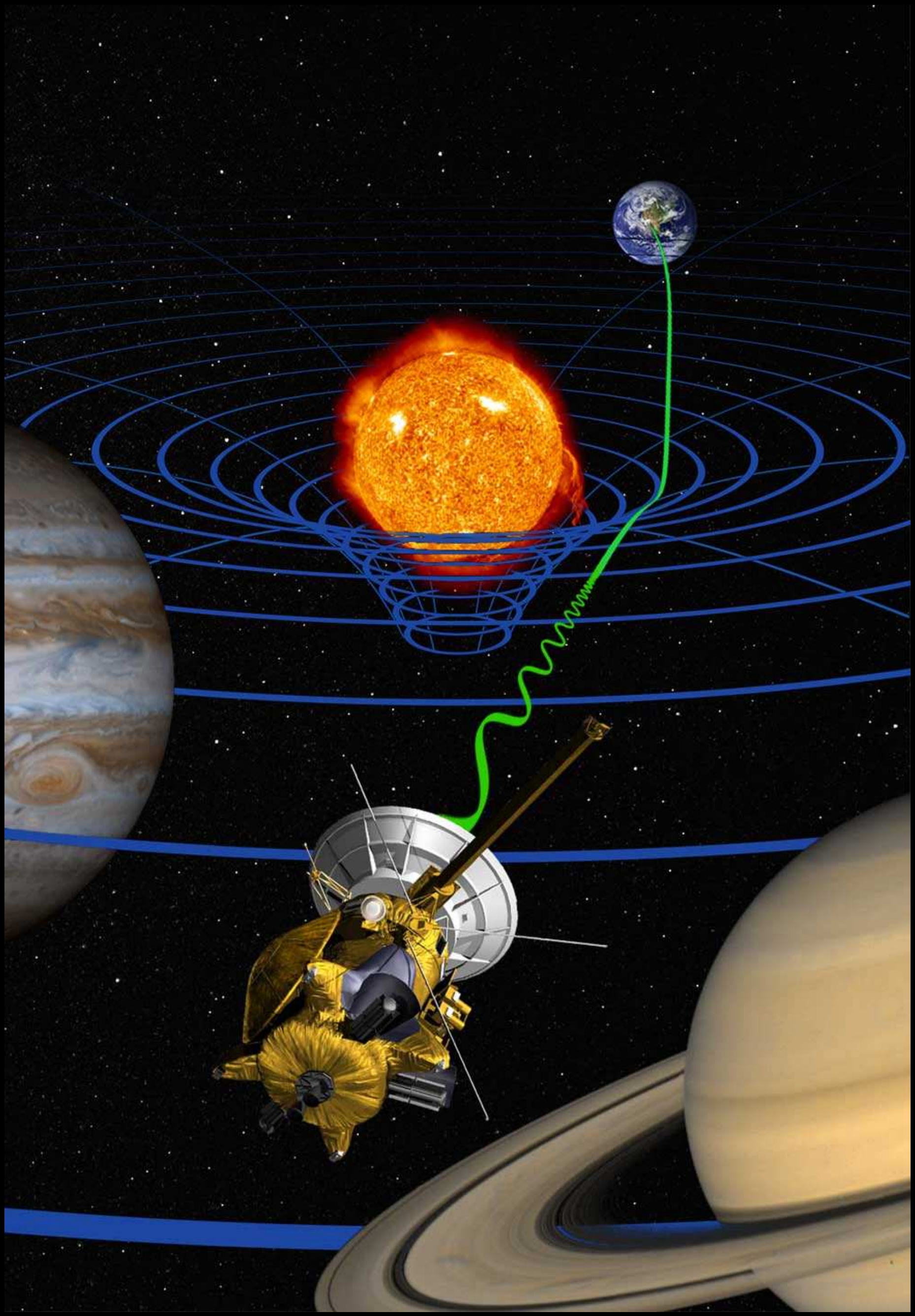


Testing General Relativity with Interplanetary Spacecraft

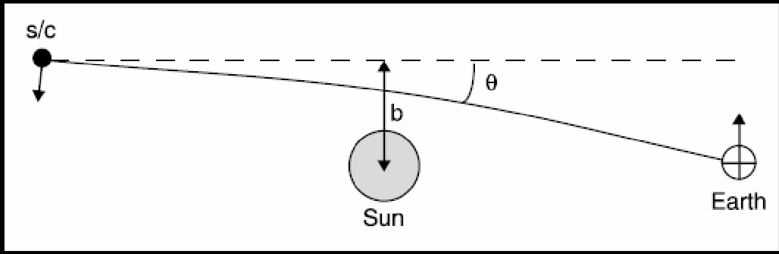
Luciano Iess

Dipartimento di Ingegneria
Aerospaziale ed Astronautica
Università La Sapienza
Rome, Italy

Testing gravitational theories in the solar system

Deflection of light

$$\theta_{gr} = 2(1+\gamma)\frac{M_{sun}}{b} = 4\times10^{-6}(1+\gamma)\frac{R_{sun}}{b}\,\text{rad}$$



Solar Gravity

Time delay

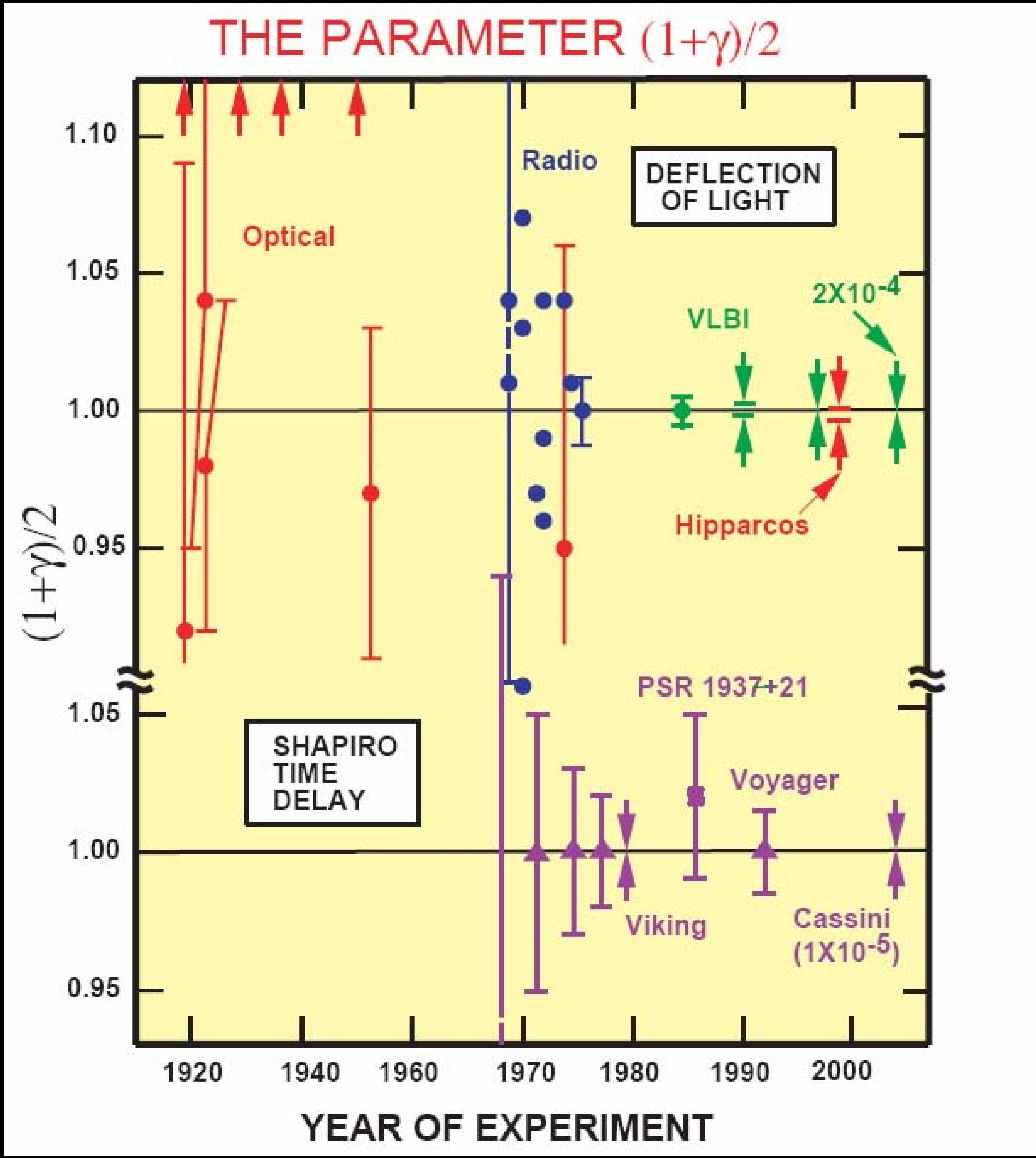
$$\Delta t = (1+\gamma)M_{sun}\ln\frac{l_0+l_1+l_{01}}{l_0+l_1-l_{01}}$$

≈ 70 km for a grazing beam

Frequency shift

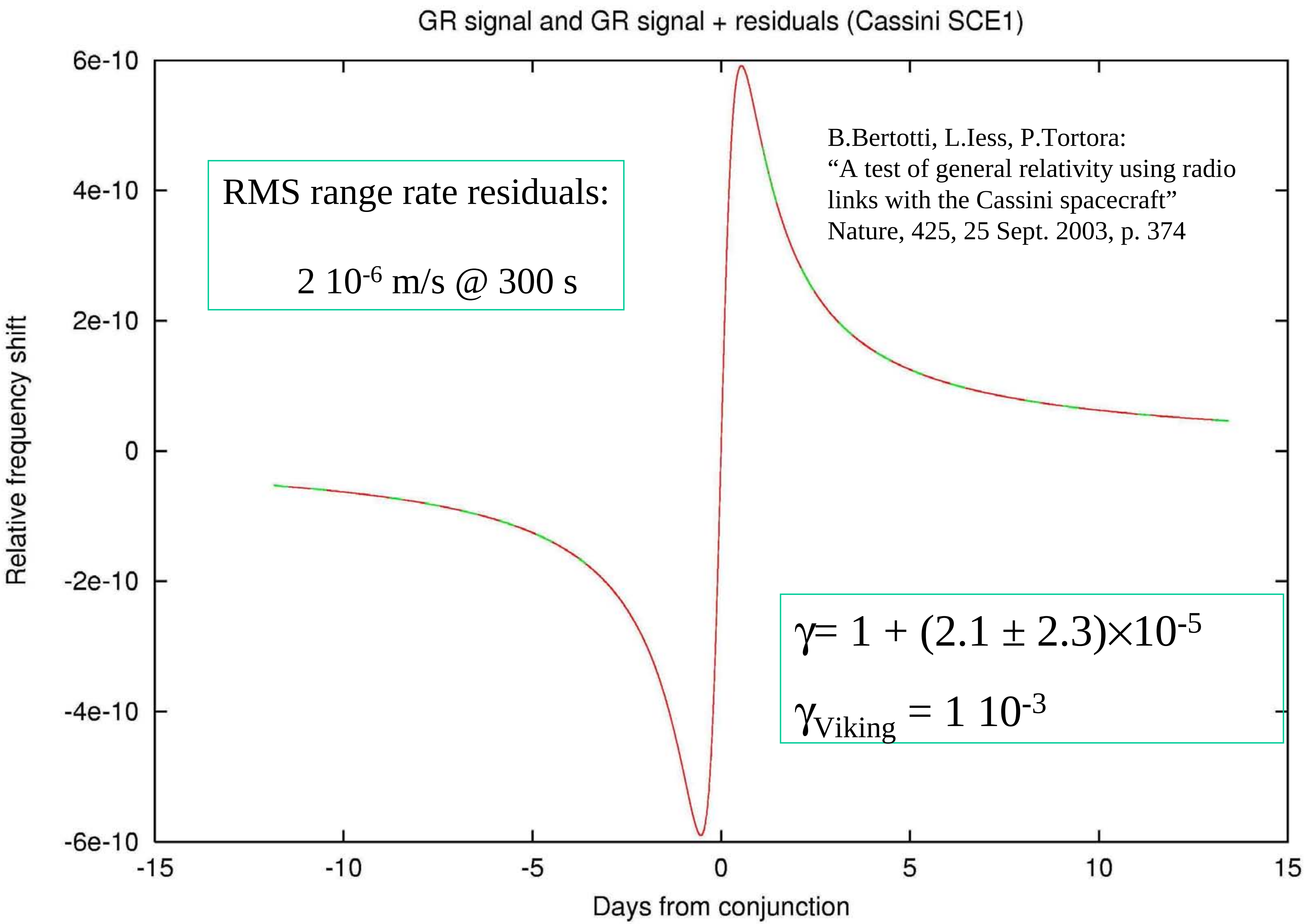
$$\frac{\Delta\nu}{\nu} = 2\frac{v_1 l_0 + v_0 l_1}{l_0+l_1}\theta_{gr} \cong 4(1+\gamma)\frac{M_{sun}}{b}\frac{db}{dt}$$

≈ 8×10^{-10} for a grazing beam



From:

Clifford M. Will,
"The Confrontation between General Relativity and Experiment",
Living Rev. Relativity, 9, (2006), 3.
http://www.livingreviews.org/lrr-2006-3



RMS range rate residuals:

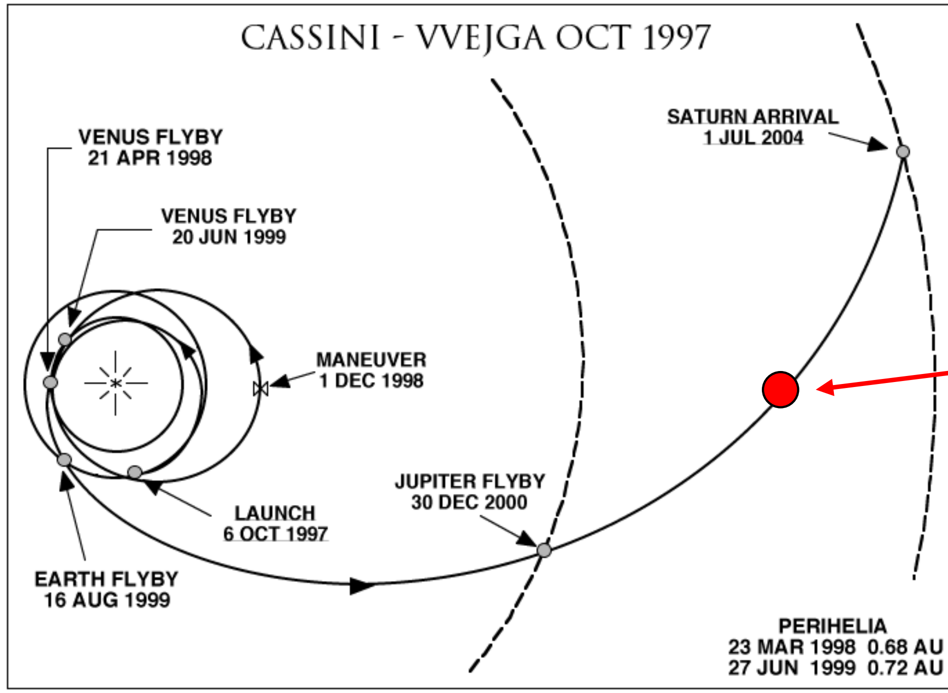
$2 \; 10^{-6}$ m/s @ 300 s

B.Bertotti, L.Iess, P.Tortora:
"A test of general relativity using radio links with the Cassini spacecraft"
Nature, 425, 25 Sept. 2003, p. 374

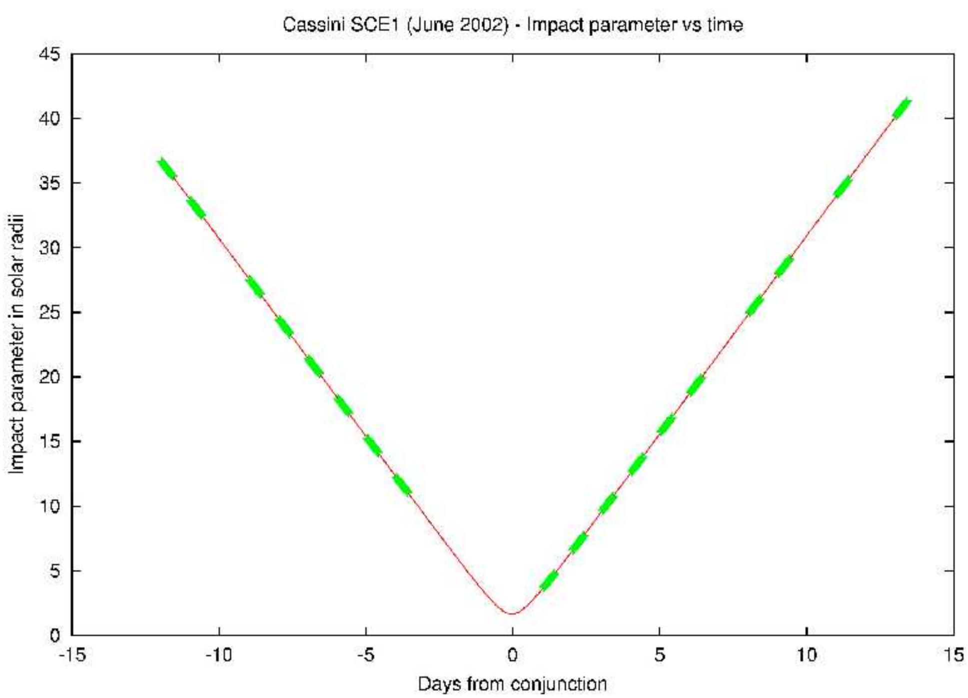
$\gamma = 1 + (2.1 \pm 2.3)\times10^{-5}$

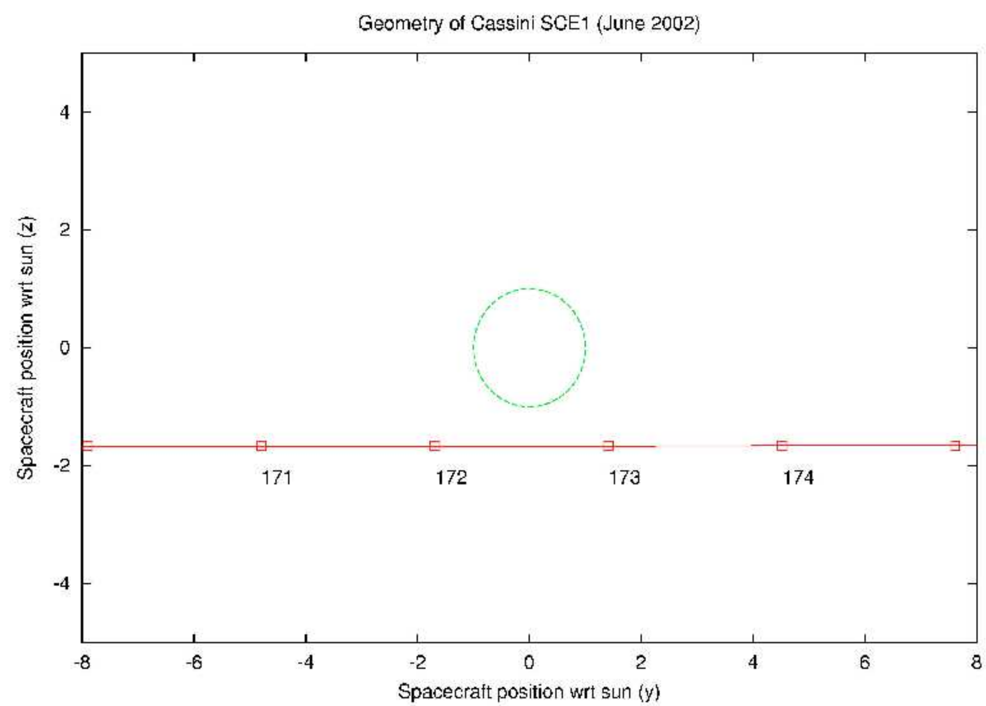
$\gamma_{Viking} = 1 \; 10^{-3}$

DSS25 and Cassini

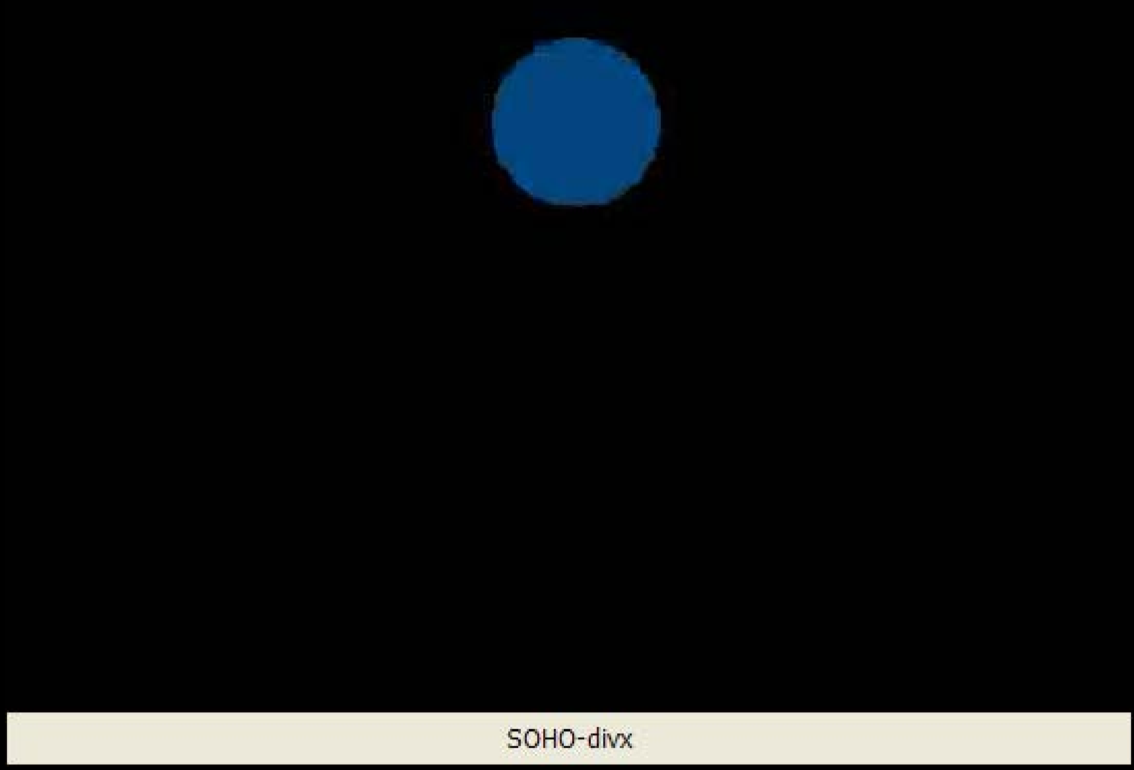


SCE1
30 days
coverage
from DSN





The trajectory of Cassini in the sky during SCE1



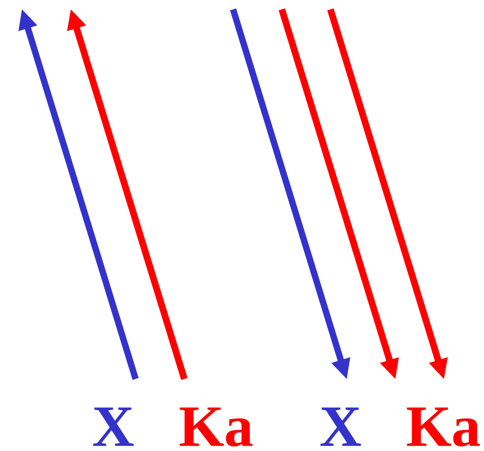
LASCO images - SOHO

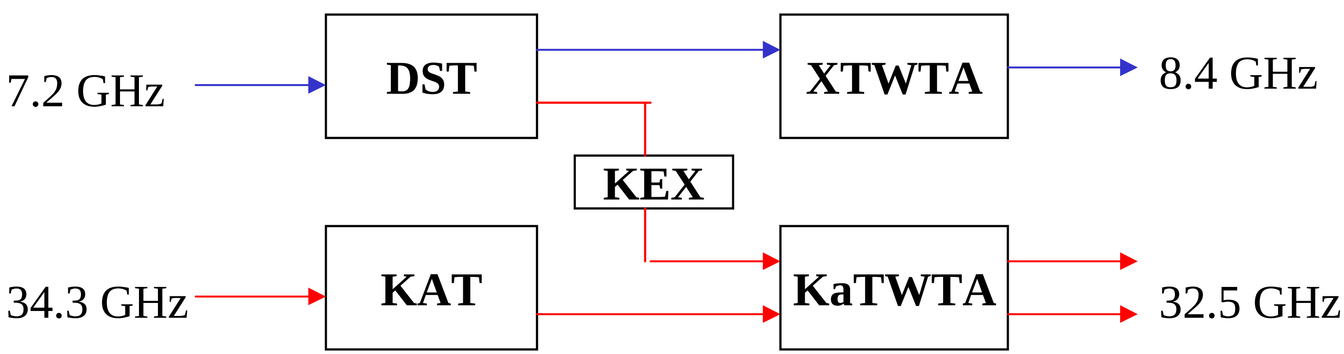
Plasma noise cancellation

Multifrequency radio link

Best accuracies:

$\Delta f/f = 10^{-14}$ at 10^3-10^4s (conjunctions) → 1.5 10^{-6} m/s
$\Delta f/f = 3 10^{-15}$ at 10^3-10^4s (oppositions) → 4.5 10^{-7} m/s





DSS 25 - Goldstone

Doppler only!

Cancellation of plasma noise with a multifrequency link

$$\Gamma_{XX} = \Gamma_{nd} + \Gamma_{\uparrow} + \frac{1}{\alpha_{XX}^2}\Gamma_{\downarrow}$$

X/X Doppler/range observable

$$\Gamma_{XK} = \Gamma_{nd} + \Gamma_{\uparrow} + \frac{1}{\alpha_{XK}^2}\Gamma_{\downarrow}$$

X/Ka Doppler/range observable

$$\Gamma_{KK} = \Gamma_{nd} + \frac{1}{\beta^2}\Gamma_{\uparrow} + \frac{1}{\beta^2 \alpha_{KK}^{\ 2}}\Gamma_{\downarrow}$$

Ka/Ka Doppler/range observable

$$\alpha_{XX} = \frac{f_{X\_D}}{f_{X\_U}} = \frac{880}{749} \qquad \alpha_{XK} = \frac{f_{K\_D}}{f_{X\_U}} = \frac{3344}{749}$$

$$\alpha_{KK} = \frac{f_{K\_D}}{f_{K\_U}} = \frac{14}{15} \qquad \beta = f_{K\_U} / f_{X\_U}$$

Three unknown quantities:

- non-dispersive term (Γ_{nd})
- uplink plasma ($\Gamma_{\uparrow}$)
- downlink plasma ($\Gamma_{\downarrow}$)

[Bertotti, Comoretto, Iess, 1993]

Cancellation of plasma noise (cont.)

$$\Gamma_{nd} \cong \Gamma_{KK} + \frac{1}{13}\Gamma_{XX} + \frac{1}{35}\Gamma_{XK}$$

$$\Gamma_{\downarrow} \cong 0.67\Gamma_{XX} - 0.67\Gamma_{XK}$$

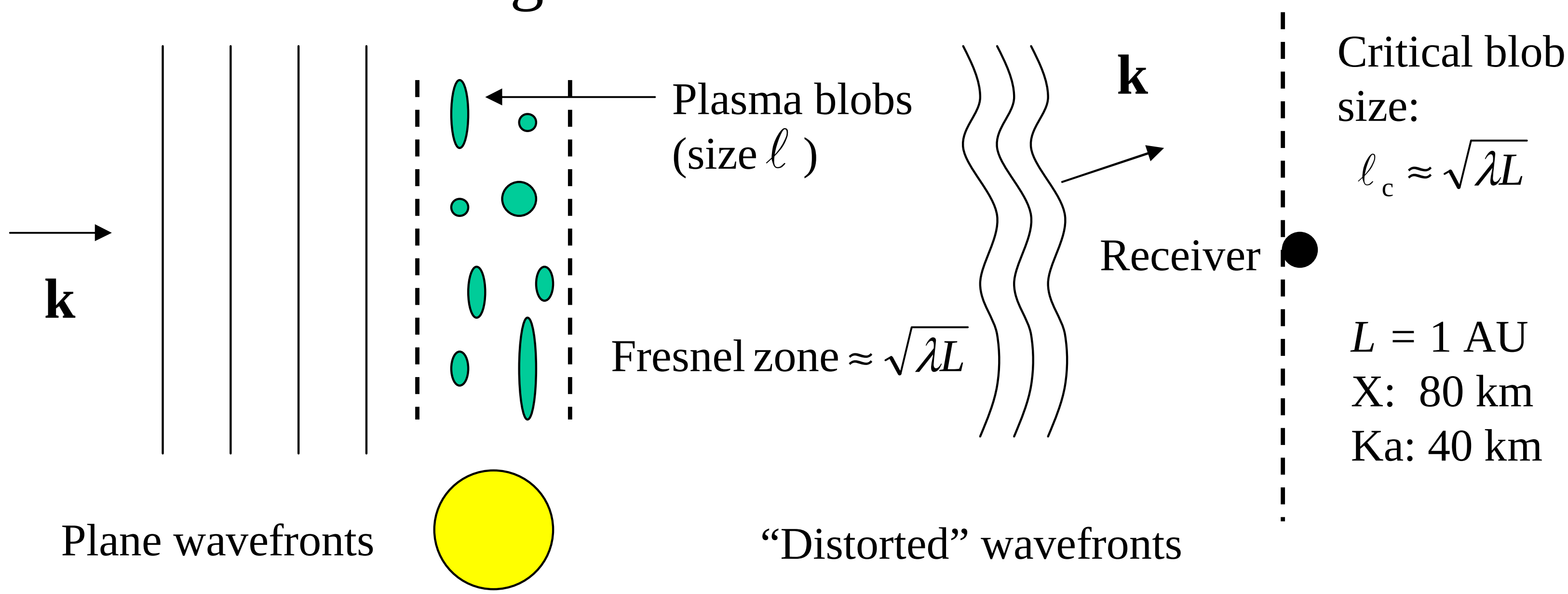
$$\Gamma_{\uparrow} \cong -1.05\Gamma_{KK} + 1.1\cdot 10^{-3}\Gamma_{XX} + 1.05\Gamma_{XK}$$

Conclusion:

The Ka/Ka link provides the crucial observable and needs the highest accuracy.

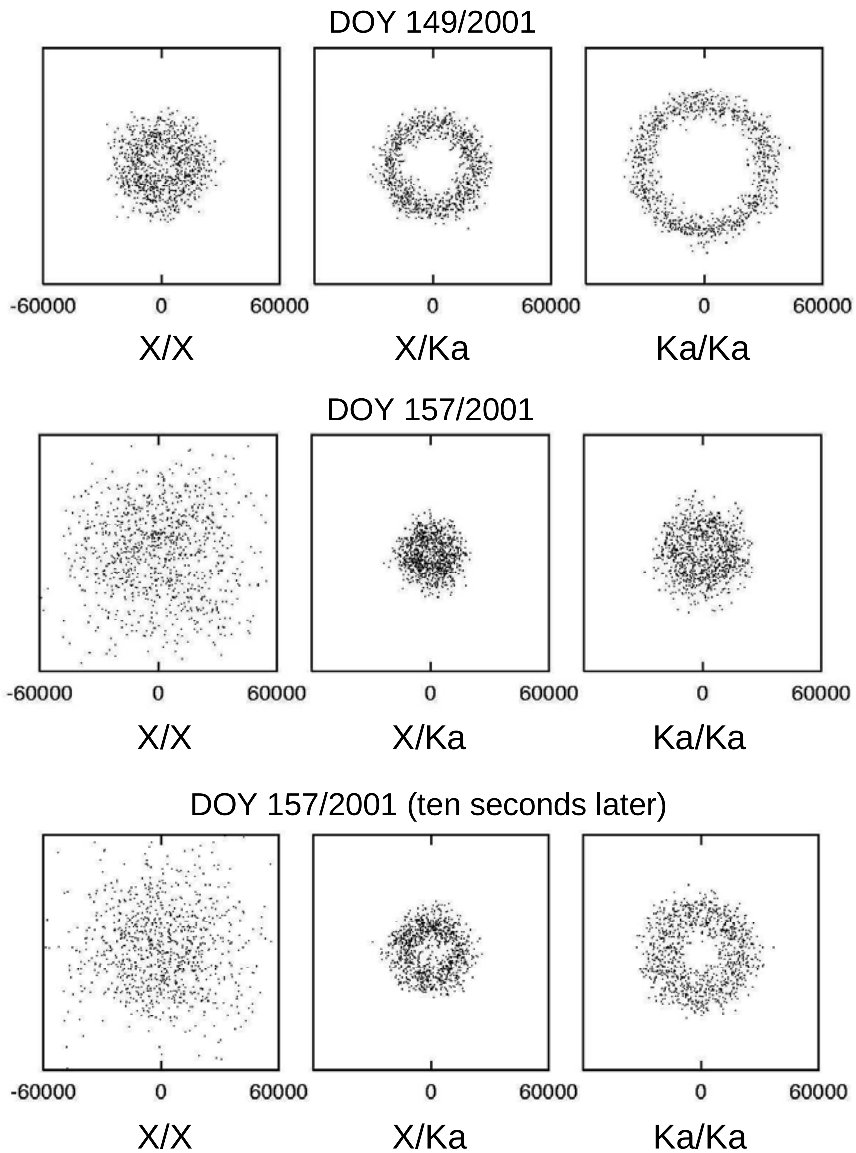
Limitations of the plasma cancellation system

- Scattering effects (strong amplitude and phase scintillation, spectral broadening, difficulty to lock the signal at very small solar elongation angles)
- Magnetic corrections to the refractive index ($\propto \omega_p^2 \Omega_c / \omega_0^3$), appreciable only within 3 solar radii
- Separation of the X and Ka radio beam due the radial dependence of the average refractive index



Physical optics effects : phasor representation of the signal

1000 I and Q samples of Cassini radio signal (sampled at 1 kHz)

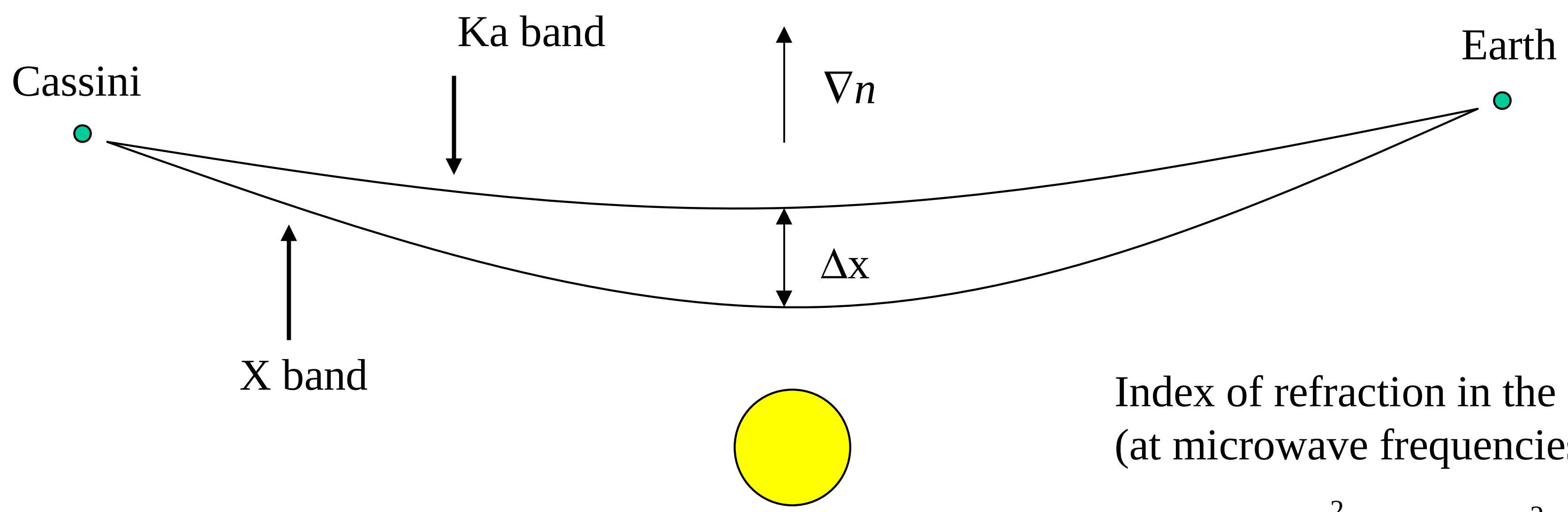


b = 25 R_s

b = 5 R_s

b = 5 R_s

Deflection of radio waves by the solar corona



Index of refraction in the corona (at microwave frequencies):

$$n = 1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2} = 1 - \frac{e^2 n_e}{2\pi m_e f_0^2}$$

Ray paths defined by the eikonal eq.

$$\left(\nabla \xi_X(\mathbf{r})\right)^2 = n_X^2$$

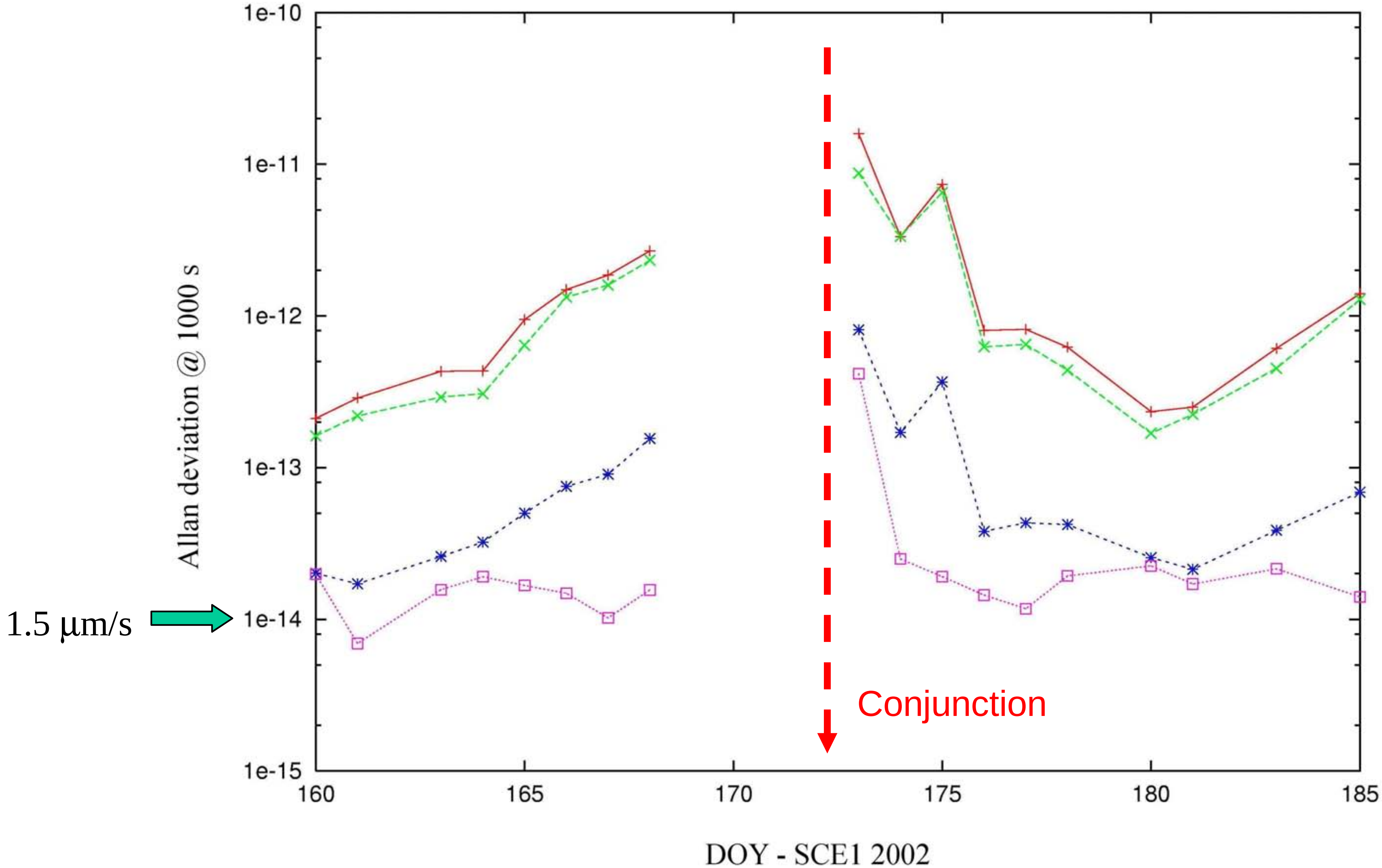
$$\left(\nabla \xi_K(\mathbf{r})\right)^2 = n_K^2$$

Compare with GR (to first order):

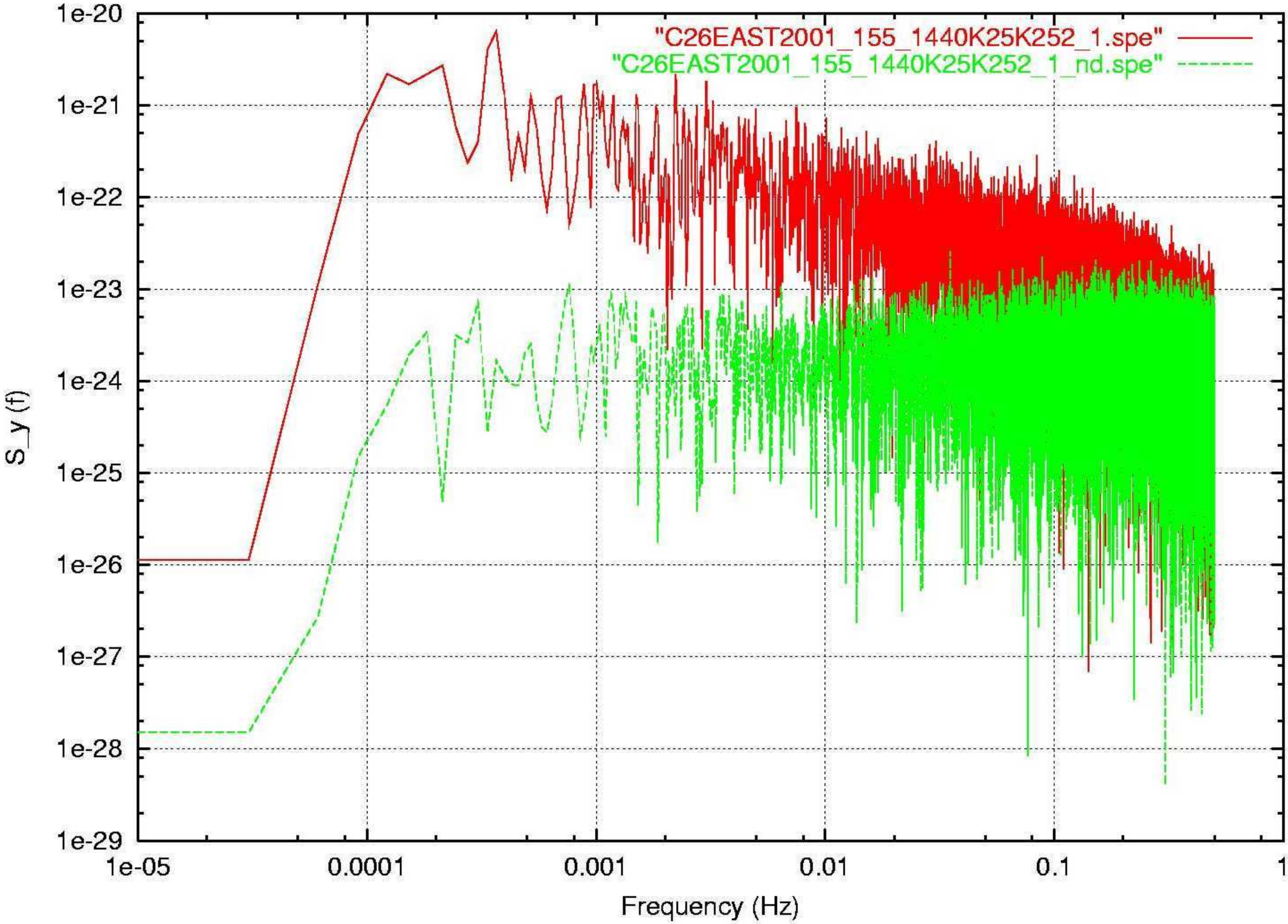
$$n = 1 + \frac{1+\gamma}{2}\frac{R_g}{r}$$

Wind speed may be estimated by correlating X and Ka band observables, if Δx is known

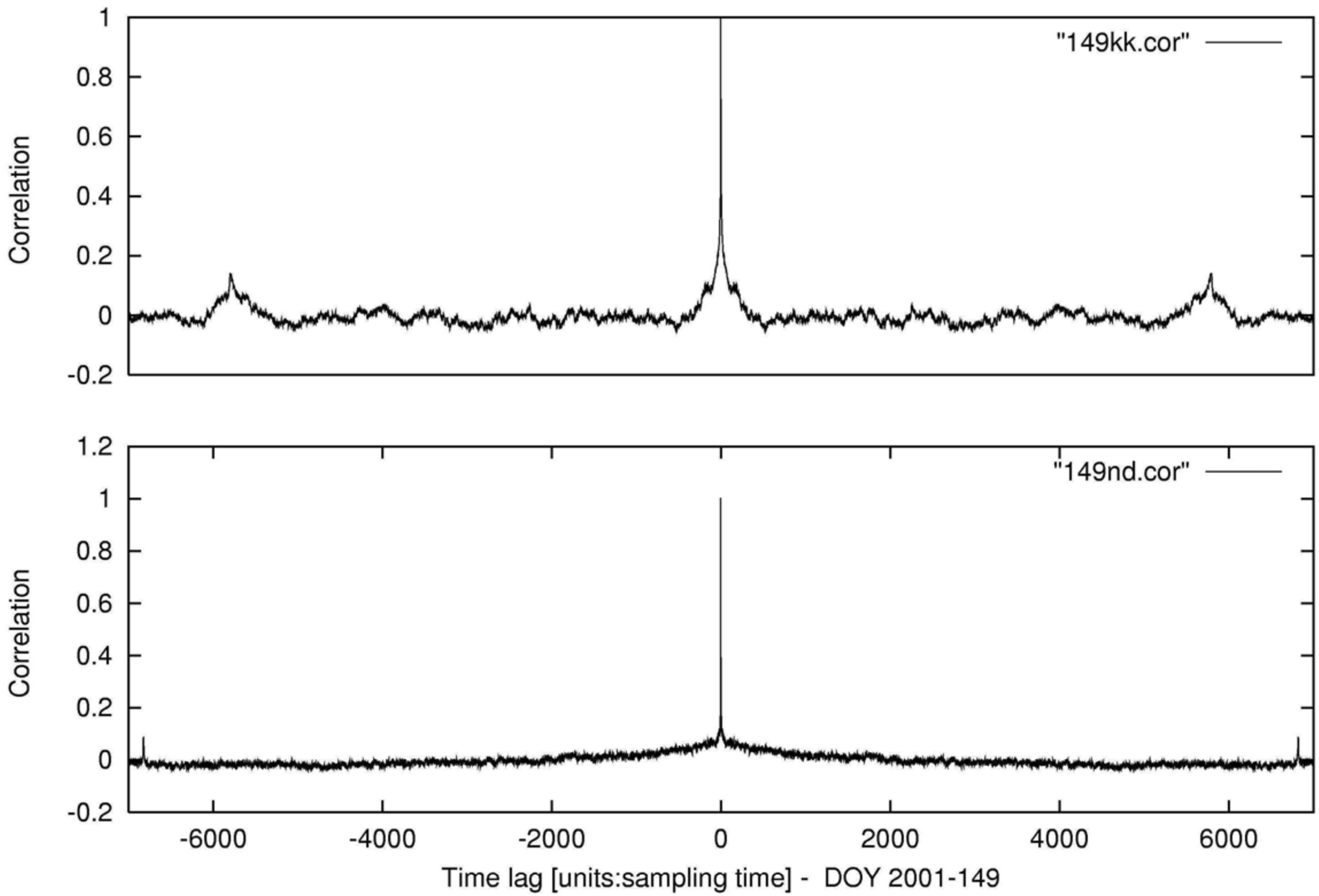
Plasma noise in the X/X, X/Ka, Ka/Ka links and the calibrated Doppler observable (daily Allan dev. @1000s, Cassini SCE1) Minimum impact parameter: 1.6 R_s (DOY 172)



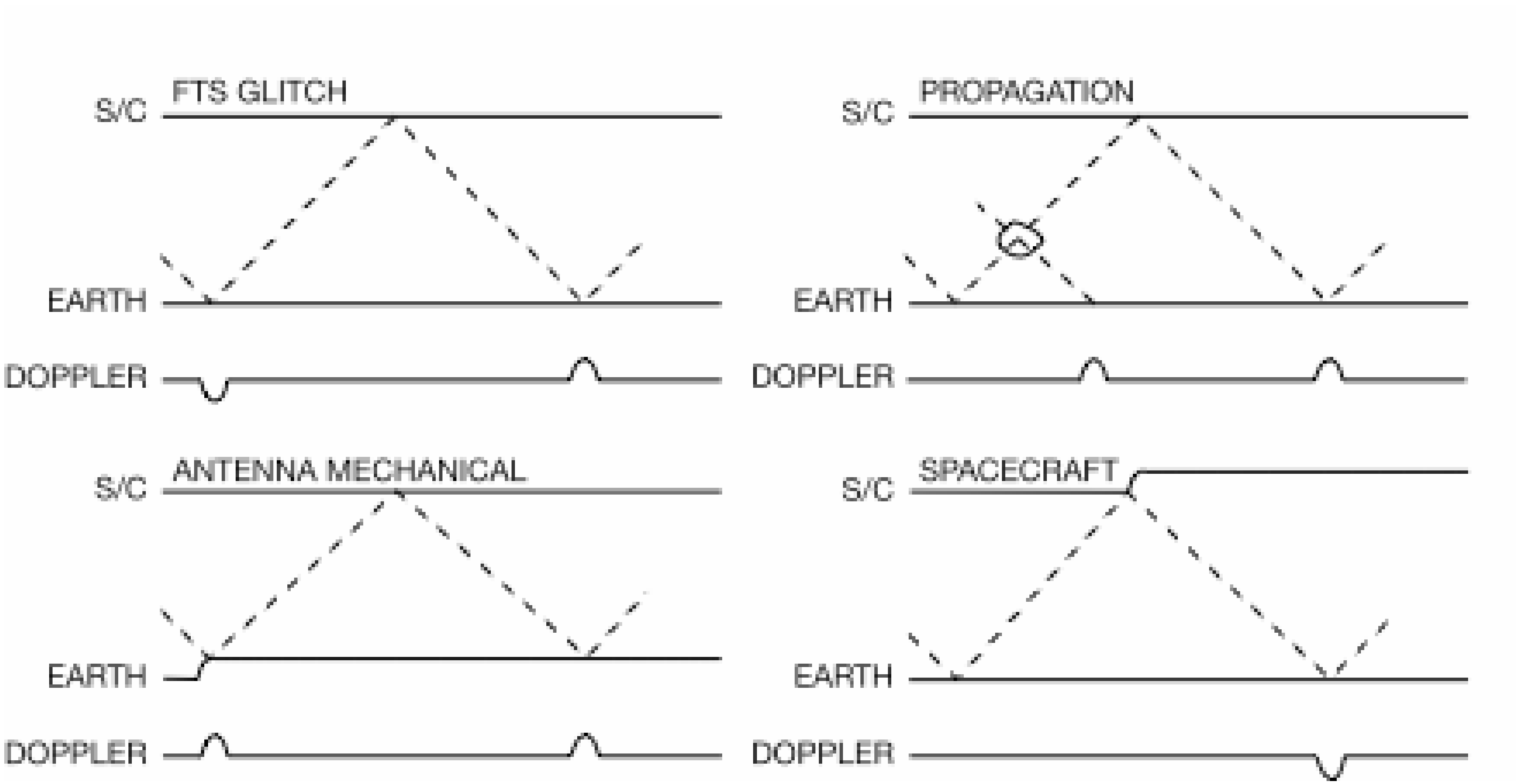
Power spectrum of relative frequency shift residuals



ACF of Doppler residuals
(Cassini DOY 2001-149)

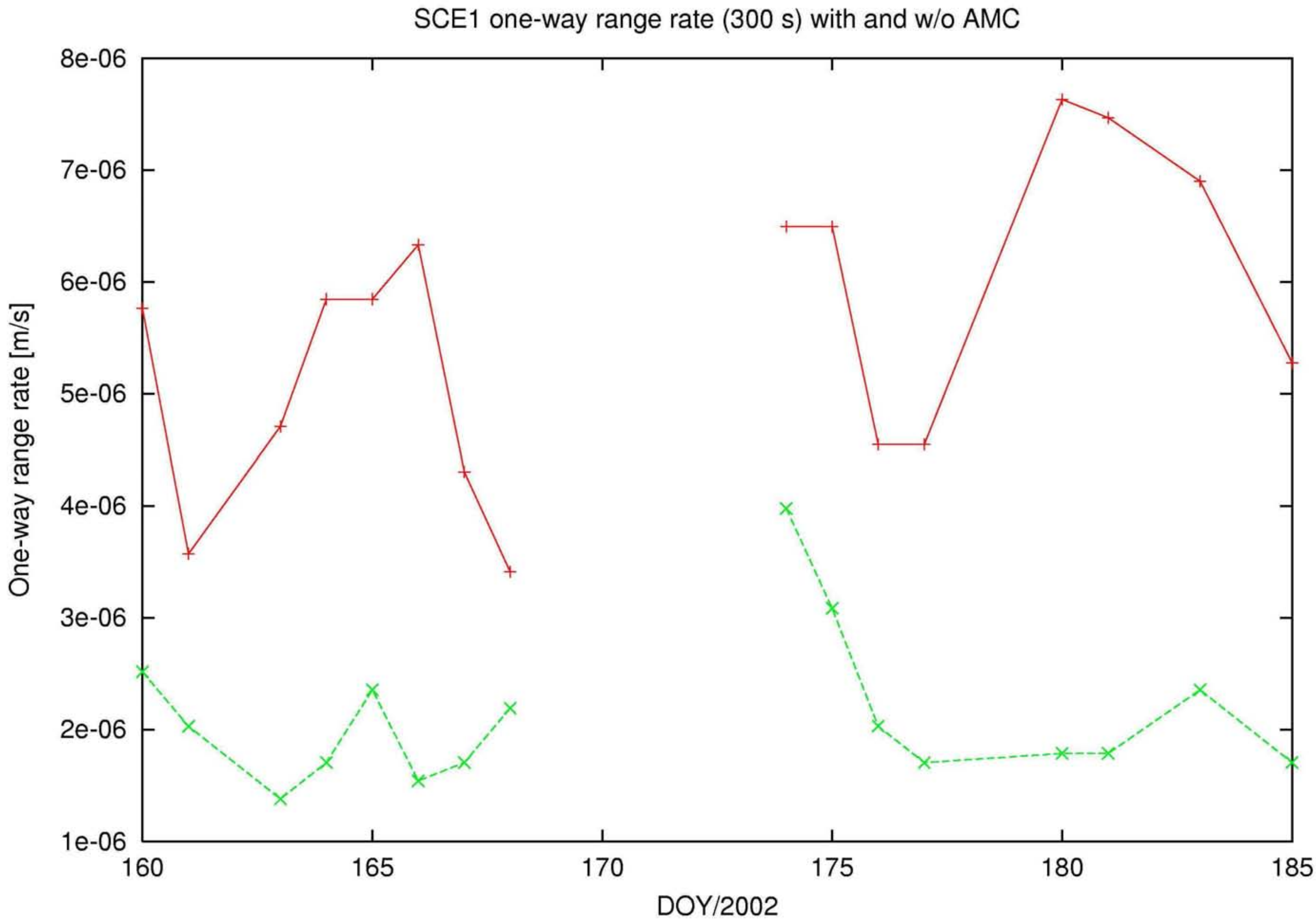


Noise Signatures in 2-way Doppler Link



The Advanced Media Calibration System for tropospheric dry and wet path delay corrections.

The 34m beam waveguide tracking station DSS 25, NASA's Deep Space Network, Goldstone, California



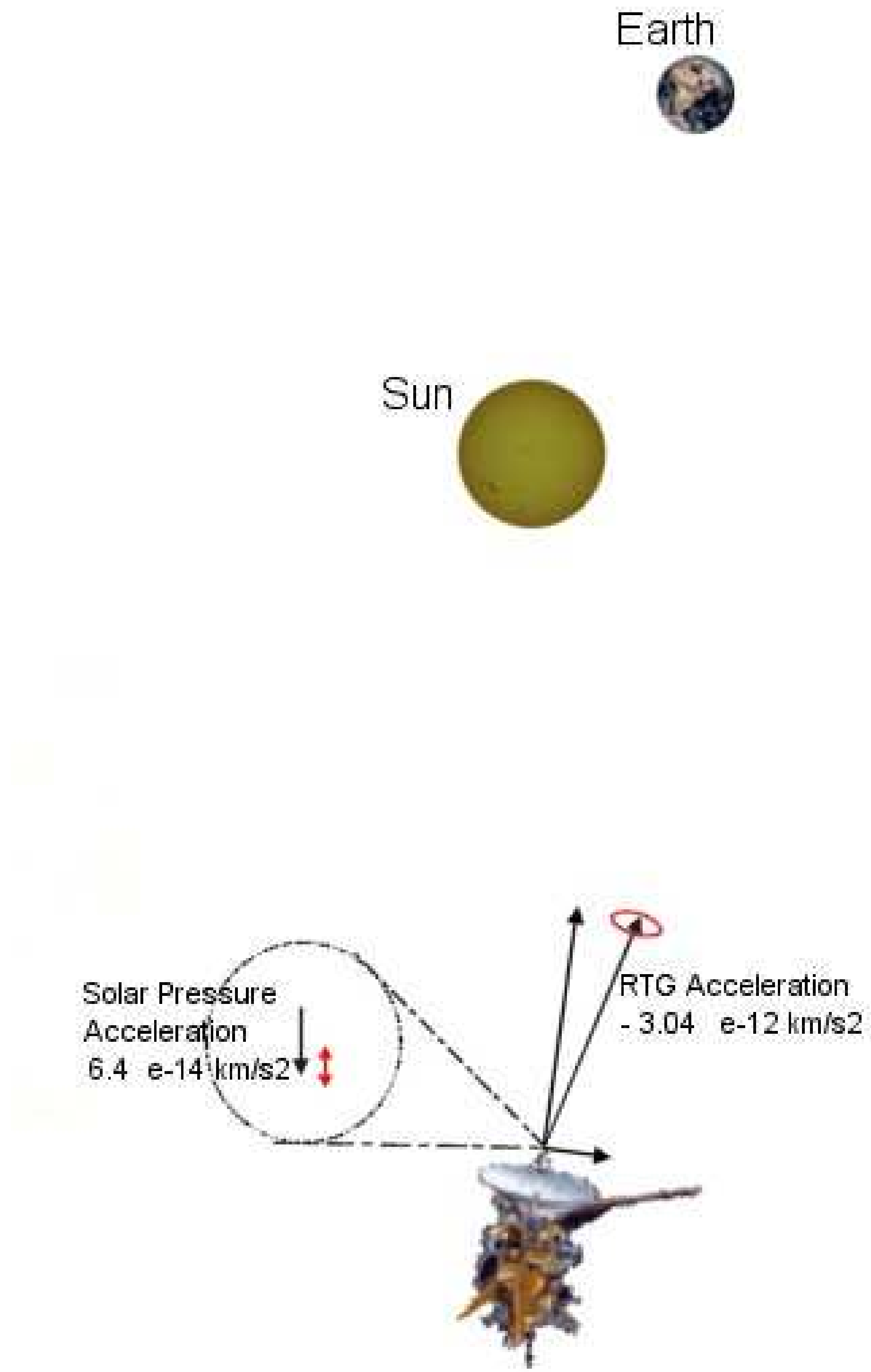
SCE1 one-way range rate (300 s) with and w/o AMC
8e-06
7e-06
6e-06
5e-06
4e-06
3e-06
2e-06
1e-06
One-way range rate [m/s]
160
165
170
175
180
185
DOY/2002

Dynamical model

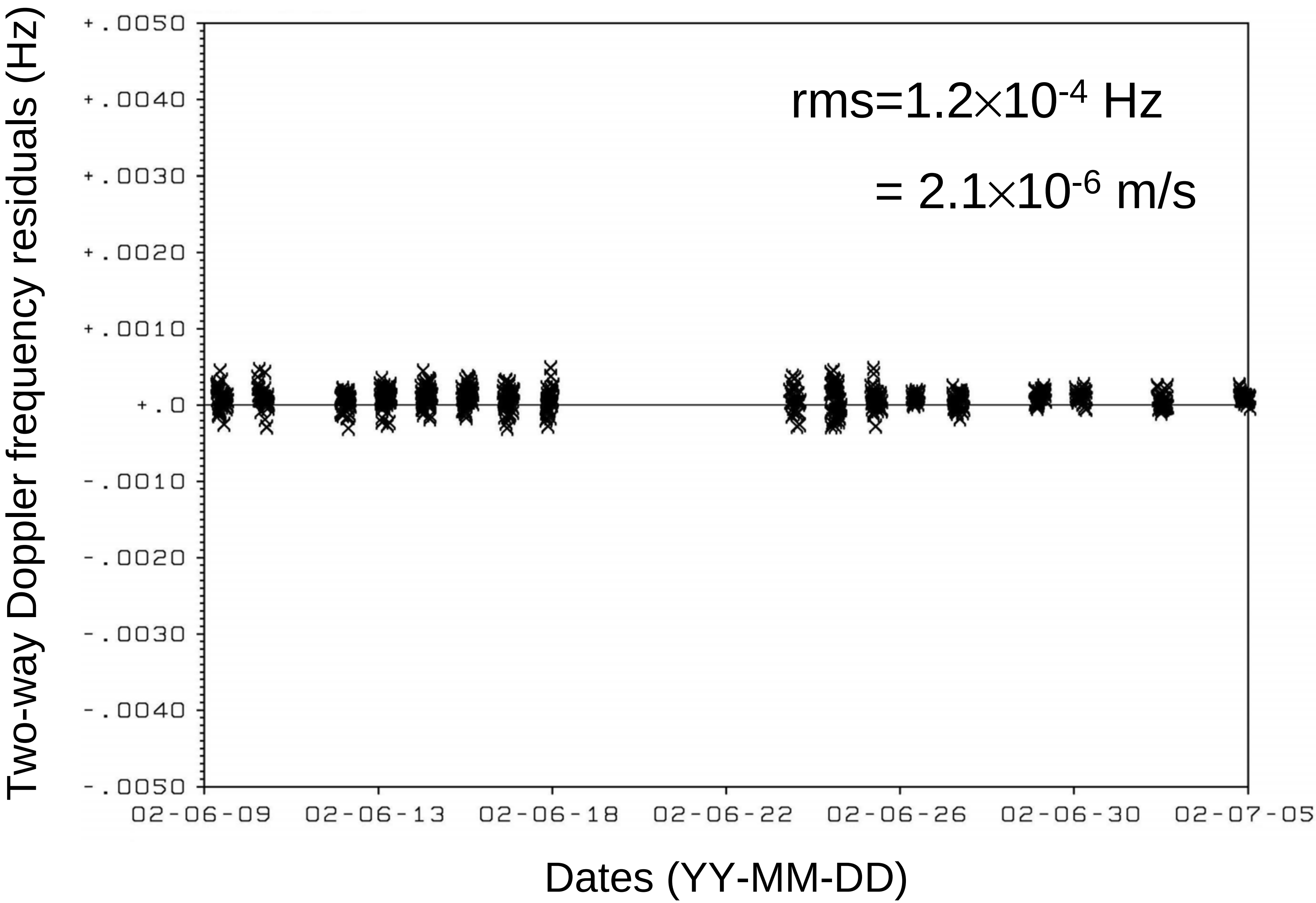
Solve-for parameters:

- Spacecraft state vector
- Specular and diffuse reflectivity of the 4m high gain antenna
- Acceleration from anisotropic thermal emission from the three RTG
- γ

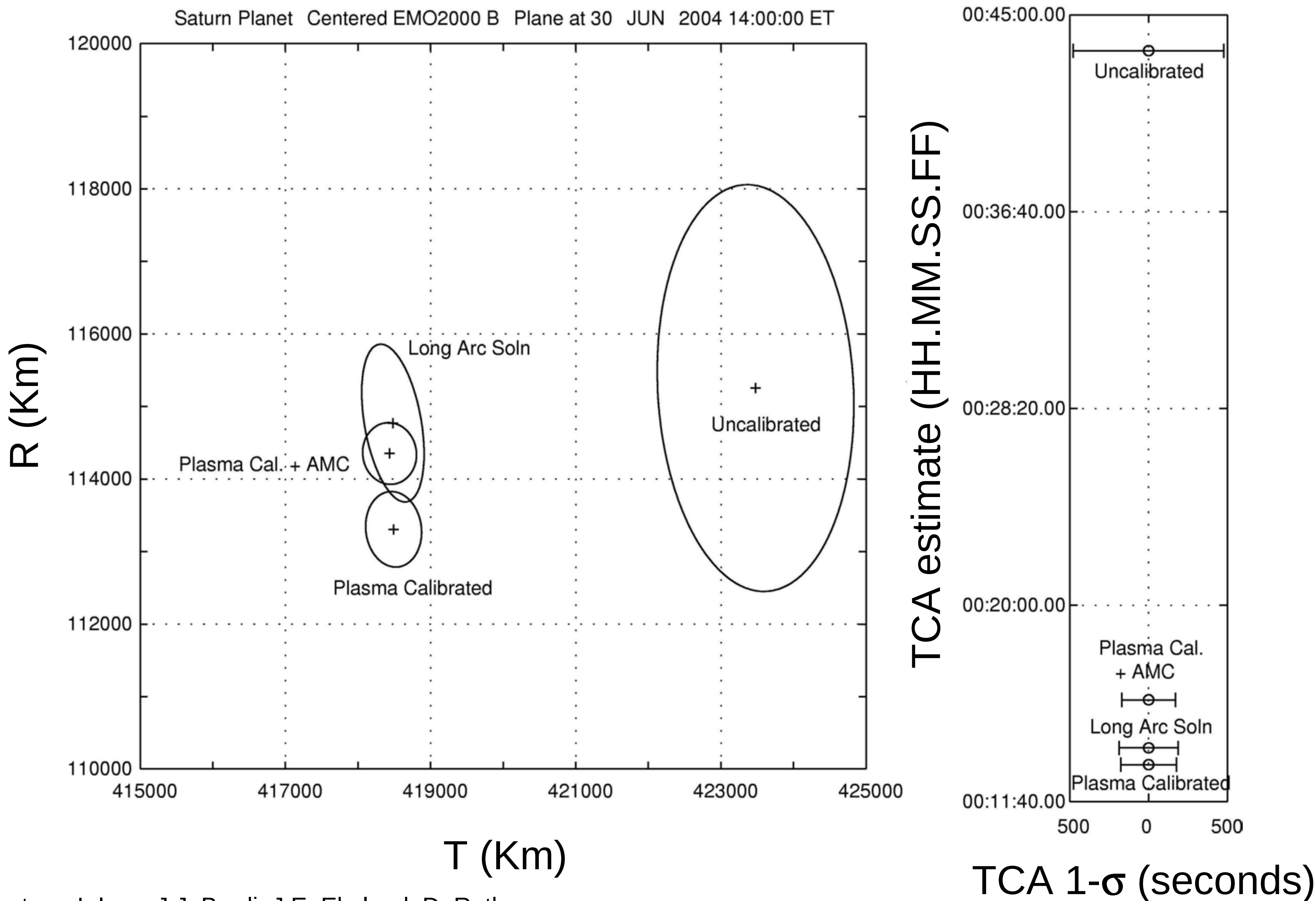
No clue of anomalous acceleration on Cassini



Pseudo X-band frequency residuals (SCE1) with plasma and tropospheric calibrations



Saturn-centered B-plane plot of the Cassini orbital solutions



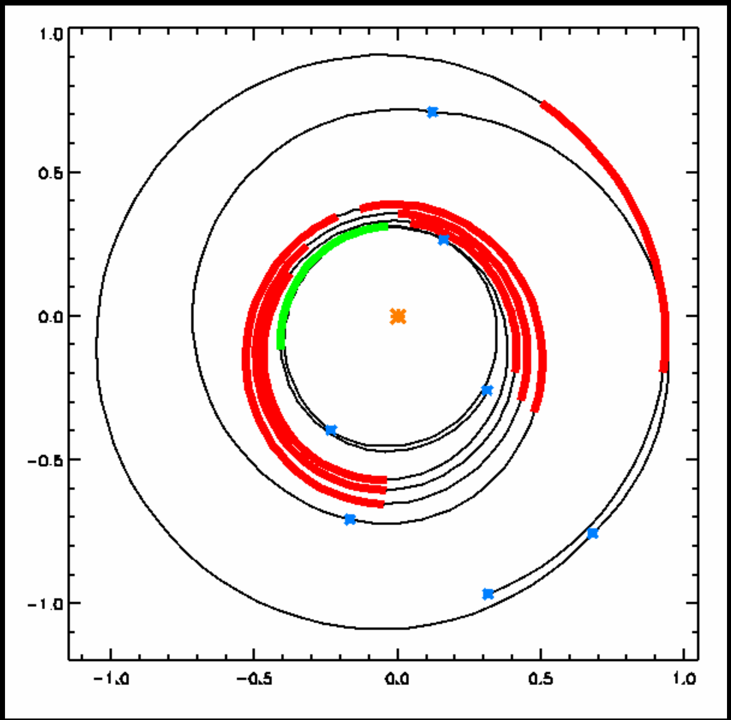
P.Tortora, L.Iess, J.J. Bordi, J.E. Ekelund, D. Roth,
J. Guidance, Control and Dynamics, 27(2), 251 (2004)

Launch on Soyuz 2-1B/Fregat-M (**13 April 2012**)
(1 August 2013 ?)

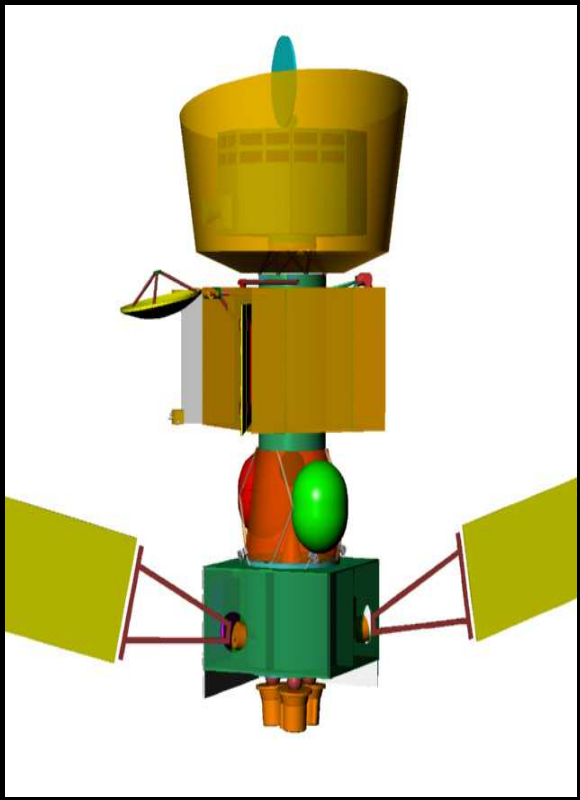
Solar Electric Propulsion

Chemical Propulsion

Arrival: 4 April 2017 (but likely later)



MMO
MPO
CPM
SEPM



MORE: Science Goals

- Spherical harmonic coefficients of the gravity field of the planet up to degree and order 25.
- Degree 2 (C_{20} and C_{22}) with 10^{-9} accuracy (Signal/Noise Ratio ~ 10^4)
- Degree 10 with SNR ~ 300
- Degree 20 with SNR ~ 10
- Love number k_2 with SNR ~ 50.
- Obliquity of the planet to an accuracy of 4 arcsec (40 m on surface – needs also SIMBIO-SYS high resolution camera)
- Amplitude of physical librations in longitude to 4 arcsec (40 m on surface – needs SIMBIO-SYS high resolution camera).
- C_m/C (ratio between mantle and planet moment of inertia) to 0.05 or better
- C/MR^2 to 0.003 or better.

MORE: Science Goals

- Spacecraft position in a Mercury-centric frame to 10 cm – 1m (depending on the tracking geometry)
- Planetary figure, including mean radius, polar radius and equatorial radius to 1 part in 10^7 (by combining MORE and BELA laser altimeter data).
- Geoid surface to 10 cm over spatial scales of 300 km.
- Topography of the planet to the accuracy of the laser altimeter (in combination with BELA).
- Position of Mercury in a solar system barycentric frame to 1 m.
- PN parameter γ, controlling the deflection of light and the time delay of ranging signals to $2.5*10^{-6}$
- PN parameter β, controlling the relativistic advance of Mercury's perihelion, to $5*10^{-6}$ [now $5*10^{-4}$]
- PN parameter η (controlling the gravitational self-energy contribution to the gravitational mass to $2*10^{-5}$ [now $5*10^{-4}$]
- The gravitational oblateness of the Sun (J_2) to $2*10^{-9}$ [now $1*10^{-7}$ – indirect]
- The time variation of G ($d(\ln G)/dt$) to $2*10^{-13}$ years^{-1} [now $1*10^{-12}$]

Fighting Noise

- **Dynamical noise and non-gravitational accelerations**
- **Propagation noise (solar corona, interplanetary plasma, troposphere)**
- **Spacecraft and ground instrumentation**

Dynamical noise must be reduced to a level compatible with the accuracy of range-rate measurements:

$$\sigma_a = \frac{c}{\tau}\sigma_y = 3\times 10^{-7}\,\mathrm{cm\,s^{-2}} \quad \text{at} \;\; \tau = 1000\,\mathrm{s}$$

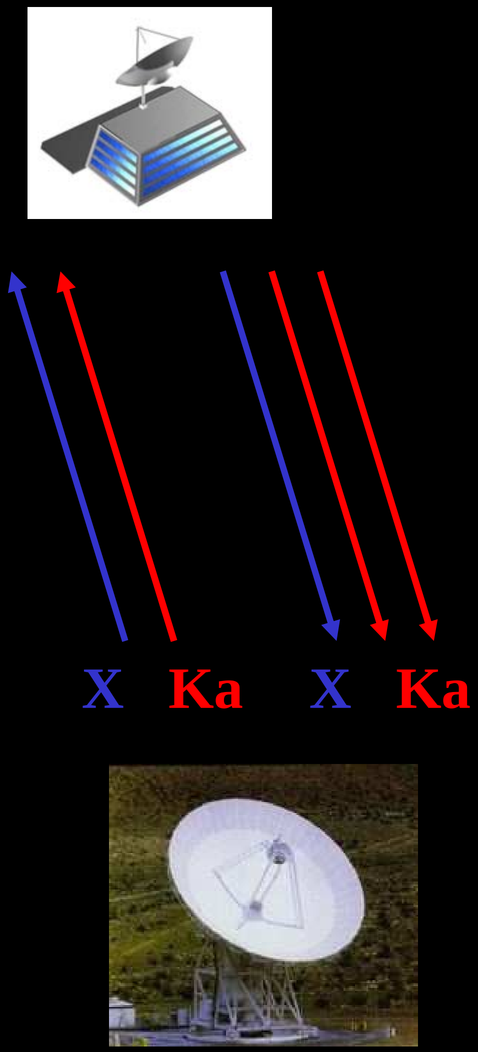
Plasma noise cancellation

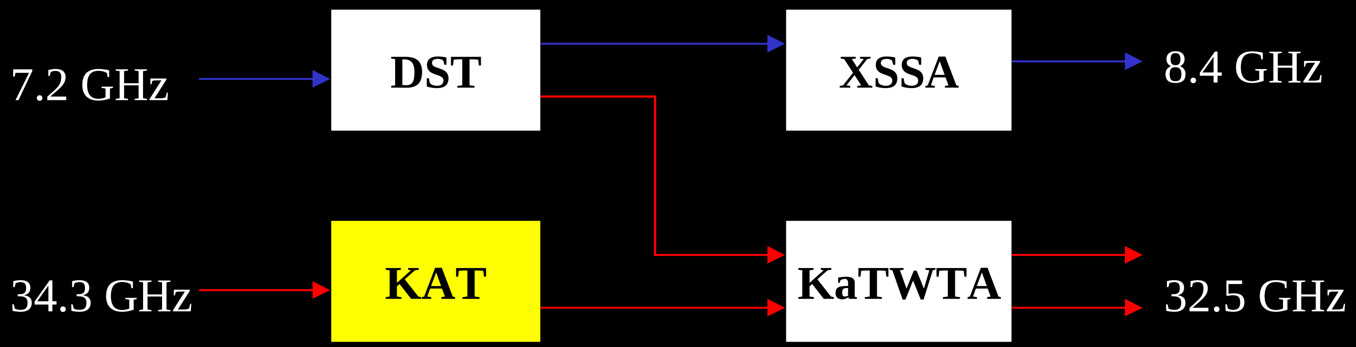
Multi-frequency radio link
(two-way)

Target accuracy:

$\Delta f/f = 10^{-14}$ at 10^3-10^4s
$\Delta\rho$ = 10 cm

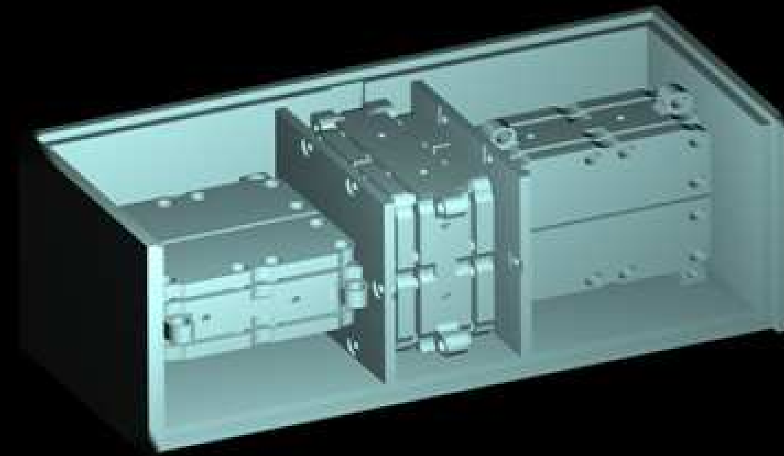
$\sigma_y = 10^{-14}$ is equivalent to a one-way range rate of 1.5 micron/s
The corresponding one-way displacement in 1000 s is 1.5 mm

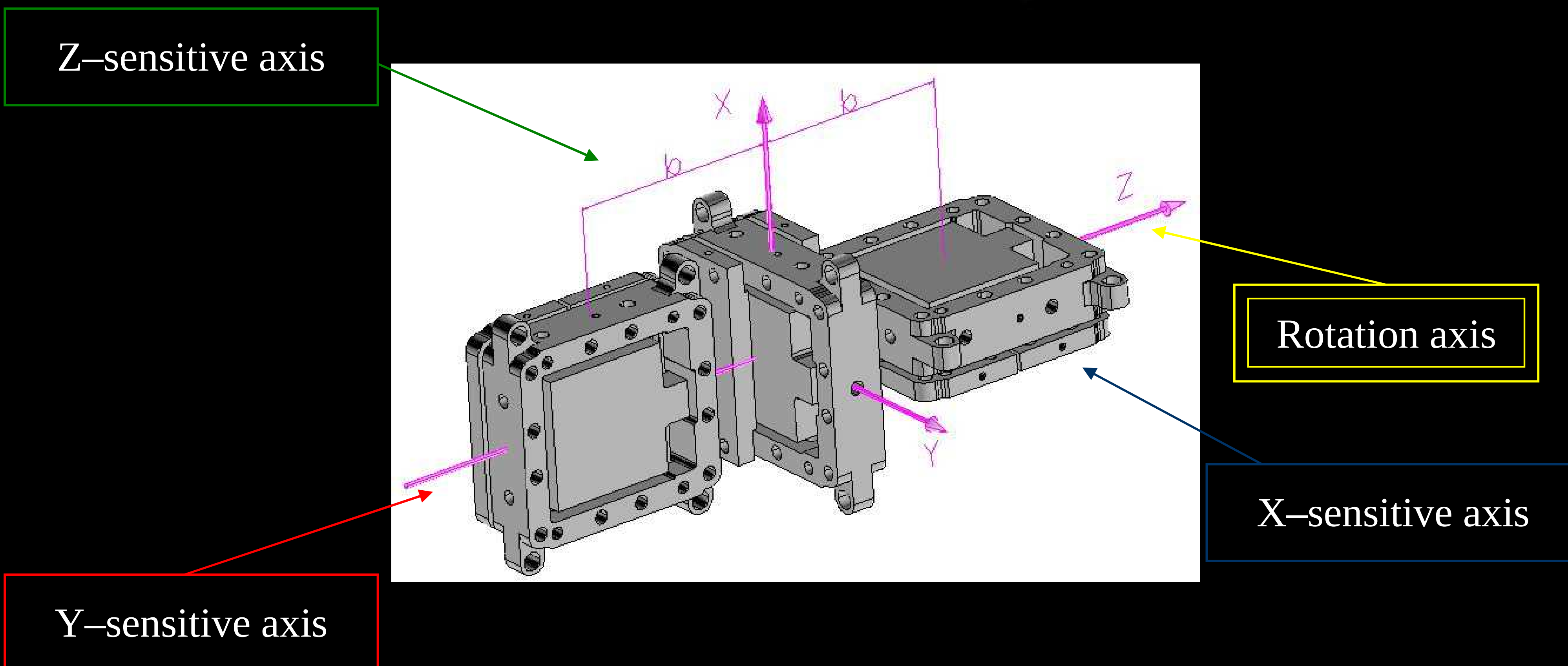




ISA

Italian Spring Accelerometer





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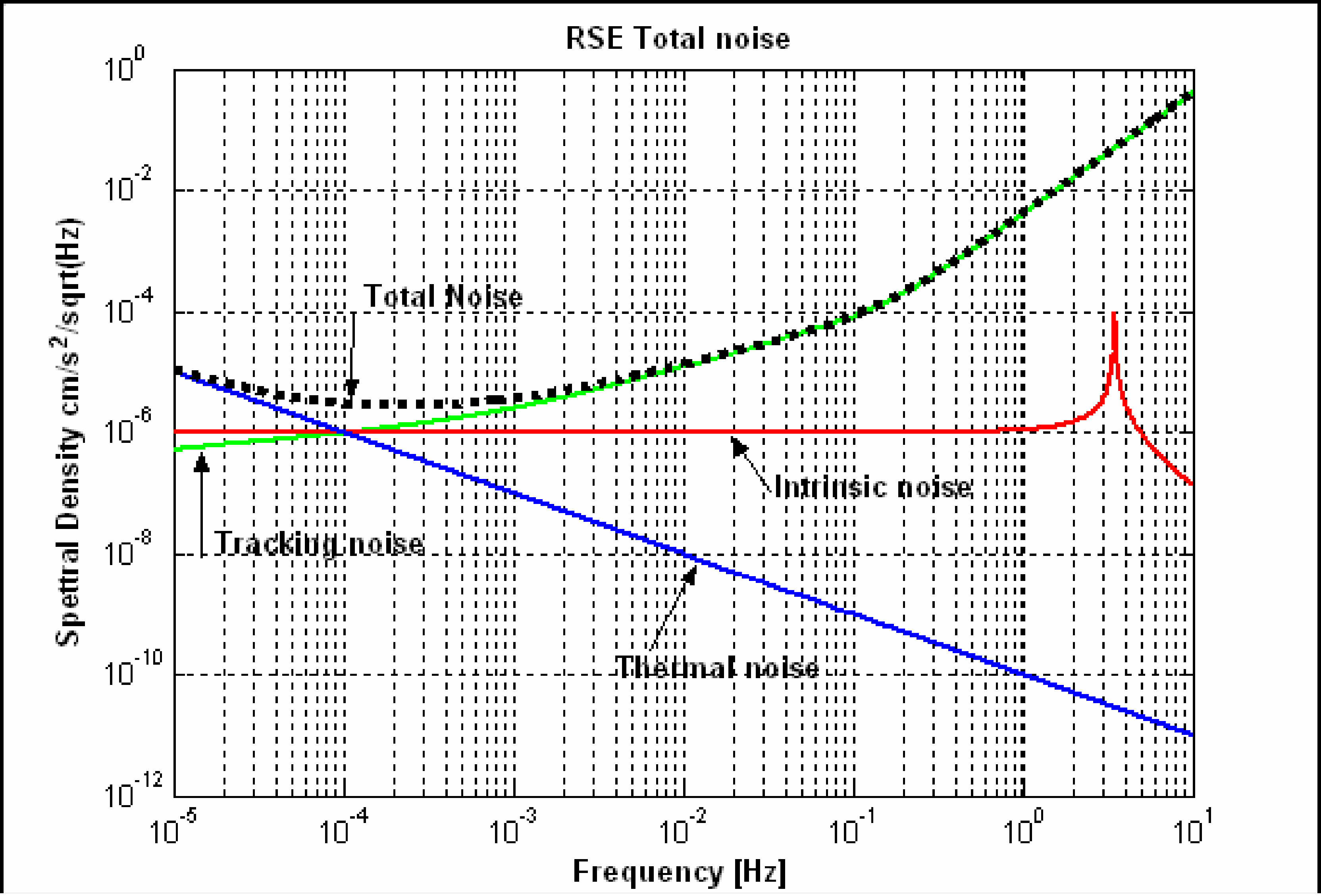
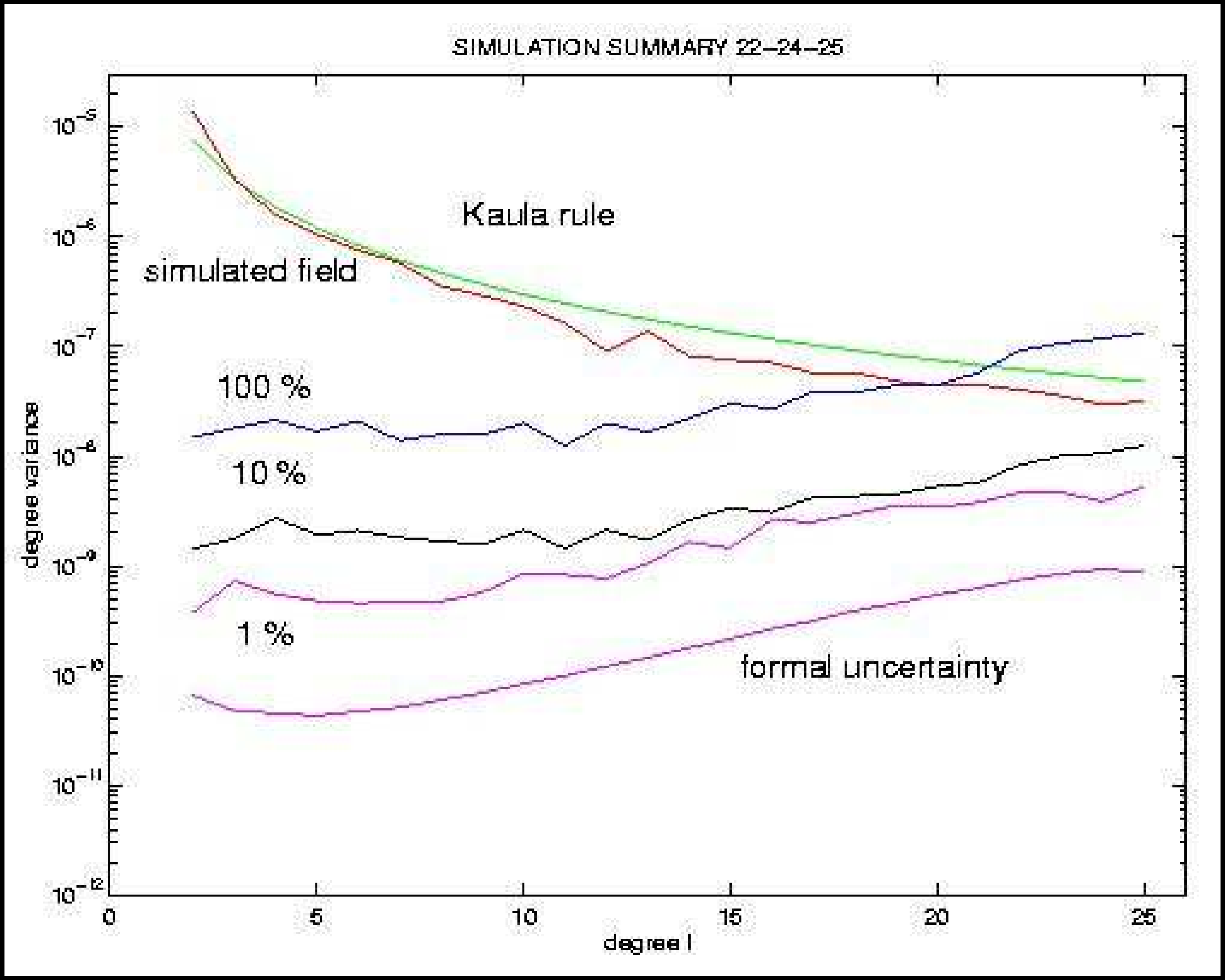


MORE OD concepts were tested by detailed numerical simulations at the Univ. of Pisa.

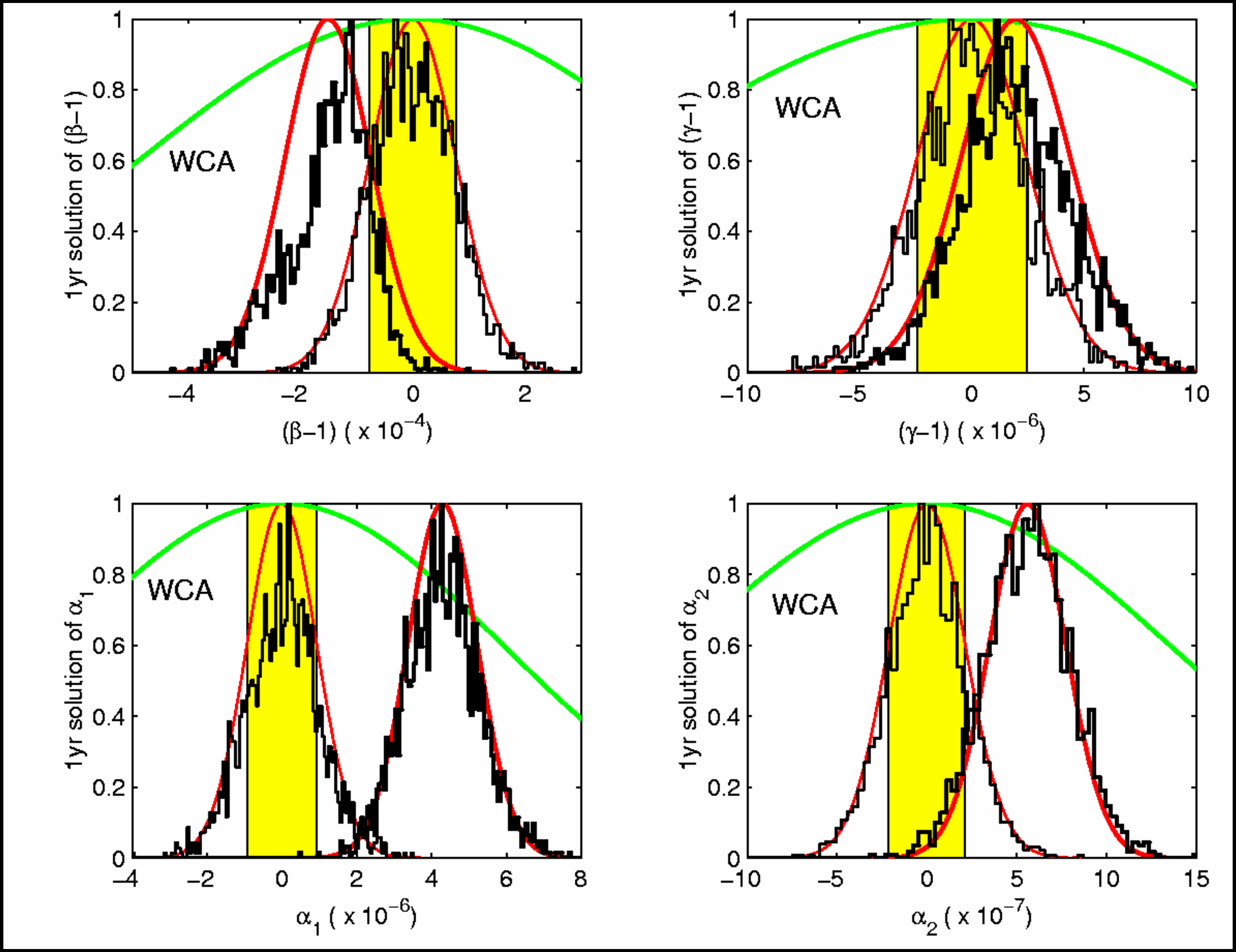


Simulations provide requirements on accelerometer and radio system for all radio science experiments.

Software used is a prototype for the operational MORE data processing.

Noise model controlled via a namelist file with 35 adjustable parameters (23 for Doppler and 12 for ranging)

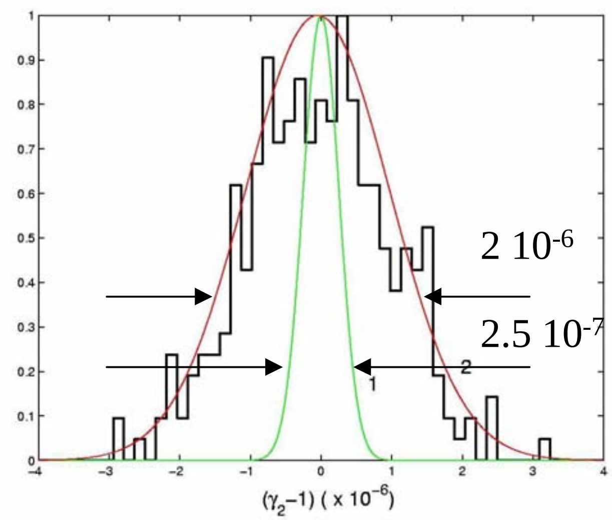
Simulations for PPN parameters β, γ, α_1, α_2

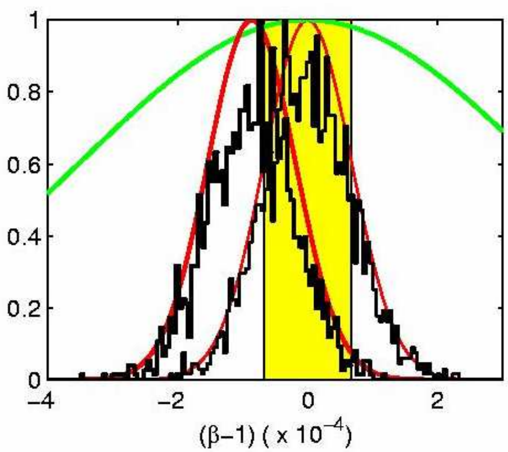


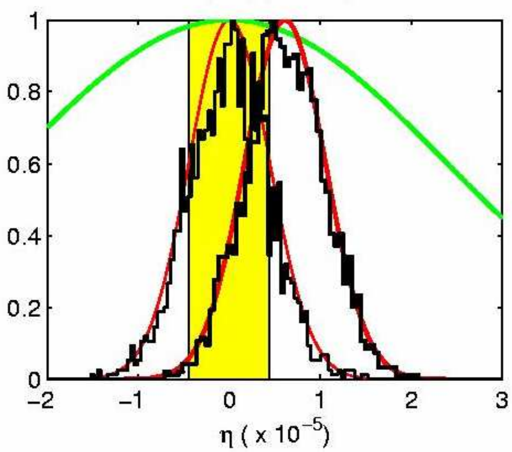
2000 simulations of 1y experiment
No preferred frame – η free

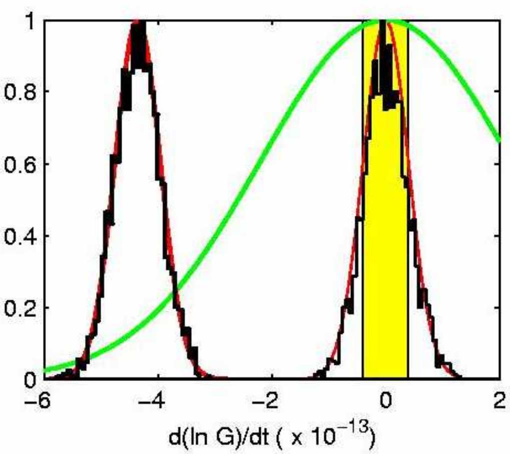
Correlation ellipses

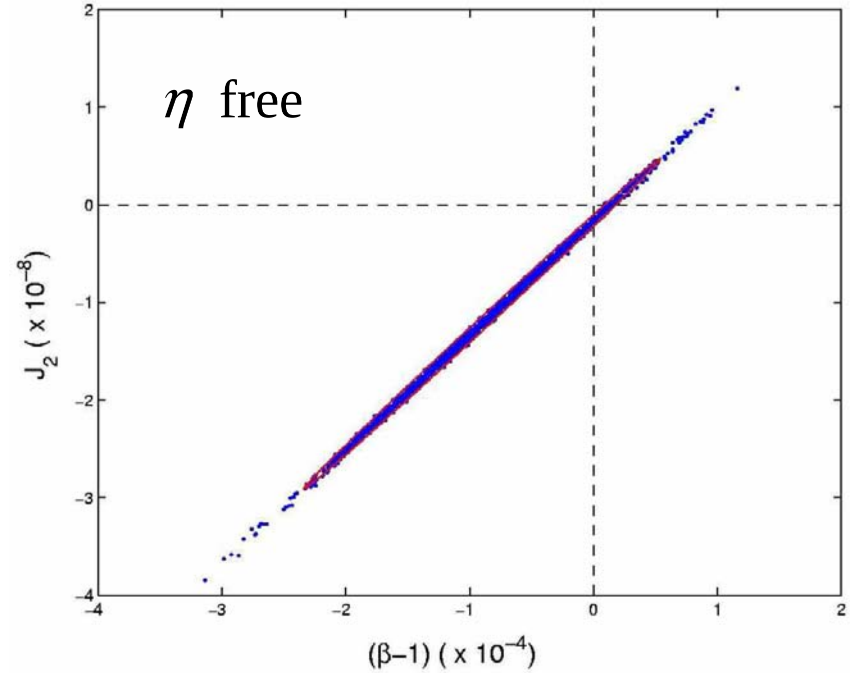
Cruise SCE →

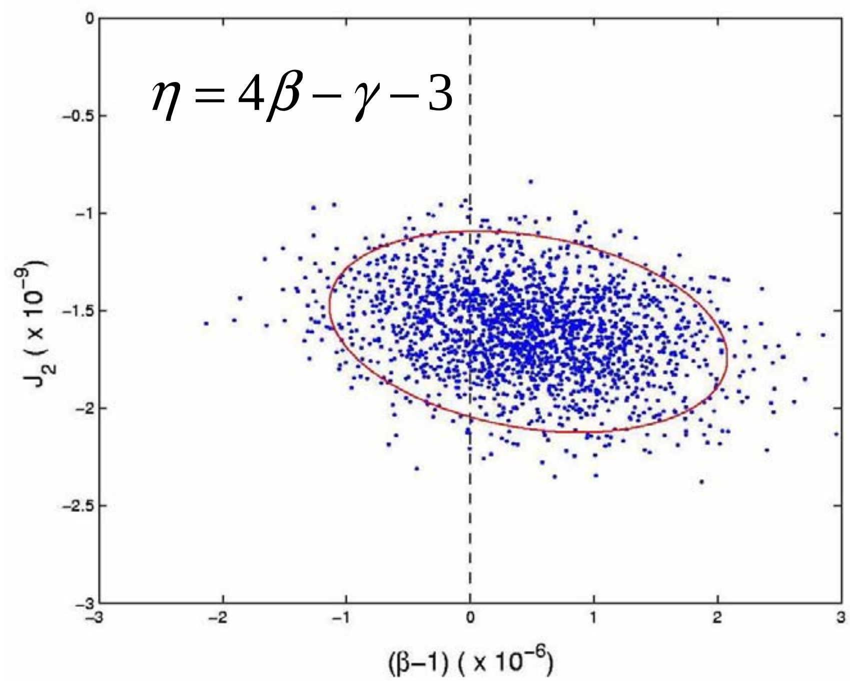












Parameter	Present accuracy	MORE
γ	2×10^{-5}	2×10^{-6}
β	1×10^{-4}	2×10^{-6}
η	5×10^{-4}	8×10^{-6}
$J_2^{\odot}$	4×10^{-8}	2×10^{-9}
$\dot{G}/G$	$9 \times 10^{-13}\ \mathrm{yr}^{-1}$	$3 \times 10^{-13}\ \mathrm{yr}^{-1}$

Current accuracies of selected PN parameters and values expected from the BepiColombo MORE experiment. Metric theories of gravity with no preferred frame effects are assumed.

Milani et al. Phys. Rev. D, **66**, 082001 (2002).