Spin dynamics in a doped ferromagnetic Bose-Hubbard insulator

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Spin-charge separation in a non-linear system

Bosons with spin tend to form completely polarized ground states. In such states ubiquitous spin-charge separation does not occur because of

- non-linear dispersion relation $\omega(k) = ck^2$
- kinematic constraint $\varrho(x) = s_z(x)$

In this work we demonstrate how to separate spin and charge in a doped Bose-Hubbard insulator and calculate the propagator of transverse spin excitations.

Recent work on 1-d Bose-ferromagnets

M. B. Zvonarev, V. V. Cheianov, and T. Giamarchi, Phys. Rev. Lett. 99, 240404 (2007)

S. Akhanjee, Y. Tserkovnyak, Phys. Rev. B 76 140408 (2007)

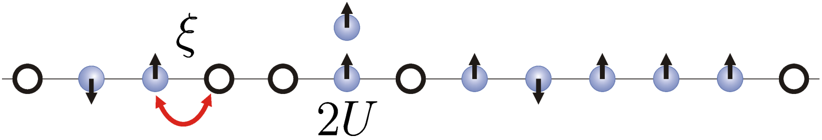
K. A. Matveev, A. Furusaki, arXiv:0808.0681

A. Kamenev, L.I. Glazman, arXiv:0808.0479

K. A. Matveev, A. Furusaki, and L. I. Glazman, Phys. Rev. B 76, 155440 (2007)

The spinor Bose-Hubbard model

A system of spin s (for simplicity $s = 1/2$) Bose particles on a 1-d lattice with nearest neighbor hopping and on-site repulsion.



ξ = hopping matrix element, U = repulsion strength

The Hamiltonian

The Fock space is generated by Bose fields $b_{\sigma,j}$, $b^\dagger_{\sigma,j}$, where $j = 1, \ldots, M$ and $\sigma = \uparrow, \downarrow$. The Hamiltonian is

$$H = T + V$$

where

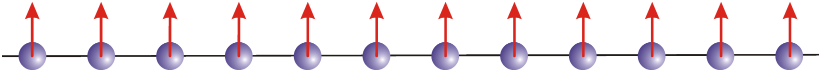
$$T = -\xi \sum_{j=1}^{M} \sum_{\sigma} [b^\dagger_{j,\sigma} b_{j+1,\sigma} + b^\dagger_{j+1,\sigma} b_{j,\sigma}]$$

and

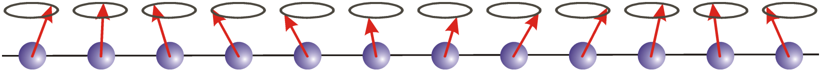
$$V = U \sum_{j=1}^{M} \varrho_j(\varrho_j - 1)$$

Spinor BH insulator: ground state and excitations

For integer filling factor ν and for large enough U the system is incompressible. For $\nu = 1$ this happens if $U > 4.3\xi$
Ground state is ferromagnetic



Excitations are transverse spin waves



Spinor BH insulator: low energy dynamics

Low energy degrees of freedom

$$\vec{s}(j) = \frac{1}{2} b^{\dagger}_{j,\lambda} \vec{\sigma}_{\lambda\mu} b_{j,\mu}, \qquad [s_\alpha(i), s_\lambda(j)] = i\delta_{ij}\epsilon_{\alpha\lambda\mu} s_\alpha(i)$$

For $E \ll U$ the dynamics is described by the Hamiltonian

$$H = -2J \sum_j \vec{s}(j) \cdot \vec{s}(j+1)$$

The evolution of spin in linear response theory is defined by

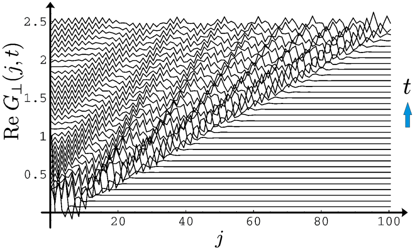
$$G_\perp(j,t) = \langle \Uparrow | s_+(j,t) s_-(0,0) | \Uparrow \rangle$$

For the Heisenberg Hamiltonian

$$G_\perp(j,t) = e^{-i\frac{\pi}{2}j} e^{2iJt} \mathcal{J}_j(2Jt)$$

where $\mathcal{J}_j(x)$ is the Bessel function of the first kind.

Transverse spin propagator: properties



- Decays rapidly for $j > 2Jt$
- Exhibits rapid oscillations as a function of j

The dispersion relation $\omega(k) = 2J(1 - \cos k) \Rightarrow$ the maximal group velocity $v_{\max} = 2J$.

Spin Dynamics in the Doped BHI

THE t-J APPROXIMATION

The Hamiltonian is

$$H = T + U$$

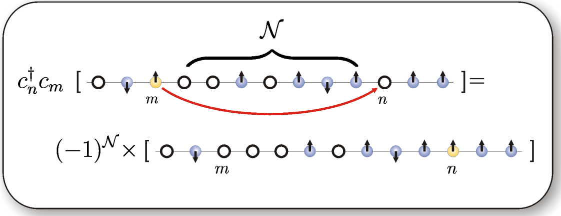
For $U \to \infty$ multiple occupancy is excluded. Denote by $\mathcal{P}$ the projector onto the space of excluded multiple occupancy. Then to the second order in T/U

$$H_{tJ} = \mathcal{P} T \mathcal{P} - \sum_a \mathcal{P} \frac{T|a\rangle\langle a|T}{E_a} \mathcal{P}$$

What are good variables?

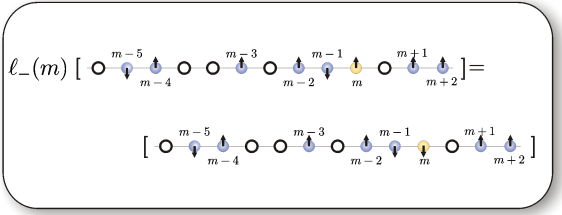
Nested variables

Spinless fermions
$c_j \, , \, c_j^\dagger$



$+$

nested spin $\vec{\ell}(m)$



The t-J Hamiltonian

In nested variables

$$H_{tJ} = T + \frac{\xi^2}{2U} \sum_{j=1} Q_j [\vec{\ell}(\mathcal{N}_j) + \vec{\ell}(\mathcal{N}_j + 1)]^2$$

where

$$\vec{\ell}(\mathcal{N}_j) = \sum_m \vec{\ell}(m) \int_0^{2\pi} \frac{d\lambda}{2\pi} e^{-\imath\lambda[m-\mathcal{N}_j]}, \qquad \mathcal{N}_j = \sum_{i \le j} \varrho_i$$

and

$$Q_j = c_j^\dagger c_{j-1}^\dagger c_{j+1} c_j + c_{j+2}^\dagger c_{j+1}^\dagger c_{j+1} c_j + 2 c_{j+1}^\dagger c_j^\dagger c_{j+1} c_j$$

Low doping

In the limit $1 - \nu \ll 1$ one can neglect fluctuations of charge

$$H_{tJ} = -\xi \sum_j (c_j^\dagger c_{j+1} + \mathrm{h.c.}) - 2J \sum_m \vec{\ell}(m) \cdot \vec{\ell}(m+1)$$

where

$$J = \frac{\xi^2}{2\pi U}(2\pi\nu - \sin 2\pi\nu)$$

Surprisingly, this result is correct even in the limit $\nu \to 0$.

The Hamiltonian shows spin-charge separation!

Separation of variables

Neglecting fluctuations of charge

$$H_{tJ} = H_{\rm spin} + H_{\rm charge} \quad \text{and} \quad |\Uparrow\rangle = |\Uparrow\rangle_{\rm spin} \otimes |{\rm FS}\rangle_{\rm charge}.$$

The local spin is

$$\vec{s}(j) = \varrho_j \vec{\ell}(\mathcal{N}_j) \approx \vec{\ell}(\mathcal{N}_j)$$

where

$$\vec{\ell}(\mathcal{N}_j) = \sum_m \vec{\ell}(m) \times \int_0^{2\pi} \frac{d\lambda}{2\pi} e^{-\imath\lambda[m-\mathcal{N}_j]}, \qquad \mathcal{N}_j = \sum_{i\le j} \varrho_i$$

The spin correlation function

The main result is

$$G_{\perp}(j,t) = \int_{-\pi}^{\pi} \frac{d\lambda}{2\pi} G_H(\lambda,t) D_{\nu}(\lambda;j,t)$$

where

$$G_H(\lambda,t) = e^{-2iJt(1-\cos\lambda)}$$

and

$$D_{\nu}(\lambda;j,t) = \langle \mathrm{FS}| e^{i\lambda \mathcal{N}_j(t)} e^{-i\lambda \mathcal{N}_0(0)} |\mathrm{FS}\rangle$$

Determinant representation

There exists a representation of

$$D_\nu(\lambda; j, t) = \langle \mathrm{FS}| e^{i\lambda \mathcal{N}_j(t)} e^{-i\lambda \mathcal{N}_0(0)} |\mathrm{FS}\rangle$$

in terms of a Fredholm determinant of an integrable kernel:

V. E. Korepin, N. M. Bogoliubov, and A. G. Izergin, QISM and Correlation Functions, Cambridge University Press, (1993),
F. Ghmann, A.G. Izergin, V.E. Korepin, A.G. Pronko, Int. J. Mod. Phys. B, 12 (1998) 2409;
V.Cheianov and M. Zvonarev, J. Phys. A:Math. Gen. 37, 2261-2297 (2004)

Asymptotic Formulae

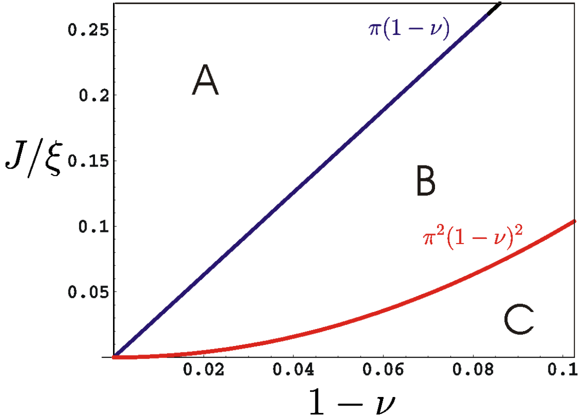
In the region $\pi(1-\nu)j \ll 1$ and $\xi\pi^2(1-\nu)^2 t \ll 1$

$$D_\nu(\lambda; j, t) = 1$$

Outside this region

$$D_\nu(\lambda; j, t) = e^{i\lambda j} e^{-\frac{\lambda^2}{4\pi^2}\ln\frac{|j^2 - v_F^2 t^2|}{v_F^2 t_c^2}}, \qquad t_c = \frac{e^{-1-\gamma}}{2\pi^2(1-\nu)^2\xi}$$

Parametric regions



REGION C: $J/\xi \ll \pi^2(1-\nu)^2$

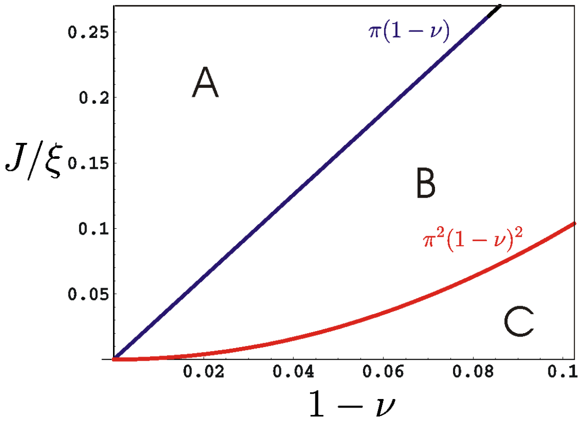
In this region there exists a time window

$$\frac{1}{\xi\pi^2(1-\nu)^2} \ll t \ll \frac{1}{2J}$$

Inside this window

$$G_\perp(j,t) = \sqrt{\frac{\pi}{2\ln t/t_c}}\, e^{-\frac{(\pi j)^2}{2\ln t/t_c}}$$

Parametric regions



Region A: $J/\xi \gg \pi(1-\nu)$

In this region and for

$$t \gg \frac{1}{\pi^2(1-\nu)^2\xi}$$

the correlator is

$$G_\perp(j,t) = \frac{G_H(\lambda_s,t)D_\nu(\lambda_s,j,t)}{2\sqrt{\pi i J t \cos\lambda_s}}, \qquad \lambda_s = \arcsin\frac{j}{2Jt}$$

This expression has a singularity at $j = v_F t$, where $v_F = \pi(1-\nu)$ is the sound velocity since

$$D_\nu(\lambda; j,t) \propto e^{-\frac{\lambda^2}{4\pi^2}\ln|j^2 - v_F^2 t^2|}$$

Conclusions

- We have demonstrated that by a proper choice of variables one can separate spin and charge in the doped Bose-Hubbard insulator
- Due to a non-local nature of the new variables there is no spin-charge separation in the local spin dynamics
- We found an explicit expression for the spin propagator in terms of a Fredholm Determinant. Depending on the parametric regime the spin propagator shows logarithmic diffusion of spin and light-cone singularities at $j = v_F t$ due to the spin-charge mixing in the long-distance limit.