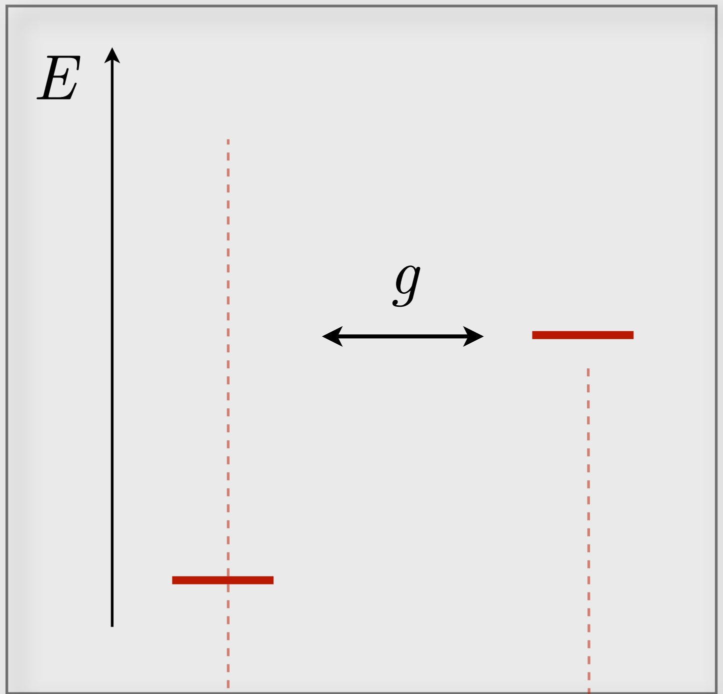
Landau-Zener (LZ) problem

- two coupled levels subject to weakly time dependent driving

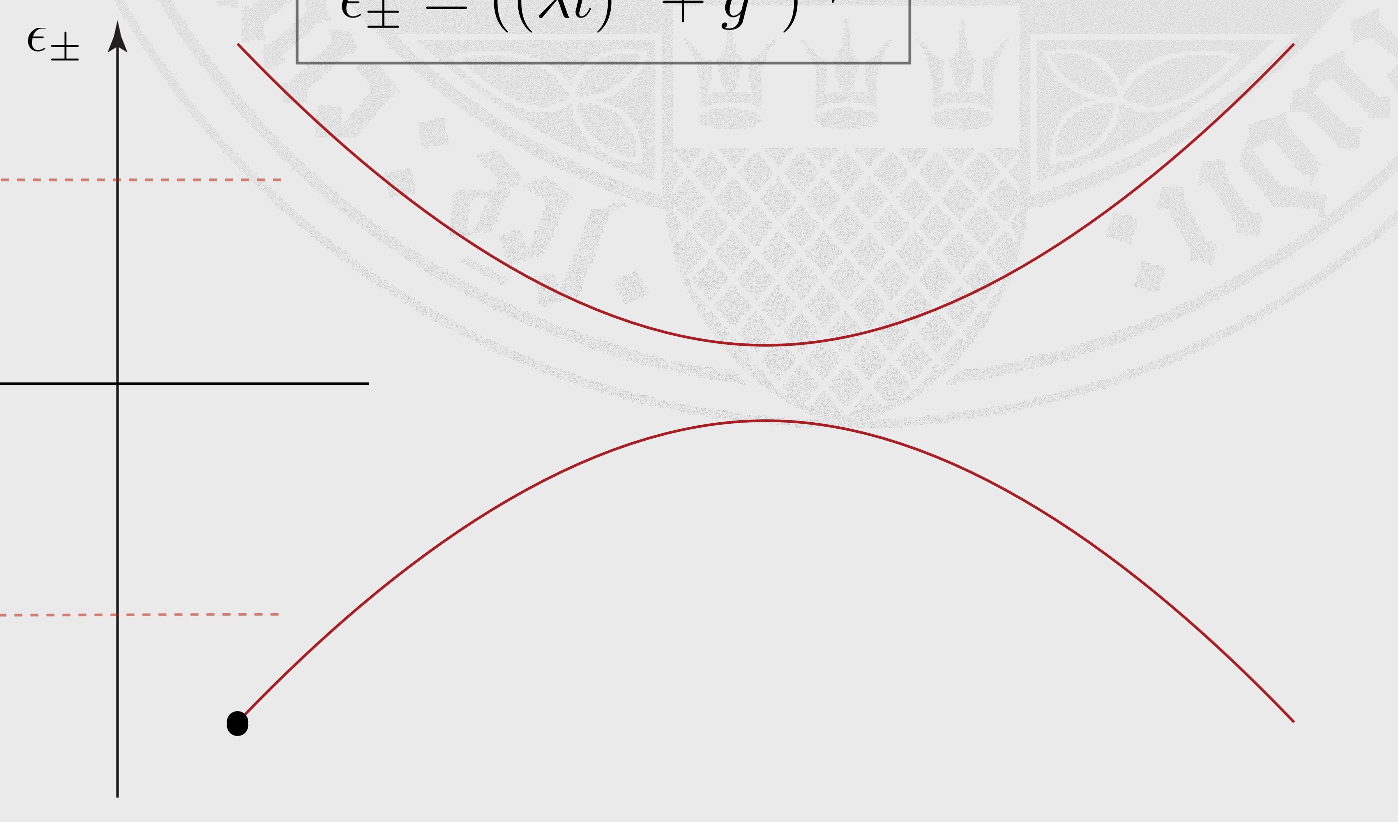


formalize by

$$\hat{H} = \begin{pmatrix} \lambda t & g \\ g & -\lambda t \end{pmatrix}$$

instantaneous levels

$$\epsilon_\pm = ((\lambda t)^2 + g^2)^{1/2}$$



$$P = 1 - e^{-\pi g^2/\lambda}$$

Landau/Zener/Stückelberg 1932

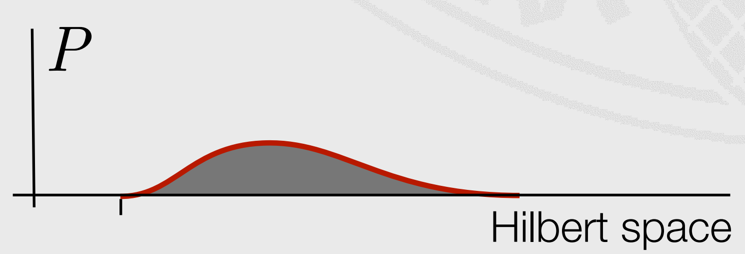
many particle Landau-Zener problem

- generalization: ⟶
  (a more structured system)

- previous studies include:
  - : a higher dimensional system (countless papers)
  - : low dimensional nonlinear (‘interacting’) systems (Wu & Niu, 00)
  - : linear many particle systems (Kayali & Synitsin, 03, Prokrovsky et al. 07)

- here: discuss a ‘generic’ (high dimensional/nonlinear) interacting variant

even at very slow driving:



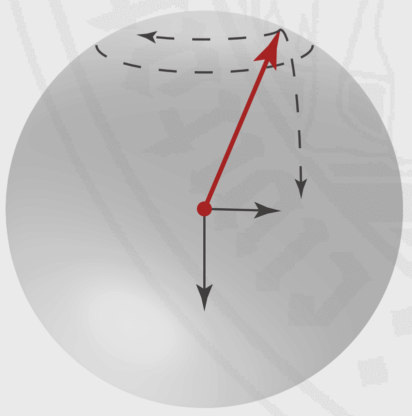
- adiabatic limit elusive
- particles broadly distributed over Hilbert space

A many particle generalization of the Landau-Zener problem

Florence, 09th Sept. 08
2nd INSTANS summer conference

Alexander Altland, Cologne University

collaboration: Victor Gurarie, University of Colorado
Anatoli Polkovnikov, Boston University
Thomas Kriecherbauer, (Mathematics) Bochum University



- model system
- driven evolution
- approach to the adiabatic limit

AA, V. Gurarie, PRL 07, AA, V. Gurarie, A. Polokovnikov, T. Kriecherbauer, unpublished

The model

- consider:

1.) a large spin $S = N/2 \gg 1$ coupled to a time dependent magnetic field of strength $-\lambda t$

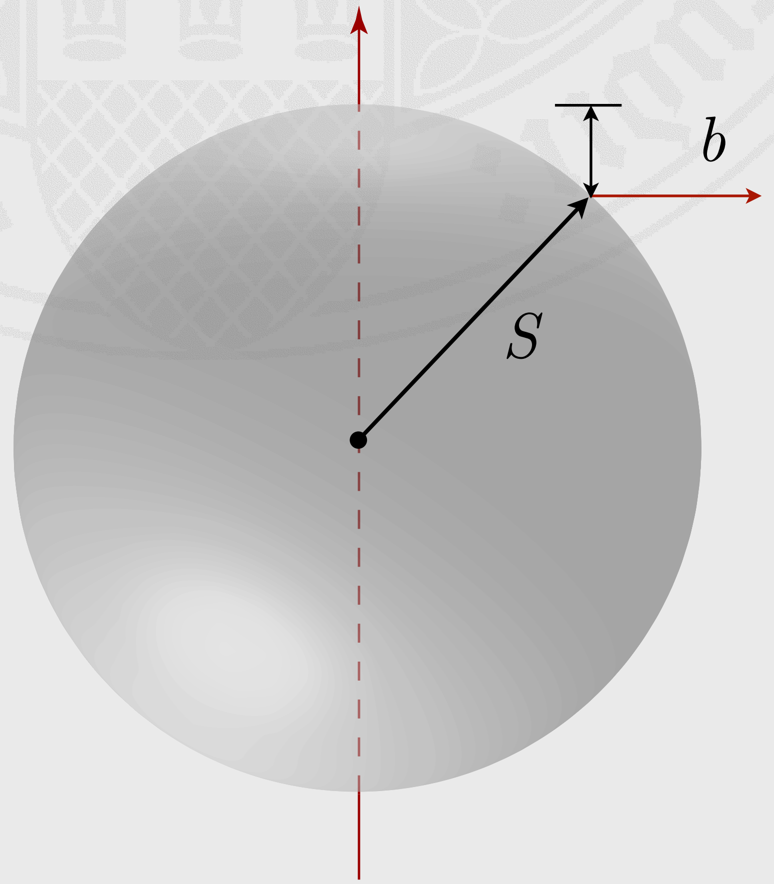
2.) a boson state kept at energy $+\lambda t$

3.) creation (annihilation) of a boson lowers (increases) spin by one.

$$\hat{H} = -\lambda t\ \hat{b}^\dagger \hat{b} + \lambda t \hat{S}_z + \frac{g}{2\sqrt{N}} \left(\hat{b}^\dagger\ \hat{S}^- + \hat{b}\hat{S}^+ \right)$$

4.) system initially prepared in its ground state:

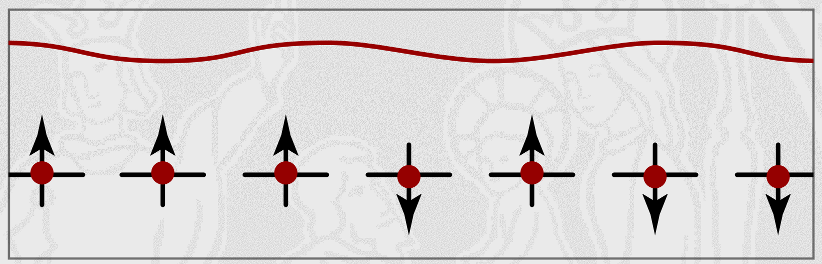
$$t \to -\infty : \quad \hat{S}_z|0,t\rangle = (N/2)|0,t\rangle$$
$$b^\dagger b|0,t\rangle = 0$$



5.) goal: compute $n_b \equiv \lim_{t\to\infty} \langle b^\dagger b \rangle$

applied relevance of the problem

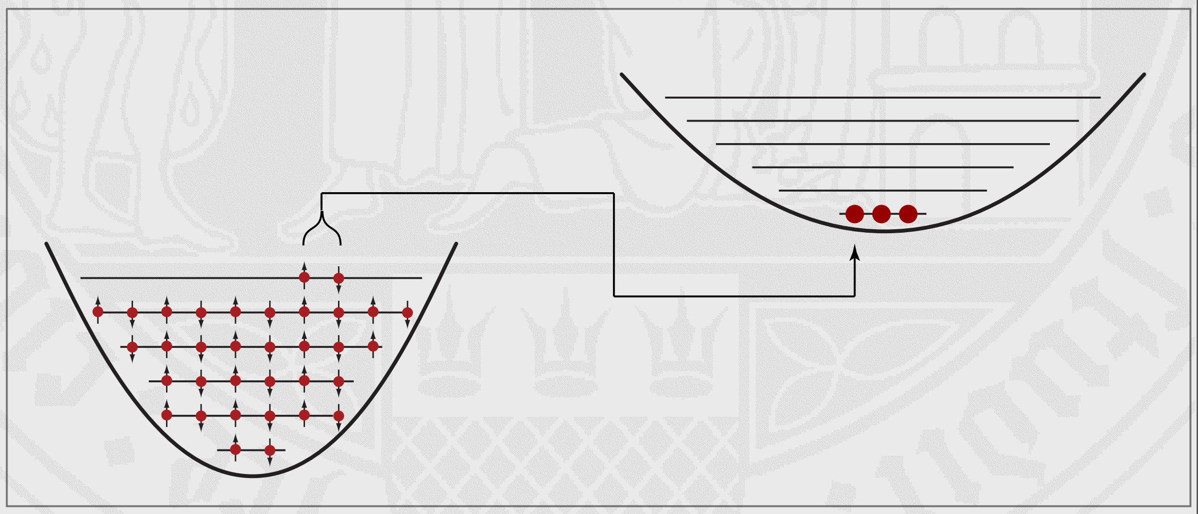
- model system equivalent to
  - system of N two level systems coupled by a bosonic mode



$$\hat{H} = -\lambda t\, \hat{b}^\dagger \hat{b} + \frac{\lambda t}{2} \sum_{i=1}^{N} \sigma_i^z + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left(\hat{b}^\dagger \sigma_i^- + \hat{b}\, \sigma_i^+ \right),$$

cf. molecular magnetism, adiabatic quantum information

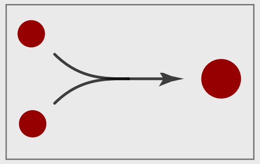
  - system of non-dispersive fermions converting to bosons



$$\hat{H} = -\lambda t b^\dagger b + \frac{\lambda t}{2} \sum_{i=1}^{N} \left(a_{i\uparrow}^\dagger a_{i\uparrow} + a_{i\downarrow}^\dagger a_{i\downarrow} \right) + \\ + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left(b^\dagger a_{i\downarrow} a_{i\uparrow} + b a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger \right)$$

realizable in fermionic condensates

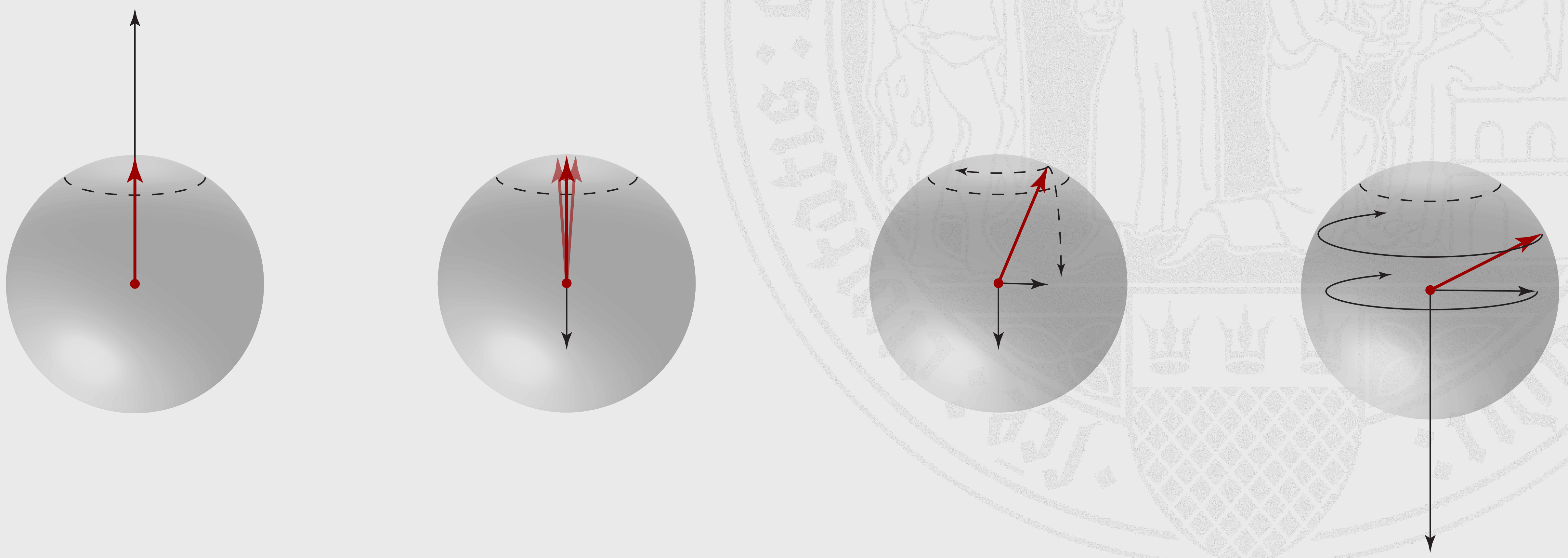
  - system of atoms converting to molecules (by tuning through a Feshbach resonance)



$$\hat{H} = -\lambda t d^\dagger d + \frac{\lambda t}{2} c^\dagger c + \frac{g}{\sqrt{N}} (d^\dagger c c + d c^\dagger c^\dagger)$$

realized in atomic condensates (JILA 05)

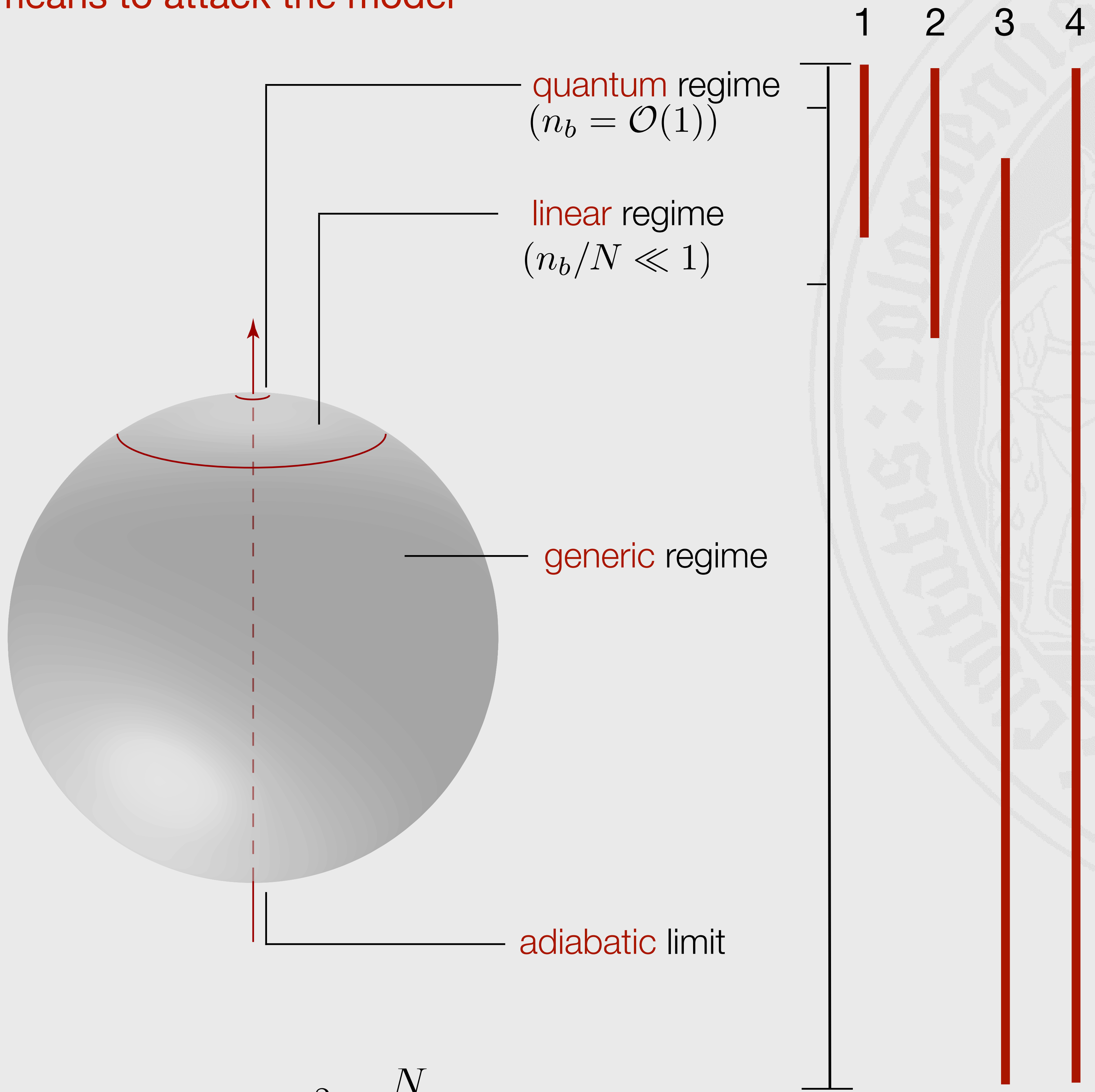
$$\hat{H} = -\lambda t\ \hat{b}^\dagger \hat{b} + \lambda t \hat{S}_z + \frac{g}{2\sqrt{N}} \left(\hat{b}^\dagger\ \hat{S}^- + \hat{b}\hat{S}^+ \right)$$



question:

What will be the distribution of polar angles at large times?

means to attack the model



1. solution of Schrödinger eq. (analytical)

2 Keldysh diagrammtic perturbation theory

3 semiclassical analysis (numerical/analytical)

4 numerical solution of Schrödinger equation (for moderate N)

semiclassical analysis with oscillator initial conditions (numerical)

$$H_{\mathrm{cl}} = -\lambda t |b|^2 + \frac{N}{2}\,\mathbf{n}\cdot\mathbf{B}$$

$$\hat{H} = -\lambda t\ \hat{b}^\dagger \hat{b} - \lambda t\ \hat{b}^\dagger_{HP}\hat{b}_{HP} + g\left(\hat{b}^\dagger\,\hat{b}^\dagger_{HP} + \hat{b}\,\hat{b}_{HP}\right).$$

Kayali & Synitsin 03

$$\mathbf{B} = \frac{g}{\sqrt{N}}\,b_1\,\mathbf{e}_1 + \frac{g}{\sqrt{N}}\,b_2\,\mathbf{e}_2 + \lambda t\,\mathbf{e}_3$$

moderately fast driving I: linearization

- for $N/2 - S_z(t) \ll N/2$: Holstein-Primakoff representation

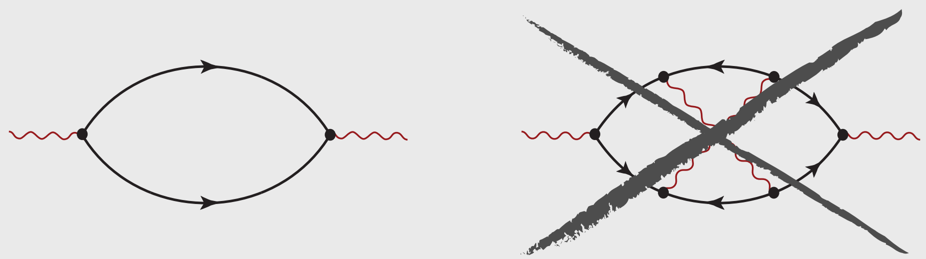
$$\hat{H} = -\lambda t \, \hat{b}^{\dagger}\hat{b} - \lambda t \, \hat{b}^{\dagger}_{HP}\hat{b}_{HP} + g\left(\hat{b}^{\dagger}\,\hat{b}^{\dagger}_{HP} + \hat{b}\,\hat{b}_{HP}\right).$$

This leads to (Kayali & Synitsin 03)

$$n_b \simeq e^{\pi g^2/\lambda} - 1, \;\; (n_b \ll N)$$

moderately fast driving II: diagramatic perturbation theory

▷ alternative strategy: apply Keldish diagrammatic perturbation theory to derive kinetic equation for $n_b(t)$. Use largeness of N to stabilize RPA approximation for self energy diagrams.



stabilized by:

1. $N \gg 1$
2. $N \gg \exp(\pi g^2/\lambda)$

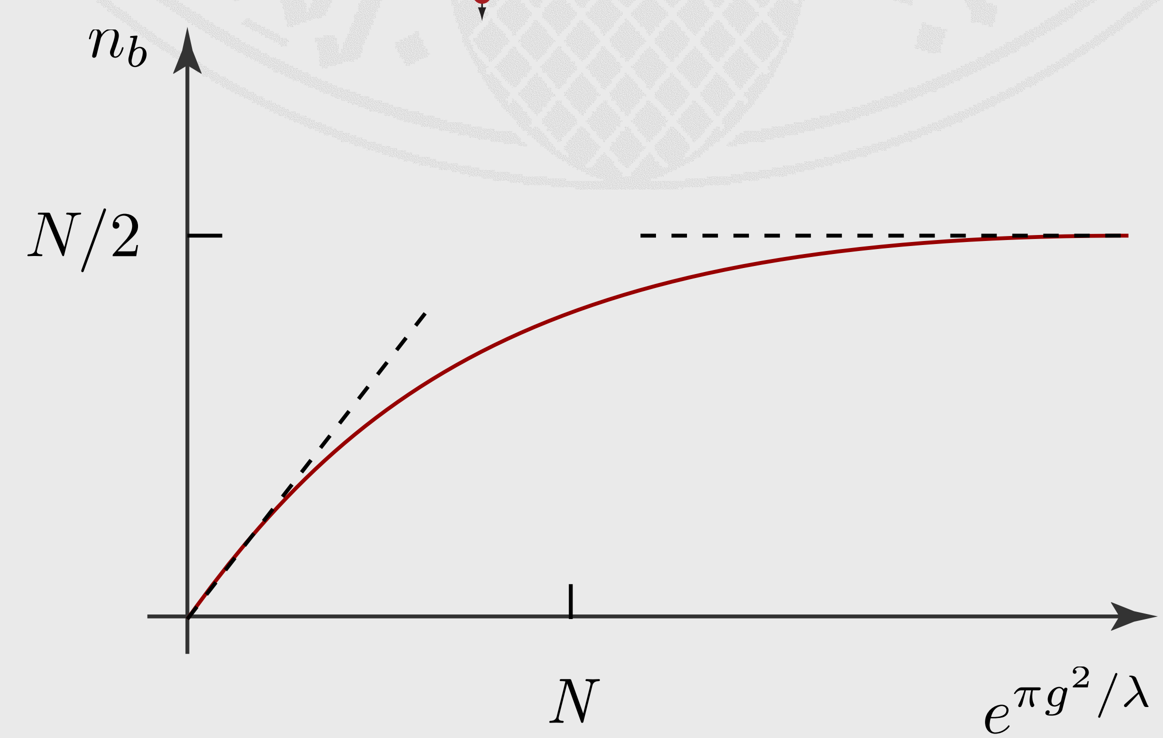
▷ rate equation

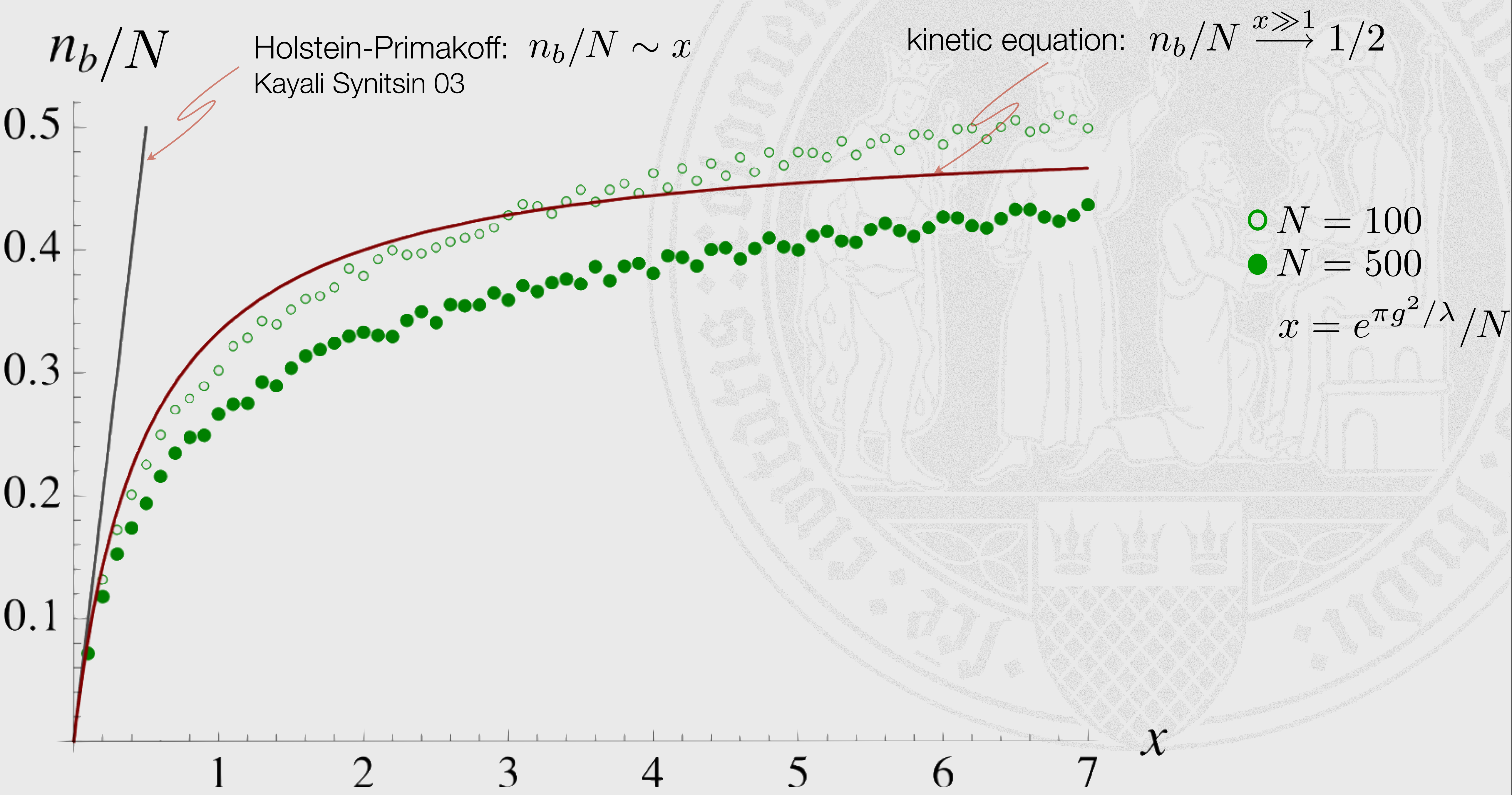
$$d_t n_b = 2\pi g^2 \delta(2\lambda t)\left(n_f^2\,(1+n_b) - n_b\,(1-n_f)^2\right)$$

where $n_b + N n_f = N$

▷ result:

$$n_b \equiv n_b(t\to\infty) = \frac{N\left(e^{\frac{\pi g^2}{\lambda}} - 1\right)}{2e^{\frac{\pi g^2}{\lambda}} + N}$$





$$\exp(\pi g^2/\lambda) \lesssim N$$

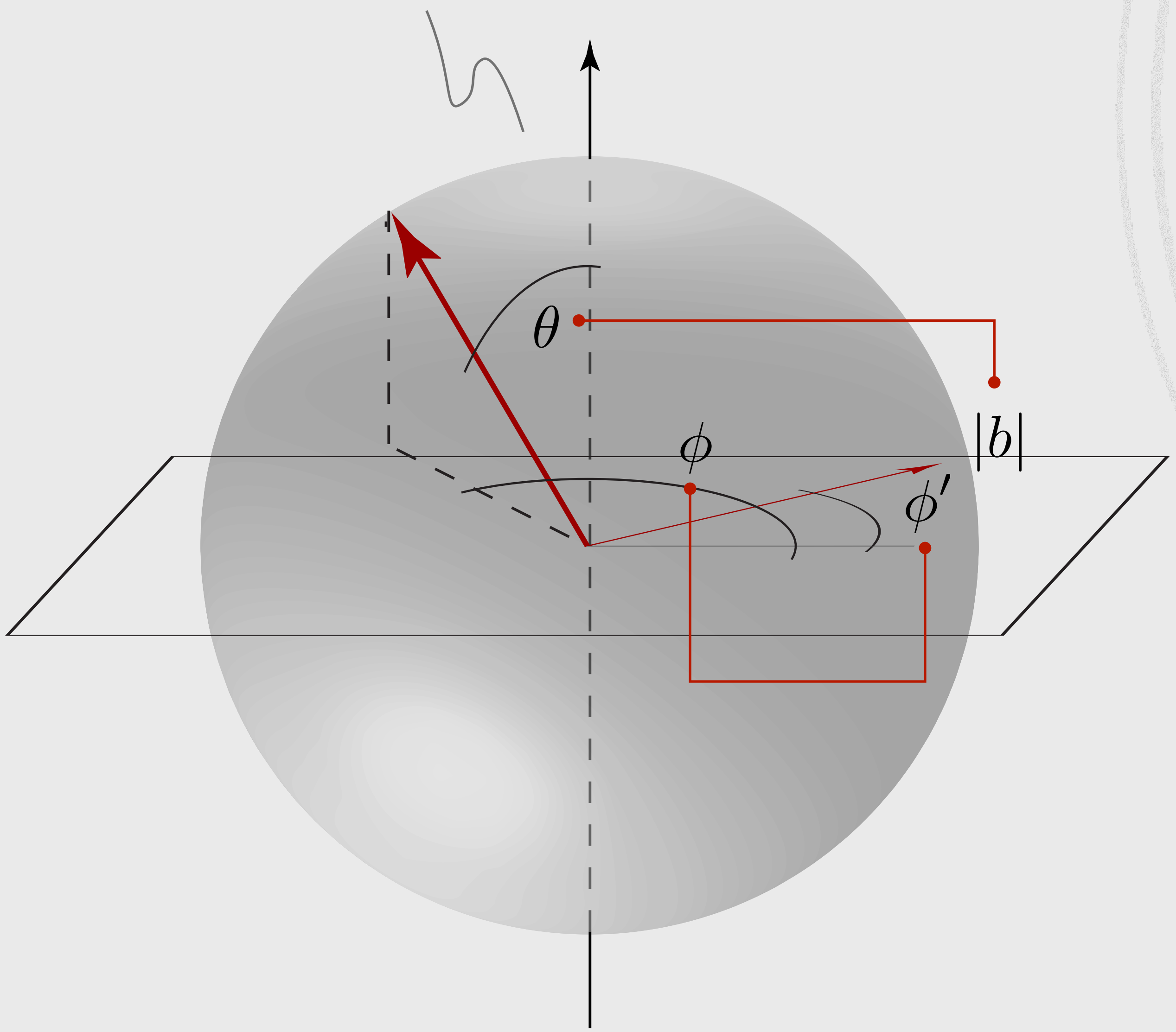
slow driving I: semiclassical analysis

- consider the classical Hamiltonian

$$H_{\mathrm{cl}} = -\lambda t |b|^2 + \frac{N}{2}\,\mathbf{n}\cdot\mathbf{B}$$

$$\mathbf{B} = \frac{g}{\sqrt{N}}\, b_1\,\mathbf{e}_1 + \frac{g}{\sqrt{N}}\, b_2\,\mathbf{e}_2 + \lambda t\,\mathbf{e}_3$$

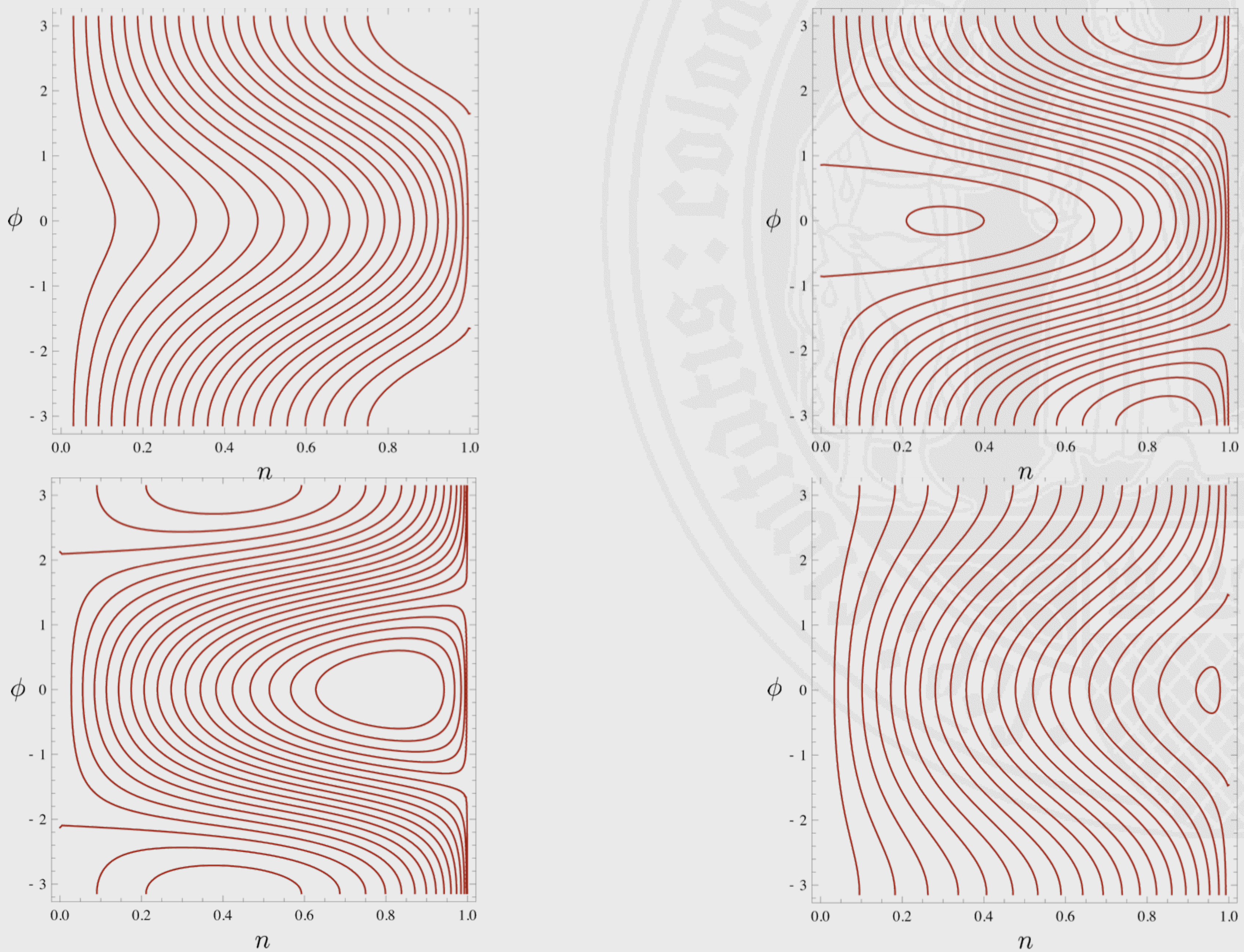
Holstein-Primakoff regime



$$\begin{aligned}\partial_t \theta &= g\sqrt{2}(1-\cos\theta)^{1/2}\sin\Delta\phi,\\ \partial_t \Delta\phi &= \frac{g}{\sqrt{2}}\frac{\sin\theta + 2(1-\cos\theta)\cot\theta}{(1-\cos\theta)^{1/2}}\cos\Delta\phi - 2\lambda t\end{aligned}$$

slow driving II: phase portraits

- phase space portrait in plane (n, ϕ), $\quad n \equiv n_b/N = (1 - \cos(\theta))/2$



- topology of trajectories changes in the driving process

- characteristic of trajectories: action integrals

$$S = \frac{1}{2\pi} \oint d\phi\, n \overset{n \simeq \text{const.}}{\simeq} n$$

slow driving III: adiabatic invariants

- Landau Lifshitz vol 1, par 48/49:

  1. in the adiabatic limit, the action of trajectories is conserved (in spite of the curves themselves changing)
  2. changes in the action are given by

action angle variables

reduced action

$$\Delta I = \dot{\gamma} \int \frac{dw}{\omega} \left(\frac{\partial \Lambda}{\partial w} \right)_I , \qquad \Lambda = \left(\frac{\partial S}{\partial \gamma} \right)_{I,\phi}$$

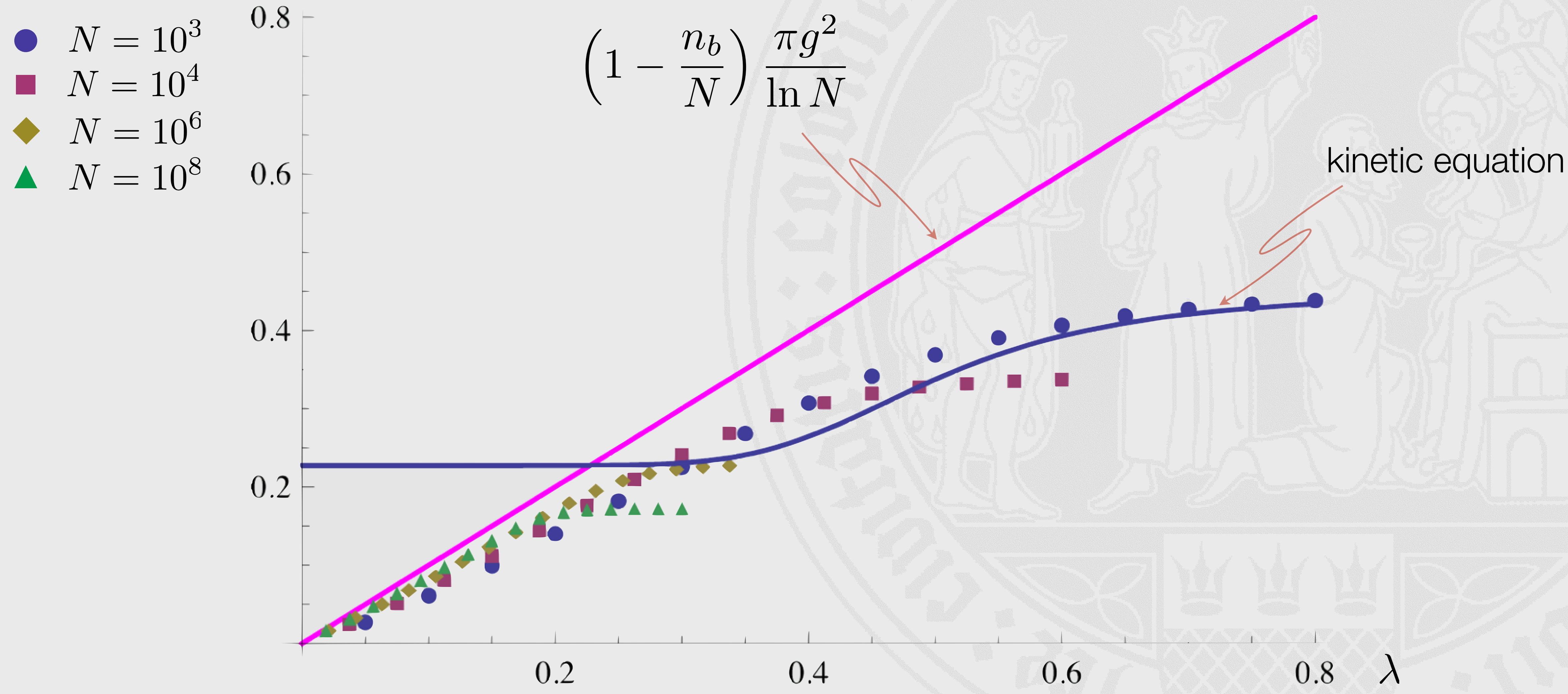
frequency of revolutions (vanishes at topological transitions)

- for conventional systems (no topological changes), variation of action exponentially small in driving rate. Here: at topological transitions, singularities

- prediction: for $\exp(\pi g^2/\lambda) \gg N$

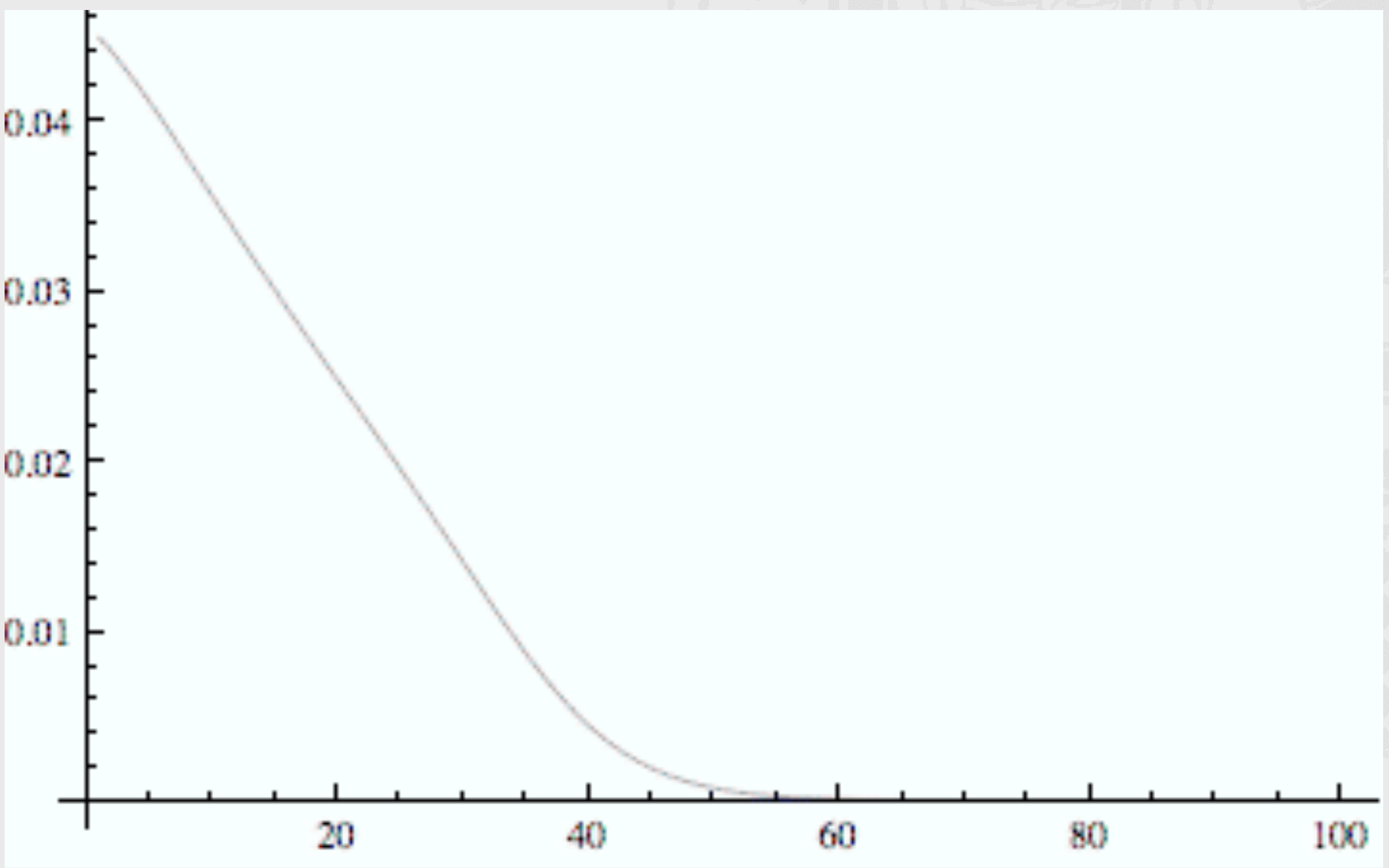
$$N^{\frac{N}{N-n_b}} \sim e^{\pi g^2/\lambda} \longrightarrow \lambda \sim \frac{\pi g^2}{\ln N} \left(1 - \frac{n}{N} \right)$$

slow driving IV: numerics and results



particle distribution

- so far considered *mean* number of produced bosons. However, broad distribution $P(n)$
- analytical solution in Holstein-Primakoff regime: $P(n) \simeq \dfrac{e^{-n/n_b}}{n_b}$

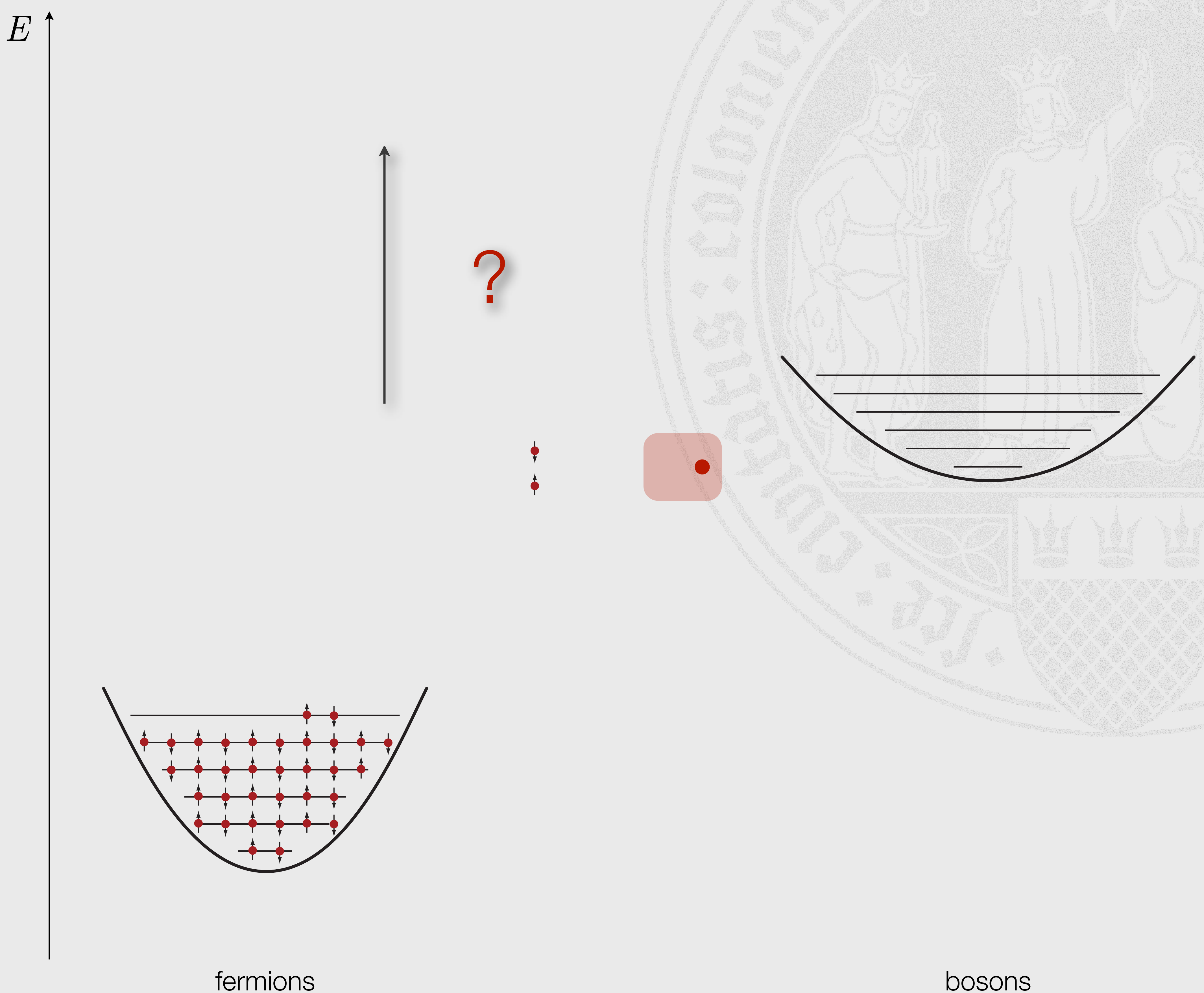


- interpretation: initial quantum fluctuations get strongly amplified.

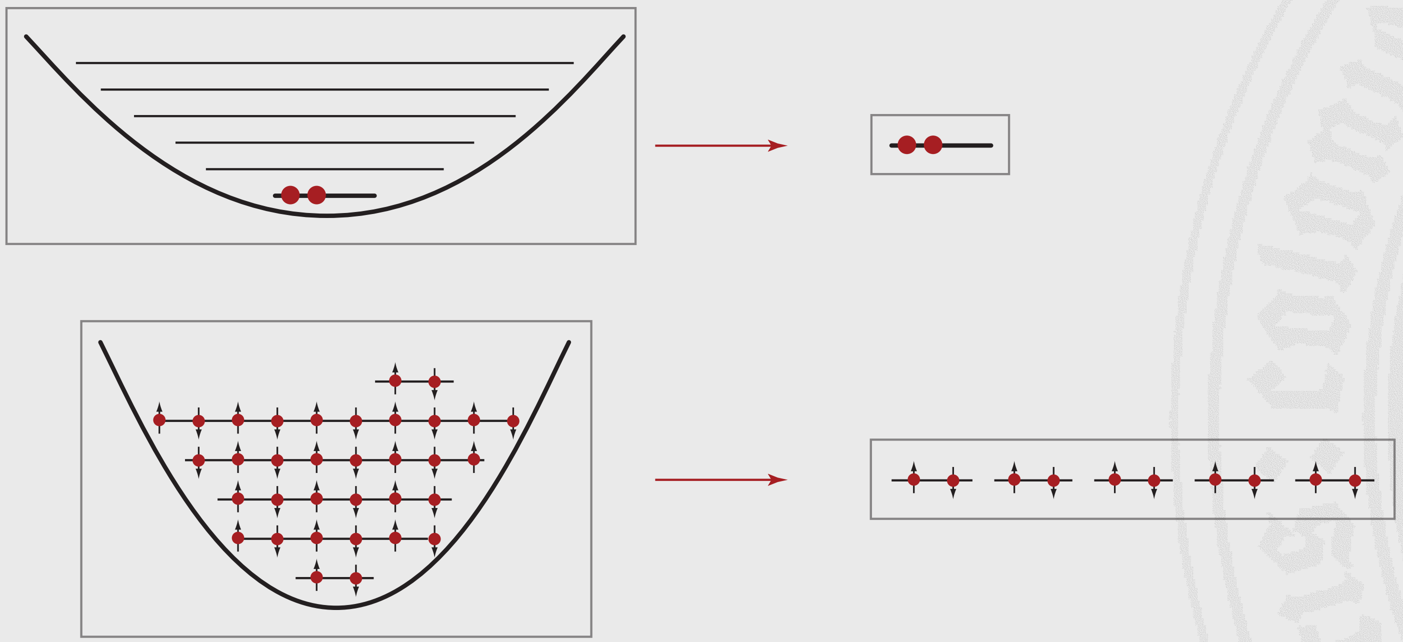
summary

- studied driven many particle system
- adiabatic limit only approached for thermodynamically vanishing driving rates
- at generic driving rates broad Hilbert space distributions (large quantum fluctatations/heating)

fermi-bose representation I

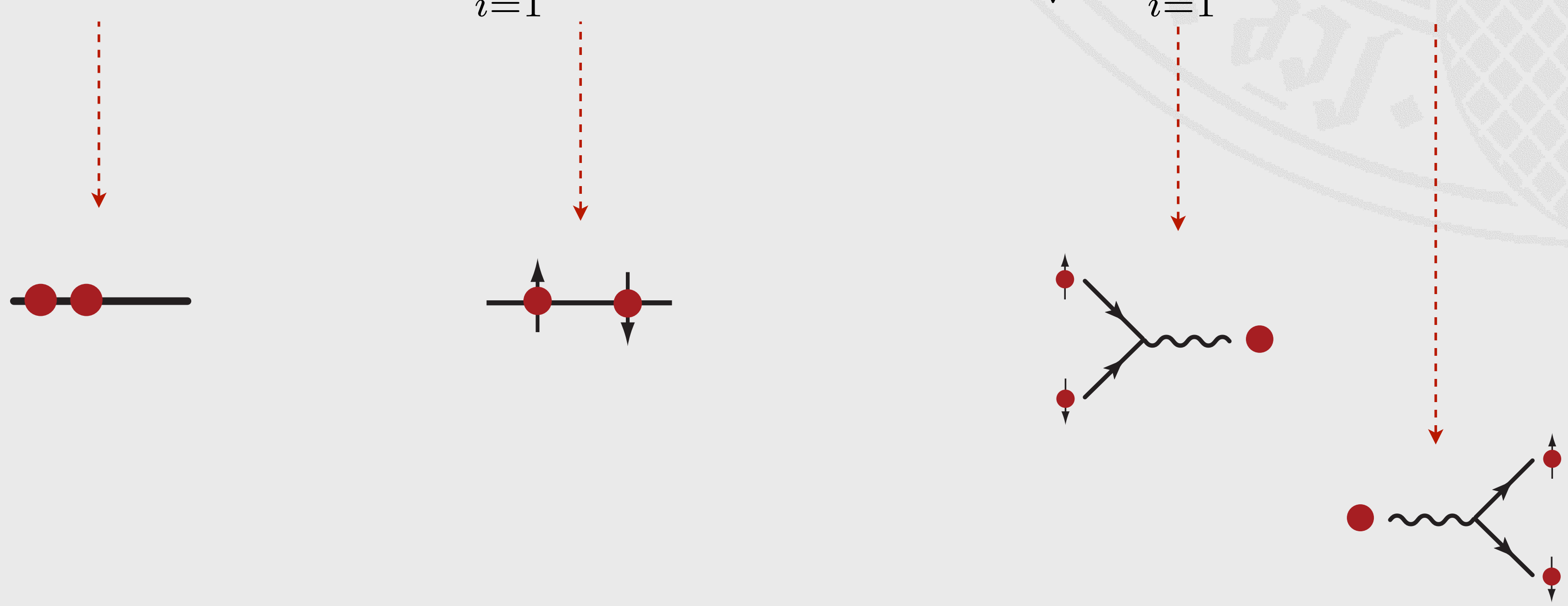


- simplify system as:



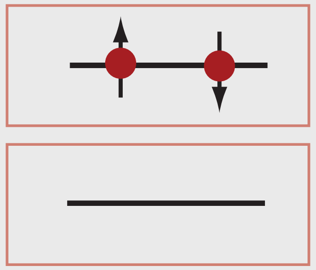
- describe system in terms of the Hamiltonian

$$\hat{H} = -\lambda t \; \hat{b}^\dagger \hat{b} + \frac{\lambda t}{2} \sum_{i=1}^{N} \left(\hat{a}^\dagger_{i\uparrow} \hat{a}_{i\uparrow} + \hat{a}^\dagger_{i\downarrow} \hat{a}_{i\downarrow} \right) + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left(\hat{b}^\dagger \, \hat{a}_{i\downarrow} \hat{a}_{i\uparrow} + \hat{b} \, \hat{a}^\dagger_{i\uparrow} \hat{a}^\dagger_{i\downarrow} \right) .$$



fermi-bose representation III

- fermionic level i hosts two configurations



(pseudo-) spin up

(pseudo-) spin down

i

$$\hat{H} = -\lambda t \; \hat{b}^\dagger \hat{b} + \frac{\lambda t}{2} \sum_{i=1}^{N} \sigma_i^z + \frac{g}{\sqrt{N}} \sum_{i=1}^{N} \left(\hat{b}^\dagger \, \sigma_i^- + \hat{b} \, \sigma_i^+ \right) ,$$

- introduce spin operators as: $\hat{S}^a = \frac{1}{2} \sum_i \sigma_i^a , \; (a = 1, 2, 3)$

$$\hat{H} = -\lambda t \; \hat{b}^\dagger \hat{b} + \lambda t \hat{S}_z + \frac{g}{2\sqrt{N}} \left(\hat{b}^\dagger \, \hat{S}^- + \hat{b} \hat{S}^+ \right)$$

where the initial configuration (all spins up) implies $\hat{S}^z |\Psi(0) = \frac{N}{2} |\Psi(0)\rangle$, i.e. the spin acts in an N-dimensional representation.