SYMMETRIES OF SUPER-LANDAU MODELS

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WORK WITH

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1° INTRODUCTION

2° FERMIONIC LANDAU MODEL

3° SUPERPLANE LANDAU MODEL $ISU(1|1)/U(1|1)$

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INTRODUCTION

BASIC MODEL FOR NON-COMMUTATIVITY IN QM

LOWEST LANDAU LEVEL

ACTION IN ONE DIMENSION - U(1) - CHERN-SIMONS

$$I = \int dA$$

$F = dA$ $\quad$ A U(1) CONNECTION OF $\frac{SU(2)}{U(1)}$

FOR NON-ANTICOMMUTATIVITY

PURE GRASSMANN $\quad SU(n|1)/U(n)$

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MIXED BOSONIC-GRASSMANN $\quad CP^{(n|m)}$

HATSUDA, ISO, UMETSU (03)
IVANOV, LM, TOWNSEND (03)

LLL - DOMINANT $\vec{B}$ - VERY BIG
PLANAR LANDAU MODEL

FOR ANTICOMMUTING COORDINATES OR MIXED

WHOSE LIMIT ? $\Rightarrow$

DYNAMICS ON SUPERMANIFOLDS

SUPER FLAG $\dfrac{SU(2|1)}{U(1)\times U(1)}$ IVANOV, LM TOWNSEND (04)

$\left\{\begin{array}{ll} \text{PLANAR SUPERLANDAU} & \dfrac{ISU(1|1)}{SU(1|1)\times Z} \\ \text{PLANAR SUPERFLAG} & \dfrac{ISU(1|1)}{U(1)\times Z} \end{array}\right.$

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STANDARD APPROACH $\Rightarrow$ NEGATIVE NORMS

WHY HEURISTICALLY!

THESE ALGEBRAS CONTAIN RELATIONS LIKE

$$\{Q, \bar{Q}\} = C$$

NOT MANIFESTLY POSITIVE DEFINITE!

ON ANOTHER HAND

PT- SYMMETRIC THEORIES

C. BENDER ....

QM MODELS WITH HAMILTONIANS

WHICH HAVE { REAL EIGENVALUES λ_A
{ COMPLETE EIGENVECTORS ϕ_A

AND WHICH ARE "HERMITIAN" UNDER

AN "INNER PRODUCT" WHICH IS NOT POSITIVE DEFINITE

$$\langle \phi_A | \phi_B \rangle = (-)^{g(A)} \delta_{AB}$$

TRANSFORMED INTO A BONA FIDE INNER PRODUCT

$$\langle\langle \phi_A | \phi_B \rangle\rangle = \langle G\phi_A | \phi_B \rangle$$

$$G \phi_A = (-)^{g(A)} \phi_A .$$

METRIC OPERATOR

IN FACT WITH A CONVENIENT METRIC OPERATOR

EVEN

COMPLEX LIOUVILLE

$$H = p^2 + m^2 e^{2ix} \qquad x \in (0, 2\pi)$$

CAN BE SHOWN TO BE HERMITIAN

T. CURTRIGHT LM (05)

FERMIONIC LANDAU MODEL

MOTION in $\mathbb{C}^{(1|1)}$ SUPERMANIFOLD (z, φ)

COMPLEX

LANDAU MODEL

$$L_b = |\dot{z}|^2 - i\varkappa(\dot{z}\bar{z} - \dot{\bar{z}}z)$$

KINETIC TERM U(1) CONNECTION

LANDAU 1930

ANALOG

?

$$L_f = \dot{\varphi}\dot{\bar{\varphi}} - i\varkappa(\dot{\varphi}\bar{\varphi} + \dot{\bar{\varphi}}\varphi)$$

HASEBE 05

E IVANOV L.M

P TOWNSEND 05

EXPECT GHOSTS - HIGHER DERIV ON G.V.

$L_b + L_F$ - SUPERPLANE LANDAU MODEL

$$L_b \Rightarrow \quad H_b = a^+a + \varkappa$$

$$a = i(\partial_{\bar{z}} + \varkappa z) \quad a^+ = i(\partial_z - \varkappa\bar{z}) \quad [a, a^+] = 2\varkappa$$

$$E = 2\varkappa(N + \tfrac{1}{2}) \quad N = 0, 1, \ldots$$

$$L_f \to \quad H_f = -\alpha^+\alpha - \varkappa \quad \alpha = \partial_{\bar{\varphi}} - \varkappa\varphi \quad \alpha^+ = \partial_\varphi - \varkappa\bar{\varphi}$$

$$\{\alpha, \alpha^+\} = -2x$$

H_ξ : $\psi(\xi, \bar{\xi})$ FOUR STATE SYSTEM

INNER PRODUCT $\langle \psi_1 | \psi_2 \rangle = \partial_\xi \partial_{\bar{\xi}} (\psi_1^* \psi_2)$

$$\|\psi\|^2 = A^* D + D^* A + B_0^+ B - B_1^* B$$

$$\psi = A + \xi B_0 + \bar{\xi} B_1 + \bar{\xi}\xi D$$

EIGEN VECTORS

$-x$ $\qquad \psi_- = A_-(1 + x\bar{\xi}\xi) + \xi B_-$

$+x$ $\qquad \psi_+ = A_+(1 - x\bar{\xi}\xi) + \bar{\xi} B_+$

NORMS: $\qquad \|\psi_-\|^2 = 2x A_-^* A_- + \bar{B}_- B_-$

<u>NEGATIVE SIGN</u> $\qquad \|\psi_+\| = -2x A_+^* A_+ - \bar{B}_+^* B_+$

BIORTHOGONAL SYSTEMS A SEQUENCE OF ELEMENS $\{\psi_j\}$ & LINEAR FUNCTIONALS $\{\Lambda_k\}$ BIORTHOGONAL : $\Lambda_k(\psi_j) = \delta_{jk}$

? GOURSAT, BANACH, MORSE & FESHBACH, GOHBERG & KREIN..

LOOK DIFFERENT FUNCTIONALS

$$|\psi^s_A\rangle \qquad g(A) = \begin{cases} 0 & A = a \quad \text{POSITIVE NORM} \\ 1 & A = \alpha \quad \text{NEGATIVE NORM} \end{cases}$$

OPERATOR G – "METRIC"

$$G\,|\psi^s_A\rangle = (-)^{g(A)}\,|\psi^s_A\rangle = \frac{-H_f}{x}\,|\psi^s_A\rangle$$

NEW INNER PRODUCT

$$\langle\langle \psi^s_A \,|\, \psi^s_B \rangle\rangle = \langle G\,\psi^s_A \,|\, \psi^s_B \rangle$$

$|\psi^s_A\rangle$ EIGENVECTORS $\hat{H}_f \Rightarrow [\hat{H}_f, G] = 0$

OPERATOR A CHANGE IN HERMITIAN CONJUGATION

$$A^{\ddagger} = A^{+} + G^{-1}[A^{+}, G]$$

$$(A^{\ddagger})^{\ddagger} = A$$

CONSIDER

$$[A, \hat{H}] = 0 \Rightarrow [A^{+}, H] = 0$$

NEW INNER PRODUCT $\Rightarrow [A^{\ddagger}, \hat{H}] = 0$

$\Rightarrow [A^{+}, G]$ NEW INTEGRAL OF MOTION

FERMIONIC LANDAU $\quad G = -\frac{H_{f}}{x}$

NO NEW INTEGRALS OF MOTION

However

ORIGINAL NORM

$$\varsigma^{+} = \bar{\varsigma}$$

$$\partial_{\varsigma}^{+} = \partial_{\bar{\varsigma}}$$

NEW NORM

$$\varsigma^{\ddagger} = \frac{1}{x}\frac{\partial}{\partial \varsigma}$$

$$\bar{\varsigma}^{\ddagger} = \frac{1}{x}\frac{\partial}{\partial \bar{\varsigma}}$$

SUPERPLANE LANDAU MODEL

$$L = L_b + L_F = \quad : \; C^{(1|1)}$$

$$= |\dot z|^2 - i\varkappa(\dot z\bar z - \dot{\bar z} z) + \dot\zeta\dot{\bar\zeta} - i\varkappa(\dot\zeta\bar\zeta + \dot{\bar\zeta}\zeta)$$

$$ISU(1|1)/SU(1|1)\times Z$$

$SU(1,1)$

$$Q = z\partial_\zeta - \bar\zeta\partial_{\bar z} \qquad Q^+ = \bar z\partial_{\bar\zeta} + \zeta\partial_z$$

LIKE SUSY $\quad \{Q, Q^+\} = C = z\partial_z + \zeta\partial_\zeta - \bar z\partial_{\bar z} - \bar\zeta\partial_{\bar\zeta}$

MAGNETIC TRANSLATIONS & SUPER TRANSLATIONS

$$P = -i(\partial_z + \varkappa\bar z) \qquad P^+ = -i(\partial_{\bar z} - \varkappa z)$$

$$\Pi = \partial_\zeta + \varkappa\bar\zeta \qquad \Pi^+ = \partial_{\bar\zeta} + \varkappa\zeta$$

$$[P, P^+] = \{\Pi, \Pi^+\} = 2\varkappa$$

$$[Q, P] = i\Pi \qquad \{Q^+, \Pi\} = iP$$

$$[C, P] = -P \qquad \{C, \Pi\} = -\Pi$$

SPACE OF STATES CONSTRUCTED WITH THE HELP

$$a, a^{+}, \alpha, \alpha^{+}$$

DIRECT PRODUCT STRUCTURE $\Rightarrow$ NEGATIVE NORMS

$$G = -\frac{H_{F}}{x}$$

$$Q \longrightarrow Q^{\mp} = Q^{+} - \frac{i}{x} a^{+}\alpha$$

NEW CHARGE:

$$S = a^{+}\alpha \overset{\text{MODEL}}{\Leftarrow} = -2ix\,Q^{+} - P\,\Pi^{+}$$

$$\boxed{-(S)^{\mp}} = S^{+} = a\alpha^{+} \overset{\text{MODEL}}{\Leftarrow} = 2ix\,Q - P^{+}\Pi$$

IN FACT GIVEN THE ALGEBRA OF ISU(1|1)

$$Q, Q^{+}, C, Z, P, P^{+}, \Pi, \Pi^{+}$$

ON ITS ENVELOPING ALGEBRA OPERATORS

$$S, S^{+} \qquad W = P^{+}P + \Pi^{+}\Pi - 2xC \overset{\text{MODEL}}{\Rightarrow} H$$

$$\{S, S^{+}\} = -2x\,W \qquad \{S, Q\} = iW$$

SUGAWARA TYPE CONSTRUCTION

W LIKE C NOT POSITIVELY DEFINED!

IN GENERAL $\overset{\text{MODEL}}{\Rightarrow} \geq 0$

SYMMETRY OF QUANTUM MODEL

$$Q_1 = -\frac{i}{2x} P^{\dagger}\Pi \quad , \quad S$$

$$\{Q_1, Q_1^{\dagger}\} = c + \frac{1}{2x} H = \frac{1}{2x}(P^{\dagger}P + \Pi^{\dagger}\Pi)$$

$$[Q_1, G] = 0 \qquad \{Q_1, {S \atop S^{\dagger}}\} = 0$$

$$\{S, S^{\dagger}\} = 2x\, W \geq 0 \text{ ON OUR MODEL}$$

$\boxed{\text{N=2 SUPERSYMMERY}}$

? REPRENTATIONS OF THE ALGBRA

WHERE W NOT positively DEFINED

$\boxed{?}$

ALSO OUR STARTING MODEL

QUADRATIC = MANY THINGS HAPPEN
IN QUADRATIC MODELS.

PLANAR SUPERFLAG

PROMOTES Q FROM STABILITY GROUP

$$ISU(1|1)/SU(1|1)\times Z \Rightarrow \text{COSET}$$

$$ISU(1|1)/U(1)\times Z$$

IN FACT THIS IS THE CONTRACTION

$$\frac{SU(2|1)}{U(1)\times U(1)} \quad \text{SUPER FLAG}$$

THE TERM WITH $\dot{\varsigma}\dot{\bar{\varsigma}} \Rightarrow \bar{\xi}\xi\dot{\varsigma}\dot{\bar{\varsigma}}$

$$\mathcal{L} = (1+\bar{\xi}\xi)|\dot{z}|^2 + (\bar{\xi}\dot{z}\dot{\varsigma} - \xi\dot{z}\dot{\bar{\varsigma}}) + \bar{\xi}\xi\dot{\varsigma}\dot{\bar{\varsigma}}$$

$$-i\kappa(\dot{z}\bar{z} - \dot{\bar{z}}z + \dot{\varsigma}\bar{\varsigma} + \dot{\bar{\varsigma}}\varsigma) + iM(\bar{\xi}\dot{\xi} + \xi\dot{\bar{\xi}})$$

ADDITIONAL GOLDSTONE FIELD

$$\delta\xi = \epsilon$$

$$\delta z = c - \bar{\epsilon}z$$

$$\delta\varsigma = \delta - \epsilon z$$

$M=0$ ξ AUXILIARY $\Rightarrow$ SUPERPLANE

QUANTIZATION GUPTA-BLEULER $\Longrightarrow$ 80's

CASALBUONI 76.

Azcárraga et al!
LUSSANNA
Azcárraga Lukierski
Balachandran al

$$\hat{H} = -K_1 \nabla_z \nabla_{\bar{z}} + \varkappa \quad \| \quad H = 4\varkappa M \quad \text{GAUGE INV.}$$

$$\nabla_z = \partial_z - \varkappa \bar{z} + \varkappa \xi^2 (\bar{\xi}_1 - \bar{z}\bar{\xi}_2) \qquad \begin{cases} \varsigma = \xi^1 - z\xi^2 \\ \xi = \xi^2 \end{cases}$$

$$\nabla_{\bar{z}} = \partial_{\bar{z}} + \varkappa z + \varkappa \bar{\xi}_2 (\xi^1 - z\xi^2)$$

$$[\nabla_z, \nabla_{\bar{z}}] = \frac{2\varkappa}{K_1} \qquad K_1 = 1 + \bar{\xi}_2 \xi^2$$

4 like $[a, a^+] = 1$

PHYSICAL WAVE VECTORS

$$\psi(z, \xi^i, \bar{z}\,\bar{\xi}_i) = K_1^M e^{-\varkappa K_2} \phi(z, \bar{z} - \xi^2(\bar{\xi}_1 - \bar{z}\bar{\xi}_2), \xi^i)$$

LIKE CHIRAL SUPERFIELDS

$$K_2 = \bar{z}z + (\xi^1 - z\xi^2)(\bar{\xi}_1 - \bar{z}\bar{\xi}_2)$$

INFINITE SET OF EIGENVECTORS

$$\psi^{(N)} = (\nabla_z)^N \left[K_1^M e^{-\varkappa K_2} \phi^{(N)}(z, \xi^i) \right]$$

$$H \psi^{(N)} = 2\varkappa \left(N + \frac{1}{2}\right) \psi^{(N)}$$

Norm

$$|||\Psi^{(N)}|||^2 =$$

$$(2x)^N N! \left\{ (2M-N)\left[2x \|A^{(N)}\|^2 + \|\Psi_1^{(N)}\|^2 \right] + 2x \|\Psi_2^{(N)}\|^2 + \|F^{(N)}\|^2 \right\}$$

$$\|f\|^2 = \int dz\, d\bar{z}\; e^{-2x|z|^2} |f(z)|^2$$

$$\Phi^{(N)}(z,\xi^i) = A^{(N}(z) + (\xi^1 - z\xi^2)\Psi_1^{(N)}(z) + \xi^2 \Psi_2^{(N)}(z) + \xi^1\xi^2 F(z)$$

$2M - N < 0$ Negative NORMS

$M > 0$ $(2M-N) > 0$. GOOD.

$2M - N$ NULL VECTORS $\Leftrightarrow$ GAUGE INVARIANCE!

$$G = \begin{cases} -1 + 2\xi^2\left(\dfrac{\partial}{\partial \xi^2} + z\dfrac{\partial}{\partial \xi^1}\right) & 2M-N<0 \\ 1 & M>0 \quad 2M-N>0 \end{cases}$$

$G \neq 1 \Rightarrow$ INTEGRAL OF MOTION VALID ON THE WHOLE SPACE

OPERATOR

$$\mathbb{W} = H - 4M\varkappa - \varkappa$$

Quantum SYMMETRY

$$\{Q_1 , Q_1^{\ddagger}\} = C_{sh} + \frac{1}{2\varkappa}\mathbb{W} \geq 0$$

$$S_{sh}^{\ddagger} = \begin{cases} S_{sh}^{\dagger} & M>0 \quad 2M-N>0 \\ -S_{sh}^{\dagger} & 2M-N<0 \end{cases}$$

$$\{S , S^{\ddagger}\} = 2\varkappa|\mathbb{W}|$$

SUPERSYMMETRY LIKE ALGEBRA

INTERPLAY BETWEEN CENRAL CHARGES AND THE DEFINITION OF THE HERMITIAN CONJUGATE LEADS TO A CORRESPONDING CONSISTENT ALGEBRAIC STRUCTURE WITH POSITIVE DEFINITE NORMS!

SUPERFLAG $\quad SU(2|1)/U(1)\times U(1)$

$$Z^M = z, \xi^i \qquad \bar{Z}_M = \bar{z}, \bar{\xi}_i \qquad i = 1,2$$

"MAGNETIC" FIELDS

$$\frac{1}{N}\mathcal{A} = -i\, dZ^M \partial_M K_2 \qquad \frac{1}{M}\mathcal{B} = i\, dZ^M \partial_M \ln K_1$$

$$K_1 = 1 + \bar{\xi}_1 \xi^1 + \bar{\xi}_2 \xi^2 \qquad K_2 = 1 + \bar{z}z + (\xi^1 - z\xi^2)(\bar{\xi}_1 - \bar{z}\bar{\xi}_2)$$

QUANTUM HAMILTONIAN

$$\hat{H}_N = -K_2^2 K_1 \nabla_z^{(N)} \nabla_{\bar{z}}^{(N)} + N$$

PHYSICAL SUPERFIELDS DEFINED BY EQUATIONS

(ODD) $\quad \hat{\varphi}^i |\psi\rangle = 0 \quad i = 1,2 \quad$ DIFFERENTIAL OPERATORS

WHOSE SOLUTIONS

$$\psi = K_1^M K_2^{-N} \phi(z, \bar{z}_{sh}, \xi^1, \xi^2)$$

$$\bar{z}_{sh} = \bar{z} - (\xi^2 + \bar{z}\xi^1)(\bar{\xi}_1 - \bar{z}\bar{\xi}_2)$$

EIGENVECTORS

$$\psi_N^{(\ell)} = K_2^{-N} K_1^M \tilde{\nabla}_z^{2(N+1)} \cdots \tilde{\nabla}_z^{2(N+\ell)} \phi^{(N+\ell, M-\frac{\ell}{2})}(z, \xi^i)$$

$$\tilde{\nabla}_z^{2N} = \partial_z - \frac{2N\bar{z}_{sh}}{1+z\bar{z}_{sh}}$$

$$H\,\psi_N^{(\ell)} = \left[(2\ell+1)N + \ell(\ell+1)\right]\psi_N^{(\ell)}$$

$$\ell = 0, 1, \ldots$$

$$\langle \psi'^{(\ell')}_N \mid \psi^{(\ell)}_N \rangle \sim \langle \phi'^{(N+\ell', M-\frac{\ell'}{2})} \mid \phi^{(N+\ell, M-\frac{\ell}{2})} \rangle$$

$$= 2\,\delta_{\ell\ell'}\frac{(2N+\ell+1)!}{(2N+1)!}\int \frac{dz\,d\bar{z}}{(1+z\bar{z})^{(N+\ell+1)}}\; a$$

$$\Big\{ \left(M-\tfrac{\ell}{2}\right)(2M+2N+\ell+1)\,\bar{A}'A + \tfrac{1}{2}\bar{F}'F$$

$$+ \left(M-\tfrac{\ell}{2}\right)\bar{\psi}'^{1}\psi_1 + \bar{\psi}'^{2}\psi_2$$

$$+ \frac{N+\ell+1}{1+z\bar{z}}\left(\bar{\psi}'^{2} + \bar{z}\bar{\psi}'^{1}\right)\left(\psi_2 + z\psi_1\right) \Big\}$$

$$\phi^{N+\ell, M-\frac{\ell}{2}}_{(z,\xi^i)} = A^{N+\ell, M-\frac{\ell}{2}} + \xi^i\psi_i^{(N+\ell, M-\frac{\ell}{2})} + \xi^1\xi^2 F^{(N+\ell, M-\frac{\ell}{2})}$$

POSITIVITY OF THE NORM OF THE PHYSICAL VECTOR DICTATED BY THAT OF THE ANALYTICAL SUPERFIELD OF ARBITRARY N, M charges.

THE SIGN OF SF COMPONENTS IN THE NORM

A , F $\Rightarrow$ $M(2M+2N+1)$; $+$

ψ^s_1 & ψ^s_2 DIFFICULT TO DIAGONALIZE
AS ANALYTIC FIELDS

SU(2) HELPS REPRESENTATION CONTENT

SOLUTIONS OF $J_- \phi^{(N,M)} = 0$

$$\phi^{(N,M)}_{ew} = a_{N,-N} + (\xi^1 - z\xi^2)\psi^s_{N-\frac{1}{2},-N+\frac{1}{2}} + \xi^2 \psi^s_{N+\frac{1}{2},-N+\frac{1}{2}}$$
$$+ \xi^1 \xi^2 f_{N,-N}$$

$$\| \phi^{(N,M)}_{ew} \|^2 = \frac{\pi}{2N+1} \left\{ 2M(2M+2N+1)\bar{a}_{-N} a_{-N} + \bar{f}_{-N} f_N \right\}$$

$$+ \frac{\pi(2M+2N+1)}{2N+1} \overline{\psi^s}_{-(N+\frac{1}{2})} \psi^s_{-(N+\frac{1}{2})} + \frac{\pi M}{N} \psi^s_{-(N-\frac{1}{2})} \psi^s_{-(N-\frac{1}{2})}$$

THEN BY CLASSROOM ANGULAR MOMENTUM ALL STATES obtAINED BY REPEATED APPLICATION OF J_+ SAME SIGN $\Rightarrow$

$M < -N - \frac{1}{2}$ COEFF BOTH ψ^s's NEGATIVE ; A,F positive

GENERIC CASE $-N - \frac{1}{2} < M < 0$

$\Rightarrow$ $|a_{-N}|^2$ & $|\psi^s_{-(N-\frac{1}{2})}|^2$ NEGATIVE NORM

FOR GENERIC CASE METRIC OPERATOR

$$G = \frac{1}{N+\frac{1}{2}}\left[J^2 + (B-2M)^2 - 2(N+\tfrac{1}{2})^2 \right]$$

$$iJ_+ = -2Nz + z^2\partial_z - \xi^1 \frac{\partial}{\partial \xi^2}$$

$$iJ_- = \partial_z + \xi^2 \frac{\partial}{\partial \xi^1}$$

$$J_3 = z\partial_z - N + \frac{1}{2}\left(\xi^1 \frac{\partial}{\partial \xi^1} - \xi^2 \frac{\partial}{\partial \xi^2}\right)$$

$$B = 2M + N + \frac{1}{2}\left(\xi^1 \frac{\partial}{\partial \xi^1} + \xi^2 \frac{\partial}{\partial \xi^2}\right)$$

THE ONLY GENERATORS THAT CHANGE

$$\Pi = \frac{\partial}{\partial \xi^1} \qquad Q = -\frac{\partial}{\partial \xi^2}$$

$$\Pi^+ = 2M\xi^1 + \xi^1\xi^2 \frac{\partial}{\partial \xi^2} - (\xi^1 - z\xi^2) z\partial_z + 2N(\xi^1 - z\xi^2)$$

$$Q^+ = -2M\xi^2 - \xi^2\xi^1 \frac{\partial}{\partial \xi^1} + (\xi^1 - z\xi^2)\partial_z$$

$$\mathcal{O}_G = [G, \mathcal{O}\} \qquad \mathcal{O}_G^{\ddagger} = [G, \mathcal{O}^+\}$$

$$\boxed{SU(2|1) \to SU_G(2|1)}$$

NOT SUSY like

RESCALINGS & NEW HYPERCHARGE

$$B_G = 2M + 2N + 1 - B$$

THERE REMAINS TO OBTAIN THE OTHER GENERATORS

$$\vartheta \qquad \tilde{\vartheta} = \vartheta + \frac{1}{2} S_\vartheta^+$$

$$S_\vartheta = -G[G, \vartheta^+]$$

THESE OPERATORS $\tilde{\vartheta}^+ = \vartheta^+$

THE FULL ALGEBRA $\tilde{\vartheta}, \vartheta_G$

THE CLOSURE

CONCLUSIONS

USING PT-METHODS ⇒ QM MODELS

WHICH CARRY UNITARY REPRESENTATIONS OF

SUPERALGEBRAS

MODIFYING THE INVARIANT NORM NOT ONLY DOES NOT

LEAD LOSS OF SYMMETRY IT UNCOVERS

HIDDEN SYMMETRIES

THE INTERESTING QUESTION :

RELATIVISTIC PARTICLES , STRINGS

AND OF COURSE DOES THIS HAPPEN !