A non-Fermi Liquid for htsc

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Outline

based on arXiv:0805.4182 with E. Kapit and
JHEP 10 (2007) 027 with M. Neubert

Outline

- High Tc phenomenology
- Mott-Hubbard insulator
- Basic requirements of any theory
- Our field theory model
- Anti-ferromagnetic phase
- d-wave superconducting phase
- pseudgogap?

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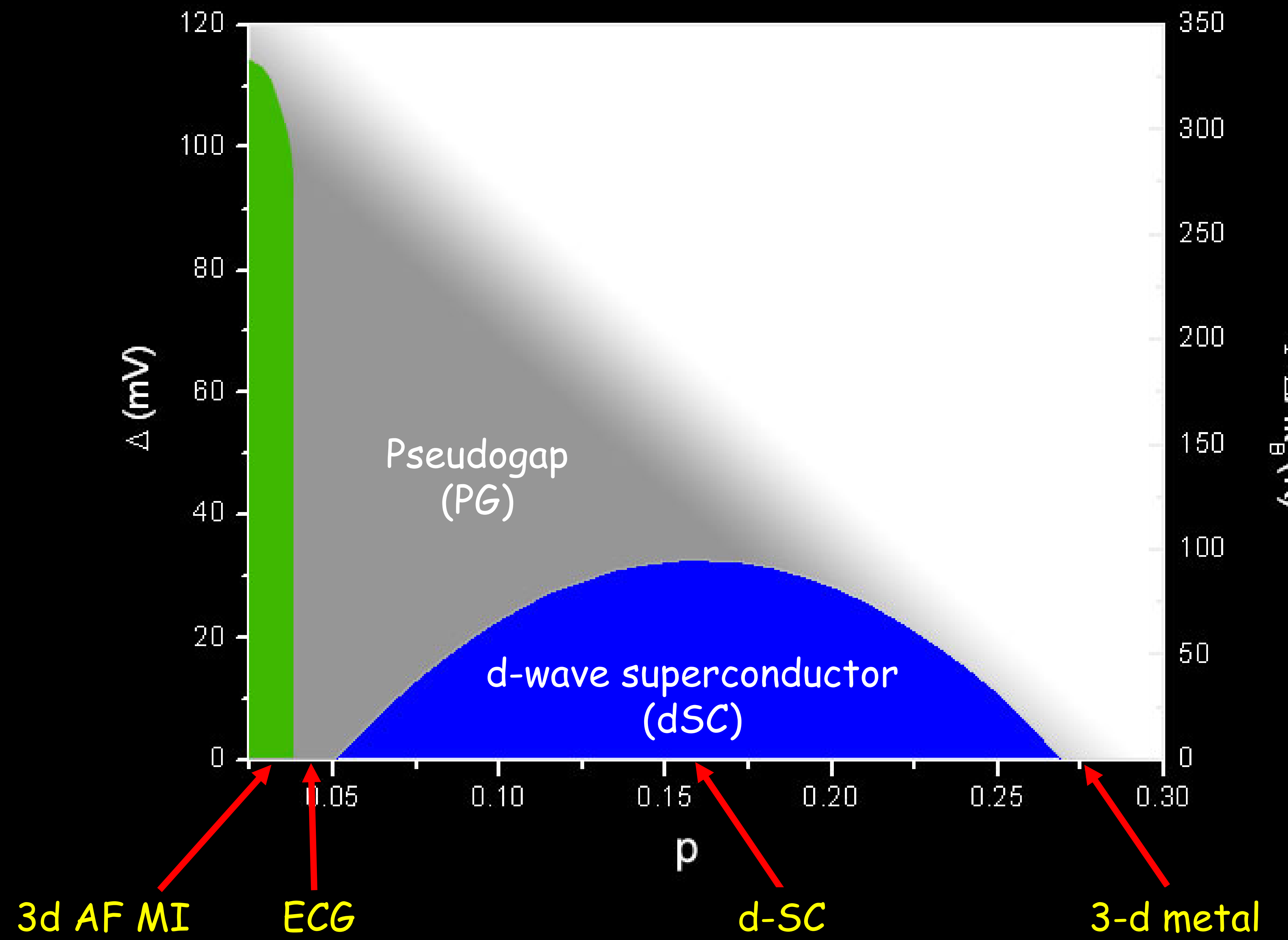
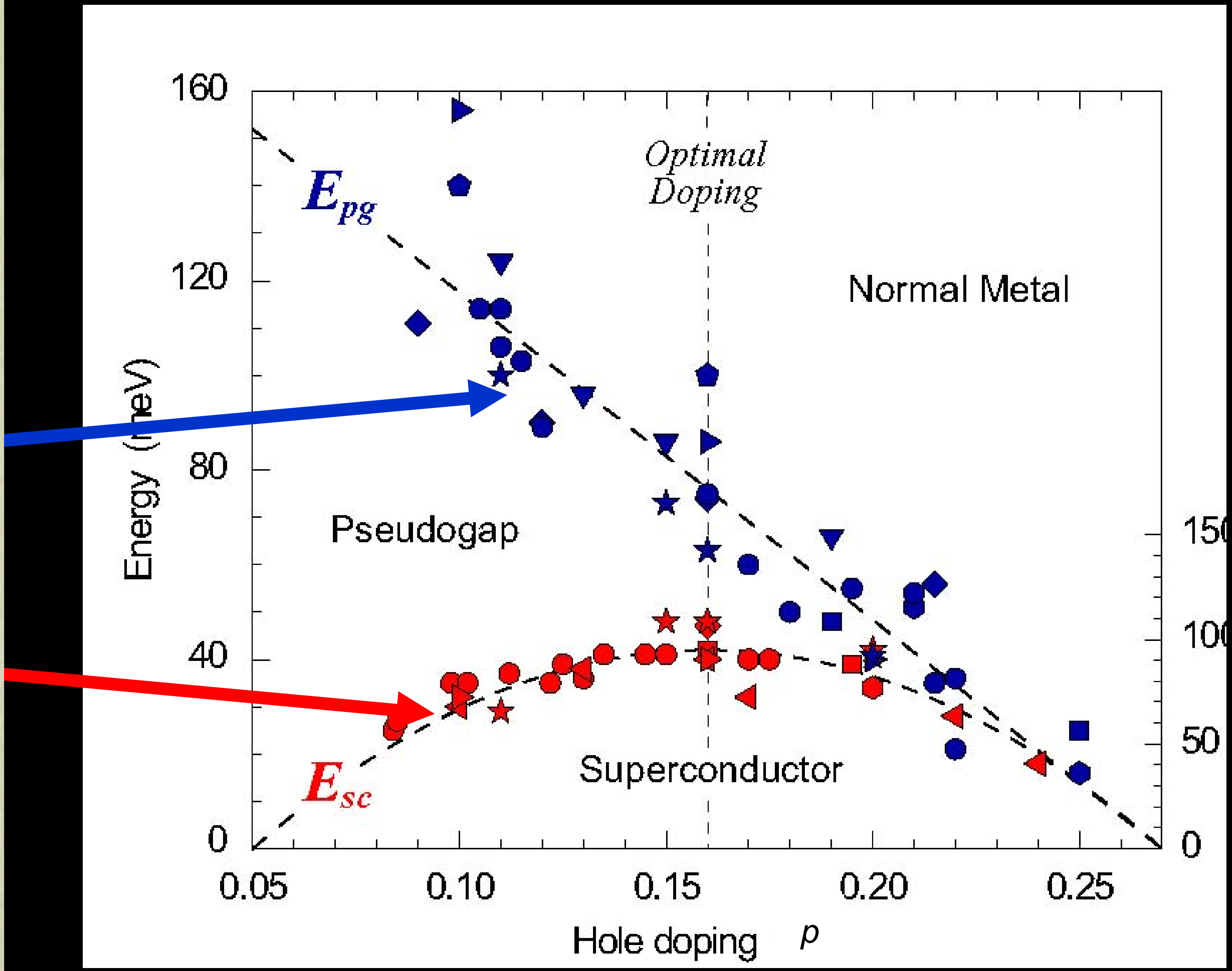
Schematic phase diagram of hole-doped cuprates



(courtesy of Seaumus Davis)



S. Huefner, M.A. Hossain,
A. Damascelli & G.A. Sawatzky
cond/mat 0706-4282

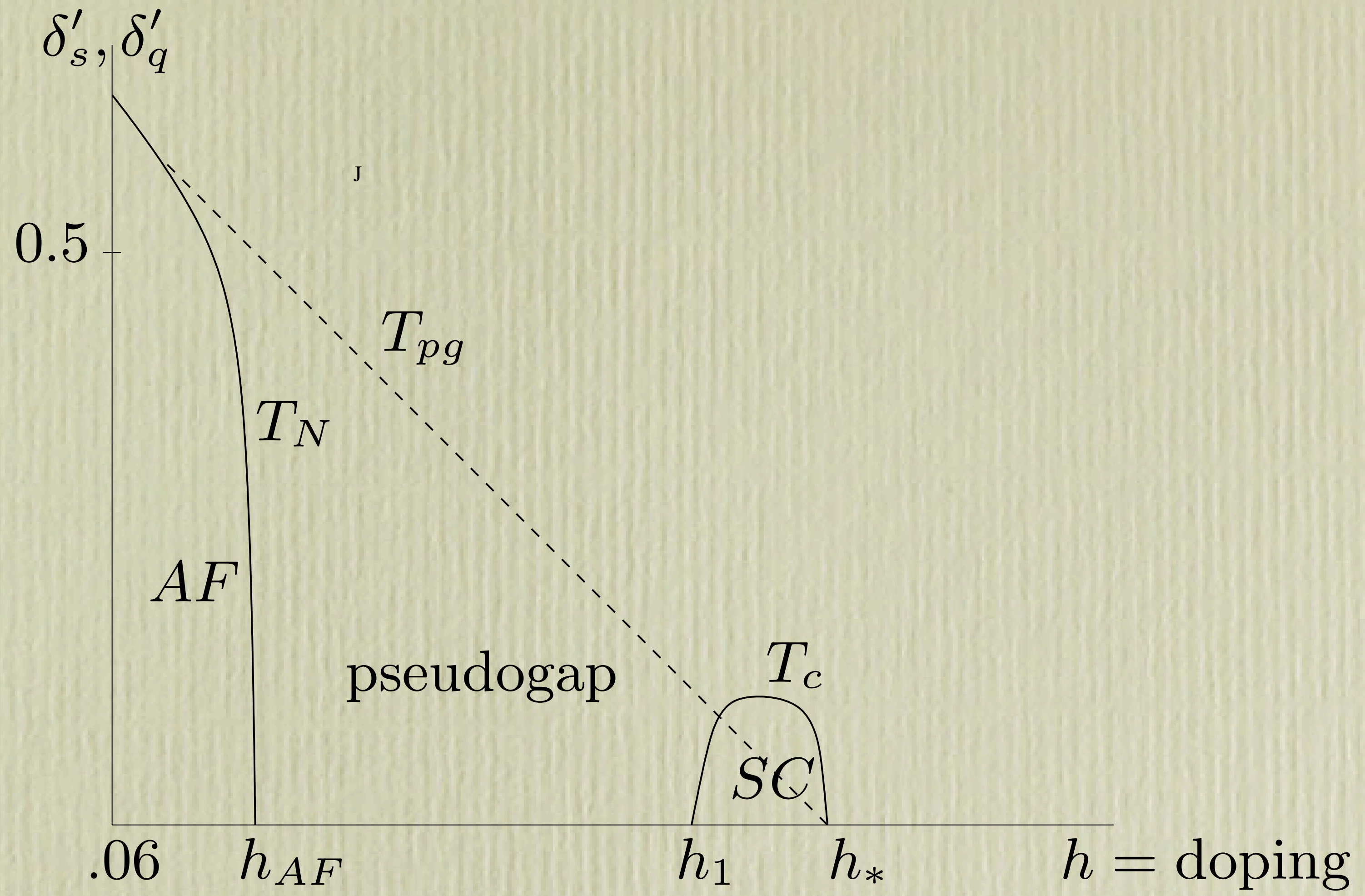
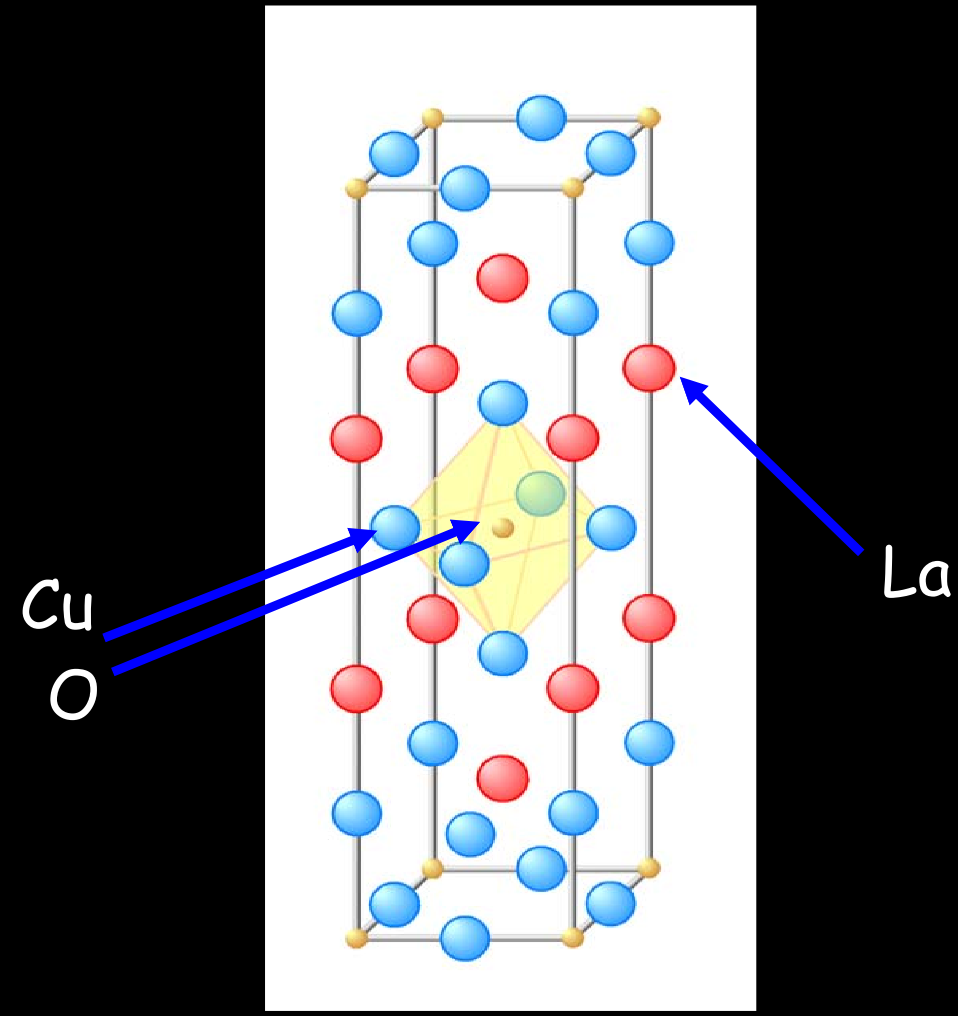


FIG. 1: Calculated phase diagram as a function of hole doping based on a single parameter $0 < \gamma < 1$, set equal to 1 (infinitely strong coupling at short distances). What is shown are solutions δ'_s, δ'_q of the AF and d-wave gap equations below, which are proportional to the critical temperature. T_{pg} is simply the RG scale of the coupling g. The AF transition point at $h_{AF} = \frac{3}{4\pi^2}$ is first order. The SC transition at $h_* = \frac{3}{2\pi^2}$ is second-order and corresponds to the fixed point of the renormalization group. $h_1 =\approx 0.13$ is not universal.

$La_{2-x}Ba_xCuO_4$

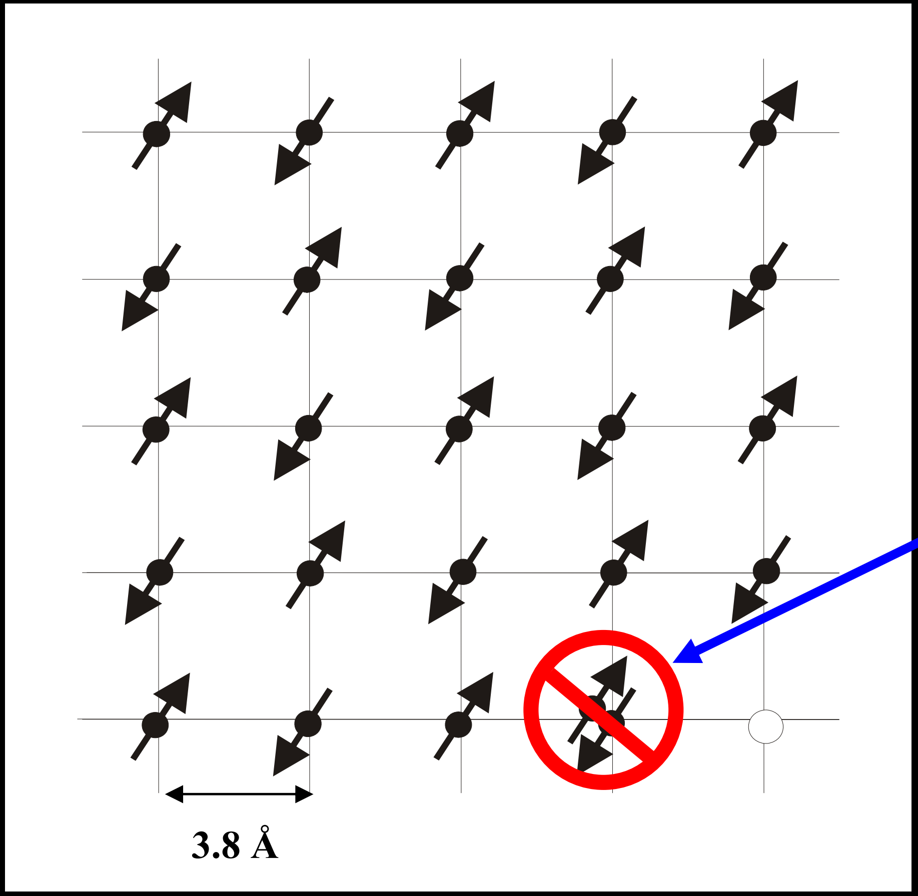


Antiferromagnetic

Mott Insulator

Z. Phys. Rev. **B 64** 189 (1986)

Mott Insulator: Repulsive Coulomb U~3eV

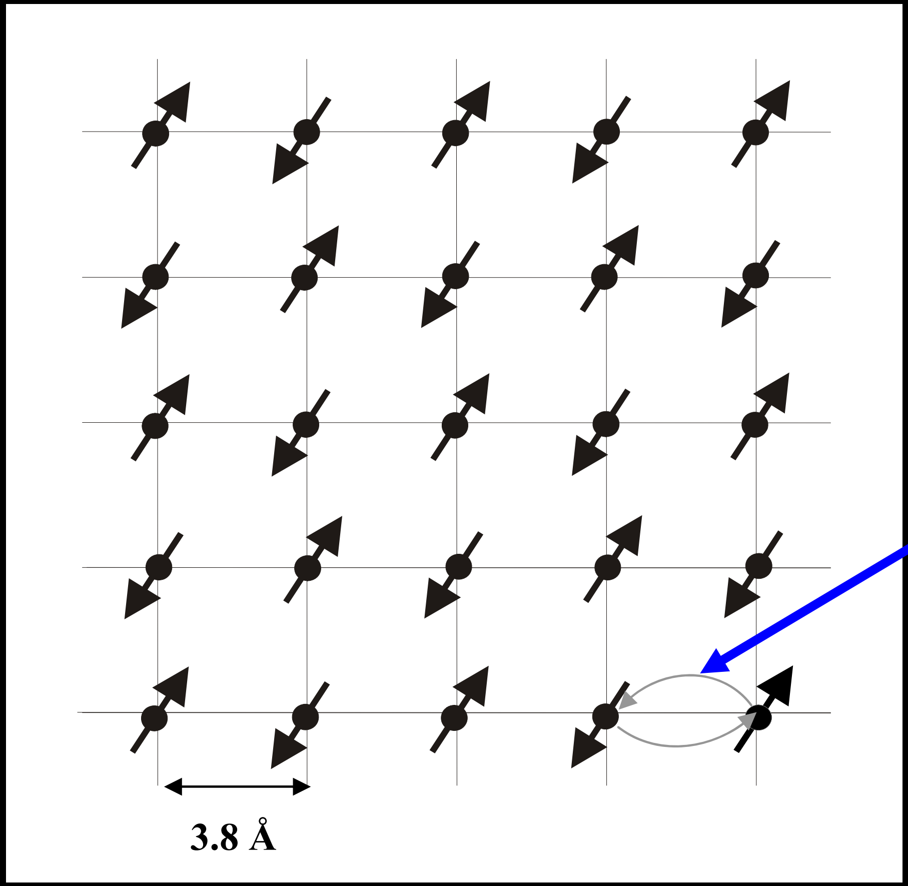


No double occupancy allowed..

N.F. Mott, *Proc. Phys. Soc A62*, 416 (1949)

(courtesy Seamus Davis)

Antiferromagnetic: Superexchange J~0.14eV



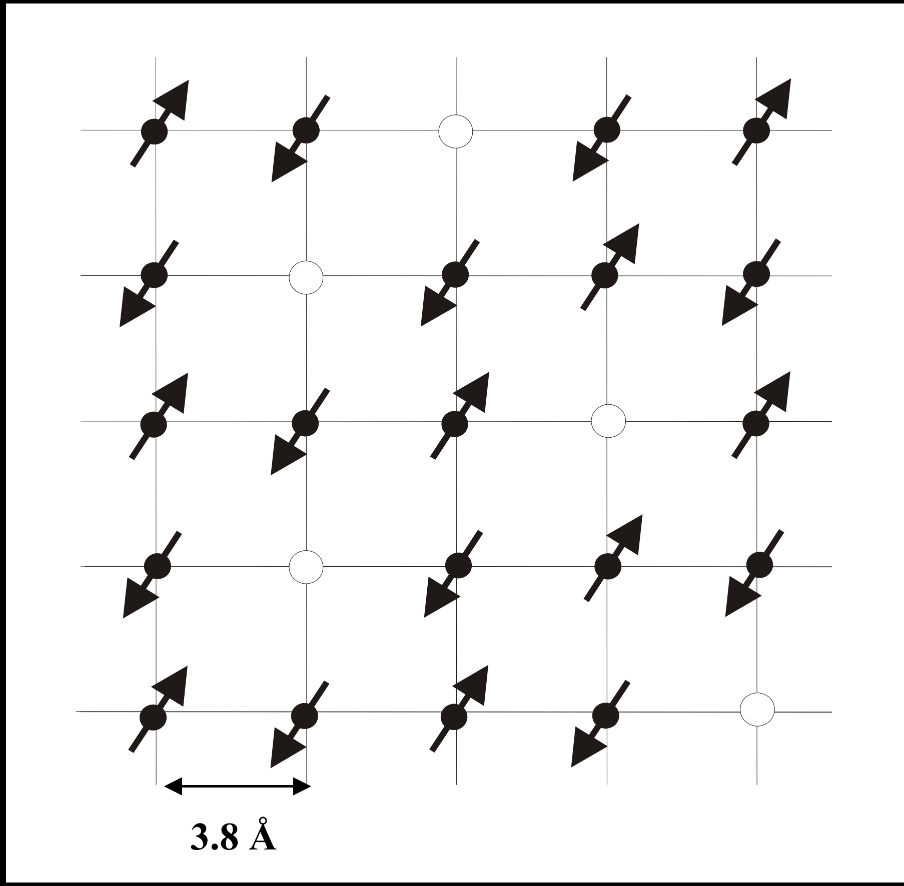
..except as a virtual process.

$$H = \frac{4t^2}{U} \sum_{<i,j>} \vec{S}_i \bullet \vec{S}_j - \frac{1}{4} n_i n_j$$

P. W. Anderson, ***Phys. Rev. 115***, 2 (1959)

AF order preferred since it allows virtual hopping

Holes introduced $\Longrightarrow$ carriers become mobile



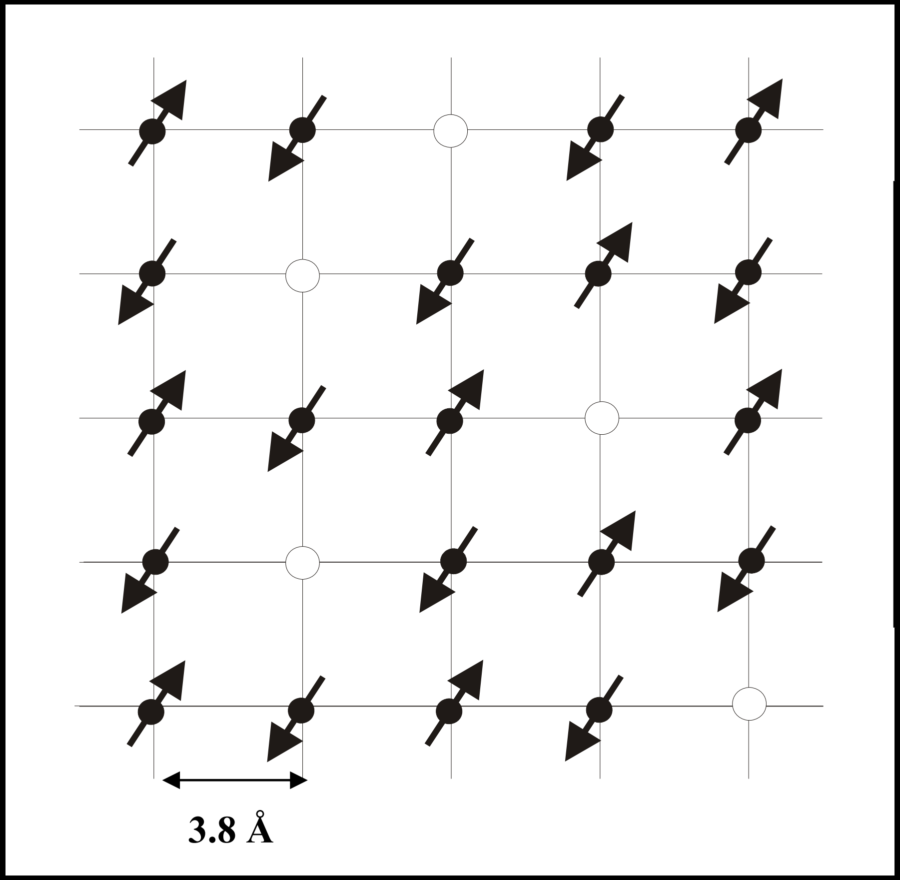
Dopant density p
= number of holes
per CuO_2 plaquette

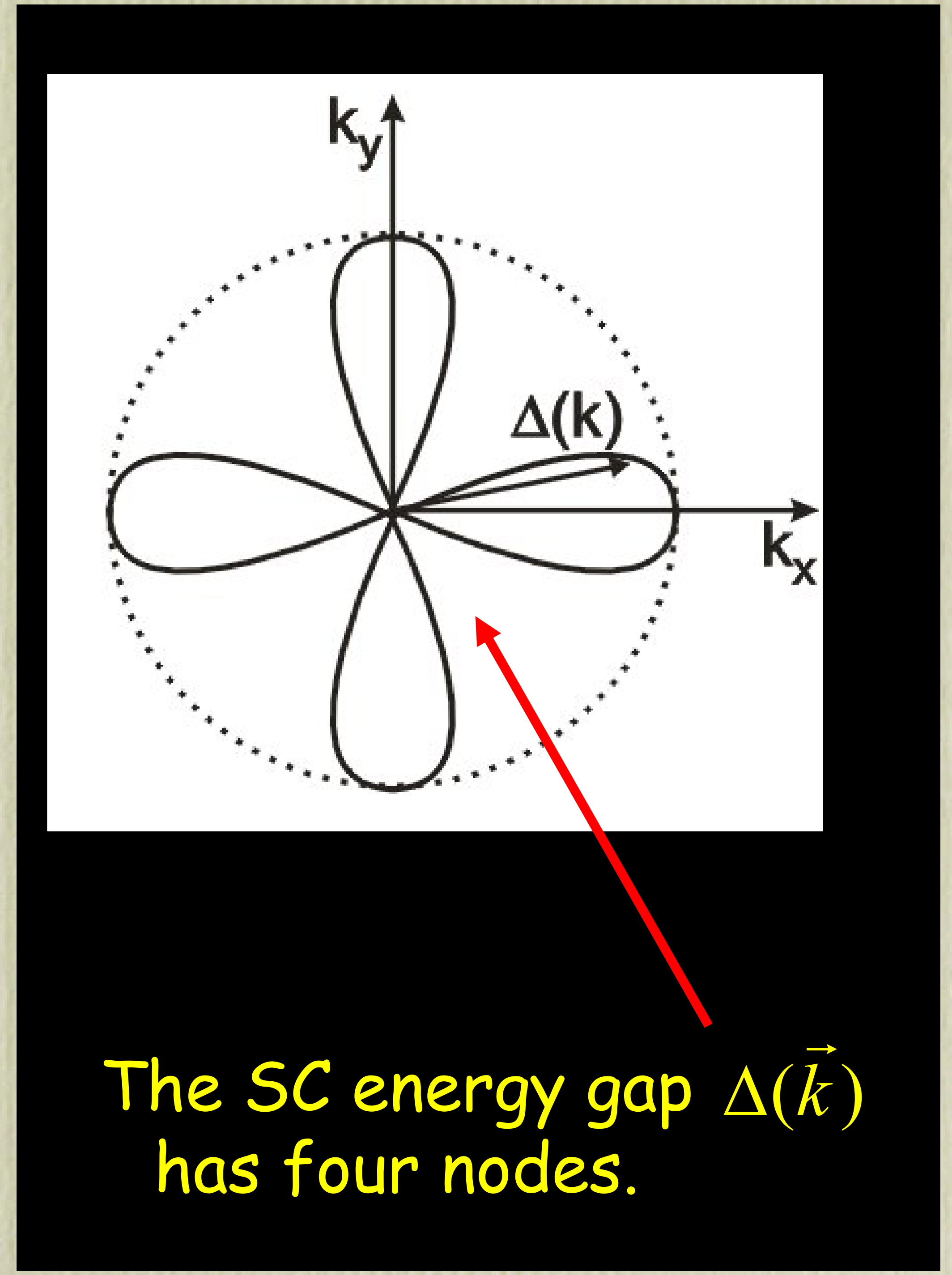
$$H = -t\sum_{<i,j>}(c_{i\sigma}^{\dagger}c_{j\sigma} + h.c.) \quad + \quad U\sum_{i} n_{i\uparrow}n_{i\downarrow}$$

J. Hubbard, ***Proc. Roy. Soc A276***, 238 (1963)

hopping destroys the AF order

How could this state become superconducting?



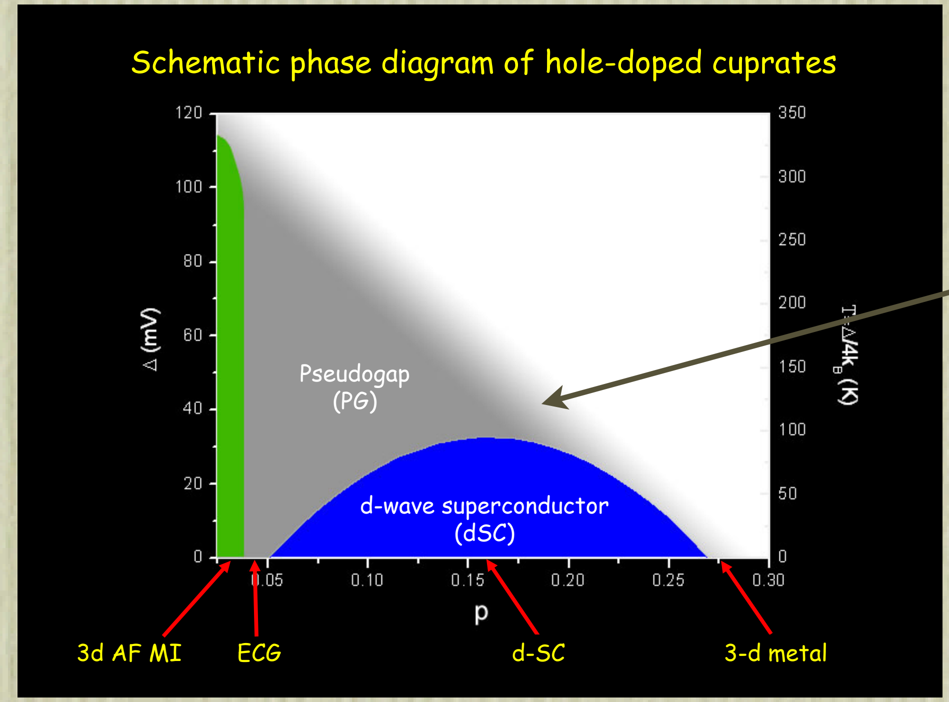


The SC gap has d-wave symmetry

Basic requirements of any theory

- non-Fermi liquid in 2 spatial dimensions
- strong coulomb repulsion with AF phase
- d-wave pairing instability
- pseudogap
- an estimate of Tc

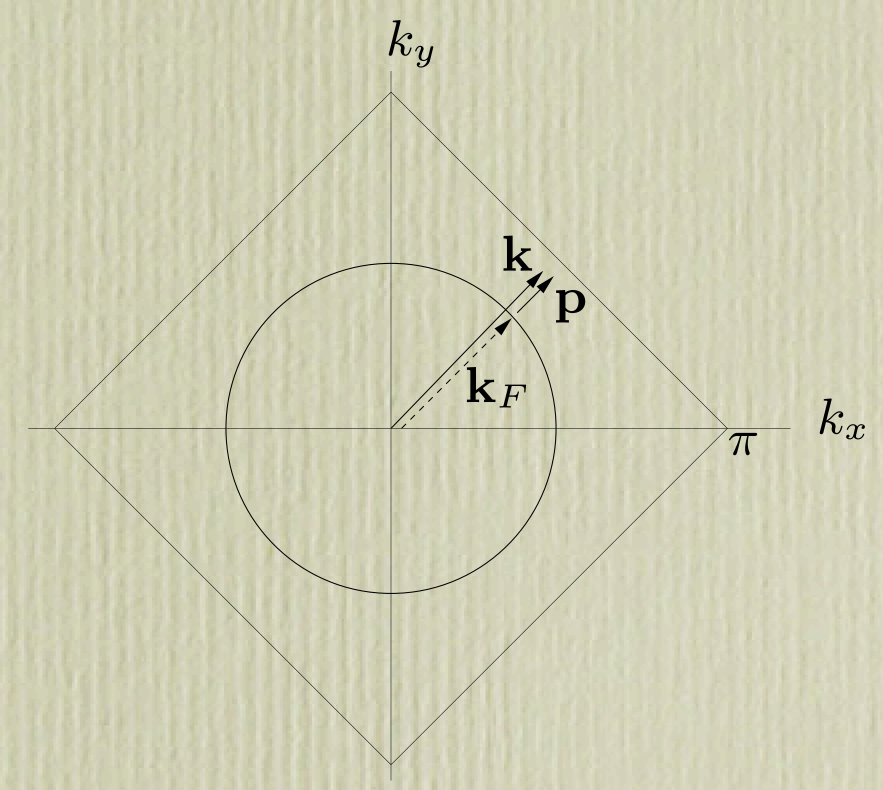
Where to begin?



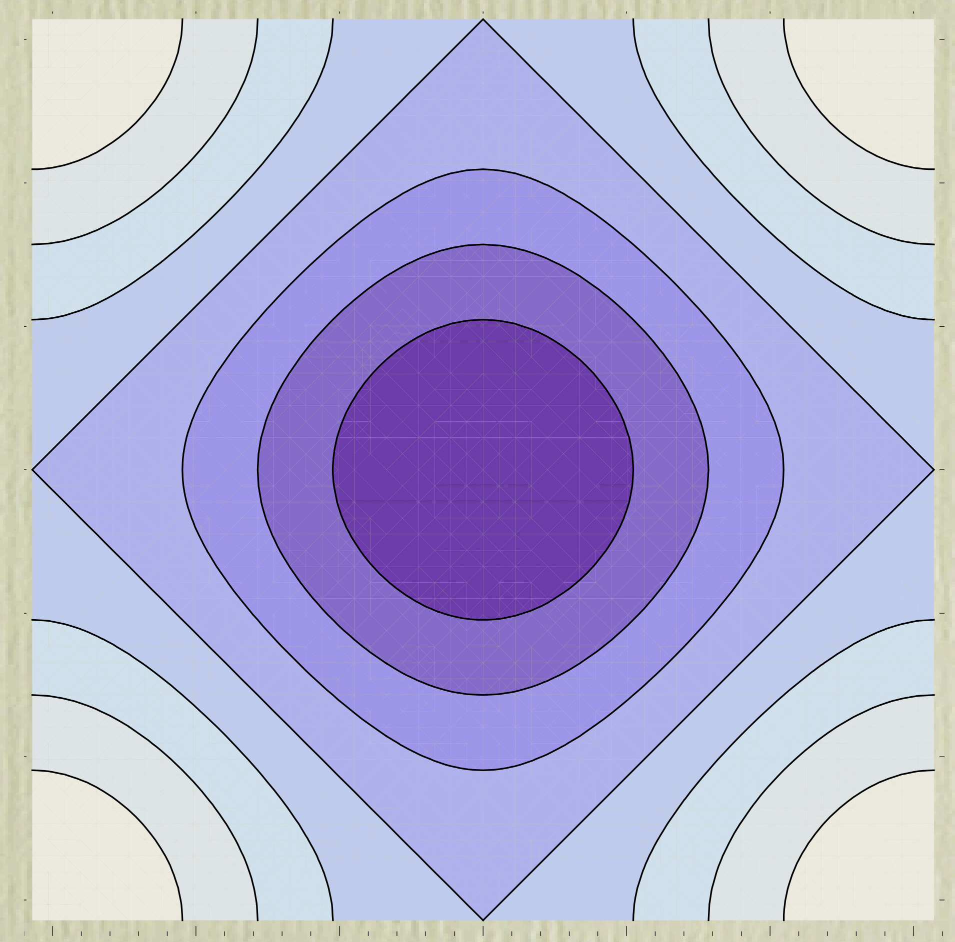
Here......

* non-Fermi liquids are rare
* assume a rotationally invariant continuum description
* intrinsically 2d theory with only quartic interactions
* starting from most entropic states

Expansion around Fermi Surface



$$H = \int_{|\mathbf{p}|<\Lambda_c} (d^2\mathbf{p}) \left[(v_F|\mathbf{p}| - \widehat{\mu}) a^\dagger_{\mathbf{p}} a_{\mathbf{p}} + (v_F|\mathbf{p}| + \widehat{\mu}) b^\dagger_{\mathbf{p}} b_{\mathbf{p}} \right] \tag{1}$$



Requiring a field theory description with relevant interactions points to a unique theory with the action for fermionic fields:

$$S = \int d^2x dt \left(\sum_{\alpha=\uparrow,\downarrow} \left(\partial_\mu \chi_\alpha^- \partial_\mu \chi_\alpha^+ + m^2 \chi_\alpha^- \chi_\alpha^+\right) - 8\pi^2 g \, \chi_\uparrow^- \chi_\uparrow^+ \chi_\downarrow^- \chi_\downarrow^+ \right)$$

*The free theory has the correct spectrum with a relativistic dispersion relation.

* Interaction is a unique dimension 2 operator, i.e. relevent.

Unitarity, spin statistics!?

- spin is a flavor here and thus does not need to be embedded in the Lorentz group.
- pseudo-hermiticity: $H^\dagger = CHC$
- C distinguishes particles and holes:

$$CaC = a, \; CbC = -b$$

$$\chi^+ = C(\chi^-)^\dagger C$$

$$
\begin{aligned}
\chi^-(\mathbf{x},t) &= \int \frac{d^2\mathbf{p}}{(2\pi)^2\sqrt{2\omega_\mathbf{p}}}\left(a_\mathbf{p}^\dagger e^{-ip\cdot x} + b_\mathbf{p} e^{ip\cdot x}\right) \\
\chi^+(\mathbf{x},t) &= \int \frac{d^2\mathbf{p}}{(2\pi)^2\sqrt{2\omega_\mathbf{p}}}\left(-b_\mathbf{p}^\dagger e^{-ip\cdot x} + a_\mathbf{p} e^{ip\cdot x}\right)
\end{aligned}
$$

SO(5) symmetry

N-component version has Sp(2N) symmetry
Sp(4) = SO(5)

5-vector of bilinears can serve as order parameters for both spontaneous symmetry breaking of spin SU(2) (AF) and the charge U(1) for superconductivity

$$\vec{\Phi} = (\vec{\phi}, \phi_e^+, \phi_e^-) = (\chi^- \vec{\sigma} \chi^+ / \sqrt{2}, \chi_\uparrow^+ \chi_\downarrow^+, \chi_\downarrow^- \chi_\uparrow^-)$$

Renormalization group

Λ_c =upper cutoff Λ = running RG scale

$$g(\Lambda) = \Lambda \widehat{g}(\Lambda)$$

$$-\Lambda \frac{d\widehat{g}}{d\Lambda} = \widehat{g} - 8\widehat{g}^2$$

fixed point: $\widehat{g}_* = 1/8$

Define: $x = 1/\widehat{g}$ $x_* = 8$

Integrate:

$$\frac{\Lambda}{\Lambda_c} = -\frac{1}{\gamma}\left(\frac{x}{x_*} - 1\right)$$

Initial condition: $\gamma = (x_* - x_0)/x_*$ $g(\Lambda_c) = \Lambda_c \widehat{g}_0$

Hole Doping

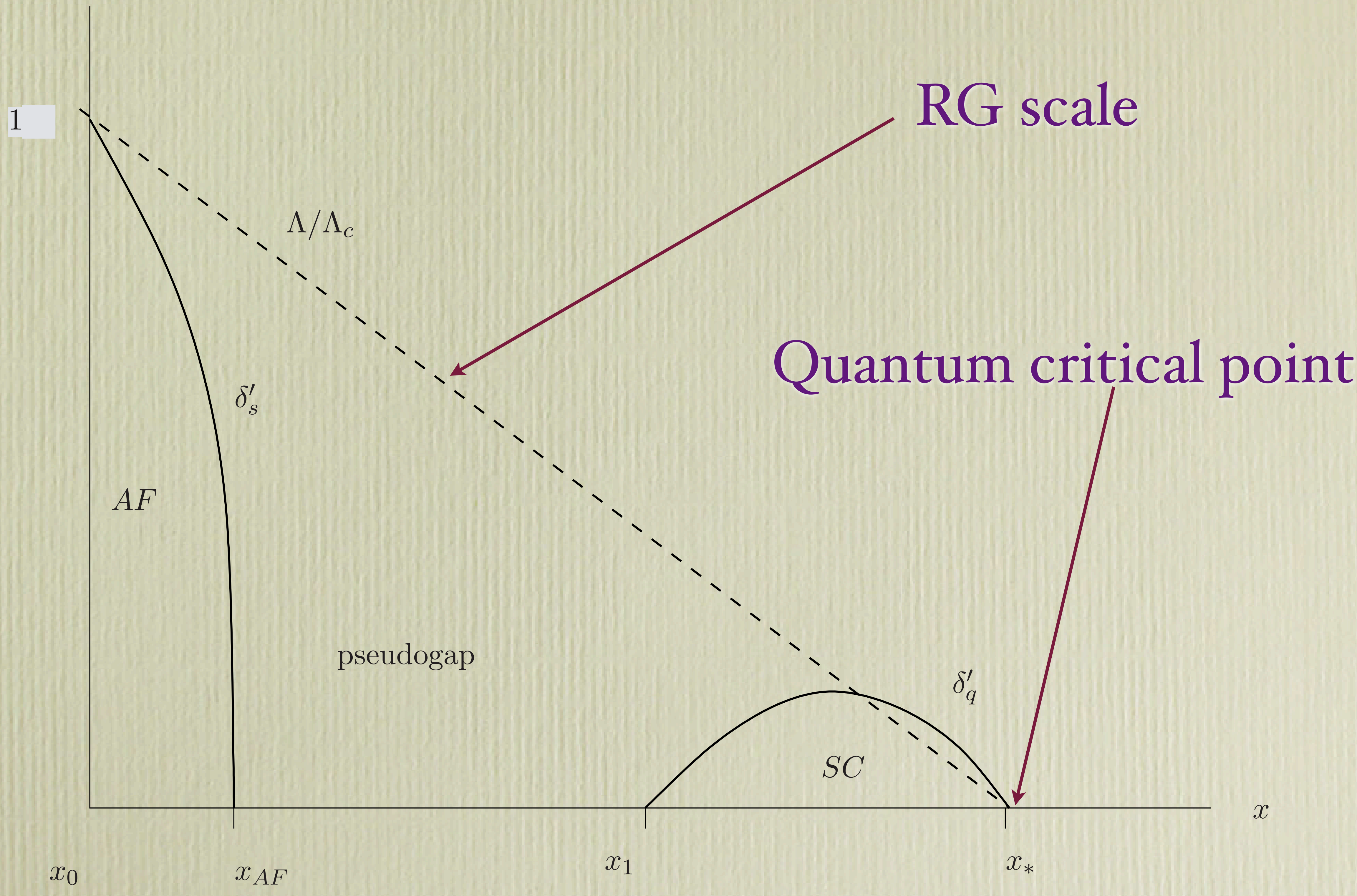
Since running the RG depopulates the spectrum, a measure of hole doping is:

$$h = \text{hole doping} \propto \left(1 - \frac{\Lambda}{\Lambda_c}\right)$$

A physical way to set the scale is through the 1-point function:

$$h = \langle \chi^- \chi^+ \rangle / \Lambda_c \approx \frac{1}{\pi^2}\left(1 - \frac{\Lambda}{\Lambda_c}\right) = \frac{1}{\pi^2}\left(\frac{x - x_0}{x_* - x_0}\right)$$

The point: hole doping linear in x, at least to lowest order


RG scale
Quantum critical point
1
Λ/Λ_c
δ'_s
AF
pseudogap
δ'_q
SC
x
x_0
x_{AF}
x_1
x_*

d-wave Superconductivity

Warm up: s-wave. To study spontaneous symmetry breaking of U(1), consider VEVs:

$$q^{\pm} = \langle \chi_{\uparrow}^{\pm} \chi_{\downarrow}^{\pm} \rangle$$

Mean field gap equation:

$$q^{\pm} = -8\pi^2 g \int \frac{d\omega d^2\mathbf{k}}{(2\pi)^3} \frac{q^{\pm}}{(\omega^2 + \mathbf{k}^2)^2 + q^+ q^-}$$

As expected: no solutions for positive g (repulsive).

Solution in 3d for negative g: $\Delta = \sqrt{q} = \Lambda_c e^{1/g}$

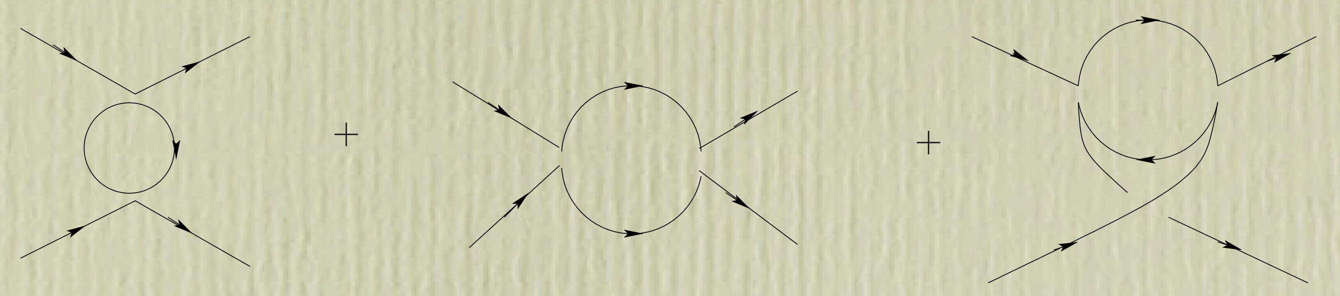
Beyond mean field: include 1-loop scattering

$$q(\mathbf{k}) = -\int \frac{d\omega\, d^d\mathbf{k}'}{(2\pi)^{d+1}}\, G(\mathbf{k},\mathbf{k}')\, \frac{q(\mathbf{k}')}{(\omega^2+\mathbf{k}'^2)^2 + q(\mathbf{k}')^2}$$

Expand in circular harmonics:

$$\begin{aligned} G(\mathbf{k},\mathbf{k}') &= \sum_{\ell=0}^{\infty} G_\ell(k,k') \cos \ell(\theta-\theta') \\ q(\mathbf{k}) &= \sum_{\ell=0}^{\infty} q_\ell(k) \cos \ell\theta \end{aligned}$$

1- loop scattering

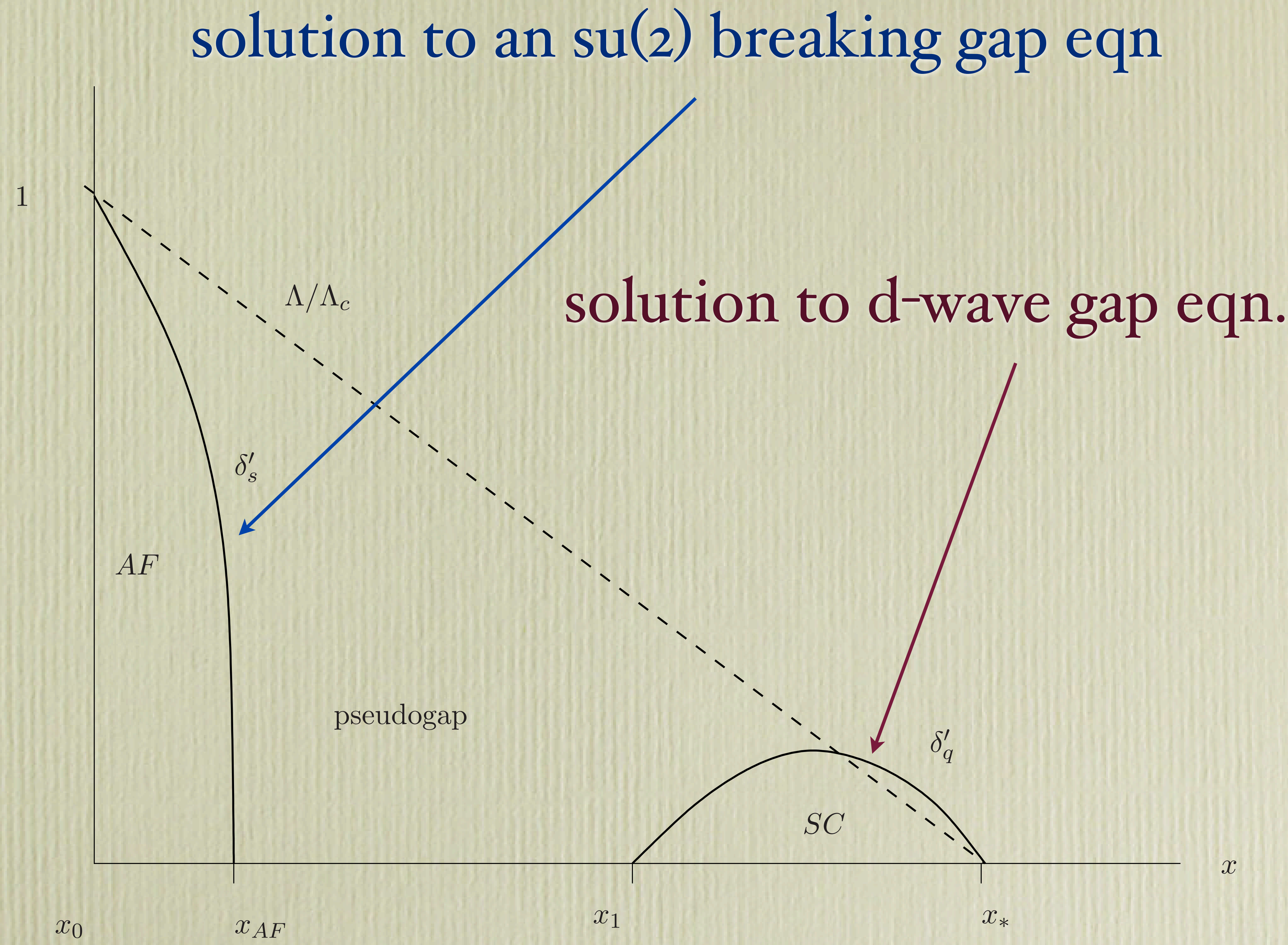


gives:

$$G_2(k,k') = -8\pi^2 g_2 k^2 k'^2 \qquad g_2 = \frac{4}{25}\frac{\widehat{g}^2}{\Lambda^3}$$

$$q(\mathbf{k}) = \delta_q^2\, k^2 \cos 2\theta = \delta_q^2\, (k_x^2 - k_y^2)$$

$$\delta_q^4 = 2g_2 \int_0^{\Lambda_c} d\omega\, dk^2 \left(1 - \frac{\omega^2 + k^2}{\sqrt{(\omega^2+k^2)^2 + \delta_q^4 k^4}}\right)$$


solution to an su(2) breaking gap eqn
solution to d-wave gap eqn.
1
Λ/Λ_c
δ'_s
AF
pseudogap
δ'_q
SC
x
x_0
x_{AF}
x_1
x_*

The pseudogap?

- previously identified as the running RG scale.
- possible manifestation:
    - dynamically generated relativistic mass.

$$m \propto \Lambda$$

Tc

the scale of Tc is set by the lattice spacing and universal nodal Fermi velocity. (X. J. Zhou et. al. 2003)

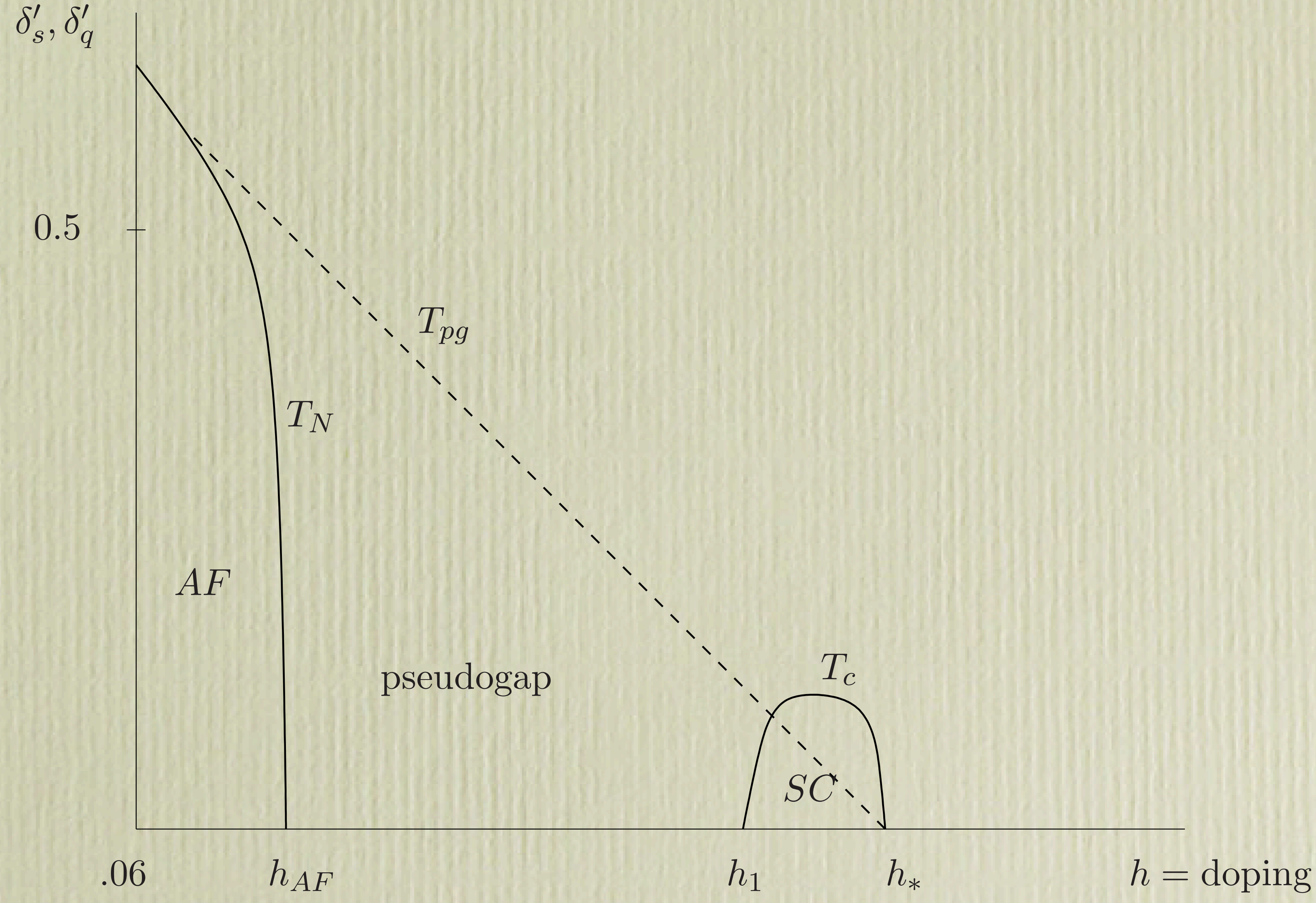
$$T_c = c_{SC}\,\frac{v_F}{a} \cdot 650K$$

LSCO: $120K < T_c < 160K$

(Here temperature was mimicked as a mass term, a better treatment is required.)

Conclusions and open problems

- a simple model that appears to capture the main features of HTSC in a calculable way
- Relation to Hubbard model, lattice effects?
- calculation of the non-Fermi liquid properties, such as specific heat, conductivity, etc.
- manifestation of the pseudogap, need to include temperature properly.



δ_s', δ_q'
0.5
T_{pg}
T_N
AF
pseudogap
T_c
SC
.06
h_{AF}
h_1
h_*
$h =$ doping