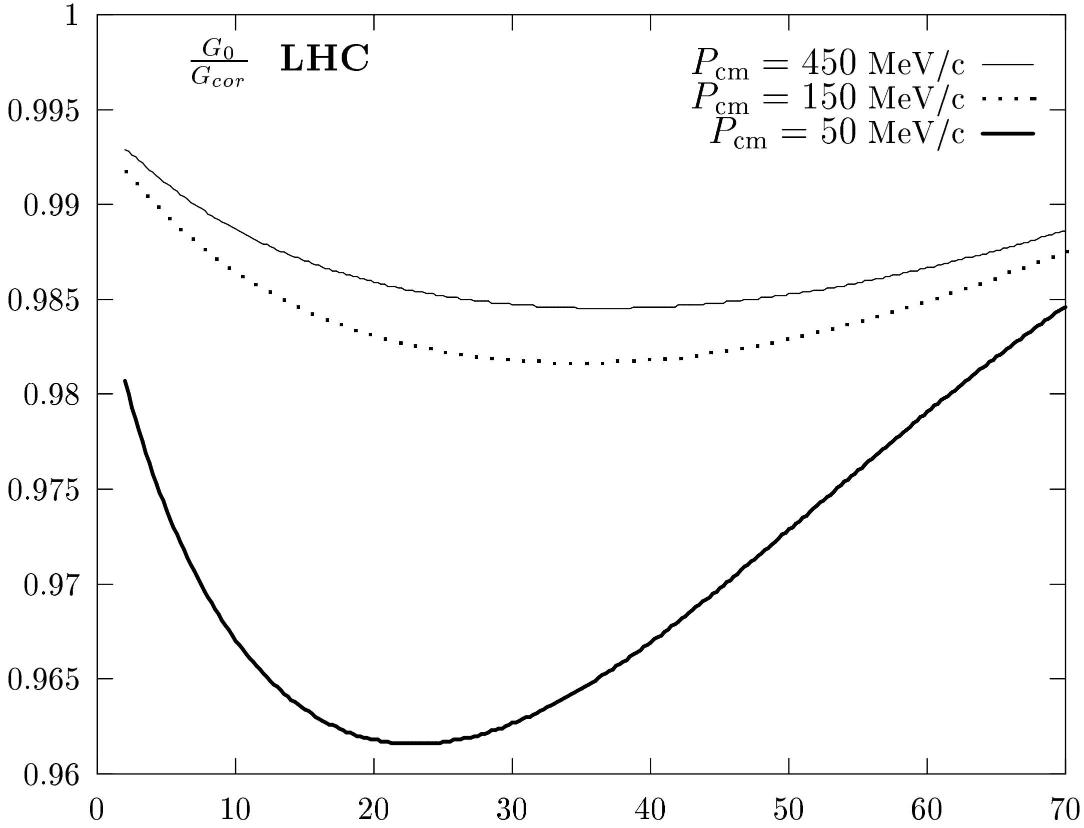
$$\boxed{U_{\mathrm{expan}}(r) = \frac{\alpha}{r}\left(\frac{\overline{r}}{r+\overline{r}}\right)^b \ .}$$



The potential energy $U_{\mathrm{expan}}(r)$ versus radius.
$n_{\mathrm{f}} = 0.2$ fm^{-3}, $T_{\mathrm{f}} = 120$ MeV, $R_{\mathrm{f}} = 10.0$ fm, $|\mathbf{K}| = 50$ MeV/c, $|\mathbf{q}| = 100$ MeV/c. The Coulomb potential energy $U_{\mathrm{Coul}}(r) = \alpha/r$ is plotted as dashed curve.



$N_\pi = 1000$, $T_{\mathsf{f}} = 190$ MeV, $R_{\mathsf{f}} = 7$ fm



$N_\pi = 8000$, $T_{\mathsf{f}} = 190$ MeV, $R_{\mathsf{f}} = 7$ fm

Summary and conclusions

- In our first approach, it was assumed the constant density of secondary particles which fill all space. In this case for RHIC and LHC experiments, the screening radius of the Coulomb interaction at the freeze-out density and temperature could be comparable with the source size and therefore the factorization is no longer valid.

- In our second model, we reduce the time evolution of two interacting pions to the stationary problem at the price of overestimation the density of the environment in which two probe particles move.

- Kinematics of an expanding fireball reveals an important regulating parameter that is the ratio of the relative velocity of the detected pions and their center-of-mass velocity in the rest frame of a fireball, $v_{\mathrm{rel}}/v_{\mathrm{cm}} \approx Q/P_{\mathrm{cm}}$.

• Due to our estimations a distortion of the Gamov factor, which is taken as an indicator of the final state Coulomb interaction, is practically negligible even if we overestimate the density of the secondary particles. Account for these interactions is of great importance for interpretation of the interferometry experiments on SPS (CERN), RHIC (BNL) and future experiments on LHC (CERN). Work along further investigation of the effect of high multiplicities on the electromagnetic final state interactions is in progress.