Small, Medium and Giant Magnons

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D.Astolfi, V.Forini, G.Grignani and G.Semenoff, hep-th/0702043

B.Ramadanovic and G.Semenoff, arXiv:0803.4028 [hep-th]

G.Grignani and G.Semenoff, to appear

The AdS/CFT correspondence asserts an exact duality

IIB string on $AdS_5 \times S^5 \quad \leftrightarrow \quad \mathcal{N} = 4$ **Yang-Mills**

N units of 5-form flux on $S^5 \quad \leftrightarrow \quad$ SU(N) gauge group

radius of curvature $R^4/\alpha'^2 \quad = \quad g^2_{YM} N \equiv \lambda$ 'tHooft coupling

closed string coupling $4\pi g_s \quad = \quad g^2_{YM}$

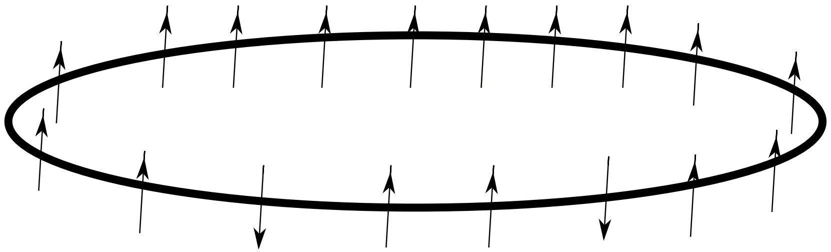
Energies of strings $\quad \leftrightarrow \quad$ conformal dimensions of operators

Free strings on $AdS_5 \times S^5 \quad \leftrightarrow \quad$ limit $N \to \infty$, $\lambda = g^2_{YM} N$ fixed

Weak coupling sigma model $\quad \leftrightarrow \quad$ **strong gauge theory**

$$S = \frac{\sqrt{\lambda}}{4\pi} \int |g| g^{ab} \partial_a X^\mu G_{\mu\nu} \partial_b X^\nu \;\leftrightarrow\; S = \frac{N}{4\lambda} \int d^4x \; \mathrm{Tr} F^2_{\mu\nu}$$

Finding spectrum of **planar** $\mathcal{N}=4$ Yang-Mills has a **spin-chain analogy**: (**J.Minahan, K.Zarembo hep-th/0212208**)



For example: scalar fields of $\mathcal{N}=4$ super-conformal YM: $\Phi^1, ..., \Phi^6$

$$Z = \Phi^1 + i\Phi^2 \quad , \quad \Phi = \Phi^3 + i\Phi^4 \quad , \quad \Psi = \Phi^5 + i\Phi^6$$

Large N planar limit ($N \to \infty$, $\lambda = g_{YM}^2 N$ fixed) : conformal dimensions of composite operators

$$\mathrm{Tr}\,[Z(0)Z(0)\Phi(0)Z(0)\Phi(0)Z(0)...] \quad J\ Z's + M\ \Phi's$$

YM interactions: $\Delta = J + M + \lambda(\text{one loop}) + \lambda^2(\text{two loops}) + \ldots$
Resolving degeneracy $\sim$ solving $PSU(2,2|4)$ spin chain with long ranged interactions

Ferromagnetic ground state of the spin chain:

$$\mathrm{Tr} Z^J$$

$\frac{1}{2}$-BPS operator, dimension $\Delta = J$ protected by supersymmetry

Symmetry of ground state $SU(2|2) \times SU(2|2) \times R^1 \subset SU(2,2|4)$

One flipped spin is a **"Magnon"** – short multiplet of this residual symmetry algebra

$$\mathrm{Tr} Z^{J-1} D_\mu Z \quad , \quad \mathrm{Tr} Z^J \Phi_i$$

$$\mathrm{Tr} Z^J \chi_\alpha^\beta \quad , \quad \mathrm{Tr} Z^J \chi_{\dot\alpha}^{\dot\beta}$$

with $\Delta = J + 1$

Because of cyclicity of the trace, they have zero magnon momentum

$$\sum_k e^{ipk} \mathrm{Tr} Z^k \Phi Z^{J-k} \sim \delta(p)$$

Two magnons

$$\sum_{k_1,k_2=0}^{J-1} e^{ip_1 k_1 + ip_2 k_2} \mathrm{Tr} Z Z ... \Phi_{k_1} ... \Phi_{k_2} ... Z \;\sim\; \delta(p_1 + p_2)$$

$$\Delta - J = 2 + \lambda(\mathrm{one-loop}) + \lambda^2(\mathrm{two-loop}) + \ldots$$

Two magnons at one loop

$$H_{\text{one loop}} = \frac{\lambda}{8\pi^2} \sum_i (1 - P_{i,i+1})$$

$$\sum_{1 \le k_1 < k_2 \le L} \psi(k_1, k_2) \mathrm{Tr} ZZ...\Phi_{k_1}...\Phi_{k_2}...Z \quad L = J + 2$$

$$\psi(k_1, k_2) = e^{ip_1 k_1 + p_2 k_2} + S(p_1, p_2) e^{ip_2 k_1 + p_1 k_2}$$

$$E = L + \frac{\lambda}{2\pi^2} \left(\sin^2 \frac{p_1}{2} + \sin^2 \frac{p_2}{2} \right) + ...$$

$$S = \frac{e^{ip_1 + ip_2} - 2e^{ip_1} + 1}{e^{ip_1 + ip_2} - 2e^{ip_2} + 1}$$

Periodic boundary conditions

$\psi(k_1, k_2) = \psi(k_2, k_1 + L) \rightarrow$ "Bethe equations"

$$e^{iLp_1} = S(p_1, p_2) \quad , \quad e^{iLp_2} = S(p_2, p_1)$$

Cyclicity of the trace implies $p_1 + p_2 = 0$

- The spin chain is thought to be integrable and solvable using a Bethe Ansatz
  N.Beisert, B.Eden, M.Staudacher hep-th/0610251
- Problem is simpler in the large volume limit.
  - planar Yang-Mills theory $N \to \infty$, $\lambda = g^2_{\mathrm{YM}} N$ fixed
  - infinite volume $J \to \infty$ with magnon momenta and λ fixed
- Bethe Ansatz has distinct quasi-particles. In infinite volume limit, integrability implies scattering with a factorized S-matrix.
- quasi-particle is a magnon
- 2-body S-matrix almost completely determined by (super-)symmetry: **N.Beisert hep-th/0603038,0606214**
- once infinite J spectrum is known – reconstruct finite J

In the SU(2) sector, the spin chain Hamiltonian is "known" to four loops

$$H = \sum_{n=0}^{\infty} \left(\frac{\lambda}{16\pi^2} \right)^n H_n$$

Permutation operator:

$$\{a, b, c, ...\} = \sum_{p=1}^{L} \mathcal{P}_{p+a}\mathcal{P}_{p+b}\mathcal{P}_{p+c}\cdots \quad , \quad \mathcal{P}_k = P_{k,k+1}$$

$$\begin{aligned} H_0 &= \{\} \quad , \quad H_1 = 2\{\} - 2\{1\} \\ H_2 &= -8\{\} + 12\{1\} - 2\left(\{1,2\} + \{2,1\}\right) \\ H_3 &= 60\{\} - 104\{1\} + 4\{1,3\} + 24\left(\{1,2\} + \{2,1\}\right) \\ &- 4i\epsilon_2\left(\{1,2,3\} + \{2,1,3\}\right) - 4\left(\{1,2,3\} + \{3,2,1\}\right) \end{aligned}$$

$$
\begin{aligned}
H_4 &= (-560-4\beta)\,\{\} + (1072+12\beta+8\epsilon_{3a})\,\{1\} \\
&+ (-84-6\beta-4\epsilon_{3a})\,\{1,3\} \\
&- 4\{1,4\} + (-302-4\beta-8\epsilon_{3a})\,(\{1,2\}+\{2,1\}) \\
&+ (4\beta+4\epsilon_{3a}+2i\epsilon_{3c}-4i\epsilon_{3d})\,\{1,3,2\} \\
&+ (4\beta+4\epsilon_{3a}-2i\epsilon_{3c}+4i\epsilon_{3d})\,\{1,1,3\} \\
&+ (4-2i\epsilon_{3a})\,(\{1,2,4\}+\{1,4,3\}) \\
&+ (4+2i\epsilon_{3a})\,(\{1,3,4\}+\{2,1,4\}) \\
&+ (96+4\epsilon_{3a})\,(\{1,2,3\}+\{3,2,1\}) \\
&+ (-12-2\beta-4\epsilon_{3a})\,\{2,1,3,2\} \\
&+ (18+4\epsilon_{3a})\,(\{1,3,2,4\}+\{2,1,4,3\}) \\
&+ (-8-2\epsilon_{3a}-2i\epsilon_{3b})\,(\{1,2,4,3\}+\{1,4,3,2\}) \\
&+ (-8-2\epsilon_{3a}+2i\epsilon_{3b})\,(\{2,1,3,4\}+\{3,2,1,4\}) \\
&- 10\,(\{1,2,3,4\}+\{4,3,2,1\}) \quad , \quad \beta = 4\zeta(3)
\end{aligned}
$$

Recent computations of the spectrum of short operators suggest that the BES Bethe Ansatz is valid only in the $J \to \infty$ limit.

F. Fiamberti, A. Santambroggio, C. Seig, D. Zanon, "Wrapping at four loops" ARXIV:0712.3522

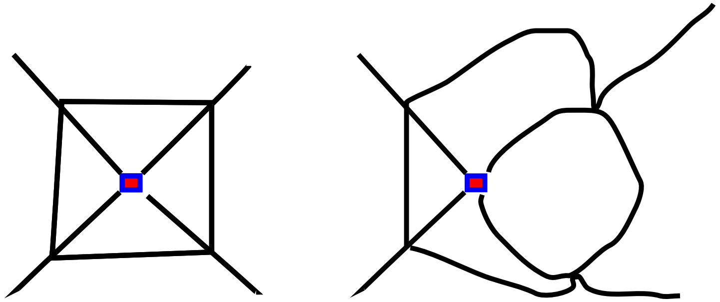
$$\Delta^K = 4 + 12\left(\frac{\lambda}{16\pi^2}\right) - 48\left(\frac{\lambda}{16\pi^2}\right)^2 + 336\left(\frac{\lambda}{16\pi^2}\right)^3 - (2584 - 384\zeta(3) + 1440\zeta(5))\left(\frac{\lambda}{16\pi^2}\right)^4 + \ldots$$

C. Keeler and N.Mann, "Wrapping interactions and the Konishi Operator", ARXIV:0801.1661

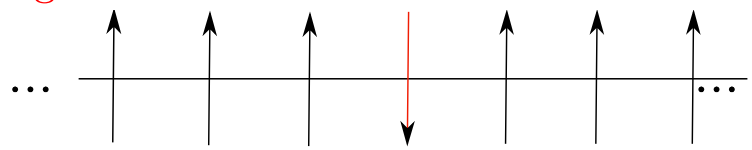
$$\Delta^K = 4 + 12\left(\frac{\lambda}{16\pi^2}\right) - 48\left(\frac{\lambda}{16\pi^2}\right)^2 + 336\left(\frac{\lambda}{16\pi^2}\right)^3 - (2607 + 28\zeta(3) + 140\zeta(5))\left(\frac{\lambda}{16\pi^2}\right)^4 + \ldots$$

Deviations from the large spin limit are due to “wrapping interactions”.

J.Ambjorn, R.Janik, Ch.Kristjansen, hep-th/0510171



Magnon with $p_{\rm mag} \neq 0$



$$\sum_x e^{ipx} ...ZZZ\Phi ZZZ...$$

infinitely long spin chain – isolate a single magnon

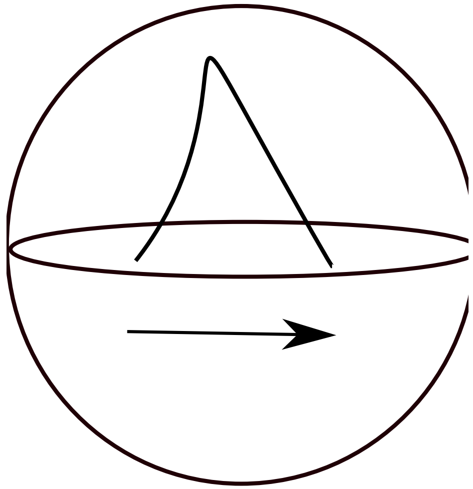
$$E = \Delta - J = \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p_{\rm mag}}{2}} \quad , \quad p_{\rm mag} = \text{magnon momentum}$$

- Compatible with perturbative YM to three loops
- all-loops integrability Ansätze at large J
- agrees with BMN limit
- Beisert: magnon are $\frac{1}{2}$–BPS states of centrally extended superalgebra $SU(2|2) \times SU(2|2) \times R^3$
- **Strong coupling limit $\lambda \to \infty$ from string dual $\longrightarrow$**

Hofman-Maldacena hep-th/0604135 identified string dual:

Giant Magnon:

Soliton solution of classical string sigma model on $R^1 \times S^2$



angle coordinate open $\phi(r) - \phi(-r) = p_{\rm mag}$ $\quad \phi'$, all others periodic

$J \;\; (\; = \; -i\partial/\partial\phi) \;\; \to \; \infty \quad \theta(\pm r) \to \pi/2$

$E = \frac{\sqrt{\lambda}}{\pi} \left| \sin \frac{p_{\rm mag}}{2} \right| \quad \leftarrow \quad \sqrt{1 + \frac{\lambda}{\pi^2} \sin^2 \frac{p_{\rm mag}}{2}}$ at large λ

What about corrections to the large J limit?

Finite size corrections?

- finite size and strong coupling from string – apparently yes! **Arutyunov, Frolov, Zamaklar hep-th/0606126**

$$E = \frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{p_{\text{mag}}}{2}\right| \cdot \left[1 - \frac{4}{e^2}\sin^2\frac{p_{\text{mag}}}{2} e^{-\mathcal{R}} + \ldots\right]$$

- Hubbard model matches exponent, $\mathcal{R} = 2\pi J/\sqrt{\lambda}|\sin p_{\text{mag}}/2| + a p_{\text{mag}} \cot p_{\text{mag}}/2$ but not pefactor
- Bethe Ansatz – maybe? – the integrable **Hubbard model** agrees with perturbation theory to a few loops, then is extrapolated to large λ and large J,

$$E_{\text{H}} = \frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{p_{\text{mag}}}{2}\right| \cdot \left[1 - \frac{2\pi^2}{\lambda\sin^2 p_{\text{mag}}/2} e^{-2\pi J/\sqrt{\lambda}|\sin p_{\text{mag}}/2|} + \ldots\right]$$

- Perturbative gauge theory – none! – at least for $J > \#$loops.

- Finite size classical Giant Magnon found by
Arutyunov, Frolov, Zamlakar hep-th/0606126

$$
\begin{aligned}
E &= \frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{p_{\rm mag}}{2}\right| \cdot \Big[1 - \frac{4}{e^2}\sin^2\frac{p_{\rm mag}}{2} e^{-\mathcal{R}} - \\
&- \frac{4}{e^4}\sin^2\frac{p_{\rm mag}}{2}\Big(\mathcal{R}^2(1+\cos p) + 2\mathcal{R}(2+3\cos p_{\rm mag} + \\
&+ a p_{\rm mag}\sin p_{\rm mag}) + 7 + 6\cos p_{\rm mag} + 6 a p_{\rm mag}\sin p_{\rm mag} + \\
&+ a^2 p_{\rm mag}^2 (1-\cos p_{\rm mag})\Big) e^{-2\mathcal{R}} + \ldots\Big]
\end{aligned}
$$

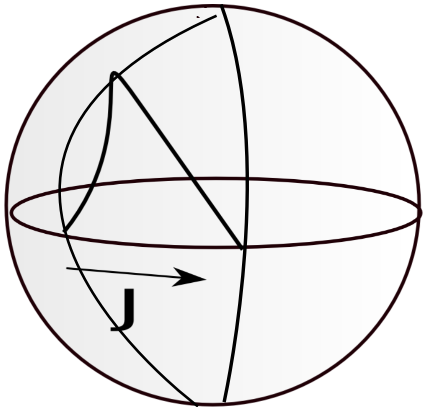
$\mathcal{R} = 2\pi J/\sqrt{\lambda}|\sin p_{\rm mag}/2| + a p_{\rm mag}\cot p_{\rm mag}/2$

- **but depend on gauge-fixing parameter** a

- There is no state of $N = 4$ SYM dual to a single giant magnon with $J < \infty$.
Gauge theory dual of finite size giant magnon?

Orbifold $AdS_5 \times S^5 \rightarrow AdS_5 \times S^5/Z_M$

Identify longitude on 2-sphere by the action of a discrete group Z_M: $\phi \rightarrow \phi + 2\pi/M$



Non-interacting strings:

- choose subset of momenta $J =$ integer $\cdot M$ (rather than $J =$integer in un-orbifold)
- Include wrapped strings $\Delta\phi = 2\pi m/M$

Giant magnon = wrapped closed string

Open ends of magnon are identified identified: $p_{\rm mag} = 2\pi m/M$

Giant magnon is a physical state on orbifold

D.Astolfi, V.Forini, G.Grignani and G.Semenoff hep-th/0702043

Finite size corrections are computable by asymptotic expansion in J (and identical to **Arutyunov, Frolov, Zamlakar hep-th/0606126** in $a = 0$ gauge)

$$\Delta - J = \frac{\sqrt{\lambda}}{\pi}\left|\sin\frac{p_{\rm mag}}{2}\right|\left[1 - 4\sin^2\frac{p_{\rm mag}}{2}e^{-2-2\pi\frac{J}{\sqrt{\lambda}|\sin p_{\rm mag}/2|}} + \ldots\right]$$

The exponential correction has been reproduced from BES by **R.Janik,T.Lukowski, ArXiv:0708:2208**
J. Minahan and O.Ohlsson Sax, “Finite size effects for giant magnons on physical strings” arXiv:0801.2064
N. Gromov, S.Shafer-Nameki, P.Viera, “Quantum wrapped giant magnon”, arXiv:0801.3671

Why orbifold?

String on flat space with magnon boundary condition:

$$X^1(\tau, \sigma + 2\pi) = X^1(\tau, \sigma) + p_{\rm mag}$$

and all other variables, including $\partial_a X^1(\tau, \sigma)$ periodic.

$$\left(\frac{\partial^2}{\partial \tau^2} - \frac{\partial^2}{\partial \sigma^2}\right) X^1 = 0 \ \rightarrow \ X^1(\tau, \sigma) = x^1 + p^1 \tau + p_{\rm mag} \frac{\sigma}{2\pi} + \text{oscillators}$$

Virasoro constraints are

$$\begin{aligned} 0 &= L_0 + \tilde{L}_0 = \frac{\alpha'}{2} p_\mu p^\mu + \frac{p_{\rm mag}^2}{4\pi^2 \alpha'} + N + \tilde{N} - 2 \\ 0 &= L_0 - \tilde{L}_0 = \frac{p^1 p_{\rm mag}}{2\pi} + N - \tilde{N} \end{aligned}$$

has no solution unless $p^1 p_{\rm mag} = 2\pi \cdot$**integer**

Indistinguishable from string where $X^1 \sim X^1 + p_{\rm mag}$
$= Z$-orbifold of flat space

IIB sigma model on $AdS_5 \times S^5$ and in the conformal gauge

$$\mathcal{L} = -\frac{\sqrt{\lambda}}{4\pi}\left\{-\left(\frac{1+\frac{\vec{Z}^2}{4}}{1-\frac{\vec{Z}^2}{4}}\right)^2 \partial_a T \partial^a T + \left(\frac{1}{1-\frac{\vec{Z}^2}{4}}\right)^2 \partial_a \vec{Z}\cdot\partial^a \vec{Z} + \left(\frac{1-\frac{\vec{Y}^2}{4}}{1+\frac{\vec{Y}^2}{4}}\right)^2 \partial_a \chi \partial^a \chi + \left(\frac{1}{1+\frac{\vec{Y}^2}{4}}\right)^2 \partial_a \vec{Y}\cdot\partial^a \vec{Y}\right\}$$

$$\chi(\tau,\sigma+2\pi) = \chi(\tau,\sigma) + p_{\rm mag}$$

If $\chi(\tau,\sigma) = \tilde{\chi}(\tau,\sigma) + p_{\rm mag}\sigma/2\pi$ with $\tilde{\chi}$ periodic,

$$\mathcal{L}[T,\vec{Z},\chi,\vec{Y}] = \mathcal{L}[T,\vec{Z},\tilde{\chi},\vec{Y}] - \frac{\sqrt{\lambda}}{4\pi}\left(\left(\frac{p_{\rm mag}}{2\pi}\right)^2 + \frac{p_{\rm mag}}{\pi}\tilde{\chi}'\right)\left(\frac{1-\frac{\vec{Y}^2}{4}}{1+\frac{\vec{Y}^2}{4}}\right)^2$$

additional terms symmetric under $SU(2)^2 \times R^1$

Level-matching condition

$$0 = \int_0^{2\pi} d\sigma \left\{ T'\Pi_T + Z'\Pi_Z + Y'\Pi_Y + \tilde{\chi}'\Pi_\chi \right\} + \frac{1}{2\pi} p_{\rm mag} J$$

$$\Pi_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} \quad , \quad J = \frac{\sqrt{\lambda}}{2\pi} \int_0^{2\pi} d\sigma \left(\frac{1 - \frac{\vec{Y}^2}{4}}{1 + \frac{\vec{Y}^2}{4}} \right)^2 \dot{\tilde{\chi}}$$

analogous to

$$0 = L_0 - \tilde{L}_0 = \frac{p^1 p_{\rm mag}}{2\pi} + N - \tilde{N}$$

Put in fermions

$$\psi(\tau, \sigma + 2\pi) = e^{i p_{\rm mag} \Sigma} \psi(\tau, \sigma) \quad , \quad \Sigma = {\rm diag}\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right)$$

Breaks all of the supersymmetries.

To retain some supersymmetry, second identification

$$(\chi, Y_1 + iY_2, \psi) \sim \left(\chi + p_{\rm mag}, e^{-ip_{\rm mag}}(Y_1 + iY_2), e^{ip_{\rm mag}\tilde{\Sigma}}\psi\right)$$

where $\tilde{\Sigma} = {\rm diag}(0, 0, 1, -1)$
$SU(2|1)^2 \times R^1$ superalgebra

$$\mathcal{L}[T, \vec{Z}, \chi, \vec{Y}] = \mathcal{L}[T, \vec{Z}, \tilde{\chi}, \vec{Y}] - \frac{\sqrt{\lambda}}{4\pi}\left(\left(\frac{p_{\rm mag}}{2\pi}\right)^2 + \frac{p_{\rm mag}}{\pi}\tilde{\chi}'\right)\left(\frac{1 - \frac{\vec{Y}^2}{4}}{1 + \frac{\vec{Y}^2}{4}}\right)^2 + \ldots$$

$$0 = \int_0^{2\pi} d\sigma \left\{T'\Pi_T + Z'\Pi_Z + Y'\Pi_Y + \tilde{\chi}'\Pi_\chi\right\} + p_{\rm mag}(J - J')$$

Orbifold $AdS_5 \times S^5/Z_M$ is dual to $\mathcal{N}=2$ superconformal quiver gauge theory (with $SU(2,2|2)$ superalgebra).

Begin with $\mathcal{N}=4$: Embed regular representation of Z_M into SU(N) gauge group, (we need $N = M\cdot$integer)

$$\gamma = \begin{bmatrix} 1 & 0 & 0 & ... & 0 \\ 0 & \omega & 0 & ... & 0 \\ 0 & 0 & \omega^2 & ... & 0 \\ . & . & . & ... & . \\ 0 & 0 & 0 & ... & \omega^{M-1} \end{bmatrix} \quad , \quad \omega = \exp(2\pi i/M)$$

(each entry is multiplied by $\frac{N}{M} \times \frac{N}{M}$ unit matrix)

Keep only those components of fields which are invariant under combined gauge and R-symmetry transformation

$$\{Z, \Psi, \Phi, A^\mu\} = \left\{\omega\gamma Z\gamma^{-1}, \gamma\Psi\gamma^{-1}, \gamma\Phi\gamma^{-1}, \gamma A^\mu\gamma^{-1}\right\} \quad , \quad \text{OR}$$

$$\{Z, \Psi, \Phi, A^\mu\} = \left\{\omega\gamma Z\gamma^{-1}, \omega^{-1}\gamma\Psi\gamma^{-1}, \gamma\Phi\gamma^{-1}, \gamma A^\mu\gamma^{-1}\right\}$$

For each $N \times N$ matrix field in the parent $\mathcal{N} = 4$ theory,
$M \quad \frac{N}{M} \times \frac{N}{M}$ blocks survive

$$Z = \begin{bmatrix} 0 & Z_1 & 0 & ... & 0 \\ 0 & 0 & Z_2 & ... & 0 \\ 0 & 0 & 0 & ... & 0 \\ . & . & . & ... & Z_{M-1} \\ Z_M & 0 & 0 & ... & 0 \end{bmatrix}, \ \Phi = \begin{bmatrix} \Phi_1 & 0 & 0 & ... & 0 \\ 0 & \Phi_2 & 0 & ... & 0 \\ 0 & 0 & \Phi_3 & ... & 0 \\ . & . & . & ... & . \\ 0 & 0 & 0 & ... & \Phi_{M-1} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0 & 0 & 0 & ... & \Psi_1 \\ \Psi_2 & 0 & 0 & ... & 0 \\ 0 & \Psi_3 & 0 & ... & 0 \\ . & . & . & ... & . \\ 0 & 0 & ... & \Psi_M & 0 \end{bmatrix}, \ A^\mu = \begin{bmatrix} A_1^\mu & 0 & 0 & ... & 0 \\ 0 & A_2^\mu & 0 & ... & 0 \\ 0 & 0 & A_3^\mu & ... & 0 \\ . & . & . & ... & . \\ 0 & 0 & 0 & ... & A_{M-1}^\mu \end{bmatrix}$$

Single trace operators from $\mathcal{N} = 4$ notation $\mathrm{Tr}\gamma^m \mathcal{O}$
planar $m = 0$ sector = planar $\mathcal{N} = 4$
(**M.Bershadsky,A.Johansen, hep-th/9803249**)

Field content:

- **Gauge group**

$$U(N) \rightarrow U_1\left(N/M\right) \times ... \times U_M\left(N/M\right)$$

- **Bi-fundamental fields**

$$Z \rightarrow \{Z_1, \ldots Z_M\} \quad , \quad Z_I \rightarrow U_I Z_I U^\dagger_{I+1}$$

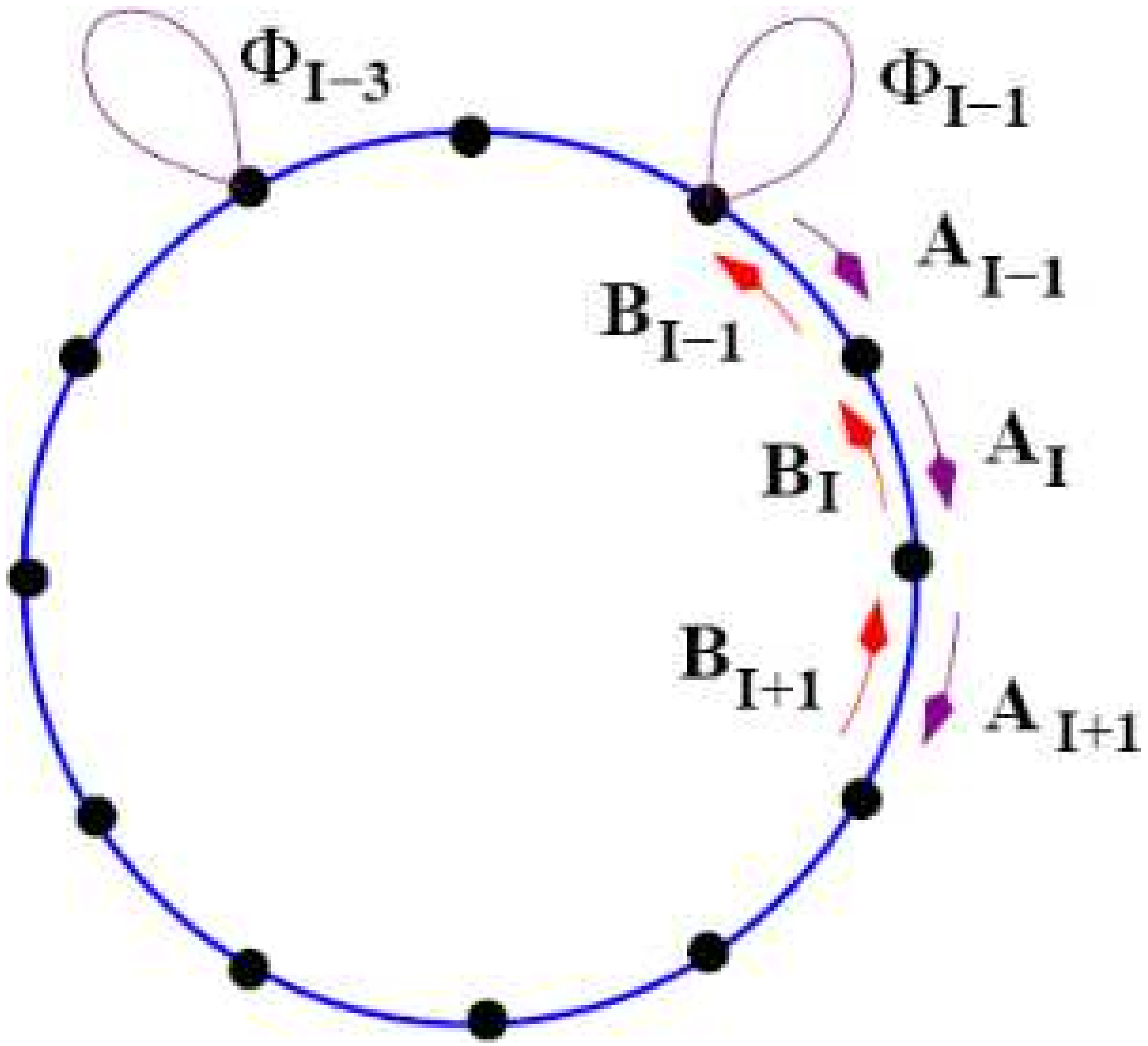
$$\Psi \rightarrow \{\Psi_1, \ldots \Psi_M\} \quad , \quad \Psi_I \rightarrow U^\dagger_{I+1} \Psi_I U_I$$

M bi-fundemental chiral hypermultiplets $\left(Z_I, \bar{\Psi}_I, \chi_{ZI}, \bar{\chi}_{\Psi I}\right)$
$SU(2) \times U(1)$ R-symmetry doublet $(Z, \bar{\Psi})$ singlet Φ

- **Adjoint fields**

$$\Phi \rightarrow \{\Phi_1, \ldots \Phi_M\} \quad , \quad \Phi_I \rightarrow U_I \Phi_I U^\dagger_I$$

M adjoint rep. vector multiplets $(A^\mu_I, \Phi_I, \psi_I, \psi_{\Phi I})$


Φ_{I-3}
Φ_{I-1}
A_{I-1}
B_{I-1}
A_I
B_I
B_{I+1}
A_{I+1}

Spin chain ground state with $\Delta - J = 0$

$$\mathrm{Tr}\gamma^m Z^J = M\delta_{m,0}\mathrm{Tr}[(Z_1\dots Z_M)^k] \quad , \quad J = kM$$

One-magnon state with $p_{\mathrm{mag}} = 2\pi m/M$, $J = kM$:

$$\mathrm{Tr}\gamma^m \Phi Z^{kM} = \sum_I e^{2\pi i \frac{m}{M} I}\mathrm{Tr}Z_1 ... Z_I \Phi_I Z_{I+1} ... Z_M (Z_1 ... Z_M)^{k-1}$$

Two-magnon state with $p_{\mathrm{mag}} = 2\pi m/M$, $J = kM$:

$$\sum_{IJ=0}^{kM} e^{2\pi i (p_1 I + p_2 J)/kM} \; \mathrm{Tr}Z_1 ... \Phi_I ... \Phi_J ... Z_{kM}$$

cyclic symmetry $(I, J) \to (I + M, J + M) \; \to \; p_1 + p_2 = \frac{2\pi}{M}\cdot$integer.
This is **level matching**
magnon multiplet of $SU(2|1)^2 \times R^1$ superalgebra. Magnon limit $J \to \infty$: Since $J = kM$, either $k \to \infty$ or $M \to \infty$
enhanced supersymmetry $SU(2|2)^2 \times R^1$

Weak coupling Yang-Mills:

Magnon is an $\mathcal{N}=4$ SYM multiplet even in $\mathcal{N}=2$ theory

$$\mathrm{Tr}\gamma^p \nabla_\mu Z Z^{kM-1}$$

$$\mathrm{Tr}\gamma^p \Phi Z^{kM} \quad , \quad \mathrm{Tr}\gamma^p \bar{\Phi} Z^{kM} \quad , \quad \mathrm{Tr}\gamma^p \Psi Z^{kM+1} \quad , \quad \mathrm{Tr}\gamma^p \bar{\Psi} Z^{kM+1}$$

$$\mathrm{Tr}\gamma^p \chi_{1\alpha_2} Z^{kM} \quad , \quad \mathrm{Tr}\gamma^p \chi_{\dot{1}\dot{\alpha}_2} Z^{kM}$$

$$\mathrm{Tr}\gamma^p \chi_{2\alpha_2} Z^{kM+1} \quad , \quad \mathrm{Tr}\gamma^p \chi_{\dot{2}\dot{\alpha}_2} Z^{kM-1}$$

is a 16-dimensional supermultiplet with

$$\Delta = J + 1 + \frac{\lambda}{2\pi^2} \sin^2 \left[\frac{1}{2} \frac{2\pi p}{M} \right] + \ldots$$

.

$$p_{\mathrm{mag}} = 2\pi \frac{p}{M}$$

The spectrum of the operator

$$\mathrm{Tr}\gamma^m Z^{kM}\Phi$$

is

$$E = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{\pi m}{M}} + ?$$

Can one compute wrapping interactions? Simplest case $(M = 2, m = 1)$ begins at 3 loops

$$\mathrm{Tr}A_1\Phi_2 A_2 - \mathrm{Tr}A_1 A_1 \Phi_2$$

$$E = \sqrt{1 + \frac{\lambda}{\pi^2}} + ?$$

String Loops: diRisi,Grignani,Orselli,Semenoff hep-th/0409315

k=1

$$\mathrm{Tr}\gamma^m \Phi Z^M = \sum_I e^{2\pi \frac{m}{M} iI} \mathrm{Tr} Z_1 ... Z_{I-1} \Phi_I Z_I ... Z_M$$

is an exact eigenstate of the full dilatation operator.

k=2

$$\left(\mathrm{Tr}\gamma^m \Phi Z^{2M}\right) \quad \pm \quad \left(\mathrm{Tr}\gamma^m \Phi Z^M\right)\left(\mathrm{Tr} Z^M\right)$$

are eigenstates with eigenvalues

$$\Delta - J = 1 + \frac{\lambda(1 \pm M/N)}{2\pi^2} \sin^2 \frac{p_{\rm mag}}{2} + ...$$

k=3

States are

$\{\mathrm{Tr}\gamma^m Z^{3M}, \mathrm{Tr}\gamma^m Z^{2M}\mathrm{Tr}Z^M, \mathrm{Tr}\gamma^m Z^M \mathrm{Tr}Z^{2M}, \mathrm{Tr}\gamma^m Z^M \mathrm{Tr}Z^M \mathrm{Tr}Z^M\}$,

eigenvalues are

$$\Delta - J = 1 + \frac{\lambda(1 \pm 2M/N)}{2\pi^2} \sin^2 \frac{p_{\mathrm{mag}}}{2} + ...$$

$$\Delta - J = 1 + \frac{\lambda(1 \pm M/N)}{2\pi^2} \sin^2 \frac{p_{\mathrm{mag}}}{2} + ...$$

This string loop correction might be computable from string theory.

Penrose limit + light-cone gauge:

$$\mathcal{L} = -\frac{\sqrt{\lambda}}{4\pi}\left\{-\left(\frac{1+\frac{\vec{Z}^2}{4}}{1-\frac{\vec{Z}^2}{4}}\right)^2 \partial_a T \partial^a T + \left(\frac{1}{1-\frac{\vec{Z}^2}{4}}\right)^2 \partial_a \vec{Z}\cdot\partial^a \vec{Z} \right.$$

$$\left. + \left(\frac{1-\frac{\vec{Y}^2}{4}}{1+\frac{\vec{Y}^2}{4}}\right)^2 \partial_a \chi \partial^a \chi + \left(\frac{1}{1+\frac{\vec{Y}^2}{4}}\right)^2 \partial_a \vec{Y}\cdot\partial^a \vec{Y}\right\}$$

$$T = X^+ = p^+\tau\ ,\ \chi = X^+ - \frac{2}{\sqrt{\lambda}}X^-$$

$$\vec{Y} \to \vec{Y}/\lambda^{\frac{1}{4}}\ ,\ \vec{Z} \to \vec{X}/\lambda^{\frac{1}{4}}$$

Flat space limit $\lambda \to \infty$

Plane wave background IIB string

$$\mathcal{L} = -\frac{1}{4\pi}\left\{-4p^+\dot{X}^- + \partial_a\vec{Y}\cdot\partial^a\vec{Y} + \partial_a\vec{Z}\cdot\partial^a\vec{Z} + (p^+)^2(Y^2+Z^2)\right\}$$
$$-\frac{ip^+}{2\pi}\left(\bar{\psi}\partial_+\psi + \psi\partial_-\bar{\psi} + 2ip^+\bar{\psi}\Pi\psi\right)$$

$\Pi = \mathrm{diag}(1,-1)$

with periodic null coordinate

$$X^- \sim X^- + \frac{\sqrt{\lambda}}{2}p_{\mathrm{mag}}$$

light-cone hamiltonian

$$p^- = \frac{1}{2p^+}\sum_{n=-\infty}^{\infty}\sqrt{n^2+(p^+)^2}\left(\alpha_n^{\alpha_1\dot\alpha_1\dagger}\alpha_{n\alpha_1\dot\alpha_1} + \alpha_n^{\alpha_2\dot\alpha_2\dagger}\alpha_{n\alpha_2\dot\alpha_2}\right.$$
$$\left. + \beta_n^{\alpha_1\dot\alpha_2\dagger}\beta_{n\alpha_1\dot\alpha_2} + \beta_n^{\alpha_2\dot\alpha_1\dagger}\beta_{n\alpha_2\dot\alpha_1}\right)$$

level-matching condition

$$\frac{pp^+}{2\pi} = \sum_{n=-\infty}^{\infty} n\left(\alpha_n^{\alpha_1\dot\alpha_1\dagger}\alpha_{n\alpha_1\dot\alpha_1} + \alpha_n^{\alpha_2\dot\alpha_2\dagger}\alpha_{n\alpha_2\dot\alpha_2} + \right.$$
$$\left. + \beta_n^{\alpha_1\dot\alpha_2\dagger}\beta_{n\alpha_1\dot\alpha_2} + \beta_n^{\alpha_2\dot\alpha_1\dagger}\beta_{n\alpha_2\dot\alpha_1}\right)$$

no solution unless $pp^+ = 2\pi\cdot$integer – DLCQ $P^+ = 2\pi/p_{\rm mag}$

Mukhi,Rangamani,Verlinde hep-th/0204147

one-oscillator states - magnon supermultiplet

$$\alpha^{\dagger}_{\alpha_1\dot{\alpha}_1}|p^+> \ , \ \alpha^{\dagger}_{\alpha_2\dot{\alpha}_2}|p^+> \ , \ \beta^{\dagger}_{\alpha_1\dot{\alpha}_2}|p^+> \ , \ \beta^{\dagger}_{\alpha_2\dot{\alpha}_1}|p^+> \qquad (1)$$

degeneracy attributed enhancement of the supersymmetry
broken by $1/\sqrt{\lambda}$ corrections:

$$\sqrt{1+\lambda\frac{m^2}{M^2}} \pm \frac{1}{2\sqrt{\lambda}}\frac{\lambda\frac{m^2}{M^2}}{\sqrt{1+\lambda\frac{m^2}{M^2}}}$$

$SU(2|2)$ **algebra**

$$\left[\mathcal{R}^{\alpha_1}{}_{\beta_1}, \mathcal{J}^{\gamma_1}\right] = \delta^{\gamma_1}_{\beta_1}\mathcal{J}^{\alpha_1} - \frac{1}{2}\delta^{\alpha_1}_{\beta_1}\mathcal{J}^{\gamma_1}$$

$$\left[\mathcal{L}^{\dot\alpha_2}{}_{\dot\beta_2}, \mathcal{J}^{\dot\gamma_2}\right] = \delta^{\dot\gamma_2}_{\dot\beta_2}\mathcal{J}^{\dot\alpha_2} - \frac{1}{2}\delta^{\dot\alpha_2}_{\dot\beta_2}\mathcal{J}^{\dot\gamma_2}$$

$$\left[\mathcal{Q}^{\dot\alpha_2}{}_{\alpha_1}, \mathcal{S}^{\beta_1}{}_{\dot\beta_2}\right] = \delta^{\beta_1}_{\alpha_1}\mathcal{L}^{\dot\alpha_2}{}_{\dot\beta_2} + \delta^{\dot\alpha_2}_{\dot\beta_2}\mathcal{R}^{\beta_1}{}_{\alpha_1} + \delta^{\beta_1}_{\alpha_1}\delta^{\dot\alpha_2}_{\dot\beta_2}\mathcal{C}$$

$$\left\{\mathcal{Q}^{\dot\alpha_2}{}_{\alpha_1}, \mathcal{Q}^{\dot\beta_2}{}_{\beta_1}\right\} = \epsilon^{\dot\alpha_2\dot\beta_2}\epsilon_{\alpha_1\beta_1}\mathcal{P}$$

$$\left\{\mathcal{S}^{\alpha_1}{}_{\dot\alpha_2}, \mathcal{S}^{\beta_1}{}_{\dot\beta_2}\right\} = \epsilon_{\dot\alpha_2\dot\beta_2}\epsilon^{\alpha_1\beta_1}\mathcal{K}$$

Plane-wave superalgebra:

$$\mathcal{R}^{\alpha_1}{}_{\beta_1} = \sum_n \{\alpha_n^{\dagger\alpha_1\dot{\gamma}}\alpha_{n\beta_1\dot{\gamma}_1} + \beta_n^{\dagger\alpha_1\gamma_2}\beta_{\beta_1\gamma_2}\}$$

$$-\frac{1}{2}\delta^{\alpha_1}_{\beta_1}\sum_n \{\alpha_n^{\dagger\gamma_1\dot{\gamma}_1}\alpha_{n\gamma_1\dot{\gamma}_1} + \beta_n^{\dagger\gamma_1\gamma_2}\beta_{\gamma_1\gamma_2}\}$$

$$\mathcal{L}^{\dot{\alpha}_2}{}_{\dot{\beta}_2} = \sum_n \left\{\alpha_n^{\dagger\gamma_2\dot{\alpha}_2}\alpha_{n\gamma_2\dot{\beta}_2} + \beta_n^{\dagger\dot{\alpha}_2\dot{\gamma}_1}\beta_{\dot{\gamma}_1\dot{\beta}_2}\right\}$$

$$-\frac{1}{2}\delta^{\dot{\alpha}_2}_{\dot{\beta}_2}\sum_n \{\alpha_n^{\dagger\gamma_2\dot{\gamma}_2}\alpha_{n\gamma_2\dot{\gamma}_2} + \beta_n^{\dagger\dot{\gamma}_1\dot{\gamma}_2}\beta_{\dot{\gamma}_1\dot{\gamma}_2}\}$$

$$\mathcal{Q}^{\alpha_1}{}_{\dot{\beta}_2} = \frac{1}{\sqrt{8p^+}}\sum_n \Big\{ \Omega^+ \left(\alpha_n^{\dagger\alpha_1\dot{\gamma}_1}\beta_{n\dot{\gamma}_1\dot{\beta}_2} - i\alpha_n^{\alpha_1\dot{\gamma}_1}\beta^{\dagger}_{n\dot{\gamma}_1\dot{\beta}_2}\right) +$$

$$+ \Omega^- \left(i\beta_n^{\dagger\alpha_1\gamma_2}\alpha_{n\gamma_2\dot{\alpha}_2} + \beta_n^{\alpha_1\gamma_2}\alpha^{\dagger}_{n\gamma_2\dot{\alpha}_2}\right)\Big\}$$

$$\mathcal{S}^{\alpha_2}{}_{\dot{\beta}_1} = \frac{1}{\sqrt{8p^+}}\sum_n \Big\{ \Omega^- \left(\alpha_n^{\dagger\alpha_2\dot{\gamma}_2}\beta_{n\dot{\gamma}_2\dot{\beta}_1} - i\alpha_n^{\alpha_2\dot{\gamma}_2}\beta^{\dagger}_{n\dot{\gamma}_2\dot{\beta}_1}\right) +$$

$$+ \ \Omega_n^+ \left(i\beta_n^{\dagger\alpha_2\gamma_1} \alpha_{n\gamma_1\dot{\alpha}_1} + \beta_n^{\alpha_2\gamma_1} \alpha^\dagger_{n\gamma_1\dot{\alpha}_1} \right) \Big\}$$

where $\Omega_n^\pm = \sqrt{\omega_n - p^+} \pm \frac{n}{|n|}\sqrt{\omega_n + p^+}$ and $\omega_n = \sqrt{(p^+)^2 + n^2}$

$$\mathcal{P} = -i\frac{\sqrt{\lambda}p_{\text{mag}}}{4\pi} \leftarrow \frac{\sqrt{\lambda}}{4\pi}\left(e^{-ip_{\text{mag}}} - 1\right)$$

$$\mathcal{K} = i\frac{\sqrt{\lambda}p_{\text{mag}}}{4\pi} \leftarrow \frac{\sqrt{\lambda}}{4\pi}\left(e^{ip_{\text{mag}}} - 1\right)$$

N.Beisert hep-th/0603038,0606214

B.Ramadanovic, G.S. arXiv:0803.4028 [hep-th]

G.Arutyunov, S.Frolov, J.Plefka, M.Zamaklar hep-th/0609157

G.Arutyunov, S.Frolov, M.Zamaklar hep-th/0612229

Concluding remarks:

- Orbifold is interesting.
- Integrability: Bethe equations at weak coupling
  B.Cheng, X.Wang and Y.S.Wu **hep-th/0403004**
  P.DiVecchia, A.Tanzini **hep-th/0405262**
  K.Ideguchi **hep-th/0408124**
  N.Beisert, R.Roiban **hep-th/0510209**
- Work in progress: semi-classical quantization of the giant magnon: bosonic and fermionic zero modes of sigma model in magnon background
  J. Minahan, **hep-th/0701005**
  Orbifold magnon has zero or four fermion zero modes $\rightarrow$ eight zero modes in magnon limit
  supersymmetry enhanced in the magnon limit

Thank you!