Quantum many particle systems in one dimensional optical potentials

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Mesoscopics
Theory group

Materials and Technologies
for Information and communication Sciences

Outline

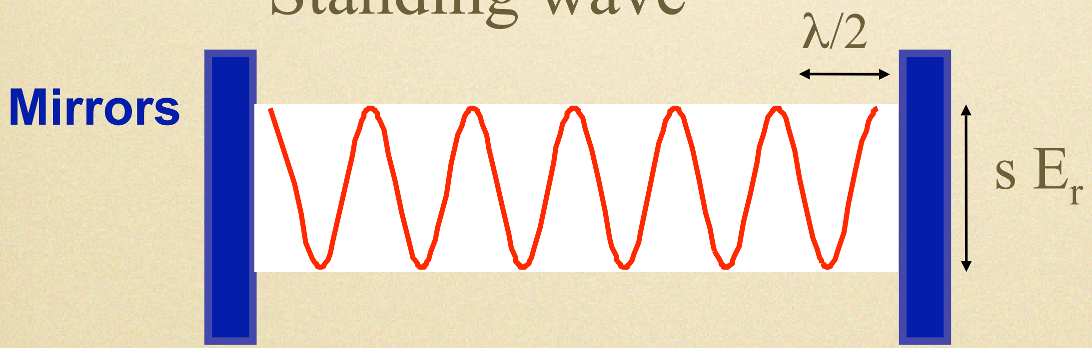
- General ideas.
- **Part I:** Quantum many particles in ring-shaped optical potentials.
  - **Fermions:** Boundary twist and persistent current in Hubbard models.
- **Part II:** Lattice regularizations of the Bose gas.

Optical Lattices

Dipole force on a two level atom from a far off-resonance laser beam:

$$V(x) \propto \frac{\Gamma}{\Delta} I(x)$$

Standing wave



$$E_r = \frac{\hbar^2 k^2}{2m}$$

$$U(x) = sE_r \cos^2(2x\pi/\lambda)$$

General Hamiltonian

$$H_0 = \int d\vec{r}\hat{\Psi}^+(\vec{r}) \left[\frac{\hbar^2}{2m}\nabla^2 - V_{latt}(\vec{r})\right] \hat{\Psi}(\vec{r}) = \sum_{i,j} t_{ij} b_i^\dagger b_j$$

$$H_{int} = \int d\vec{r} d\vec{r}'\, \hat{\Psi}^+(\vec{r})\hat{\Psi}^+(\vec{r}')V_{int}(\vec{r}-\vec{r}')\hat{\Psi}(\vec{r}')\hat{\Psi}(\vec{r}) = \sum_{ijkl} t_{ij;kl} b_i^\dagger b_j^\dagger b_k b_l$$

$$V_{int}(\vec{r}) \sim a_s \delta(\vec{r}) + g\frac{1 - 3\cos^2\theta}{|r|^3}$$

$$\boxed{t_{ij} \propto \varepsilon^{|i-j|^2} \qquad t_{ij;kl} \propto \varepsilon^{\gamma/2}}$$

$$\varepsilon \doteq \exp\left(-a^2/4l_{opt}^2\right) \ll 1\,, \qquad l_{opt} \sim a_s \left(\frac{E_r}{V_0}\right)^{\frac{3}{4}}$$

$$\gamma = (i-j)^2 + (i-k)^2 + (i-l)^2 + (j-k)^2 + (j-l)^2 + (k-l)^2$$

Effective model.

$$H_b = -\epsilon \sum_i n_i - t \sum_{<i,j>} \hat{b}_i^\dagger \hat{b}_j + U_0 \sum_i \hat{n}_i(\hat{n}_i - 1)$$

Jaksch, Bruder, Cirac, Gardiner, Zoller 1998;
Review: Lewenstein, Sanpera, Ahufinger, Damski, Sen De, Sen, Advances. in Phys. (2007)

$$+U_1 \sum_{<i,j>} \hat{n}_i \hat{n}_j - t_c \sum_{<i,j>} \hat{b}_i^\dagger (\hat{n}_i + \hat{n}_j) \hat{b}_j + t_p \sum_{<i,j>} (\hat{b}_i^\dagger)^2 (\hat{b}_j)^2 + \ldots$$

$$t_c = U_0 \varepsilon^{3/2} + I_{dd} \varepsilon$$

$$U_1 = 2t_p = (U_0 + I_{dd})\, \varepsilon^2$$

$$I_{dd} = \frac{m\mu_0\mu^2}{2a\pi^3\hbar^2} \qquad U_0 = (2/\pi)^{3/2} a_{BB} a s^{1/4} / l_\perp^2$$

Amico, Cataliotti, Mazzarella, Pasini arXiv:0806.2378

Quantum degenerate gas
(Bosons & Fermions)

Review: Lewenstein, Sanpera, Ahufinger, Damski, Sen De, Sen, Advances. in Phys. (2007)

Design of Hamiltonians in optical lattice: $(\mathcal{O}[\varepsilon^0])$

$$H = H_B + H_F + H_I$$

$$H_F = -\sum_{i,\sigma} \mu_i n_{i,\sigma} - \sum_i t(\sigma)\left(c_i^+ c_{i+1} + h.c.\right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$H_B = -\sum_i \varepsilon_i N_i - \sum_i J\left(a_i^+ a_{i+1} + h.c.\right) + V \sum_i N_i (N_i - 1)$$

$$H_I = \gamma \sum_i N_i n_i$$

Highly controllable systems:

- **Feasible optical & magnetic Manipulations** W. Hänsel et al. Nature 413, 498 (2001); H. Ott et al. PRL 87, 230401 (2001)
- **New opportunities to study open problems in condensed matter** (Feynman, 1982-1986).
- Possibly: implementations for quantum computation (low decoherence rate) Survey: Cirac, Duan, Zoller (2001) ; Garcia-Ripoll, Cirac, Zoller (2004).

Part I: Why ring shaped potentials?

- **General**:

  i) simple way to implement traslational invariance;

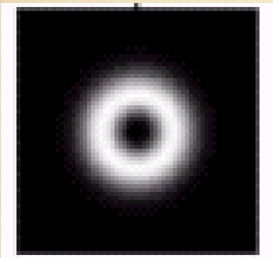
  ii) physical quantities approach to the thermodynamic limit in a fast (exponential) way [...Barber and Fisher PRL 1972...]

  Therefore: many studies for finite rings.

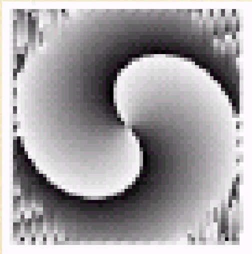
- Applications where the "topology" is crucial (Ex: "persistent currents" **in mesoscopics:** ....

Physical realization of the ring: Laguerre-Gauss + Plane wave

Amico, Osterloh, Cataliotti PRL 2005.



Intensity

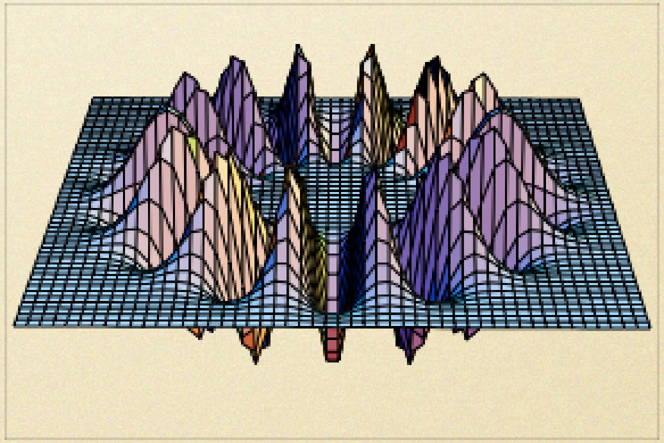


Phase

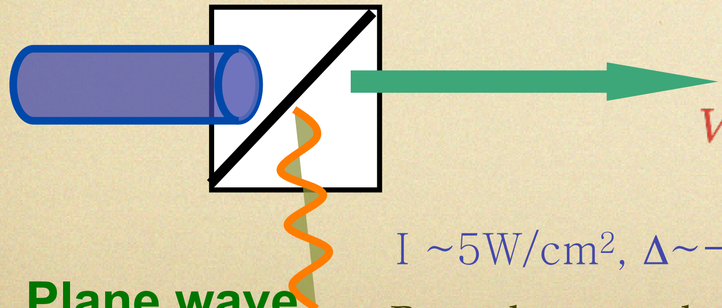
$$E(r,\varphi)=E_0 f_{p,l}(r)e^{il\varphi}e^{i(\omega t-kz)}$$

Chavez-Cerda JOB 2002.

$$f_{p,l}(r)=(-)^p\sqrt{\frac{2p!}{\pi r_0^2(p+|l|)}}\,\xi^{|l|}L_p^{|l|}(\xi^2)e^{-\xi^2} \quad ; \quad \xi=\sqrt{2}r/r_0$$



Very far-off-below resonant Laguerre-Gauss laser beam



Plane wave

$$V_{latt}(\varphi,z)=E_0^2\left[1+f_{pl}^2+2f_{pl}cos(l\varphi)\right]\cos^2(kz)$$

I ~5W/cm^2, Δ~-10^6 MHz, Barrier~5μK

Remark: $t_z << t_\phi$ by focusing the LG: Ex. L=15, waist/λ=100, t_z / t_ϕ<1/100

Optical ferris wheel for ultracold atoms

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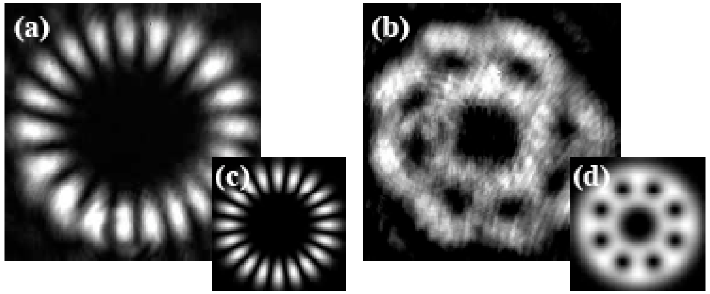


Fig. 3. (color online) Observed intensity distribution for the bright (a) and dark (b) lattice on an area of $3 \times 3\,\mathrm{mm}^2$ and the corresponding theoretical distributions (c) and (d). The bright lattice is generated from LG beams $\ell_1 = -\ell_2 = 10$ of equal intensity and the dark lattice from $\ell_1 = 3, \ell_2 = 11$ with $I_2 \approx \sqrt{\ell_2/\ell_1} I_1$. As an illustration of a rotating lattice we have made movies of the experiments e.g. (link $\ell_1 = -\ell_2 = 10$).

Optics Express, Vol. 15, Issue 14, pp. 8619-8625

Persistent currents: boundary twist.

$$\Psi(x_1,..x_n + L,..x_N) = e^{i\Phi_\sigma}\Psi(x_1,..,x_n,..x_N)$$

Boundary twist may set a current prop. to grad[Φ_σ(r)].

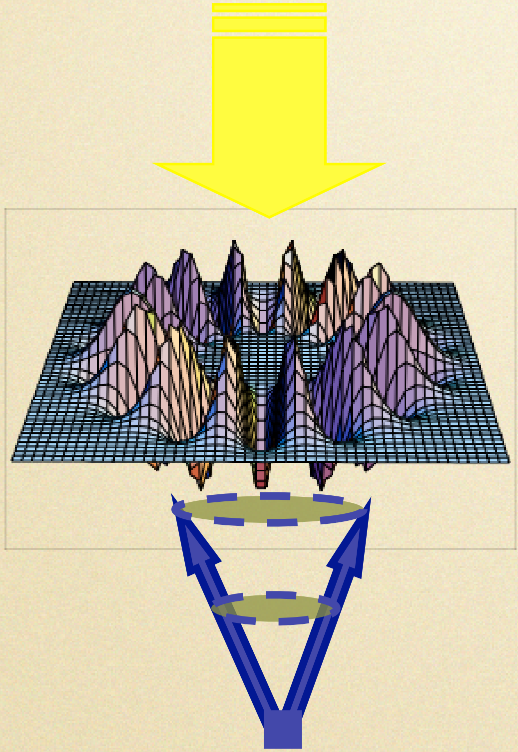
Khon PR 1964; Shastry, Sutherland, PRL1990; Zotos, Prelovsek , (Kluwer 2003).

See also Loss, Goldbart and Balatsky PRL 1990.

Realization of the boundary twist

Amico, Osterloh, Cataliotti PRL 2005.

Gaussian laser beam with a very different frequency of the beams generating the lattice: $A_E(m_F)$.



$$\Psi(x_1,..x_n+L,..x_N) = e^{i\Phi_\sigma}\Psi(x_1,..,x_n,..x_N)$$

$$\Phi_\sigma = m_F \pi \cos\theta + A_E(m_F)$$

Φ_σ is tunable: Generalization of the phase imprinting (Lenhardt et al PRL 2002)

Conical shaped magnetic field.

Berry phase on the hyperfine states m_F: $m_F \pi \cos\theta$ $\qquad \tan\theta = B_\phi / B_z$

Fermionic atoms:

Hubbard rings with correlated hopping.

$$H = \sum_i \left(t_i(\sigma) c_i^+ c_{i+1} + h.c. \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$t_i(\sigma) = -t \exp\left[i\gamma_i(\sigma) + i \sum_j \left(\alpha_{ij}(\sigma) n_{j,-\sigma} + A_{ij}(\sigma) n_{j,\sigma} \right) \right]$$

Key: Equivalent to ordinary Hubbard model with boundary twist $\Psi' = e^{i\Phi_\sigma} \Psi$

$$\phi(\sigma) = \sum_j \alpha_{jm}(\sigma)$$

$$\phi_{+-}(\sigma) = \sum_j \gamma_j(\sigma) + A_{j,j}(\sigma)$$

$$\phi_{++}(\sigma) = \sum_{j \neq m, m-1} A_{j,m}(\sigma) + A_{m,m-1}(\sigma) + A_{m-1,m+1}(\sigma)$$

$$\boxed{\Phi_\sigma = \phi(\sigma) + \phi_{+-}(\sigma) + \phi_{++}(\sigma)}$$

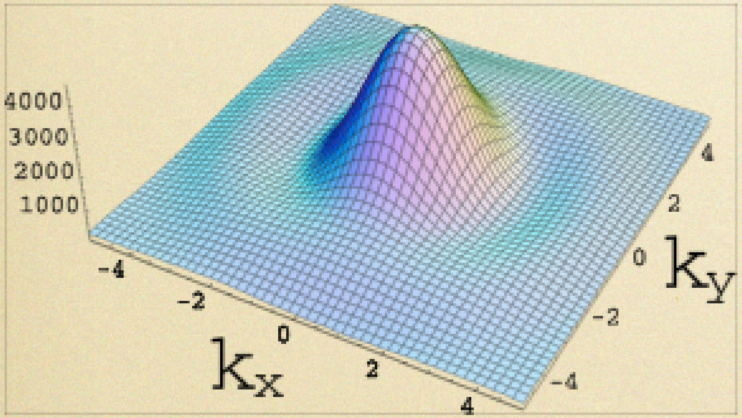
Schulz. Shastry, PRL 1998; Amico, Osterloh, Eckern NPB 2000

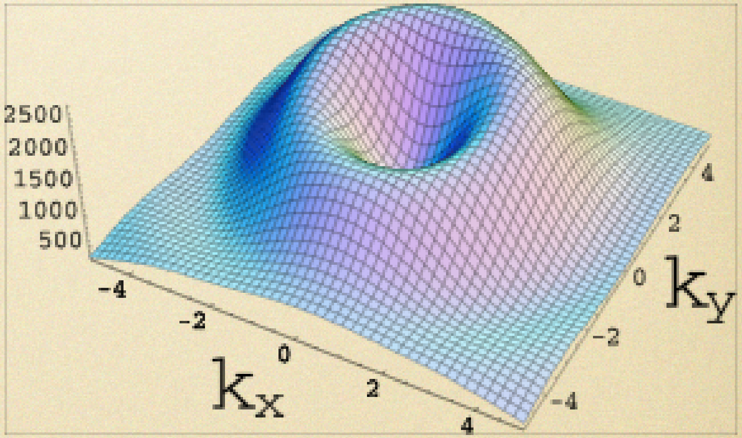
Persistent current in atomic rings with Hubbard interaction

Amico, Osterloh, Cataliotti PRL 2005.

$$\left|\Psi(k_x,k_y)\right|^2 = \left|w(k_x,k_y)\right|^2 \sum_{i,j=1} e^{i\mathbf{k}\cdot(\mathbf{x}_i-\mathbf{x}_j)} \sum_{k_\phi} e^{ik_\phi(\phi_i-\phi_j)} \left\langle n_{k_\phi} \right\rangle$$

N/L=32/16





Summary

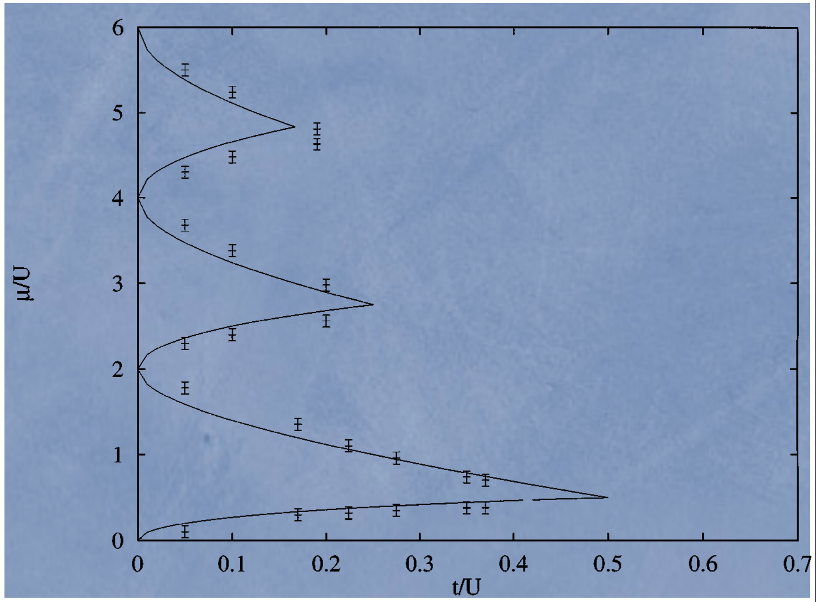
- Physical realizations of many body quantum systems with periodic b.c.. Persistent currents.
- This could represent a valid tool to study open questions in condensed matter (Persistent currents Vs Level statistics; Casimir effect, phase coherence...).

Part II: Bosonic atoms.

The Bose-Hubbard model

$$H_{BH} = U \sum_{i=-N_s}^{N_s} n_i[(n_i - 1) - \mu] - t(a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i)$$

- Density of bosons per site: $D = \dfrac{N}{(2N_s+1)\Delta}$
- Filling factor: $\nu = \dfrac{N}{(2N_s+1)} = D\Delta$



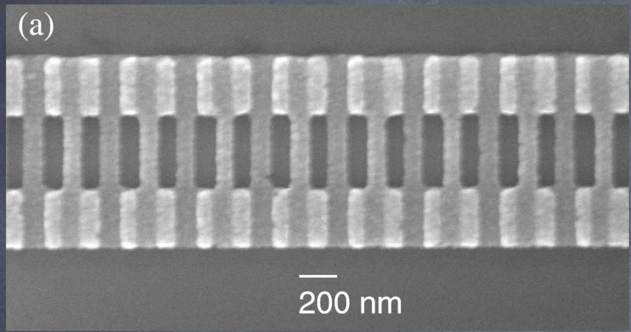
Commensurate filling:

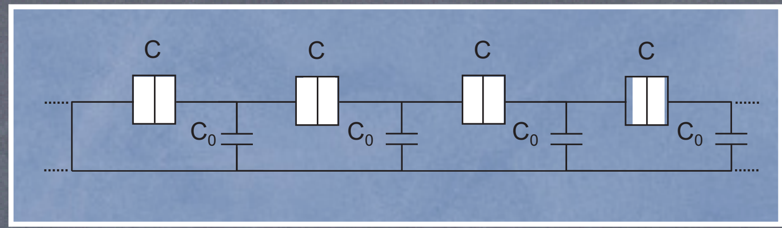
Insulator-Superfluid T=0 phase transition.

Haldane, PLA 1980;
Fisher, Weichman, Grinstein Fisher, PRB 1989;
Review: Fazio and van der Zant Phys. Rep. 2001.

See f.i. Kuehner and Monien 1999.
[From Amico, Penna, PRL 1998]

Other realization: 1d-Josephson junctions.





Delsing, Claeson, Likharev, Kuzmin, PRB 1990
Chow, Delsing, Haviland, PRL 1998

Electrostatic energy of Cooper pairs in each island: $\frac{e^2}{2C}\sum_i n_i(n_i-1)$

Josephson Energy: $E_J(\Phi)\sum_i a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i$

Review: Fazio, Van der Zant, Phys. Rep. 2001

Faliure of Coordinate Bethe Ansatz: Example N=3

$$|\Psi\rangle = \sum_{1\le j_1 \le j_2, \le \ldots \le j_N \le L} \psi(j_1,\ldots,j_N|\pi) a^\dagger_{j_1} a^\dagger_{j_2} \ldots a^\dagger_{j_N} |0\rangle$$

$$\psi(j_1,\ldots,j_N|\pi) = \sum_{Q\in\mathcal{S}_N} A_\pi(Q) \exp\left\{ i \sum_{l=1}^{N} k_{Q(l)} j_l \right\},$$

$$[\mathcal{H} - \mathcal{E}]\psi(j_1,j_2,j_3) = \frac{U^2}{4t} f(k_1,k_2,k_3)\delta_{j_1,j_2}\delta_{j_2,j_3}\psi(j_1,j_2,j_3)$$

$$\mathcal{E} = -2t\left[\cos(k_1) + \cos(k_2) + cos(k_3)\right]$$

$$f(k_1,k_2,k_3) = \frac{1}{\cos\frac{k_1+k_2}{2}\cos\frac{k_2+k_3}{2}\cos\frac{k_3+k_1}{2}}$$

Haldane, Choy Phys. Lett. A 1982

- Due to the multi-occupancy of the bosonic particles, the scattering is diffractive.

Remark: Level statistics is Wigner-Dyson!

(Kolovsky and Buchleitner 2004)

The dilute limit & Bose gas with δ-interaction

At small filling factors $\nu = D\Delta \to 0$ the lattice model turns into a continous integrable field theory:

$$H_{BH} = t\Delta^2 \mathcal{H}_{BG}$$

$$a_i = \sqrt{\Delta}\Psi(x), \quad n_i = \Delta\Psi^\dagger(x)\Psi(x)$$

$$x = \Delta i$$

$$\mathcal{H}_{BG} = \int dx \left[(\partial_x \Psi^\dagger)(\partial_x \Psi) + c\Psi^\dagger\Psi^\dagger\Psi\Psi - h\Psi^\dagger\Psi\right]$$

- Access to asymptotics of correlation functions of the Bose-Hubbard model in the dilute limit:

$$T = 0 \quad \Longrightarrow \langle a_i^\dagger a_j \rangle = A(i-j)^{-1/\theta} \quad \langle n_i n_j \rangle = \left(\frac{\nu}{\Delta}\right)^2 + \frac{A}{(i-j)^2} + B\frac{\cos[2\pi\nu(i-j)/\Delta]}{(i-j)^\theta}$$

$$T \neq 0 \quad \Longrightarrow \langle a_i^\dagger a_j \rangle_T = \left(\frac{v_F}{\pi T}\right)^{-1/\theta} \sinh[\pi T(i-j)/v_F]^{-1/theta} \quad \theta = 2\left(1 + \frac{4\nu t}{U}\right)$$

$$\langle n_i n_j \rangle_T = B_2\left(\frac{2\pi T}{v_F}\right)^2 e^{-2\pi T(i-j)/v_F} + B_3\left(\frac{2\pi T}{v_F}\right)^\theta e^{-\pi T\theta(i-j)/v_F} \cos[2\pi\nu(i-j)/\Delta]$$

Luttinger liquids: Haldane PRL, PLA 1981.

Recent summary: Amico and Korepin, Ann. Phys. 2004.

Integrable corrections to the Bose-Hubbard model

Lattice regularization of the Bose gas:

- R-matrix preserved & change of the trasfer matrix ⟹ 'quasi-local' Hamiltonians (Izergin-Korepin; Faddev-Takhtadjan-Tarasov).
- Modification of the R-matrix, keeping the Hamiltonian formally unaltered (quantum Ablowitz-Ladik).

Korepin, Izergin NPB 1982; Tarasov, Takhtadjan, Faddeev TMP 1983; Kundu, Ragnisco JPA 1994; Kulish LMP 1981; Bogolubov, Bullough 1992-1995; Amico and Korepin 2004.

Non-local corrections to BHM: Korepin-Itzergin model

Weak coupling limit: $c\Delta \ll 1$

$$H_{IK} = -\frac{4}{3c\Delta^3}\sum_j \frac{c\Delta}{8}(K_{j,j-1} - K_{j-1,j+1} - n_j) + \frac{(c\Delta)^2}{16}(K^2_{j,j-1} + h_j)$$

$$K_{l,m} = a_l a^\dagger_m + a_m a^\dagger_l$$

$$h_j = \sum_{\alpha,\beta=-2}^{2}\left[v_{\alpha\beta}n_{j+\alpha}n_{j+\beta} + w_{\alpha\beta}a_{j+\alpha}a^\dagger_{j+\beta} + (t_\alpha + q_\alpha n_{j+\alpha})\left(r_\beta a_{j+\beta}a^\dagger_{j+\beta-1} + s_\beta a_{j-\beta}a^\dagger_{j+\beta}\right)\right] + h.c. \,,$$

- Coupling of five neighbours: j-2...j+2

$$H_{IK} = H_{BH} - g\frac{c\Delta}{8}K_{j-1,j+1} + g\left(\frac{c\Delta}{4}\right)^2\left[K^2_{j,j-1} + h_j\right] \,,$$

- Besides for non the local terms, BH differs from IK for the quadratic hopping

Amico and Korepin, Ann. Phys. 2004

Non-local corrections to BHM: Faddev-Takhtadjan-Tarasov model

Integrable model for higher spin: $H_{FTT} = -2\kappa \sum_j \frac{\Gamma'(J_{j,j+1}+1)}{\Gamma(J_{j,j+1}+1)}$

$$J_{j,j+1}(J_{j,j+1}+1) = 2S_j \otimes S_{j+1} + 2s(s+1)$$

The FTT model is a realization of the lattice NLS with: $s = -2/(c\Delta)$

$$S_j^x = \frac{a_j^\dagger \rho_j + \rho_j a_j}{\sqrt{c\Delta}}, \quad S_j^y = i\frac{-a_j^\dagger \rho_j + \rho_j a_j}{\sqrt{c\Delta}}, \quad S_j^z = -\frac{2}{c\Delta}\left(1 + \frac{c\Delta a_j^\dagger a_j}{2}\right) \qquad \rho = \sqrt{1 + c\Delta a^\dagger a/4}$$

For large s:

$$H_{FTT} = H_{BH} + V \sum_j \left[n_j n_{j+1} + \frac{1}{2} K_{j,j+1}^2 + (n_j + n_{j+1}) K_{j,j+1} \right]$$

Amico and Korepin, Ann. Phys. 2004

Quantum Ablowitz-Ladik

$$L_i(\zeta) := \left(\begin{array}{cc} \zeta & q_i \\ -q_i^\dagger & \zeta^{-1} \end{array} \right), \quad R(\phi;\eta) := \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & b^- & c & 0 \\ 0 & c & b^+ & 0 \\ 0 & 0 & 0 & 1 \end{array} \right).$$

$$b^\pm = \frac{e^{\pm\eta}\sinh\phi}{\sinh(\phi-\eta)}, \qquad c = -\frac{\sinh\eta}{\sinh(\phi-\eta)}, \qquad \eta \text{ real}, \qquad \exp\phi = \zeta/\xi$$

$$H_{AL} = -\sum_{j=-N_s}^{N_s} \left[\boxed{q_j^\dagger q_{j+1} + q_{j+1}^\dagger q_j} - \alpha \boxed{\log(1 + q_j^\dagger q_j)} \right].$$

$$[q_i, q_j^\dagger] = (e^{2\eta} - 1)\left(1 + q_j^\dagger q_j\right)$$

Kulish, Lett. Math. Phys. 1981; Gerdikov, Ivanov, Kulish JMP 1984

$$\lim_{\zeta\to\infty} \frac{t(\zeta) - \zeta^{2N_s+1}}{\zeta^{2N_s-1}} + \lim_{\zeta\to 0} \frac{t(\zeta) - \zeta^{-2N_s-1}}{\zeta^{1-2N_s}}$$

$$\ln \det_q(T) - (2N_s + 1)\eta$$

- Therefore: α is NOT coupling constant!

Amico and Korepin, Ann. Phys. 2004

Integrable XXZ model

U_α[sl(2)]-quantum group symmetry.

$$R(\lambda) = \frac{\Gamma_\alpha[J(\alpha)+1-i\lambda]}{\Gamma_\alpha[J(\alpha)+1+i\lambda]}\frac{\Gamma_\alpha[1+i\lambda]}{\Gamma_\alpha[1-i\lambda]}$$

$$L_i(\lambda) := \frac{1}{\sin\alpha}\begin{pmatrix} \sinh(\lambda+iS^z) & S^-\sinh\alpha \\ S^+\sinh\alpha & \sinh(\lambda-iS^z) \end{pmatrix}$$

$$\Gamma_\alpha(x) = (1-\exp(\alpha))^{1-x}\prod_{n=0}^{\infty}[1-\exp(\alpha(n+1))]/[1-\exp(\alpha(n+x)]$$

Casimir of $su_\alpha(2)$: $C_S = S^\dagger S^- + \sinh(\alpha S^z)\sinh(\alpha(S^z+1))/\sinh^2(\alpha)$

$$H_{XXZ} = -\sum_i \psi_\alpha[J(\alpha)_{i,i+1}+1] + \psi_\alpha[1]$$

$$\psi_\alpha[x] = \Gamma'_\alpha(x)/\Gamma_\alpha(x)$$

$$\Delta C_S = \frac{\sinh[\alpha J(\alpha)]\sinh[\alpha(J(\alpha)+1)]}{\sinh^2\alpha}$$

Bytsko 2001

- Ground state is a singlet S_z=0

Zamolodchikov and Fateev (1981);
Sogo, Akutsu, Abe (1984);
Kirillov and Reshetikhin (1986).

Bosonic models with correlated hopping

$$q_j = \sqrt{\frac{e^{2\eta(a_j^\dagger a_j+1)}-1}{a_j^\dagger a_j+1}}\,a_j$$ with a_j true bosonic operators.

Small η expansion of the quantum Ablowitz-Ladik Hamiltonian:

$$H_{AL} \approx 2\eta\left\{\sum_j (a_j^\dagger a_{j+1} + h.c.) + \alpha n_j\right\} +$$

$$\eta^2\left\{\sum_j [(2+n_j+n_{j+1})a_j^\dagger a_{j+1} + h.c.] + 2\alpha n_j\right\} +$$

$$\eta^3\left\{\sum_j [(16(1+n_j+n_{j+1}) + 5(n_j^2+n_{j+1}^2) + 6n_j n_{j+1})a_j^\dagger a_{j+1} + h.c.] + 4\alpha n_j^2(1+n_j)\right\}$$

- All these models are solvable by algebraic BA.

The limit of large S

Amico, Cataliotti, Mazzarella, Pasini **arXiv:0806.2378.**

The limit: $\text{small}\,\alpha\,,\ \text{large}\,S\,,\ \text{small}\,\alpha S$

$$H_{XXZ} = -\sum_i \psi\left[J(\alpha)_{i,i+1} + 1\right] + \psi(1) \to H_b - N[\ln S - \psi(1)]$$

$$H_b = -\epsilon \sum_i n_i - t \sum_{<i,j>} \hat{b}_i^\dagger \hat{b}_j + U_0 \sum_i \hat{n}_i(\hat{n}_i - 1)$$

$$+U_1 \sum_{<i,j>} \hat{n}_i \hat{n}_j - t_c \sum_{<i,j>} \hat{b}_i^\dagger(\hat{n}_i + \hat{n}_j)\hat{b}_j + t_p \sum_{<i,j>} (\hat{b}_i^\dagger)^2 (\hat{b}_j)^2 + \ldots$$

- Isotropic Limit α=0: large S of Faddev-Takhtadjan-Tarasov model: $U_1 = 2U_0 = t^2 = 2t_c = 4t_p$

(Amico and Korepin, Ann. Phys. 2004)

PRL 97, 260401 (2006) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2006

Hidden Order in 1D Bose Insulators

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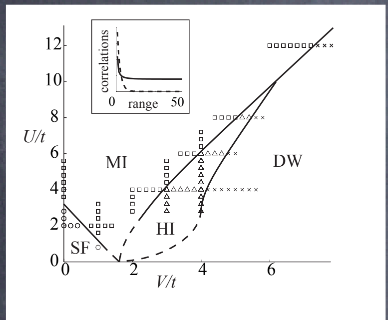
(Received 13 September 2006; published 27 December 2006)

We investigate the phase diagram of spinless bosons with long range ($\propto 1/r^3$) repulsive interactions, relevant to ultracold polarized atoms or molecules, using density matrix renormalization group. Between the two conventional insulating phases, the Mott and density wave phases, we find a new phase possessing hidden order revealed by nonlocal string correlations analogous to those characterizing the Haldane gapped phase of integer spin chains. We develop a mean field theory that describes the low-energy excitations in all three insulating phases. This is used to calculate the absorption spectrum due to oscillatory lattice modulation. We predict a sharp resonance in the spectrum due to a collective excitation of the new phase that would provide clear evidence for the existence of this phase.

See also: Berg, Dalla Torre, Giamarchi, Altman, cond-mat/08032851.

$$\Delta_c = E^{(0)}_{\delta n=1} + E^{(0)}_{\delta n=-1} - 2E^{(0)}_{\delta n=0}$$

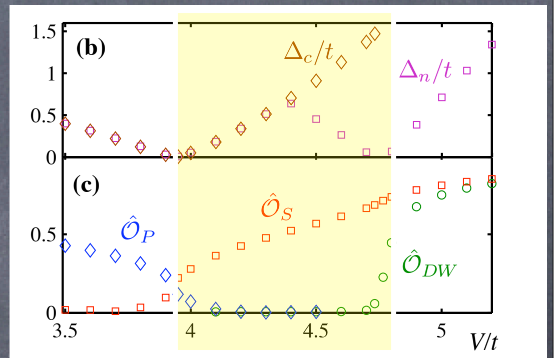
$$\Delta_n = E^{(1)}_{\delta n=0} - E^{(0)}_{\delta n=0}$$



$$\mathcal{O}_S^2 = \lim_{|i-j|\to\infty} \langle \delta n_i \left(\exp i\pi \sum_{i\le k<j} \delta n_k \right) \delta n_j \rangle$$

$$\mathcal{O}_{DW}^2 = (-)^{|i-j|} \lim_{|i-j|\to\infty} \langle \delta n_i \delta n_j \rangle$$

$$\delta n = n - \bar{n}$$



Haldane order: $|\uparrow,0,0,\downarrow,0,0,0,\uparrow,0,0,0,0,\downarrow,0,0,\uparrow\rangle$

$S_z = n - S$ $\quad |+\ 0\ 0\ -\ 0\ 0\ 0\ +\ 0\ 0\ 0\ 0\ -\ 0\ 0\ +\rangle$

Hidden order indicated by:

- Neutral/charged gaps.
- 'String-order parameter'.

Hidden order and NLσM

Amico, Cataliotti, Mazzarella, Pasini 2008

Fluctuations around the 'Neel order':

$$\vec{n}_j(\tau)^2 = 1$$

$$\vec{l}_j(\tau) \cdot \vec{n}_j(\tau) = 0$$

$$\vec{S}_{2k+1} = l_k + S n_k \qquad \vec{S}_{2k} = l_k - S n_k$$

$$\mathcal{S} = iS\omega[\Omega] + \int d\tau dx \mathcal{H}[\Omega](\tau, x)$$

$$\mathcal{H} \longrightarrow \Delta C_S^{(i,1+1)} = S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \lambda S_i^z S_{i+1}^z + \frac{\alpha^2}{2}\left((S_i^z)^2 + (S_{i+1}^z)^2\right)$$

$$\lambda = 1 + \alpha^2[S(S+1) + 1/2]$$

$$\boxed{\mathcal{L} = \frac{1}{2}\left[|\partial_x \mathbf{n} \perp|^2 + \frac{c_\perp}{2}|\partial_\tau \mathbf{n}_\perp|^2\right] + \frac{1}{2}\left[\alpha^2|\partial_x n_z|^2 + \frac{c_z}{2}|\partial_\tau n_z|^2\right] + \mu n_z^2}$$

S=1 λ-D model (From Pasini, Ph.D thesis; Campos Venuti et al 2006)

- t_p and t_c do not appear in the field theory (see Affleck NPB 1985-86).
- Integrability manifests in restrictions on the coefficients.

Hidden order and NLσM

Amico, Cataliotti, Mazzarella, Pasini 2008

Non integrable case: 1/S expansion of the λ-D model.

$$\mathcal{L} = \frac{1}{2}\left[|\partial_x \mathbf{n} \perp|^2 + \frac{c_\perp}{2}|\partial_\tau \mathbf{n}_\perp|^2\right] + \frac{1}{2}\left[\alpha^2|\partial_x n_z|^2 + \frac{c_z}{2}|\partial_\tau n_z|^2\right] + \mu n_z^2$$

$$c_z = 2\frac{1 + U_1 + U_0/2 - 2n_z^2(1-U_1)}{(2U_1 + M|\mathbf{n}_\perp|^2)(1 + U_1 - U_0 - Mn_z^2(2 - Mn_z^2)}$$

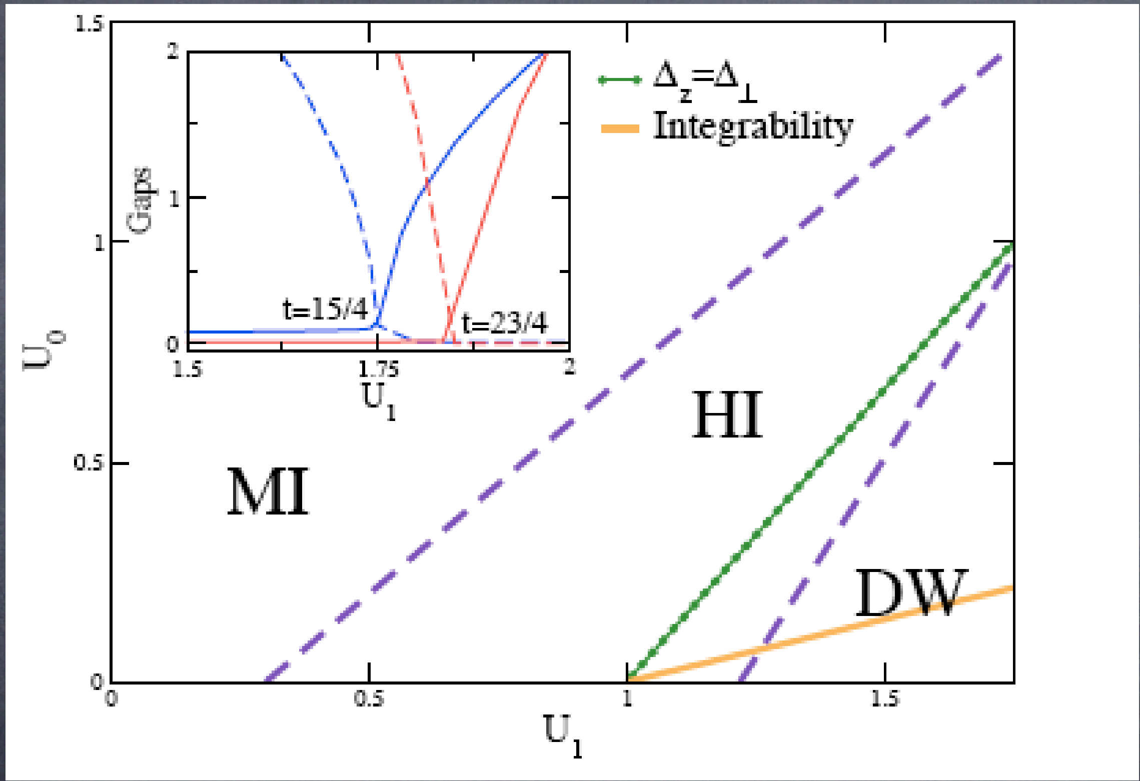
$$c_\perp = 2\frac{U_1 + |\mathbf{n}_\perp|^2(1-U_1)}{(2U_1 + M|\mathbf{n}_\perp|^2)(1 + U_1 - U_0/2 - M|\mathbf{n}_\perp|^2)}$$

$$M = 1 + S(S - 1/2)U_0 - U_1$$

Integrability: $U_1 = 1 + [S(S+1) + 1/2]U_0$

Skematic Phase Diagram

Saddle point & $\|(\delta n_j)^2 - \delta n_j \delta n_{j+1}\| \ll 1$



The two gaps play the role of the masses of the particles of an 'anisotropic Haldane triplet'.

Amico, Cataliotti, Mazzarella, Pasini 2008

Conclusions

- The effective model beyond the Bose-Hubbard. -Integrability for certain restrictions on the coefficients
- By exact means spin and bosonic paradigms are related. Charged/Neutral gap like **Singlet/triplet gaps, breaking of the $Z_2 x Z_2$ symmetry.**
- NLσM, Haldane insulator. Phase diagram.

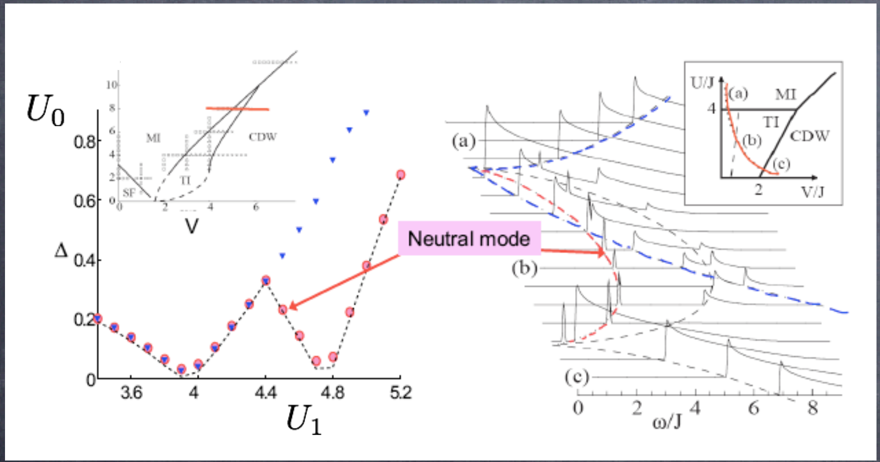
Amico, Cataliotti, Mazzarella, Pasini arXiv:0806.2378.

Suggestions for the experimental detection

Idea: apply periodic modulation of the lattice

$$H_b \rightarrow H_b + h\cos(\omega t)\sum_i b_i^\dagger b_{i+1} + h.c.$$

Lattice modulation couple to the neutral excitation.



Dalla Torre, Berg, Altman 2007.

$$I(\omega) \sim \sum_\alpha |\langle\psi_\alpha|T|\psi_0\rangle|^2\delta(\omega_{\alpha,0}-\omega)$$

Other ideas:

1. Bragg spectroscopy?
2. Spin diffusion in closed lattice?
3. ....

Spin diffusion: Open

- Open boundaries: washboard potential.

$$|+\,0\,0\,0\,-\,+\,0\,+\,0\,0\,0\,0\,-\,0\,0\,+\rangle$$

$$|+\,0\,0\,-\,0\,0\,0\,+\,0\,0\,0\,0\,-\,0\,0\,+\rangle$$

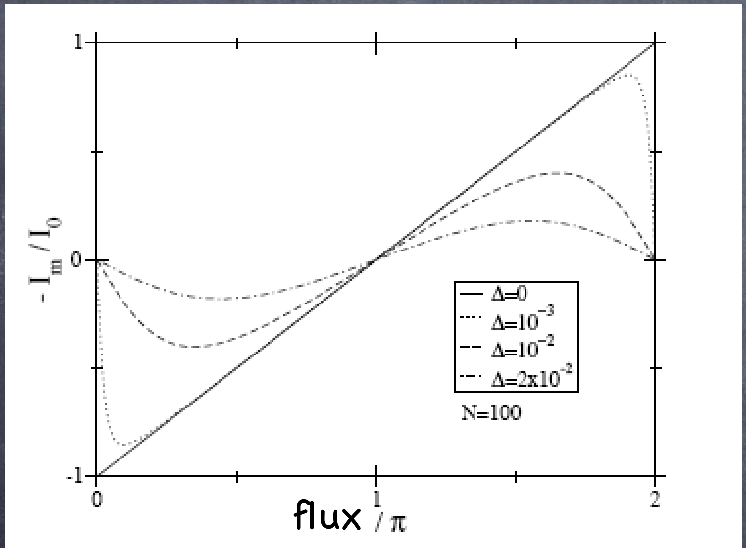
$$|+\,-\,0\,-\,0\,0\,0\,+\,0\,0\,0\,0\,-\,0\,0\,+\rangle$$

$$m\omega^2 x_M < gap$$

Current would be strongly dependent from the lenght of the chain.

Spin Diffusion: PBC

- Condensate in ring-shaped potential.



$\xi \propto 1/gap \ll L:$

The current is exponentially suppressed and becomes sinusoidal.

Magnetization current.
(Shutz, Kollar Kopietz PRB 2004)