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Planar Equivalence: an update

Gabriele Veneziano
(Collège de France)

Outline

* Large-N: history and prehistory
* Orientifold planar equivalence
  * ASV's 2003 claim
  * Arguments, counter-arguments, dust-settling
  * SUSY relics in QCD?
  * KUY's 2007 proposal
  * Further developments
* Outlook

Prehistory (1970-'74)

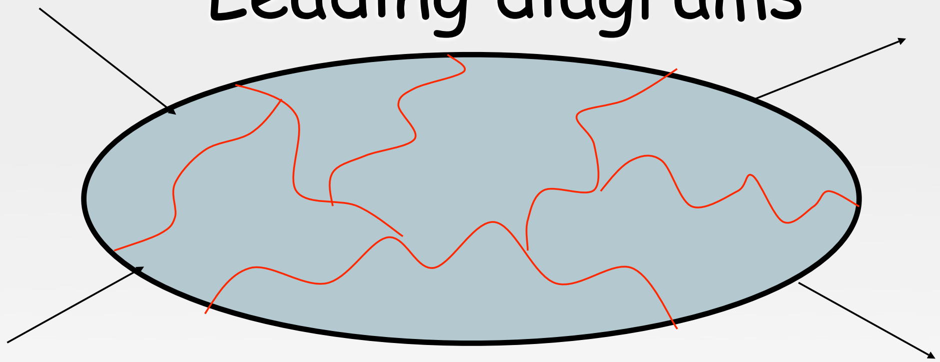
- DFSV (1970): a topological approach to unitarity in DRM/string theory
- Planar diagrams, planar unitarity => Reggeon, with $\alpha_R(0) \sim 1 - d\langle n\rangle/dy < 1$
- Cylinder topology => (bare, soft) Pomeron with $\alpha_P(0) \sim 1$
- Higher topologies => Gribov's RFT
- Hard to sell, then came QCD & 't Hooft

Large-N expansions in QCD

❶ Planar & quenched limit ('t Hooft, 1974)

$1/N_c$ expansion @ fixed $\lambda = g^2 N_c$ and N_f

Leading diagrams



Corrections: $O(N_f/N_c)$ from q-loops,

$O(1/N_c^2)$ from higher-genus diagrams

Properties at leading order

1. Resonances have zero width
2. U(1) problem not solved, WV @ NLO
3. Multiparticle production not allowed

Theoretically appealing: should give the **tree level** of **some** kind of string theory

Proven hard to solve, except in D=2....

Right after 't Hooft's paper, (GV '74) I used his trick to reinterpret/sell my previous work as a $1/N_f$ expansion

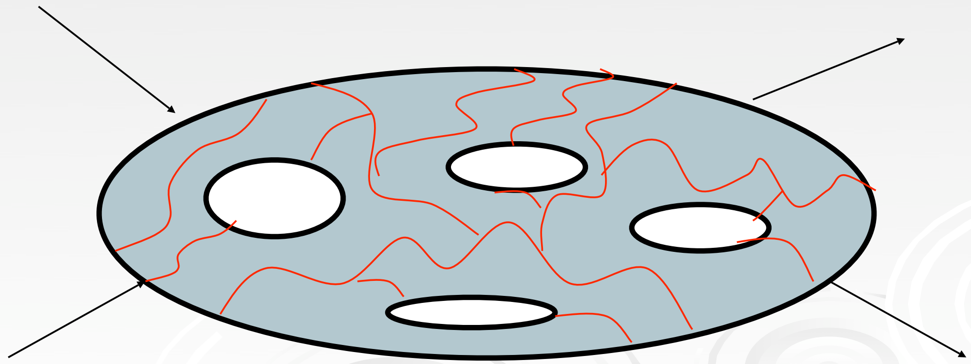
❷ Planar limit = Topological Expansion (GV, 1976)

= 1/N expansion at fixed g^2N and ($N_f/N_c \lesssim 5$)

Leading diagrams planar but include "empty" q-loops

Corrections: $O(1/N^2)$ from non-planar diagrams

First paper discussing necessity and properties of glueballs @ large N ?



Properties at leading order

1. Widths are O(1)
2. U(1) problem solved to leading order, no reason for WV to be good (small N_f/N_c ?)
3. Multiparticle production allowed

   => Bare Pomeron & Gribov's RFT

Perhaps phenomenologically more appealing than 't Hooft's but even harder to solve…

But there is a third possibility…

❸ Generalize QCD to $N \neq 3$ ($N = N_c$ hereafter) in other ways by **playing with matter rep**. The conventional way, QCD_F, is to keep the quarks in $N + N^*$ rep.

Another possibility, called for stringy reasons QCD_{OR}, is to assign quarks to the 2-index-antisymm. rep. of SU(N) (+ its c.c.)

As in 't Hooft's exp. (and unlike in TE), N_f is kept fixed ($N_f < 6$, or else AF lost at large N)

NB: For $N = 3$ this is still **good old QCD!**

Leading diagrams are planar, include “filled” q-loops since there are $O(N^2)$ quarks

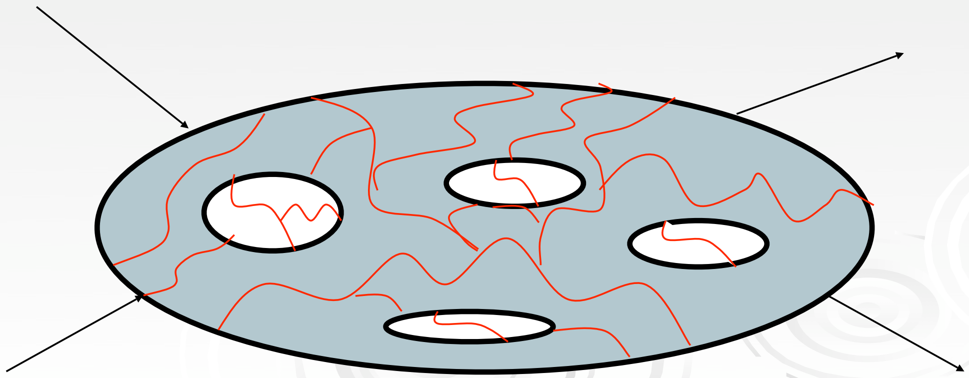
Widths are zero, U(1) problem solved, no p.pr.

Phenomenologically interesting? Don't know.

Better manageable? In some cases, I will claim…

QCD_{OR} as an interpolating theory:

1. Coincides with pure YM (AS fermions decouple) @ N=2
2. Coincides with QCD @ N=3
3. … and at large N?



ASV's 2003 claim

At large-N a bosonic sector of QCD_{OR} is equivalent to a corresponding sector of QCD_{Adj} i.e. of QCD with N_f Majorana fermions in the adjoint representation

An important corollary:

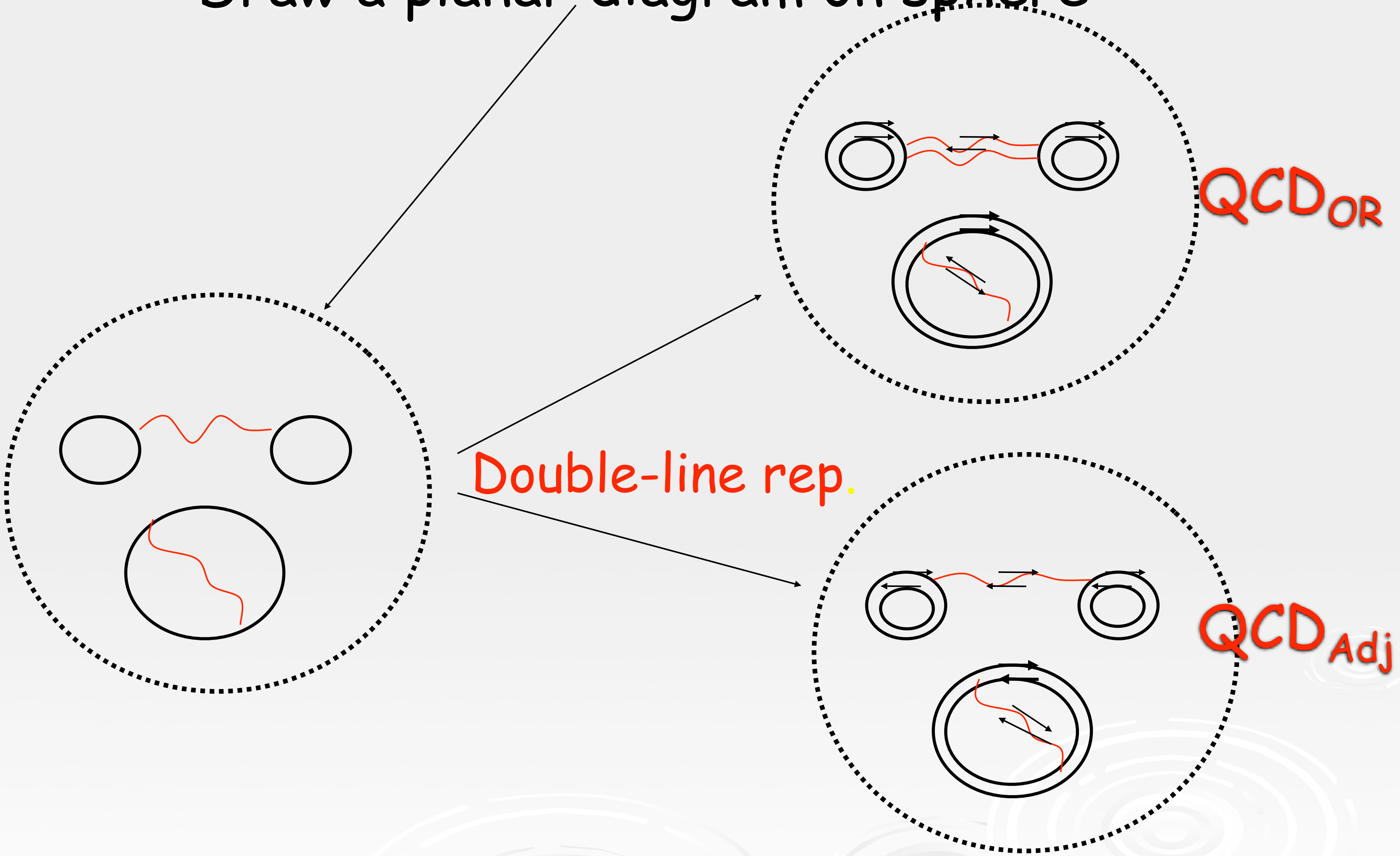
For $N_f = 1$ and $m = 0$, QCD_{OR} is planar-equivalent to supersymmetric Yang-Mills (SYM) theory

Some properties of the latter should show up in one-flavour QCD ... if N=3 is large enough

NB: Expected accuracy 1/N but improved by interpolation w/ N=2 case (Cf. N_f/N_c of 'tH!)

Perturbative arguments, checks

Draw a planar diagram on sphere



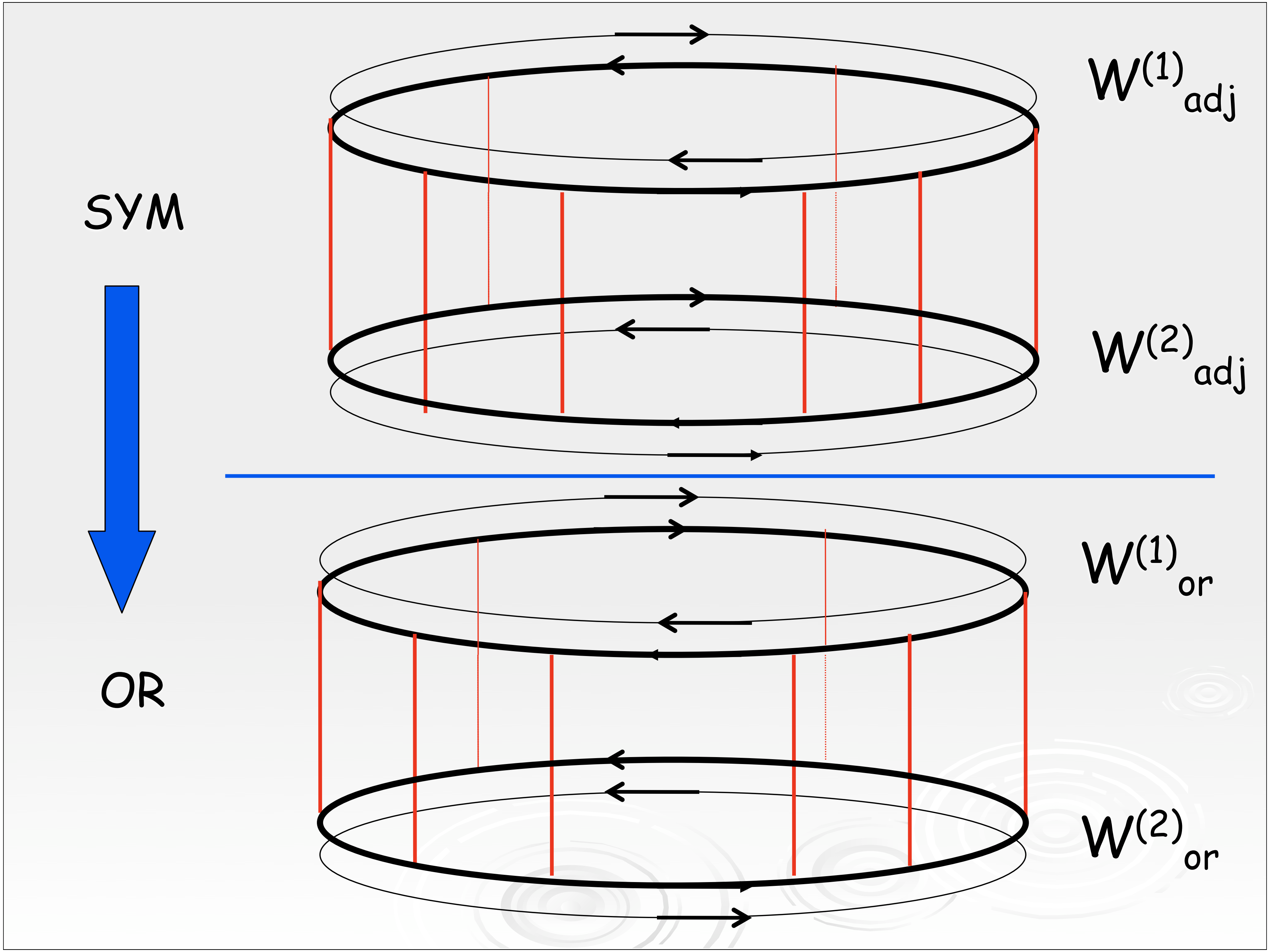
Differ by an even number of - signs...

Sketch of non-perturbative argument

(ASV '04, A. Patella, '05 + thesis '08)

- Integrate out fermions (after having included masses, bilinear sources)
- Express Trlog($\not{D}$+m+J) in terms of Wilson-loops using world-line formulation (expansion convergent?)
- Use large-N to write adjoint and AS Wilson loop as products of fundamental and/or antifundamental Wilson loops (e.g. $W_{adj} = W_F \times W_{F^*} + O(1/N^2)$)
- Use symmetry relations between F and F* Wilson loops and their connected correlators

An example: $\langle W^{(1)}\, W^{(2)} \rangle_{conn}$



SYM
OR
$W^{(1)}_{adj}$
$W^{(2)}_{adj}$
$W^{(1)}_{or}$
$W^{(2)}_{or}$

Key ingredient is C!

- Clear from our NP proof that C-invariance is necessary. Kovtun, Unsal and Yaffe have argued that it is also sufficient
- U&Y (see also Barbon & Hoyos) have also shown that C is spontaneously broken if the theory is put on $R^3 x S^1$ w/ small enough S^1. PE doesn't (was never claimed to) hold in that case
- Numerical calculations (De Grand and Hoffmann) have confirmed this, but also shown that, as expected on some general grounds (see e.g. ASV), C is restored for large radii and in particular on R^4
- Lucini, Patella & Pica have shown (analyt.lly & numer.lly) that SB of C is also related to a non-vanishing Lorentz-breaking F#-current generated at small R but disappearing as well as R is increased

Uncontroversial formulation of PE?

Provided that C is not spontaneously broken, the C-even bosonic sector of QCD_{OR} is planar-equivalent to the corresponding sector of QCD_{Adj} i.e. of QCD with N_f Majorana fermions in the adjoint representation

(NB: This should also work in the quenched approximation..)

Irrespectively of PE, it would be interesting to study (unquenched) QCD_{adj} for its own sake, e.g.

- As one varies N_f, the singlet PS mass should grow like N_f & coincide with the singlet S mass at $N_f=1$, $m=0$
- For $N_f=1$, $m \neq 0$ one should recover the behaviour of SYM when SUSY and Z_{2N} are softly broken (degeneracy of N-vacua is lifted, multiplets split etc.)

SUSY relics in one-flavour QCD

- Approximate bosonic parity doublets:

$m_S = m_P = m_F$ in SYM => $m_S \sim m_P$ in QCD

Looks ~ OK if can we make use of:

i) WV for m_P ($m_P \sim \sqrt{2}(180)^2/95$ MeV ~ 480 MeV),

ii) Experiments for m_S (σ @ 600MeV w/ quark masses)

Lattice work by Keith-Hynes & Thacker also support this approximate degeneracy

② Approximate **absence of "activity"** in certain chiral correlators

In SYM, a well-known WI gives

$$\langle \lambda\lambda(x)\lambda\lambda(y)\rangle = const.\ ,\ \langle \lambda\lambda(x)\bar{\lambda}\bar{\lambda}(y)\rangle \neq const.$$

PE then implies that, in the large-N limit:

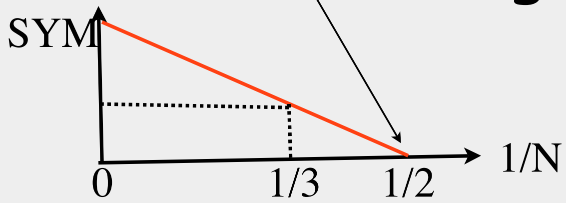
$$\langle \bar{\Psi}_R\Psi_L(x)\bar{\Psi}_R\Psi_L(y)\rangle = const.\ ,\ \langle \bar{\Psi}_R\Psi_L(x)\bar{\Psi}_L\Psi_R(y)\rangle \neq const.$$

Of course the constancy of the former is due to an exact cancellation between intermediate scalar and pseudoscalar states.

The quark condensate in N_f=1 QCD

Using $$\langle\bar\lambda\lambda\rangle_\mu = -\frac{9}{2\pi^2}\mu^3\lambda_\mu^{-2}exp\left(-\frac{1}{\lambda_\mu}\right)$$ $$\lambda_\mu = \alpha_s(\mu)N/2\pi$$

and vanishing of quark cond. at N=2, we get



$$\langle\bar\psi\psi\rangle_\mu = -\frac{3}{2\pi^2}\mu^3\lambda_\mu^{-1578/961}exp\left(-\frac{27}{31\lambda_\mu}\right)\, k(1/3)$$

1±0.3?

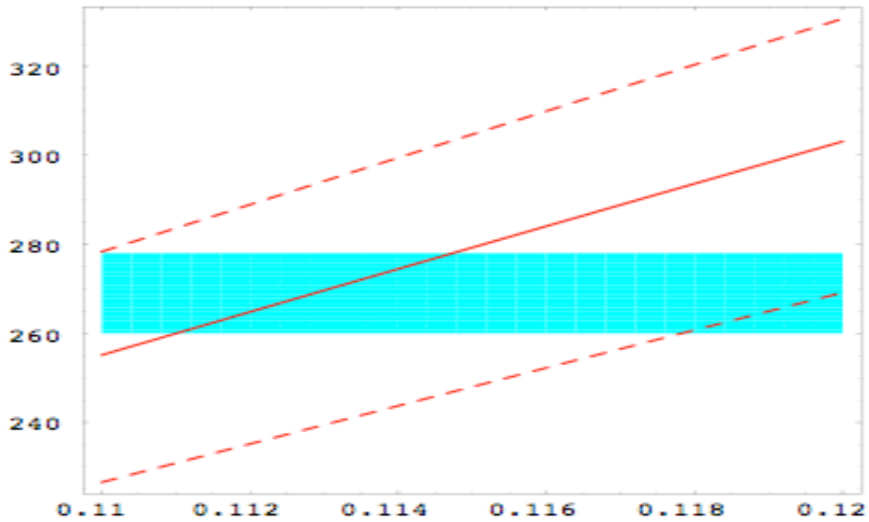
$$<(g^2)^{12/31}\bar\psi\psi> = -1.1k(1/3)\Lambda_{st}^3$$

$$\Lambda_{st} = \mu\ exp\left[-\frac{N}{\beta_0\lambda_\mu}\right]\left(\frac{2N}{\beta_0\lambda_\mu}\right)^{\beta_1/\beta_0^2}$$

N_f=1 condensate "measured"?

DeGrand, Hoffmann, Schaefer & Liu,
hep-th/0605147
(using dynamical overlap fermions and distribution of low-lying eigenmodes)

$\left(\langle\bar{\psi}\psi\rangle_{2GeV}\right)^{1/3}$



$3\alpha_s(2GeV)/2\pi$

Exact meaning of agreement still to be fully understood

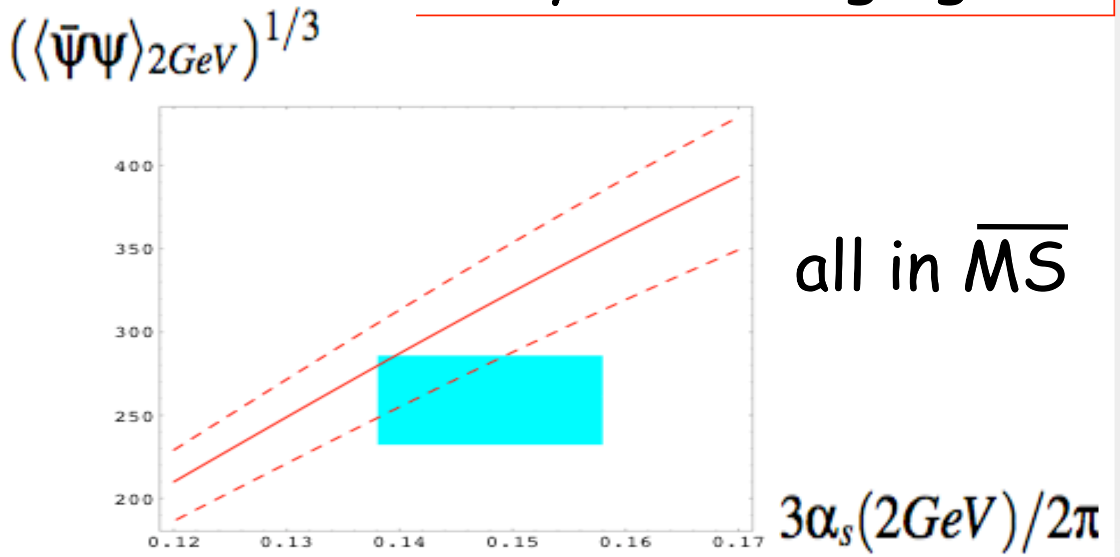
Extension to $N_f > 1$

(Armoni, G. Shore and GV, '05)

- Take OR theory and add to it n_f flavours in $N+N^*$.
- At $N=2$ it's n_f-QCD, @ $N=3$ it's $N_f(=n_f+1)$-QCD.
- At large N cannot be distinguished from OR (fits SYM β-functions even better at $n_f=2$: e.g. same β_0)
- Vacuum manifold, NG bosons etc. are different!
- Some correlators should still coincide in large-N limit. In above paper it was argued how to do it for the quark condensate

Quark condensate (ren. @ 2 GeV) vs α_s(2GeV) for N_f=3

Very encouraging!



all in $\overline{MS}$

$$\langle\bar{\psi}\psi\rangle_\mu = -\frac{3}{2\pi^2}\mu^3\lambda_\mu^{-\frac{44}{27}} exp\left(-\frac{1}{\lambda_\mu}\right)$$

Cf. $$\langle\bar{\lambda}\lambda\rangle_\mu = -\frac{9}{2\pi^2}\mu^3\lambda_\mu^{-2} exp\left(-\frac{1}{\lambda_\mu}\right)$$

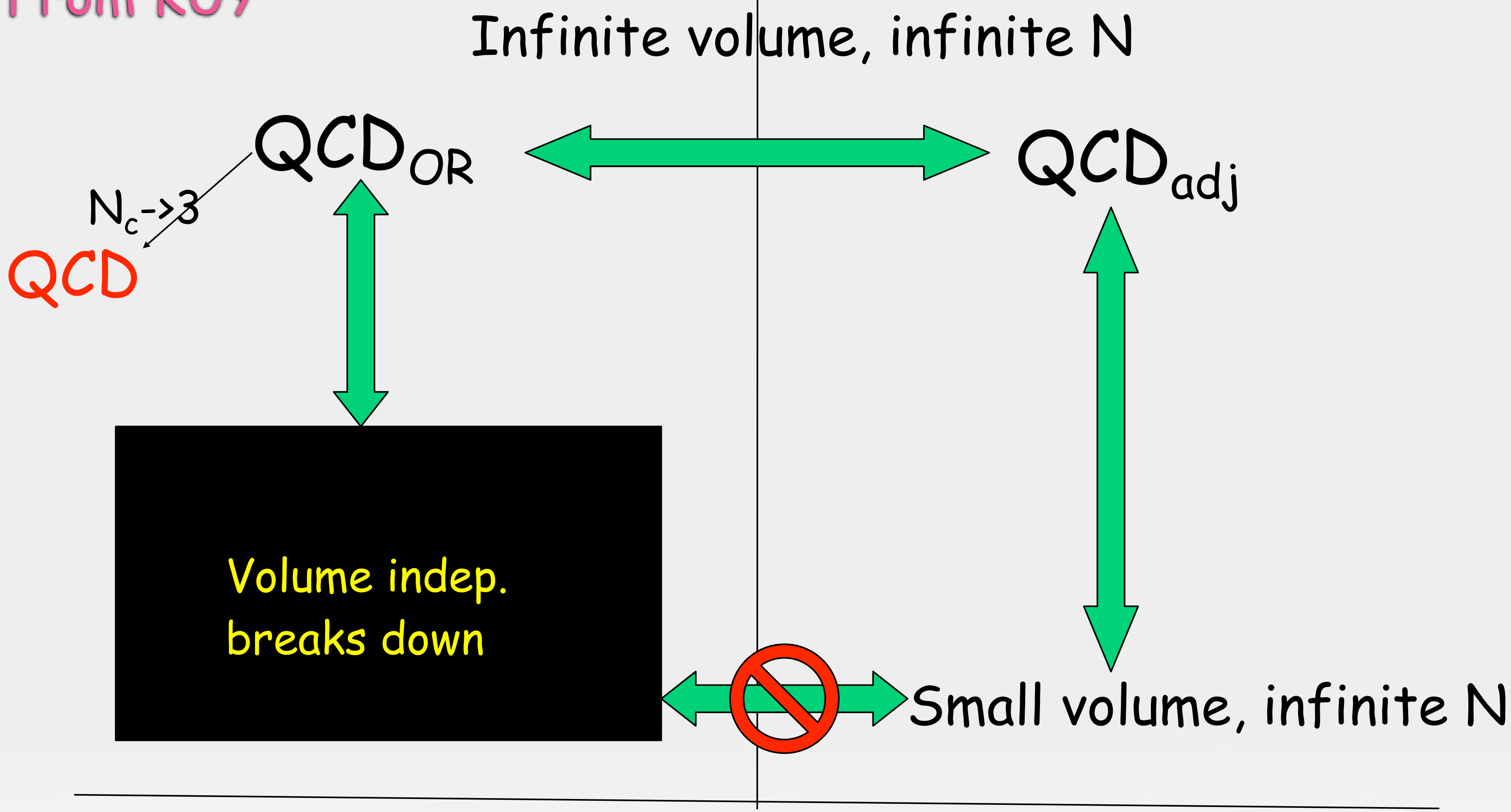
KUY's 2007 proposal

Kovtun, Unsal and Yaffe ('07) have made the interesting claim that QCD_{adj} , unlike QCD_F and QCD_{OR} , suffers no phase transition as an Eguchi-Kawai volume-reducing process is performed at large-N

If this were the case, we could get properties of QCD_{adj} at small volume by numerical methods and use them at large volume where the connection to QCD_{OR} can be established (C being nbroken there)

Finally, one would make semi-quantitative predictions for QCD itself (at different values of N_f and of the quark masses) by extrapolating to N=3

From KUY



Bottom line:

Solving QCD$_{adj}$ at infinite N and small volume should provide an $O(1/N_c)$ approximation to QCD with < 6 light flavours

Further developments

I: Emerging Center Symmetry

- Large-N emergence, in QCD_{OR}, of the Z_{2N} center symmetry of SYM (Armoni, Shifman, Unsal 0712.0672)
- Leading-N observables respect Z_{2N} in spite of the fact that the OR-theory has, at most, a Z_2

II: Lattice Evidence for T-independence at large N in confined phase of QCD.

Reviewed by:

R. Narayanan and H. Neuberger, arXiv:0710.0098 [hep-lat].

III: Quenched lattice evidence in favour of PE: the quark condensate (Armoni, Lucini, Patella, 0804.4501)

$$\frac{1}{N^2}\langle\bar{\psi}\psi\rangle_{\rm S}(m=0) = 0.2291(1) + \frac{0.4295(1)}{N} - \frac{0.925(3)}{N^2} + \ldots \ ,$$
$$\frac{1}{N^2}\langle\bar{\psi}\psi\rangle_{\rm As}(m=0) = 0.2291(1) - \frac{0.4295(1)}{N} - \frac{0.925(3)}{N^2} + \ldots \ ,$$
$$\frac{1}{N^2}\langle\lambda\lambda\rangle_{\rm Adj}(m=0) = 0.2291(1) - \frac{0.301(39)}{N^2} + \ldots \ .$$

Conclusions

- The orientifold large-N expansion is arguably the first example of large-N considerations leading to **quantitative analytic predictions** in D=4, strongly coupled, non-supersymmetric gauge theories

- Since its proposal, much progress made on

  - Tightening the non-perturbative **proof**
  - Providing **numerical checks**
  - Performing **simulations** for different N/reps.

But more work is still needed for:

- Estimating the size of 1/N corrections
- Extending the equivalence in other directions (Armoni, Israel, Moraitis, Niarcos, 0801.0762)
- Assessing the viability of the KUY proposal

One general question to end:

How come that lattice calculations become more and more complicated as we increase N when the actual dynamics should become simpler?

There must be some way to approach directly the large-N limit even numerically

My question/suggestion:

Is the time ripe for a large-N workshop at the GGI?