Universal properties of the confining string in gauge theories

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Plan of the talk

The long life of the confining string

The long life of the confining string

1969 Nambu in his reinterpretation of the Dual Resonance Model of Veneziano: the quarks inside nucleons are tied together by strings (Nielsen, Susskind, Takabayashi, 1970)

1974 Wilson puts the gauge theories on a lattice. In the strong coupling expansion the colour flux is concentrated in a confining string.The v.e.v. of a large Wilson loop γ can be written as a sum of terms associated to surfaces encircled by γ

1975 The QCD vacuum as a dual superconductor, the strings are long dual Abrikosov vortices ('t Hooft, Mandelstam and Parisi)

1980 The quark confinement is seen in lattice simulations (Creutz, Jacobs and Rebbi)

1981 Roughening transition: The confining string fluctuates as a free vibrating string (Lüscher, Münster, Symanzik, Weisz..)

The free bosonic string

The effective string picture of the Wilson loop

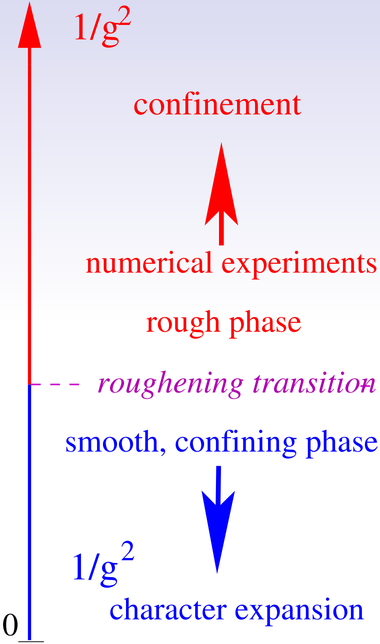
- The vacuum expectation value of large Wilson loops can be represented by the functional integral over the transverse displacements h_i of the string of minimal length

$$\langle W_f(C)\rangle = \int \prod_{i=1}^{D-2} \mathcal{D}h_i \exp\left[-\int d^2\xi\, \mathcal{L}(h_i)\right]$$

- The effective string action $S = \int d^2\xi\, \mathcal{L}(h_i)$ is largely unknown, except for its asymptotic form

$$S \to \sigma A + \frac{\sigma}{2}\int d^2\xi \sum_{i=1}^{D-2}(\partial_\alpha h_i \partial^\alpha h_i)$$

❄ *it brings about effects which are (more than) universal, i.e. independent of the gauge group*



Area law

$$\langle W_\gamma \rangle \propto R_\gamma^{\frac{D-2}{4}}\, c_\gamma\, e^{-b\,|\gamma| - \sigma\, A_\gamma}$$

A_γ = minimal area of $\Sigma : \partial\Sigma = \gamma$
R_γ= linear size of γ
c_γ = shape function
$(c_{rectangle} = [\eta(it/r)]^{-\frac{D-2}{2}})$

$$\langle W_\gamma \rangle \propto e^{-b\,|\gamma| - \sigma\, A_\gamma}$$

Universal string effects

- Two main consequences
  1. *Quantum broadening of the flux tube:* the mean area w^2 of its cross-section grows logarithmically with the interquark distance r

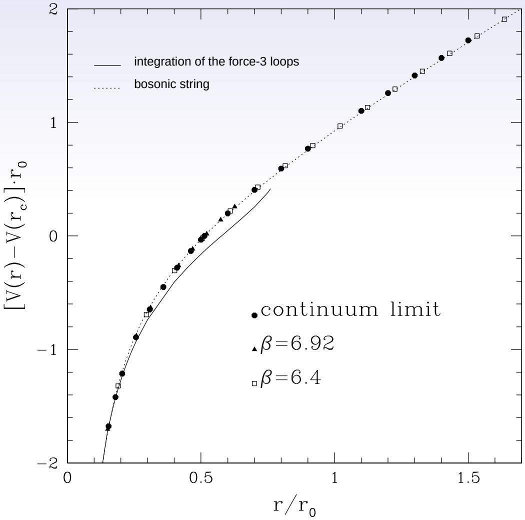
$$w^2 = \frac{1}{2\pi\sigma} \log(r\,\Lambda)$$

  2. *Lüscher term,* in the confining, static interquark potential

$$V(r) = \sigma\, r + \mu - \frac{\pi}{24}\frac{D-2}{r}$$

- The Lüscher term is simply the Casimir, or zero point energy E_o of a string of length r with fixed ends:
- ➪ normal modes: $\frac{\pi\, n}{r}, \quad n = 1, 2, \ldots$
- ➪ $E_o = (D-2)\sum_n \frac{\pi\, n}{2r} = (D-2)\,\frac{\pi}{2}\,\zeta(-1) = -\frac{\pi}{24}\frac{D-2}{r}$
- the first uncontroversial observations in the 90's

SU(3) interquark potential S Necco & R Sommer 2001



$r_o \sim 0.45 fm$

How thick are chromoelectric flux tubes?

M Lüscher , G M ünster and P Weisz, 1981

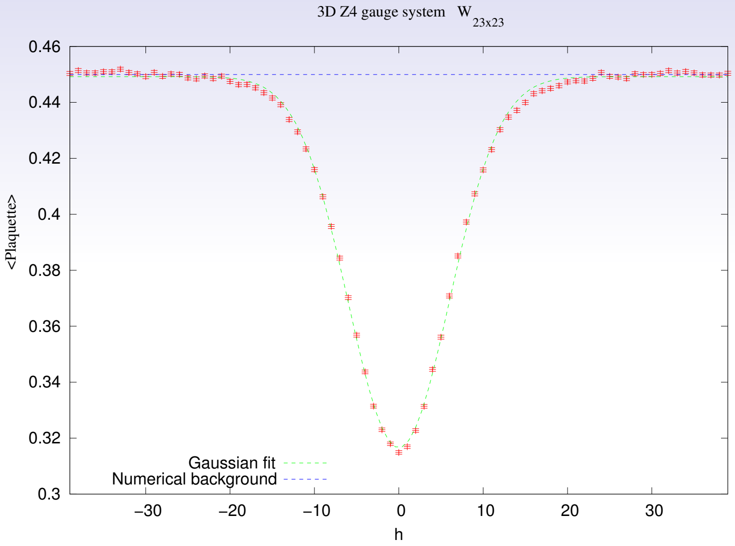
- In gauge theory one may define the density $\mathcal{P}(x)$ of the flux tube in the point x through a plaquette operator P_x

$$\mathcal{P}(x) = \frac{\langle W(C)\, P_x \rangle - \langle W(C) \rangle \langle P_x \rangle}{\langle W(C) \rangle}$$

and the mean squared width as

$$w^2 = \frac{\int h^2 \mathcal{P}(x) d^3x}{\int \mathcal{P}(x) d^3x}$$

h= distance between the plaquette and the plane of the Wilson loop


3D Z4 gauge system W 23x23
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Gaussian fit
Numerical background
–30
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–10
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10
20
30
h

flux width in the confining string picture

- On the string side

$$w^2(\xi_1, \xi_2) = \sum_{i=1}^{D-2} \langle (h_i(\xi) - h_i^{CM})^2 \rangle_{gauss}$$

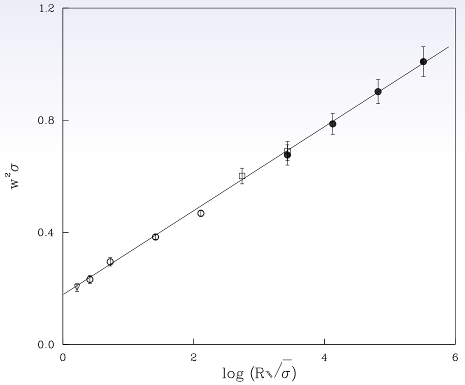
- yields logarithmic broadening with a universal slope

$$w^2 = \frac{1}{2\pi\sigma} \log(r\,\Lambda)$$

r = linear size of the loop
Λ= shape-dependent UV scale

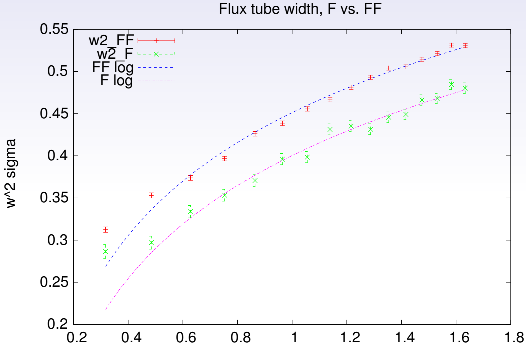
w^2 in 3 D $\mathbb{Z}_2$ gauge theory

M Caselle, FG, U Magnea,S Vinti 1995



- Logarithmic broadening is very difficult to be observed current SU(N) simulations,(so far checked compatibility only in SU(2) Bali 2004)
- in 3D $\mathbb{Z}_2$ case checked over distance scale ~ 100
- Recently observed also in 3D $\mathbb{Z}_4$ gauge theory

Flux broadening in 3 D $\mathbb{Z}_4$ S Lottini, FG, P Giudice 2007



$\sqrt{\sigma}R$

- In $\mathbb{Z}_4$ gauge theory there are two non-trivial confining repr.s
- both lead to logarithmic broadening of long flux tubes

- Notice that the Lüscher term is visible at a scale where the width of the flux tube is larger than its length!
- Contrarily to earlier belief the chromoelectric flux tube cannot be identified with the string-like degrees of freedom leading to universal quantum effects

Where are the string-like degrees of freedom?

the lesson of the gauge duals of 3D Q-state Potts models

Electric-magnetic duality in a 3D lattice

- Many lattice gauge systems in 3D have a dual description in terms of suitable 3D spin models
- Like in electric-magnetic duality, weakly coupled gauge systems correspond to strongly coupled spin systems and vice versa
- The prototype is the 3D $\mathbb{Z}_2$ gauge model, which is dual to the Ising model through the Kramers-Wannier tranformation:
  - Gauge model on a lattice $\Lambda \Leftrightarrow$ spin system on the dual lattice $\widetilde{\Lambda}$
  - $K_{gauge} = \frac{1}{2} \log \tanh K_{spin}$
- A wide class of models with a dual description in terms of a spin systems is formed by the gauge duals of the 3D Q-state Potts models

Q-state Potts models

= Spin models defined by the Hamiltonian on a cubic lattice Λ

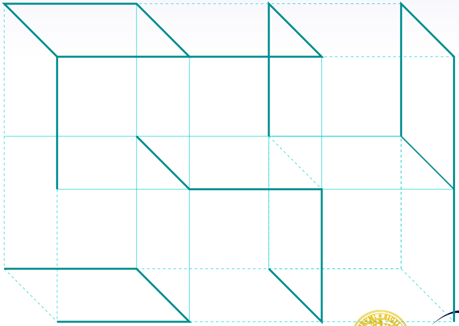
$$H = -\sum_{\langle i\, j \rangle} \delta_{\sigma_i\, \sigma_j}\,, \quad (\sigma = 1, 2 \ldots Q)$$

- ➪ Its global symmetry is the permutation group of Q elements S_Q
- ➪ In 3D it is dual to a gauge model with gauge symmetry S_Q
- ❋ The properties of the gauge theory can be read directly in the spin (or disorder parameter) formulation
- ❋ In these models the implementation of the confining mechanisms (monopole condensation & center vortices percolation) is particularly simple

Q-state Potts models admit a remarkable representation in terms of Fortuin Kasteleyn (FK) random clusters:

$$Z \equiv \sum_{\{\sigma\}} e^{-\beta H} = \sum_{G \subseteq \Lambda} v^{b_G} Q^{c_G} \quad ,$$

- each link of the lattice can be active or empty
  - $v = e^{\beta} - 1$,
  - $G =$ spanning subgraphs of Λ.
  - $b_G =$ number of links of G (active bonds –)
  - c_G number of connected components (FK clusters).
  - the FK random cluster representation allow to extend the model to any continuous Q



- All these models have a phase transition corresponding to the spontaneous breaking of the S_Q symmetry (magnetic monopole condensation) associated to the appearance of an infinite FK cluster
- much studied $Q = 2$ (Ising model) and $Q = 1$ (random percolation)
  [The partition function of the random percolation is trivial: $Z_{Q=1} = (1 + v)^N \equiv (1 - p)^{-N}$ N= total number of links; p= probability of an active link]
- The dual gauge theory is non-trivial for any $Q \geq 0$
- Any gauge-invariant quantity can be mapped exactly into a suitable observable of the Q-state Potts model

Example: Wilson loops

- The Wilson operators W_γ, are associated to arbitrary loops γ of the dual lattice $\widetilde{\Lambda}$ and their values on a graph G of active bonds are set by the following rule

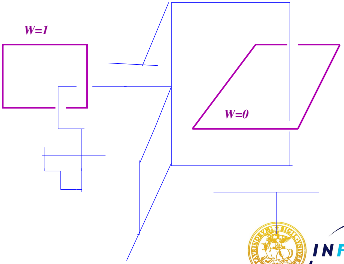
1. $W_\gamma(G) = 1$ if no cluster of G is topologically linked to γ;
2. $W_\gamma(G) = 0$ otherwise

- linking of W depends only on *closed paths*
- The area law falloff of $\langle W_\gamma \rangle$ requires an infinite cluster

$$\Downarrow$$

hence the formation of an infinite, percolating FK cluster= *magnetic monopole condensate*

$$\Rightarrow \langle W_\gamma \rangle = \frac{\sum_{G\subseteq\Lambda} W_\gamma(G)\, v^{b_G} Q^{c_G}}{Z}$$



$W_\gamma(G)$ acts as a projector on the configuration G: $W_\gamma(G) = 1$ selects only those configurations where there is at least one simply connected surface $\Sigma \subset \tilde{\Lambda}$ such that

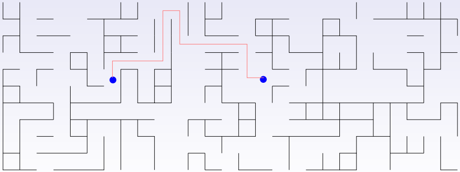
1. it does not intersect any active link of G
2. its boundary $\partial\Sigma = \gamma$

Denoting with p the occupancy probability of an active link, the total weight of Σ is $\propto \; (1-p)^{Area \; \Sigma}$

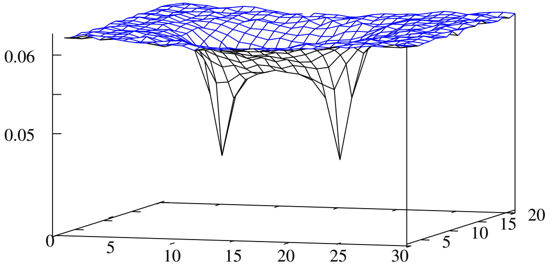
- the most favoured G 's with $W_\gamma(G) = 1$ are associated to a $\Sigma \subset \tilde{\Lambda}$ of minimal area with $\gamma = \partial\Sigma$

A two-dimensional example

A single configuration with $W = 1$



Accumulation of 10^6 configurations with W=1:



Universal shape effects in Wilson loops

- The IR Gaussian action gives rise to a universal multiplicative correction Ambjorn, Olesen & Peterson 1984

$$\langle W(r,t)\rangle = c\, \mathrm{e}^{-\sigma rt-\mu(r+t)} \left[\frac{\sqrt{r}}{\eta(it/r)}\right]^{\frac{D-2}{2}}$$

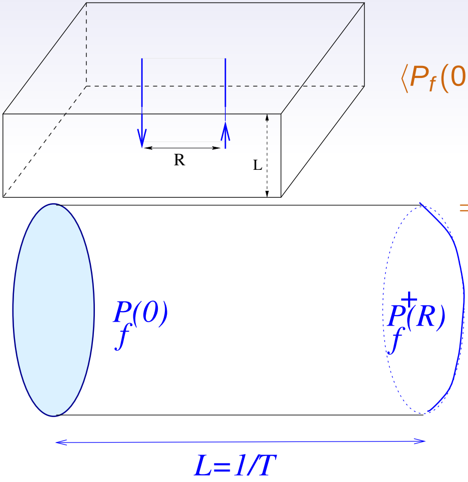
$$\eta(\tau) \equiv q^{\frac{1}{24}} \prod_{n>0}(1-q^n)\,, \quad q = e^{2i\pi\tau}$$

$\eta =$ Dedekind eta function

- $$V(r) = -\lim_{t\to\infty} \log(\langle W(r,t)\rangle) = \sigma\, r + \mu - \frac{\pi}{24}\frac{D-2}{r} + \dots$$

- *on a lattice, much easier to see universal shape effects rather than the Lüscher term*

Universal shape effects in Polyakov loop correlation function at finite T (Olesen,1985)



$$\langle P_f(0)\, P_f^\dagger(R)\rangle_T = e^{-\sigma RL - \mu L}\, \eta(iL/2R)^{2-D}$$

$$\simeq e^{-\mu L - \sigma(T) RL} \quad (2R > L)$$

$$\Rightarrow \; \sigma(T) = \sigma(0) - (D-2)\frac{\pi}{6}\, T^2$$

- Two different approaches to study shape effects

1. Use zero-momentum projection of the Polyakov loop correlators

$$\int dx_\perp \langle P(0)\, P^\dagger(x_1, x_\perp)\rangle = \sum_n |v_n|^2\, e^{-E_n|x_1|}$$

evaluate numerically the transition matrix elements v_n and the energy levels E_n of the first excited string states and compare them to the expectations of the confining string A Athenodorou, B Bringoltz, M Teper 2007)

2. Try to fit directly the predicted shape dependence to the numerical data in order to find the range of validity (Torino group)

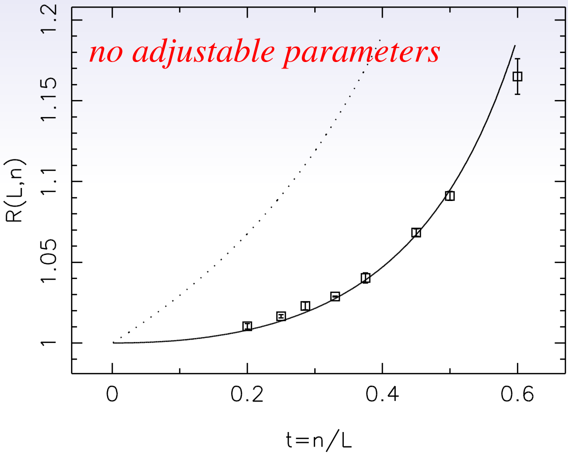
universal shape effects

- A suitable quantity which is sensible to the universal shape effects is the function $\mathcal{R}(n,L)=\exp(-n^2\sigma)\frac{\langle W(L-n,L+n)\rangle}{\langle W(L,L)\rangle}$
- asymptotically (large L and $L-n$) (Gaussian limit) $\mathcal{R}$ becomes only a function $f(t)$ of the ratio $t=\frac{n}{L}$

$$\mathcal{R}(n,L)\;\rightarrow\;f(t)=\left[\frac{\eta(i)\sqrt{1-t}}{\eta\left(i\frac{1+t}{1-t}\right)}\right]^{\frac{1}{2}}$$

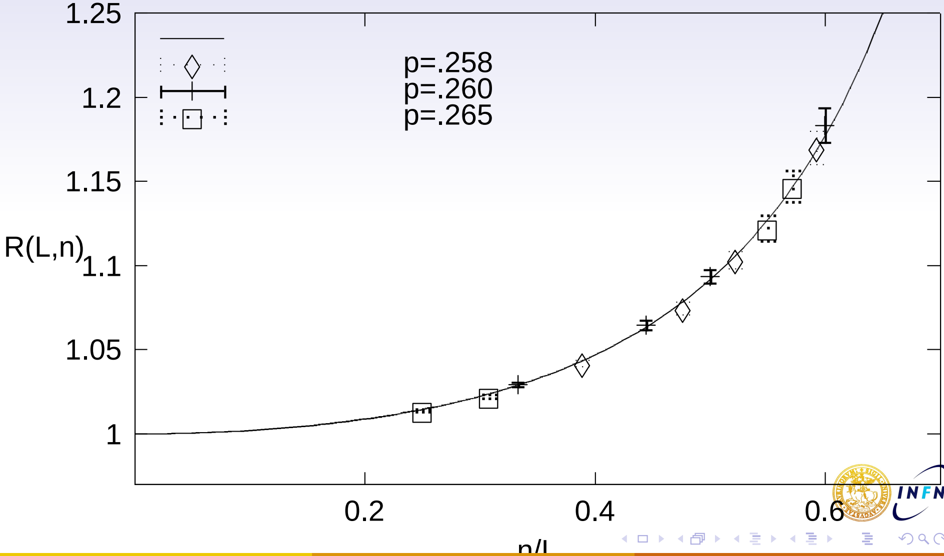
$\mathcal{R}(L, n)$ in 3D $\mathbb{Z}_2$ gauge theory

M Caselle,R Fiore,FG, M Hasenbusch, P Provero (1997)



$\mathcal{R}(L, n)$ in 3D gauge dual to random percolation (Q=1)

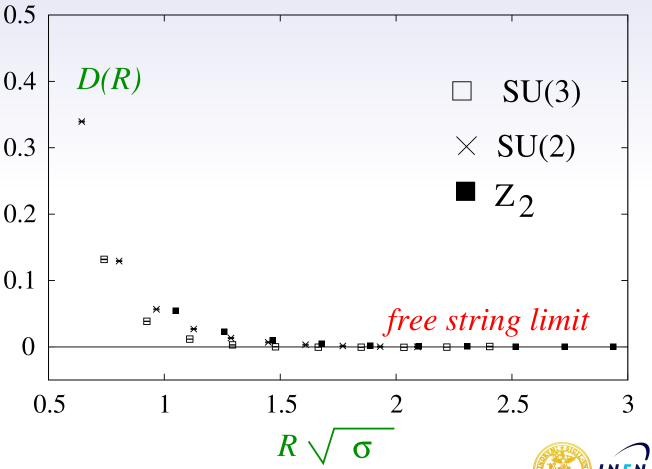
FG, S Lottini, M Panero, A Rago (2005)



Short distance behaviour of the confining string (3D)

M Caselle, M Hasenbusch & M Panero 2004

- $D(R) =$ scale-invariant combination of Polyakov correlators
- $D(R) = 0$ free bosonic string limit



Beyond the free string limit

An effective action for the confining string

- $\langle P(0)\,P^{\dagger}(R)\rangle = \int \mathcal{D}h\, e^{-S[h]}$
- The simplest choice: Nambu-Goto action:
  $S[h] = \sigma\, Area = \sigma \int d^2\xi \sqrt{1 + \partial_\alpha h_i \partial^\alpha h^i}$, however
  - The rotational invariance is spoiled by light-cone quantisation, or
  - Covariant quantisation leads to additional longitudinal oscillators outside the critical dimension of 26
  - the only degrees of freedom required by the low energy theory are the D-2 transverse oscillators
- A possible way-out (Polchinski & Strominger 1991): apply the quantisation à la Polyakov, using however the induced metric $g_{\alpha\,\beta} = \partial_\alpha h^i \partial_\beta h_i$
- The resulting non-polynomial action is rather complicated, but the first three terms in the expansion in the parameter $1/(\sigma RL)$ coincide with the ones of Nambu-Goto: Drummond 2004, Hari Dass & Matlock 2006
- $S[h] = \sigma\left[RL + \frac{1}{2}\partial_\alpha h_i \partial^\alpha h^i - \frac{1}{8}(\partial_\alpha h_i \partial^\alpha h^i)^2 + \dots\right]$

- The confining string representation of the Polyakov loop correlation function

$$\langle P(0)\,P^{\dagger}(R)\rangle_{T=1/L} = \int \mathcal{D}h\, e^{-S[h]}$$

is only expected to be valid to any finite order of the perturbation expansion in the parameter $1/(\sigma RL)$

- Decays of highly excited states through glueball radiation are not included in the string description

- ⇨ The Polyakov loop correlator and the corresponding string partition function differ by non-perturbative corrections of the order e^{-mL} (m= mass of the lightest glueball)

Open-closed string duality

- The Polyakov loops can be considered as sources of closed strings wrapping around a compact direction x_1 and transverse position $x_\perp = (x_2, \ldots, x_{D-2})$
- The zero-momentum projection of the Polyakov loop correlation function is expected do have the following spectral representation

$$\int dx_\perp \langle P(0)\, P^\dagger(x)\rangle = \sum_n |v_n|^2\, e^{-E_n|x_1|}$$

- Lüscher and Weisz (2004) showed that this implies

$$\langle P(0)\, P^\dagger(x)\rangle = \sum_{n=0}^{\infty} |v_n|^2 2R \left(\frac{E_n}{2\pi R}\right)^{\frac{D-1}{2}} K_{\frac{D-3}{2}}(E_n R)$$

which severely constrains the functional form of the Polyakov loop correlator [$K_j(x) =$ Bessel f.]

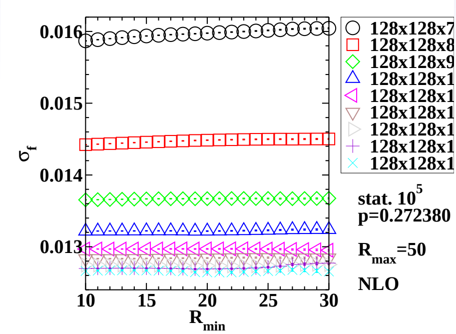
Two-loop approximation

- A systematic analysis of the most general effective string action up to $O[(\frac{1}{\sigma RL})^3]$ yields Lüscher & Weisz 2002
  $S[h] = \sigma RL + \frac{\sigma}{2}\int d^2\xi \partial_\alpha h_i \partial^\alpha h^i + S_1 + S_2$
- $S_1 = -\frac{b}{4}\int d\xi_2 [(\partial_1 h)^2_{\xi_1=0} + (\partial_1 h)^2_{\xi_1=R}]$, excluded by open-closed string duality Lüscher & Weisz, 2004
- $S_2 = \frac{1}{4}\int d^2\xi \left[c_2(\partial_\alpha h_i \partial^\alpha h^i)^2 + c_3(\partial_\alpha h_i \partial^\beta h^i)(\partial^\alpha h_j \partial_\beta h^j)\right]$
- open-closed string duality implies Lüscher & Weisz, 2004
  $(D-2)c_2 + c_3 = \frac{D-4}{2\sigma}$, $D = 3 \;\Rightarrow S_2 = -\frac{1}{8}(\partial_\alpha h_i \partial^\alpha h^i)^2$ = N-G term!
- $\langle P(0)P^\dagger(R)\rangle_{T=1/L} = e^{-\mu L - \sigma LR}\left(\eta(\tau)\, e^{-\frac{\pi^2 L\, E(\tau)}{1152\sigma R^3} + O(1/R^5)}\right)^{2-D}$
  $\simeq e^{-\mu L - \sigma(T)LR + O(1/R^3)}$
- $\tau = L/2R$, $E = 2E_4 - E_2^2$, $E_n(\tau)$ = Eisenstein series

- The T dependence of the string tension turns out to be
- $\sigma(T) = \sigma - (D-2)\frac{\pi}{6}T^2 - (D-2)^2\frac{\pi^2}{72\sigma}T^4 + O(T^5)$ which agrees with LGT in the range $T \leq \frac{1}{2}T_c$
- These are the first terms of the exact N-G result Olesen 1985
  $\sigma(T) = \sigma\sqrt{1-\left(\frac{T}{T_c}\right)^2}$ which however disagrees with LGT data near T_c
- In gauge dual of random percolation one can reach very high precision in numerical calculations
- Try to evaluate the first non vanishing correction

$\sigma(T)$ in the gauge dual of random percolation

$$\Rightarrow \quad \sigma(T=1/L)=\sigma-\frac{\pi}{6L^2}-\frac{\pi^2}{72\sigma L^4}+\frac{\pi^3}{C\sigma^2L^6}+\mathcal{O}(1/L^8)$$



- $\Rightarrow C \neq \infty$
  - C should not depend on the lattice cut-off, i.e. on the occupancy probability p nor on the kind of lattice used

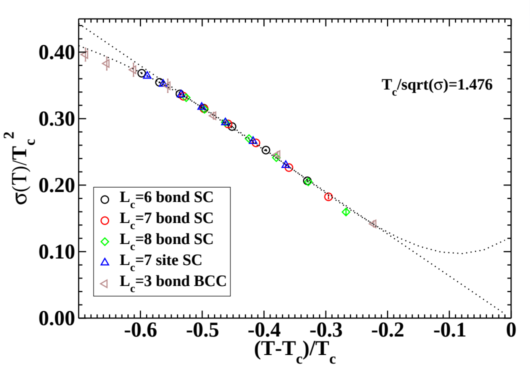
- check it for few different values of p and different lattices

  $p_1 = 0.272380$ (corresponding to $T_c = 1/6$) ➪ $C = 296 \pm 5$

  $p_2 = 0.268459$ (corresponding to $T_c = 1/7$) ➪ $C = 302 \pm 4$

  ......

- another check: The adimensional ratio $f(t) = \frac{\sigma(T)}{T_c^2}$ $\quad (t = \frac{T - Tc}{Tc})$ should not depend on p nor on the kind of lattice:



Conclusions

1. There are universal shape effects in Wilson loops and Polyakov correlators that are well understood and accurately explained in terms of an underlying confining bosonic string
2. The chromoelectric flux tube joining a quark pair cannot identified with the confining string
3. In gauge duals of Q-state Potts models it is possible to recognise stringlike degrees of freedom