Topological symmetry and (de)confinement in gauge theories and spin systems

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QCD* parts with M. Shifman

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E.Poppitz, M. Mulligan, L.G. Yaffe, O. Aharony. for useful communications.

- In zero temperature asymptotically free or super-renormalizable (non-)abelian gauge theories, is there a symmetry associated with confinement/deconfinement?
- IR gapped, IR gapless, IR CFT ? Is there a distinguishing (continuum) notion beyond perturbation theory?
- Obvious answer: No. There is no symmetry in microscopic theory related to confinement.

The theories and the goal

Polyakov

Representation : nothing, fundamental or adjoint

$$\mathrm{P}(\mathcal{R}) \text{ on } \mathbb{R}^3$$

$$\mathrm{QCD}(\mathcal{R})^* \text{ on } \mathbb{R}^3 \times S^1$$

Small non-thermal circle with stabilized center (if center is not already stable quantum mechanically)

Frustrated spin systems in d=2 space dimensions

Lattice or <u>continuum compact QED</u> d=2+1dimensions

AMBIGUOUS! (will be discussed, can be resolved.)

The goal

All these theories have long distance regimes where they are described in terms of a compact abelian gauge theory.

All has monopole-instantons. What is their IR physics?

‘t Hooft, Mandelstam, Nambu, Polyakov 70’s

Gapped, ungapped, interacting CFT

Surprisingly, all of these are possible! A sharp and useful notion of emergent (IR) topological symmetry is at work. It is the goal of this talk to make it precise.

Reminder: Abelian duality and Polyakov model

Free Maxwell theory is dual to the free scalar theory.

$$F = *d\sigma$$

The masslessness of the dual scalar is protected by a **continuous shift symmetry**

$$U(1)_{\rm flux} : \ \sigma \to \sigma - \beta$$

Noether current of dual theory:

Topological current vanishes by Bianchi identity.

$$\mathcal{J}_\mu = \partial_\mu \sigma = \tfrac{1}{2}\epsilon_{\mu\nu\rho}F_{\nu\rho} = F_\mu$$

Its conservation implies the absence of magnetic monopoles in original theory

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = 0$$

The presence of the monopoles in the original theory implies reduction of the continuous shift symmetry into a discrete one.

$$\partial_\mu \mathcal{J}_\mu = \partial_\mu F_\mu = \rho_m(x)$$

The dual theory

$$L = \frac{1}{2}(\partial\sigma)^2 - e^{-S_0}(e^{i\sigma} + e^{-i\sigma})$$

Discrete shift symmetry: $\sigma \to \sigma + 2\pi$

$U(1)_{\mathrm{flux}}$ if present, forbids (magnetic) flux carrying operators.

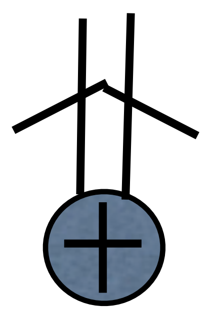
Reminder: Add a massless adjoint Dirac fermion P(adj)

Affleck, Harvey, Witten 82

Microscopic theory has U(1) fermion number symmetry.

$$\psi \to e^{i\beta}\psi, \quad \bar{\psi} \to e^{-i\beta}\bar{\psi}$$

Monopole operator:



$$e^{-S_0} e^{i\sigma} \psi\psi$$

$$\mathcal{I}_{\boldsymbol{\alpha}_i} = (\dim\ker \not{D}_{\boldsymbol{\alpha}_i} - \dim\ker \overline{\not{D}}_{\boldsymbol{\alpha}_i})$$ Jackiw-Rebbi 76, Callias 78

The invariance under U(1) fermion number symmetry demands

$$\psi \to e^{i\beta}\psi, \qquad \sigma \longrightarrow \sigma - N_f \mathcal{I}_{\alpha_1}\beta = \sigma - 2\beta$$

Symmetry of the long distance theory

$$U(1)_* : U(1) - N_f \mathcal{I}_{\boldsymbol{\alpha}_i} U(1)_{\rm flux}$$

Forbids any pure flux operators such as $$e^{iq\sigma}$$

Current:

$$K_\mu = \bar{\psi}\sigma_\mu\psi - n_f \mathcal{I}_{\alpha_1}\partial_\mu\sigma = \bar{\psi}\sigma_\mu\psi - n_f \mathcal{I}_{\alpha_1}\mathcal{J}_\mu$$

AHW concludes: Fermion number breaks spontaneously and photon is NG boson.

Current conservation = Local version of Callias index theorem

Massless Fundamental fermions and IR CFT

N_f 4-component Dirac spinors or $2n_f$ 2-component Dirac spinors:

$$\Psi^a = \begin{pmatrix} \psi_1^a \\ \bar{\psi}_2^a \end{pmatrix}, \qquad \overline{\Psi}_a = \begin{pmatrix} \psi_2^a \\ \bar{\psi}_1^a \end{pmatrix},$$

Microscopic symmetries:

$$\mathcal{G}_{\mathcal{M},\mathrm{P(F)}} = SO(3)_L \times C \times P \times T \times \mathbb{Z}_2 \times U(1)_V \times \underline{U(1)_A} \times SU(n_f)_1 \times SU(n_f)_2$$

non-anomalous on $\mathbb{R}^3$

This theory and certain frustrated spin systems in two spatial (and one time) dimensions share universal long distance physics. (to be discussed).

Microscopic symmetries identical with QCD and QCD* except the underlined symmetry.

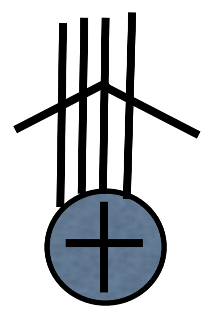
HOW DOES THIS THEORY FLOWS INTO A CFT?

Perturbation theory: 3d QED with massless fermions

$$S^{\mathrm{P(F)}}_{\mathrm{pert.}} = \int_{\mathbb{R}^3} \left[\frac{1}{4g_3^2} F_{\mu\nu}^2 \; + \; i\bar{\Psi}^a \gamma_\mu (\partial_\mu + iA_\mu)\Psi_a\right]$$

Is the masslessness destabilized non-perturbatively?

Monopole operators:



$$e^{-S_0} e^{i\sigma} \det_{a,b} \psi_1^a \psi_2^b + e^{-S_0} e^{-i\sigma} \; \det_{a,b} \bar{\psi}_1^a \bar{\psi}_2^b$$

$$U(1)_* : \psi_1 \to e^{i\beta}\psi_1, \qquad \psi_2 \to e^{i\beta}\psi_2, \qquad \sigma \longrightarrow \sigma - 2n_f\beta \; .$$

Topological symmetry: $\quad U(1)_* : U(1)_A - N_f \mathcal{I}_{\boldsymbol{\alpha}_i} U(1)_{\mathrm{flux}}$

$N_f \geq 2$ No relevant flux (monopole) operators in the original electric theory!!

IR theory quantum critical due to the absence of relevant or marginal destabilizers.

Integrate out a thin momentum slice of massless fermions: Applequist et.al 88

$$\frac{1}{g_3^2} F_{\mu\nu}^2 \rightarrow \frac{1}{g_3^2} \left(F_{\mu\nu}^2 + \frac{g_3^2 n_f}{8} F_{\mu\nu} \frac{1}{\sqrt{\Box}} F_{\mu\nu} \right)$$

Move into deep IR:

$$L \sim F_{\mu\nu} \Box^{-1/2} F_{\mu\nu} + i \bar{\Psi}^a \gamma_\mu (\partial_\mu + i \frac{1}{\sqrt{n_f}} A_\mu) \Psi_a$$

Dimensionless coupling of CFT

Big enhancement of spacetime and global symmetries

$$\mathcal{G}_{\mathrm{IR,P(F)}} \sim \quad (\text{conformal symmetry})\times$$
$$C \times P \times T \times U(1)_V \times U(1)_{\text{flux}} \times SU(2n_f)$$

Theories interpolate between weakly and strongly coupled interacting CFTs as the number of flavors is reduced.

Anticipating ourselves a bit: The same dynamics as the IR of the $SU(n_f)$ frustrated spin systems with no broken symmetries.

$N_f = 1$ Perhaps one relevant flux operator, photon remains massless due to U(1)*.

Sub-conclusions

The infrared of Polyakov models with complex fermions

Take two electric charges at $(\mathbf{x}, \mathbf{y}) \in \mathbb{R}^2$

$$V_{\mathrm{non-pert.}}(|\mathbf{x}-\mathbf{y}|) \sim \begin{cases} |\mathbf{x}-\mathbf{y}| & \text{pure Polyakov or with heavy fermions} \\ |\mathbf{x}-\mathbf{y}|^{-1} & \text{with massless fundamental fermions,} \\ \log|\mathbf{x}-\mathbf{y}| & \text{with massless adjoint fermions,} \end{cases}$$

Respectively,

P: Gapped, linear confinement $\mathbb{Z}_1$

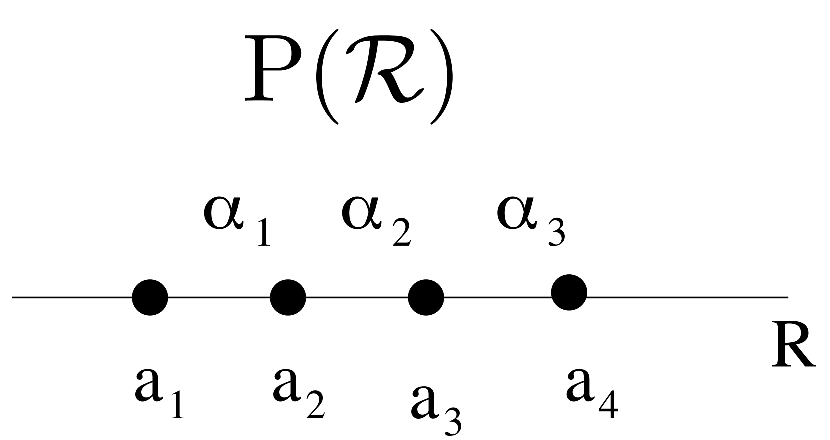
P(F): Interacting CFT, massless photon $U(1)_*$

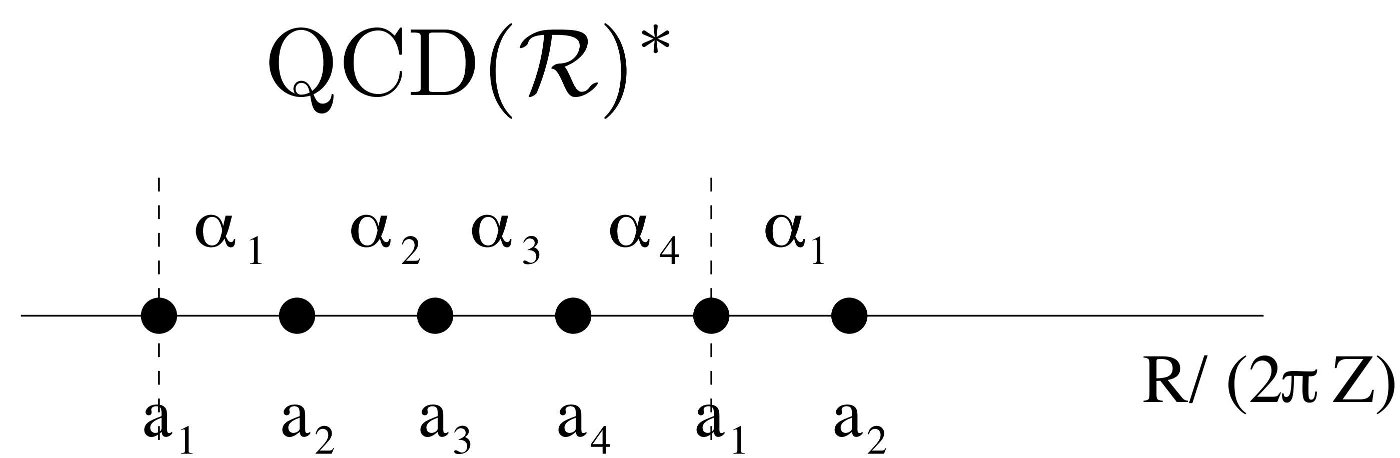
P(adj): massless photon, NG boson $\cancel{U(1)}_*$

Recall: All of these theories have monopole-instantons!

YM* and QCD*

Yaffe, M.U. 08 Shifman, M.U. 08



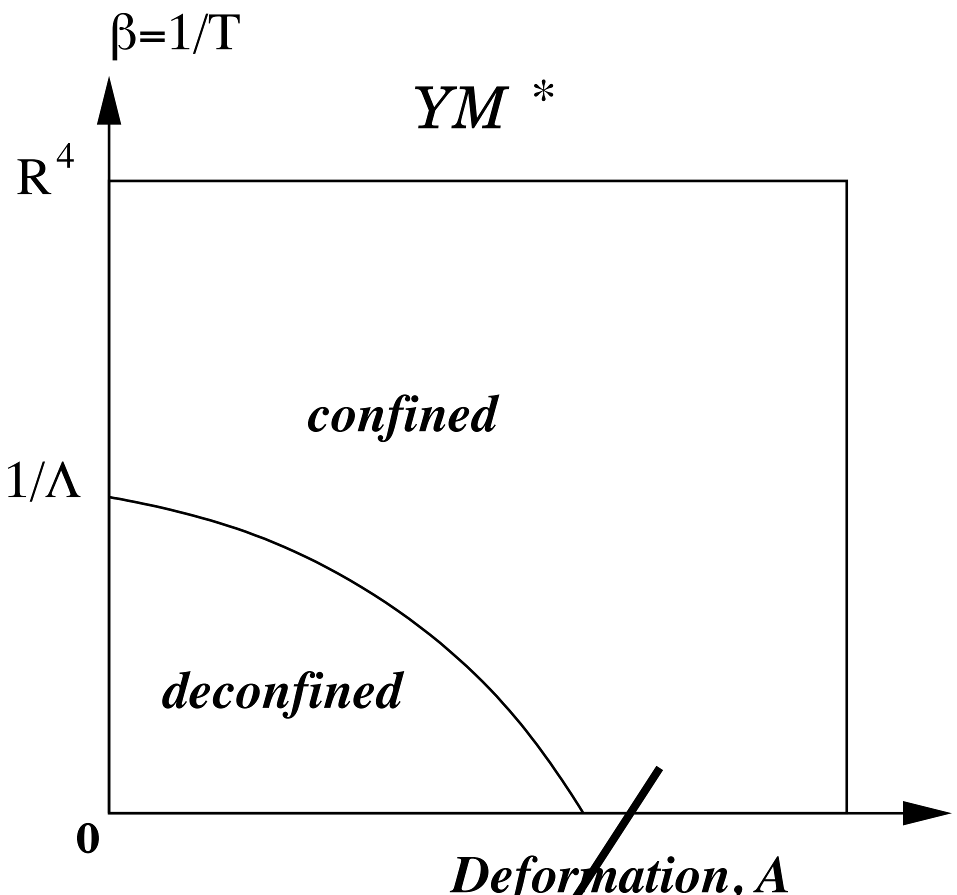


1) Alter the topology of adjoint Higgs scalar into a compact one. An extra topological excitation moves in from infinity. (associated with affine root)

2) Take YM or QCD(R) on small $S^1 \times \mathbb{R}^3$ and add center stabilizing double trace deformations. (Different theory from QCD? See below.)

1 and 2 are the same.

YM* theory at finite N



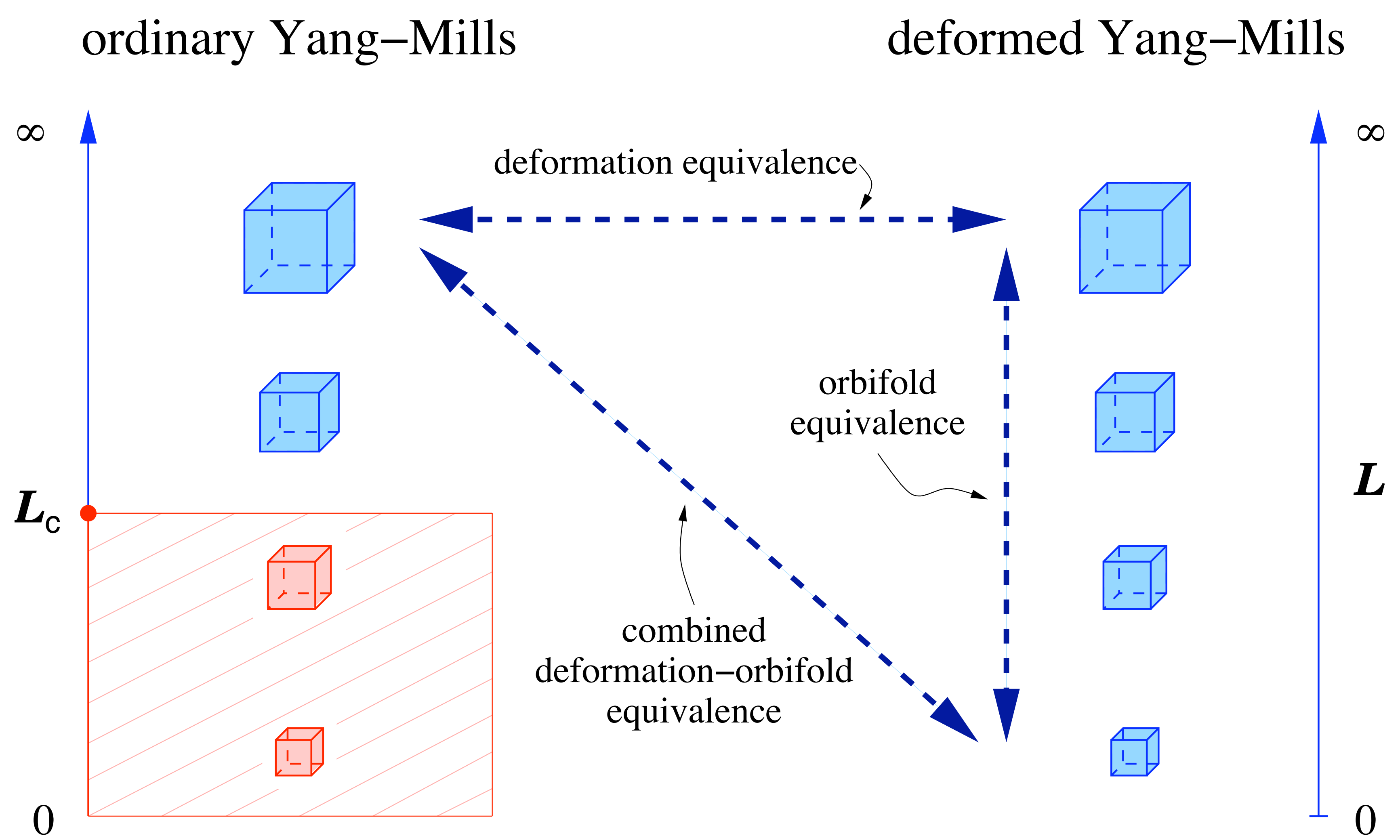
$$S^{\mathrm{YM}^*} = S^{\mathrm{YM}} + \int_{R^3 \times S^1} P[U(\mathbf{x})]$$

$$P[U] = A \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} \left|\mathrm{tr}\,(U^n)\right|^2$$

IDEA: Connect large and small circle physics in a smooth way! Then solve in regime where we have theoretical control.

$$S^{\mathrm{dual}} = \int_{R^3} \left[\frac{1}{2L}\left(\frac{g}{2\pi}\right)^2 (\partial\sigma)^2 - \zeta \sum_{i=1}^{N} \cos(\alpha_i \cdot \sigma)\right].$$

At large N, the difference of YM and YM* is sub-leading in N. Volume independence (valid EK reduction) via center stabilizing deformations.



Large N dynamics on $\mathbb{R}^4$ =Large N quantum mechanics

The notion of continuum compact 3d QED is ambiguous.

Option 1: YM noncompact adjoint Higgs field, Polyakov model

Option 2: YM compact adjoint Higgs field, QCD*

In the presence of certain number of massless complex fermions, the first class always remains ungapped and the latter develops a gap for gauge fluctuations. Why?

AHW and the first part of the talk:

$$\underbrace{SU(N)}_{\text{with noncompact scalar}} \quad \underbrace{\longrightarrow}_{\text{Higgsing}} \quad \underbrace{[U(1)]^{N-1}}_{\text{compact QED}_3} \quad \underbrace{\longrightarrow}_{\text{nonperturbative}} \quad \underbrace{U(1)}_{\text{CFT or free photon}}$$

Shifman, MU and the second part of the talk

$$\underbrace{SU(N)}_{\text{with compact scalar}} \quad \underbrace{\longrightarrow}_{\text{Higgsing}} \quad \underbrace{[U(1)]^{N-1}}_{\text{compact QED}_3} \quad \underbrace{\longrightarrow}_{\text{nonperturbative}} \quad \underbrace{\text{nothing}}_{\text{gapped gauge bosons}}$$

QCD(R)*

On locally three manifolds, there is no chiral anomaly.
On locally four manifolds, due to chiral anomaly, the axial U(1) symmetry reduce to a discrete one.

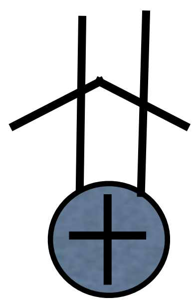
Quantum theory: $U(1)_A \rightarrow \mathbb{Z}_{2h}$

Discrete topological shift symmetry: $\psi \rightarrow e^{i\frac{2\pi}{2h}}\psi, \qquad \sigma \rightarrow \sigma - \frac{2\pi}{h}$

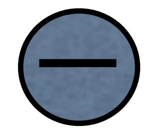
$$\underbrace{U(1)_*}_{\text{non}-\text{compact Higgs or P}(\mathcal{R})} \longrightarrow \underbrace{(\mathbb{Z}_h)_*}_{\text{compact Higgs or QCD}(\mathcal{R})^*}$$

QCD(F)* with one flavor

Shifman, M.U. 08

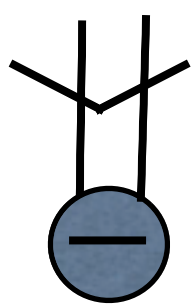


$$\mathcal{M}_1(x) = e^{-S_0} e^{i\sigma} \psi_1 \psi_2,$$

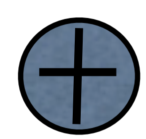


$$\mathcal{M}_2(x) = e^{-S_0} e^{-i\sigma}$$

$$(\mathbb{Z}_1)_*$$



$$\overline{\mathcal{M}}_1(x) = e^{-S_0} e^{-i\sigma} \bar{\psi}_1 \bar{\psi}_2,$$



$$\overline{\mathcal{M}}_2(x) = e^{-S_0} e^{+i\sigma}$$

Structure of the zero modes dictated by Callias index theorem, observed beautifully on lattice by Bruckmann, Nogradi, Pierre van Baal 03. BNvB also introduced the notion of zero mode hopping as the boundary conditions are changed for fermions.

IR of QCD(F)* with one flavor

$$S = \int_{\mathbb{R}^3} \Big[\frac{1}{4g_3^2} F_{\mu\nu}^2 \; + \; i\bar{\Psi}\gamma_\mu(\partial_\mu + iA_\mu)\Psi\Big]$$

$$+c_1 e^{-S_0} e^{i\sigma} \psi_1 \psi_2 + c_1 e^{-S_0} e^{-i\sigma} \; \bar{\psi}_1 \bar{\psi}_2$$

$$+c_2 e^{-S_0}(e^{-i\sigma} + e^{i\sigma}) + \ldots$$

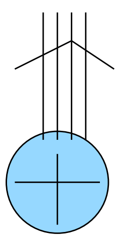
Mass gap for gauge fluctuations and fermions. Chiral condensate (which does not break any symmetry).

QCD(adj)* $\left(\int_{S^2} F,\ \int_{R^3\times S^1} F\tilde{F}\right)$

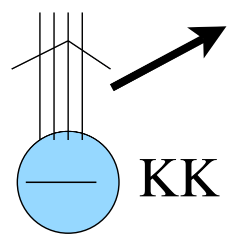
Magnetic Monopoles

Magnetic Bions

Rebbi-Jackiw fermionic zero modes

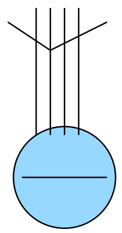

BPS

(1, 1/2)

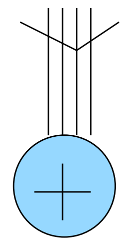

KK

(–1, 1/2)

$e^{-S_0} e^{i\sigma} \det_{I,J} \psi^I \psi^J,$


$\overline{\text{BPS}}$

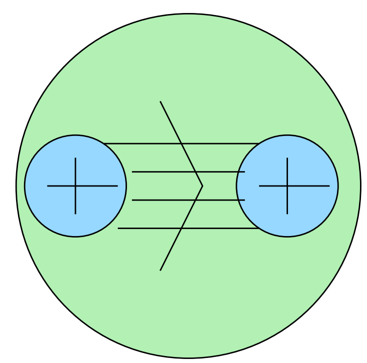
(–1, –1/2)

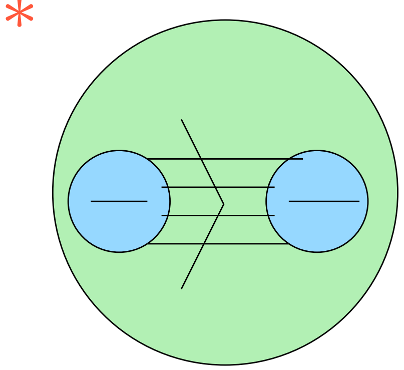

$\overline{\text{KK}}$

(1, –1/2)

$e^{-S_0} e^{i\sigma} \det_{I,J} \bar{\psi}^I \bar{\psi}^J$

$(\mathbb{Z}_2)_*$


(2,0)


(–2, 0)

$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$

Discrete shift symmetry : $\sigma \longrightarrow \sigma + \pi \qquad \psi^I \longrightarrow e^{i\frac{2\pi}{8}} \psi^I$

- Discrete topological symmetry $(\mathbb{Z}_h)_*$ forbids all pure flux operators with magnetic charges not multiple of h.

- Does this mean that any theory with a discrete topological symmetry will have a mass gap in its gauge sector and confine?

- Take a QCD(F)* theory with large number of fundamental flavors, but still asymptotically free (such as Banks-Zaks window, so that weak gauge coupling at small circle makes sense.) What happens?

With fundamentals, topological symmetry in QCD(F)* is always $(\mathbb{Z}_1)_*$

$(\mathbb{Z}_1)_*$ allows $e^{iq\sigma}$ for all q.

Can a monopole operator (whose classical dimension is +3) be irrelevant in the renormalization group sense in the presence of many massless fermions? If so,

$$(\mathbb{Z}_1)_* \to \underbrace{U(1)_{\text{flux}}}_{\text{accidental}}$$

This would be a non-perturbative confirmation of Banks-Zaks type window beyond the usual perturbation theory. (Since the non-perturbative excitations are also taken into considerations.)

The quantum scaling dimension of monopole operator receives corrections proportional to the number of flavors leading monopoles towards irrelevance in the RG sense.

Many peoples beautiful work: Hermele, Senthil,....04
using results by Kapustin, Borokhov, Wu 02 on 3d CFTs..
See the applications to quantum criticality in
Senthil, Balents, Sachdev, Vishwanath, Fisher 04..
Please see the paper for an incomplete set of references...

Why are these most interesting questions of P(F) and QCD(F)* are of relevance in condensed matter physics?

The frustrated spin systems maps into identical gauge theories in some circumstances.

From $SU(n_f)$ quantum spin systems to lattice QED$_3$

$$H = J \sum_{\langle \boldsymbol{r}, \boldsymbol{r}' \rangle} \mathrm{tr}\left[\boldsymbol{S}(\boldsymbol{r}).\boldsymbol{S}(\boldsymbol{r}')\right] + \ldots = J \sum_{a=1}^{\dim(\mathrm{adj})} \sum_{\langle \boldsymbol{r}, \boldsymbol{r}' \rangle} S^a_{\boldsymbol{r}} S^a_{\boldsymbol{r}'} + \ldots$$

Mostly antiferromagnetic exchange $J > 0$

Global spin rotation symmetry $\boldsymbol{S}(\boldsymbol{r}) \to U \boldsymbol{S}(\boldsymbol{r}) U^\dagger, \qquad U \in SU(n_f)_D$

A magnetic ground state if the effect of ellipsis negligible: (Neel order) mean field theory OK.

If ellipsis causes frustration (for example, by some double-trace deformation) of spin such that spin refuses to order, can the mean field theory be applied usefully?

Nontrivial. But possible. Initiated by Baskaran, Anderson 88, Affleck, Marston 88

"Slave-fermion mean field theory"

$$S^a_{\boldsymbol{r}}(\boldsymbol{r}) = f^\dagger_{\boldsymbol{r},\alpha} T^a_{\alpha\beta} f_{\boldsymbol{r},\beta}, \qquad \text{or} \;\; \boldsymbol{S}_{\alpha\beta} = (S^a_{\boldsymbol{r}} T^a)_{\alpha\beta} = f^\dagger_{\boldsymbol{r},\alpha} f_{\boldsymbol{r},\beta} - \frac{1}{2n_f}\delta_{\alpha\beta}$$

Apparent local gauge redundancy: $f_{\boldsymbol{r},\alpha} \longrightarrow e^{i\theta(\boldsymbol{r})} f_{\boldsymbol{r},\alpha}$

With a constraint on the occupancy of each lattice site, the fermionic Hamiltonian describes the original spin system.

Affleck, Marston 88 introduced a mean field state which satisfies:

$$\overline{\chi}_{\boldsymbol{r}\boldsymbol{r}'} = \langle f^\dagger_\alpha(\boldsymbol{r}) f_\alpha(\boldsymbol{r}')\rangle \qquad \prod_{\partial p} \overline{\chi}[\partial p] = e^{i\pi} = -1$$

Fluctuations around this mean-field is the usual Kogut-Susskind Hamiltonian for compact U(1) lattice QED_3 with massless fermions.

$$H \sim J \sum_{\langle \boldsymbol{r},\boldsymbol{r}'\rangle} \bar{\chi}_{\mathbf{r}'\mathbf{r}} f^\dagger_{\boldsymbol{r},\alpha} e^{i a_{\boldsymbol{r},\boldsymbol{r}'}} f_{\boldsymbol{r}',\alpha} + \text{h.c.} + (\text{Maxwell term})$$

$$G_{\text{discrete}} \subset SO(2)_D = \text{Diag}(SO(2)_{\text{Lorentz}} \times SU(2)_{\text{flavor}})$$ Staggering or twisting

$$\mathcal{G}_{\mathrm{QED}_3} \sim G_{\mathrm{discrete}} \times C \times P \times T \times U(1)_V \times SU(n_f)_D$$

If magnetic flux operators are irrelevant, the theory deconfines and infrared symmetry enhances drastically flowing into a scaleless theory (i.e, forgetting about J, which sets the scale in the spin Hamiltonian).

$$\mathcal{G}_{\mathrm{IR,QED}_3} \sim \quad (\text{conformal symmetry}) \times C \times P \times T \times U(1)_V \times U(1)_{\mathrm{flux}} \times SU(2n_f)$$

In the Kagome lattice, the geometric frustration of spin is large even for S=1/2.

$\mathrm{ZuCu_3(OH)_6Cl_2}$ Position of Cu ions form a Kagome lattice.

$J \sim 200K$ No ordering observed upto $30mK$ Helton, J. 07

Guess: likely a CFT of the above type.

The necessary and sufficient conditions for (de)confinement: A topological symmetry characterization.

1) The existence of continuous U(1)* topological symmetry is the necessary and sufficient condition to establish deconfinement and to show the absence of mass gap in gauge sector.

1.a) If U(1)* is spontaneously broken, photon is a massless NG boson.

1.b) If the U(1)* is unbroken, the unbroken U(1)* protects the masslessness of the photon. In some cases, infrared theory flows into an interacting CFT.

2) The existence of a discrete topological symmetry is necessary but not sufficient condition to exhibit confinement.

2.a) If the monopole (and other flux) operators are irrelevant at large distances, then there is an extra accidental continuous topological symmetry. This class of theories will deconfine and some will flow into interacting CFTs. (emergent topological symmetry)

2.b) If the monopole (and other flux) operators are relevant at large distances, then the mass gap and confinement will occur. Showing the relevance of flux operators is the sufficient condition to exhibit confinement.

1.a) P(adj) AHW 82,
1.b) P(F),
2.a) Spin liquids, quantum criticality, (critical points, critical phases)
Banks-Zaks type QCD* theories.
2.b) QCD(F/adj)*, P, YM*, (t Hooft, Mandelstam, Polyakov intuition.)

- Valid when long distance dynamics is abelian and three dimensional. Correctly characterize abelian confinement and abelian interacting CFTs. Can we use this to say something useful on non-abelian confinement and non-abelian CFTs?

- Observation: Take any non-abelian gauge theory, and push it into a regime where long distance theory abelianizes. The confined versus deconfined CFT behavior seems to be invariant under such deformations. (in a large class of theories I looked at)*. This suggests the topological symmetry may also be useful for theories which do not possess an abelian regime.

Herbertsmithite