Phase noise due to vibrations in Mach-Zehnder atom interferometers

Université Paul Sabatier and CNRS, Toulouse

Marion Jacquey
Alain Miffre
Matthias Büchner
Gérard Trénec
Jacques Vigué

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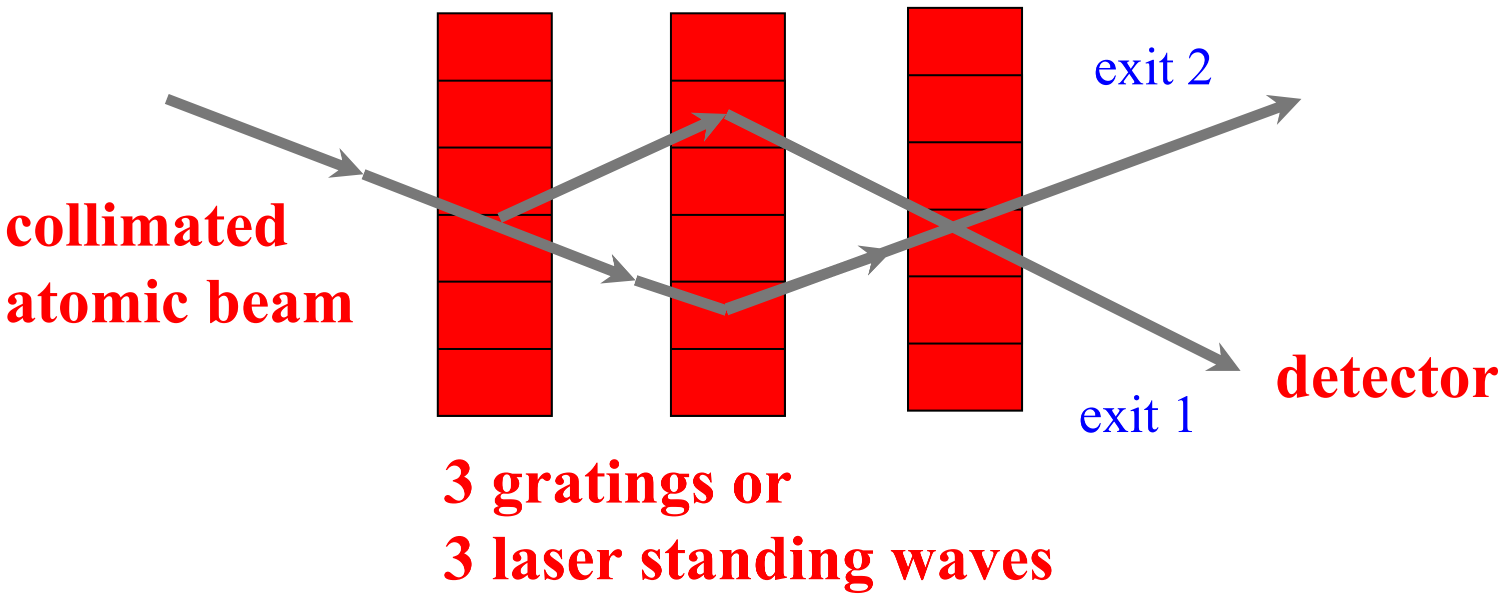
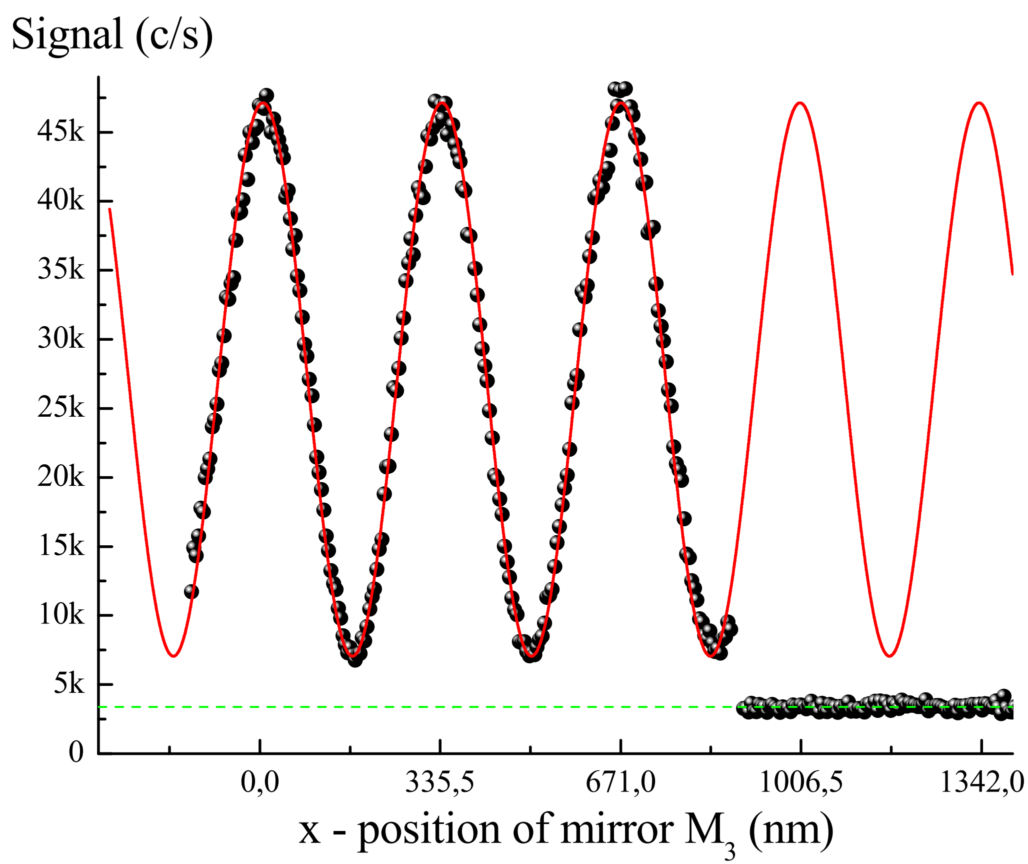
Mach-Zehnder atom interferometers operating at thermal energies



The mirrors and beam-splitters of the Mach-Zehnder optical interferometers are replaced by elastic diffraction on gratings.
In the Bragg regime, diffraction of order p can be used.

Atom interference fringes with ^{7}Li

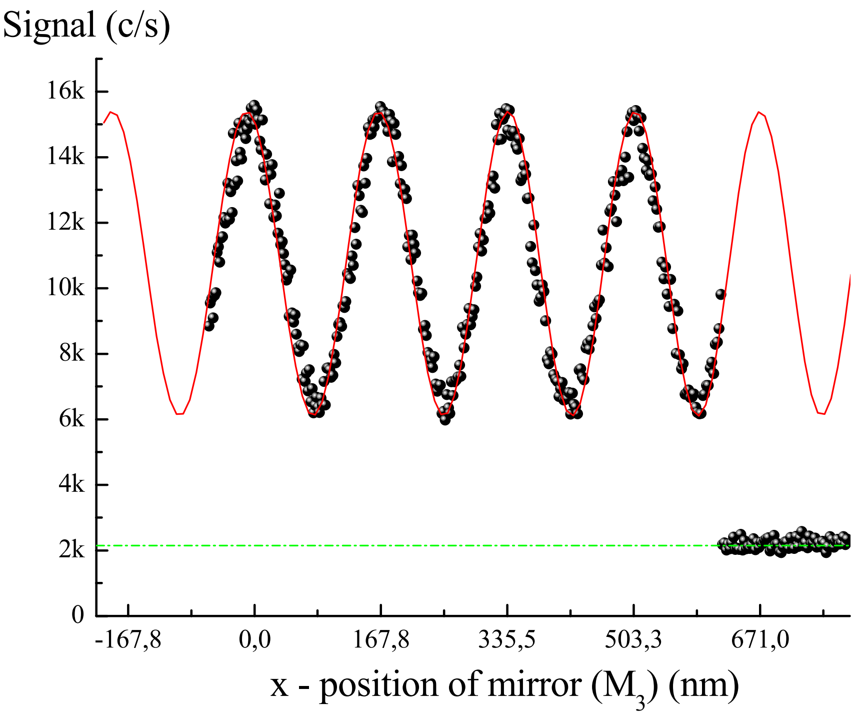


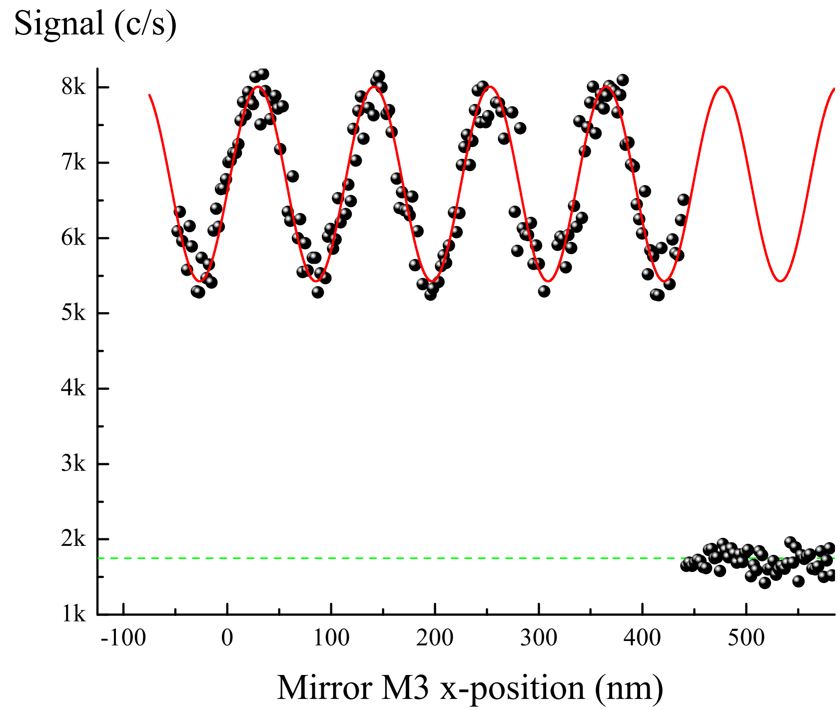
diffraction order p = 1

counting time = 0.1 s/point

fringe visibility V = 84.5 ± 1%

mean output flux I_0 = 23700 c/s





diffraction order p = 2
counting time = 0.1 s/point
fringe visibility $V = 54 \pm 1\%$
mean output flux $I_0 = 8150$ c/s

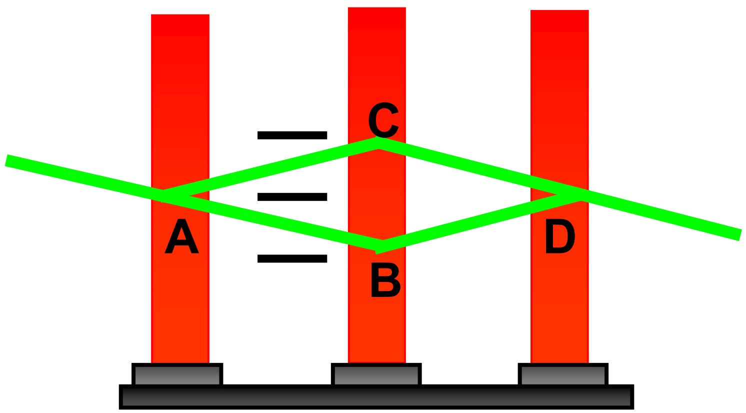
diffraction order p = 3
counting time = 0.1 s/point
fringe visibility $V = 26 \pm 1\%$
mean output flux $I_0 = 4870$ c/s

Interests of thermal atom interferometers

the two atomic beams are spatially separated:

→ one can apply a perturbation on one beam only

→ interferometric measurements of this perturbation



examples of such perturbations

an electric field → atom electric polarizability

a low-pressure gas → index of refraction for an atom wave

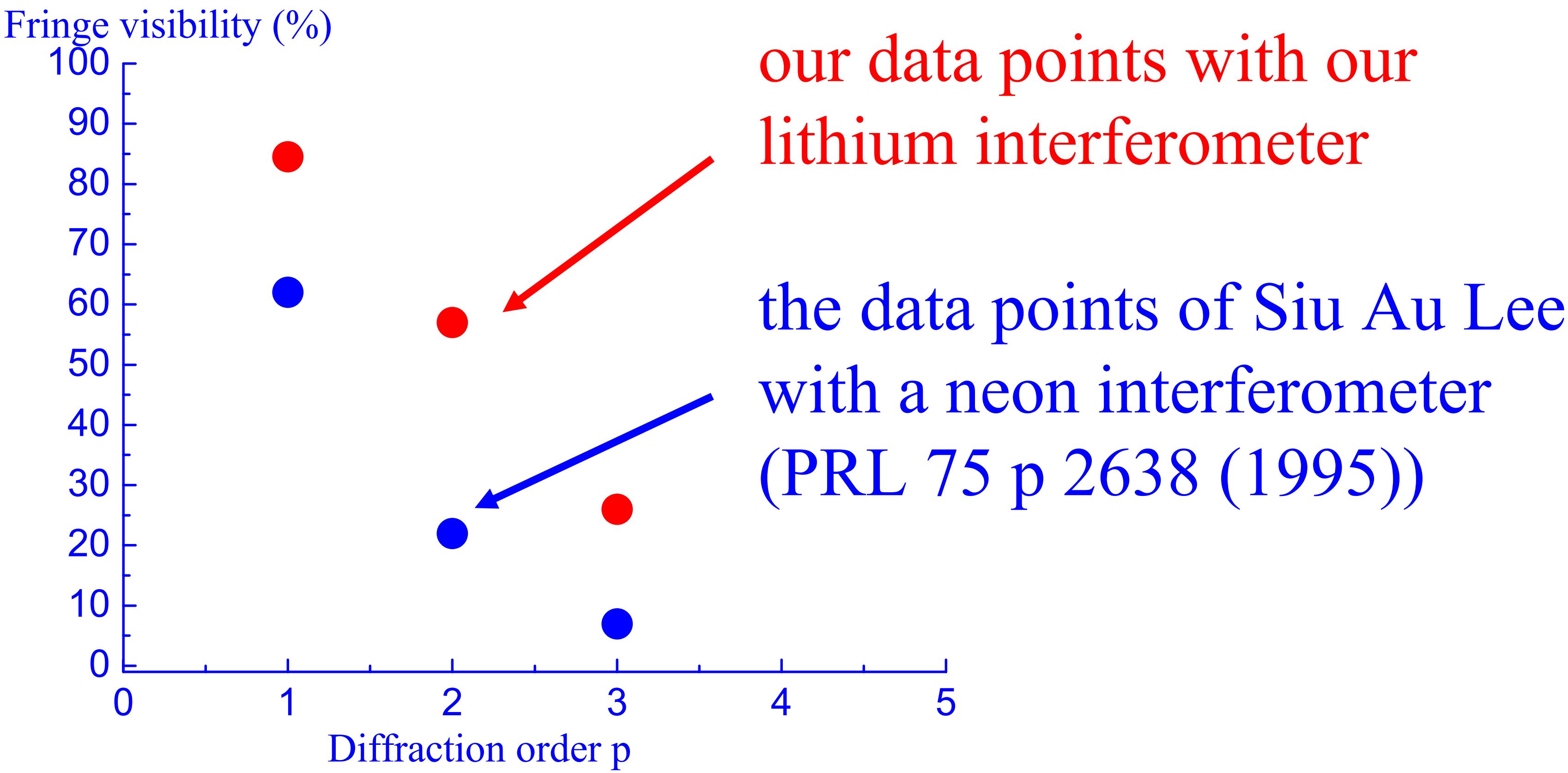
The de Broglie wavelength $\lambda_{dB} = h / (m\ v)$ is very small

→very sensitive measurements

The accuracy on a phase measurement increases with the flux I_0 and the fringe visibility V

$$\Delta\Phi_{min} \propto 1 / \sqrt{(I_0\ V^2)}$$

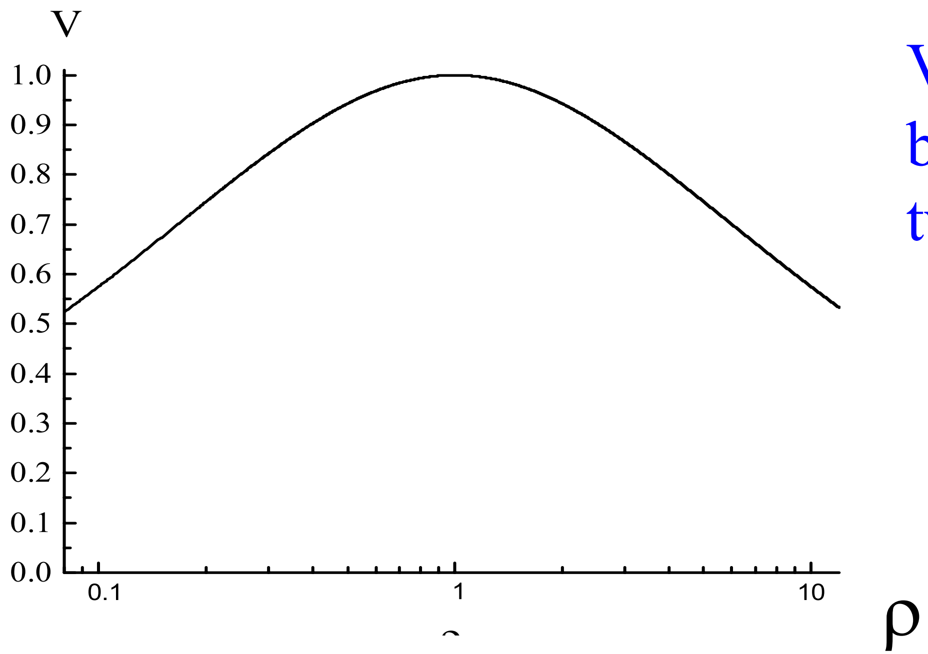
Fringe visibility as a function of the diffraction order p



Decrease of the fringe visibility:

- either an intensity mismatch
- or a phase averaging effect.

-- an intensity mismatch between the interfering beams



Visibility V as a function of the beam intensity ratio ρ for two-beam interference fringes.

-- a phase averaging effect

a phase noise $\Delta\phi$ with a Gaussian distribution

$$V = V_{max} \exp(- \langle\Delta\phi^2\rangle/2)$$

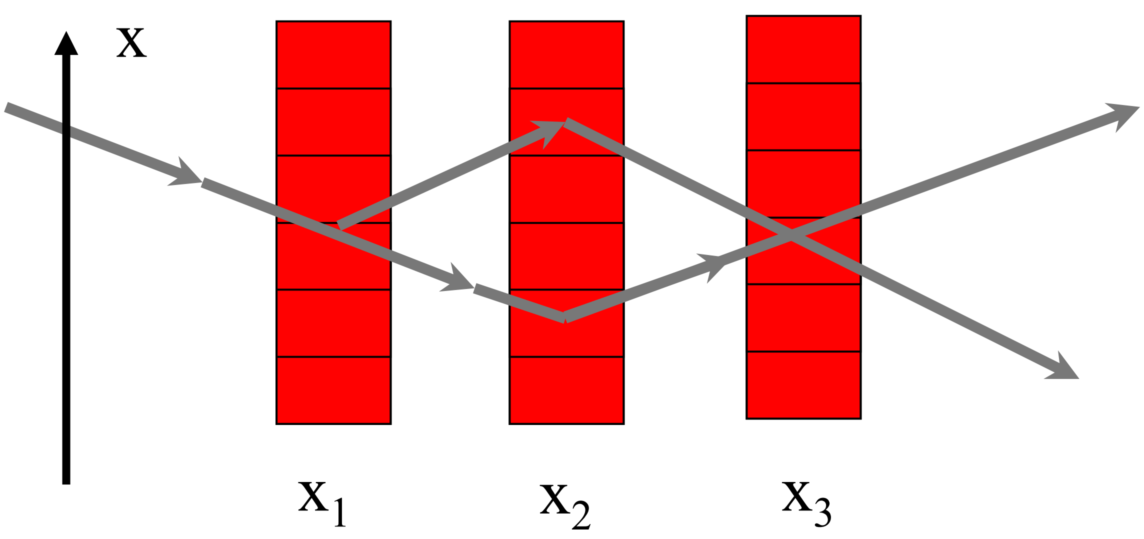
Inertial sensitivity of atom interferometers

applications by S. Chu (measurement of g), by M. Kasevich (gradient of g and gyrometer), by G. Tino (measurement of G) .

This sensitivity is due to a phase term dependent on the grating positions

$$\phi = p\, k_G\, [x_1 + x_3 - 2\, x_2]$$

p is the diffraction order.



If the gratings are moving with respect to a Galilean frame,

$$x_i \rightarrow x_i(t_i)$$

where t_i is the time at which a given atom crosses grating G_i

$$\phi = p\, k_G\, [x_1(t_1) + x_3\,(t_3) - 2\, x_2\,(t_2)]$$

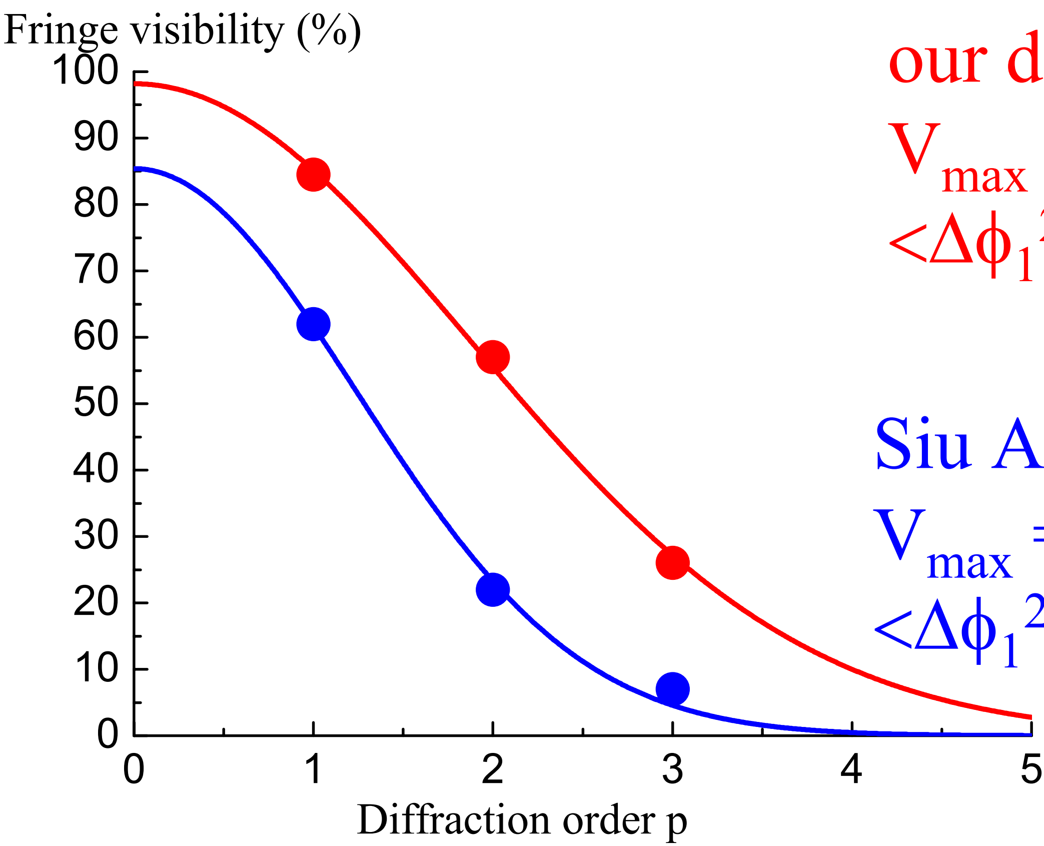
→ phase noise $\Delta\phi_p$ with $\Delta\phi_p = p\,\Delta\phi_1$

$$V = V_{max}\, \exp(-\ p^2 <\Delta\phi_1{}^2>/2)$$

→ Gaussian dependence of the visibility with the diffraction order p.

Fit of the data with

$$V = V_{max} \exp(- \; p^2 <\Delta\phi_1{}^2>/2)$$



our data points

$V_{max} = 98 \pm 1$ %

$<\Delta\phi_1{}^2> = 0.286 \pm 0.008$ rad^2

Siu Au Lee's data points

$V_{max} = 85 \pm 2$ %

$<\Delta\phi_1{}^2> = 0.650 \pm 0.074$ rad^2

Expansion of the inertial phase term in powers of the atom time of flight $T = L/u$

L intergrating distance; u atom velocity

$$\phi = p\, k_G\, [x_1(t-T) + x_3\,(t+T) - 2\, x_2\,(t)]$$

$$\phi = \phi_{bending} + \phi_{Sagnac} + \phi_{acceleration}$$

T^0 $\quad \phi_{bending} = p\, k_G\, [x_1(t) + x_3\,(t) - 2\, x_2\,(t)]$

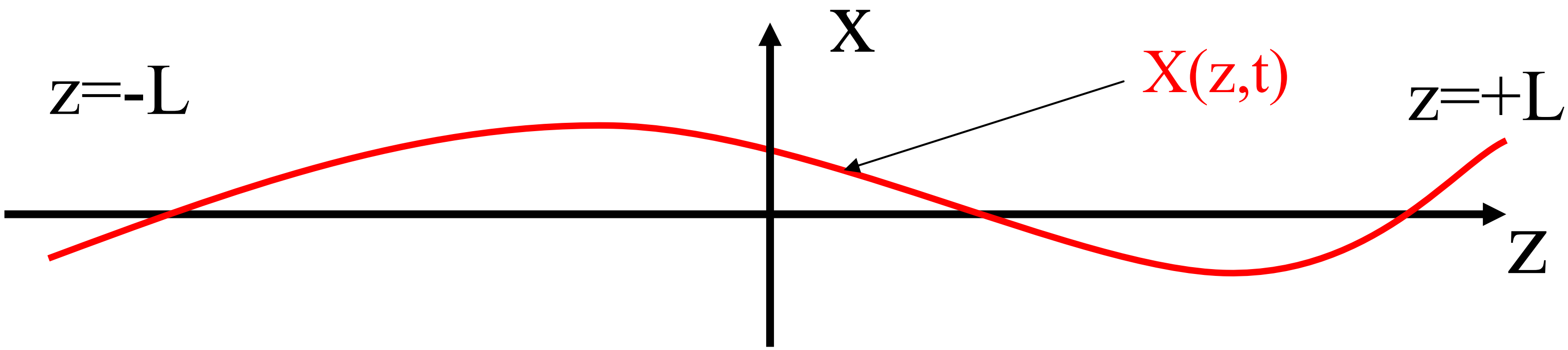
T^1 $\quad \phi_{Sagnac} = p\, k_G\, (v_{3x} - v_{1x})\, T$

T^2 $\quad \phi_{acceleration} = p\, k_G\, (a_{1x} + a_{3x})\, T^2/2$

Estimation of the phase noise from laboratory seismic noise

Model calculation of the rail supporting the three mirrors

→ rail treated as a beam of constant section with a neutral line X(z,t)



elasticity theory →

$$\rho A \frac{\partial^2 X}{\partial t^2} = -E I_y \frac{\partial^4 X}{\partial z^4}$$

ρ: density of the beam material, A: area of the beam cross-section,
E: Young's modulus of the material, $I_y = \int x^2\, dx\, dy$

The forces and torques at the two ends $\varepsilon = \pm 1$ of the beam are related to the derivatives of X(z,

$$F_{x\epsilon} = -\epsilon E I_y \frac{\partial^3 X}{\partial z^3}(z = \epsilon L)$$

$$M_{y\epsilon} = \epsilon E I_y \frac{\partial^2 X}{\partial z^2}(z = \epsilon L)$$

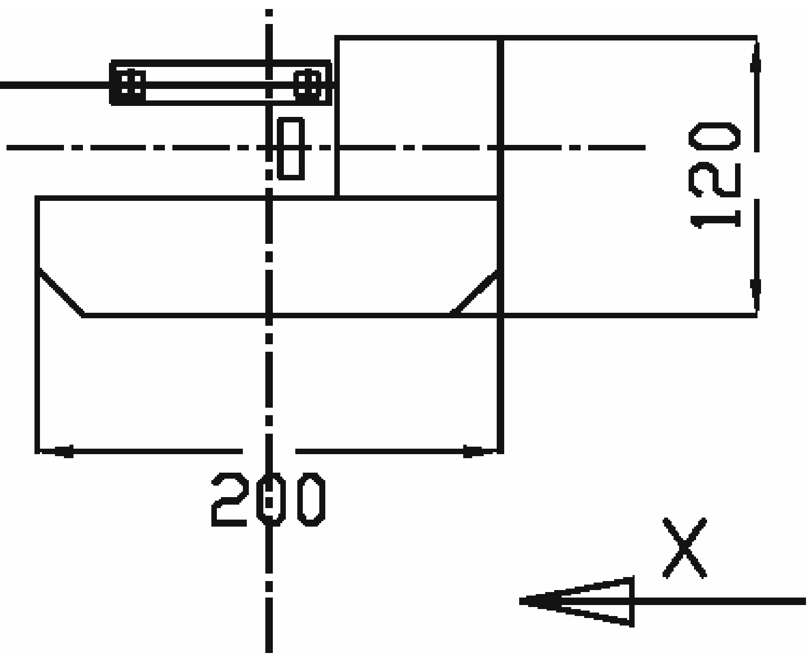
We assume that $M_{y\varepsilon} = 0$ and that the forces are the sum of an elastic term and a damping term

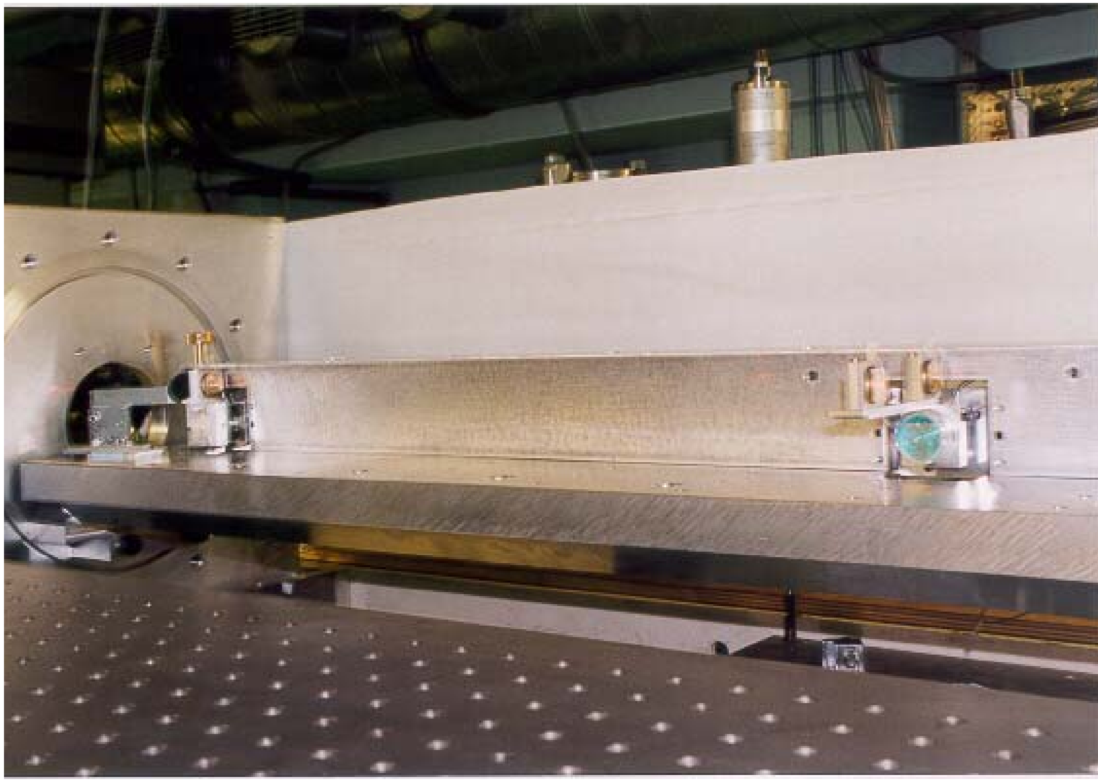
$$F_{x\epsilon} = -K_\epsilon \left[X(\epsilon L, t) - x_\epsilon(t)\right] - \mu_\epsilon \frac{\partial \left[X(\epsilon L, t) - x_\epsilon(t)\right]}{\partial t}$$

$x_\varepsilon(t)$ is the position of the support at the end ε at time t.

2 low frequency resonances (oscillation of the rail almost like a solid)
a series of high frequency resonances (flexion of the rail)

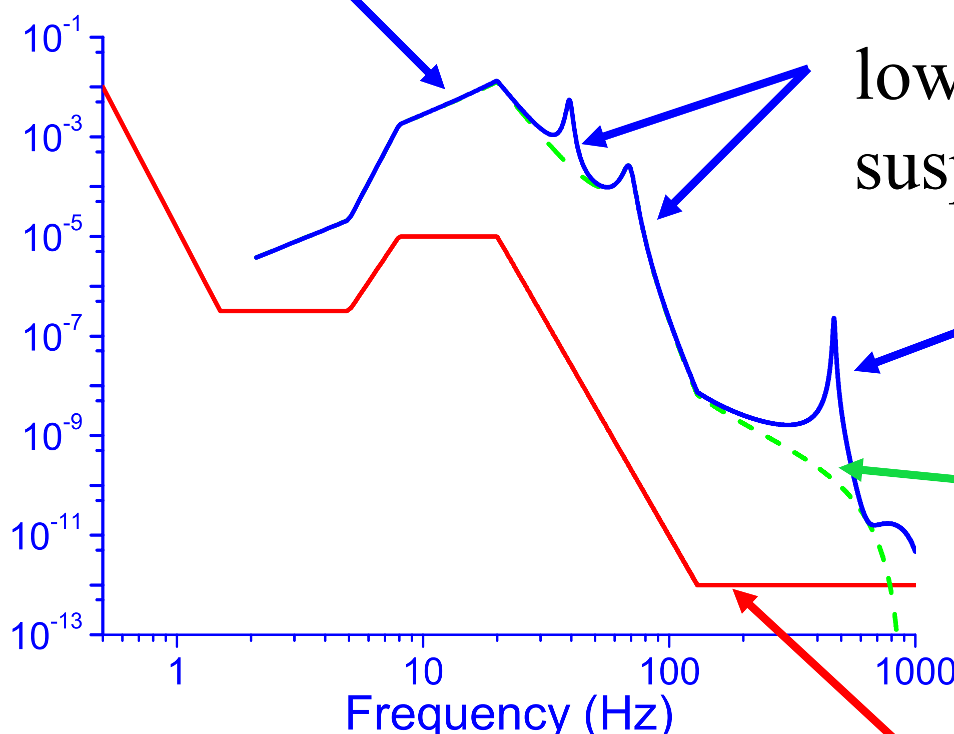
The rail of our interferometer:





- very stiff rail with a first flexion resonance at ν =460 Hz
- simple suspension on rubber blocks with resonances in the 40 - 60 Hz range.

calculated phase noise spectrum $|\phi(\nu)|^2$
in rad^2/Hz for diffraction order p=1



low frequency
suspension resonances

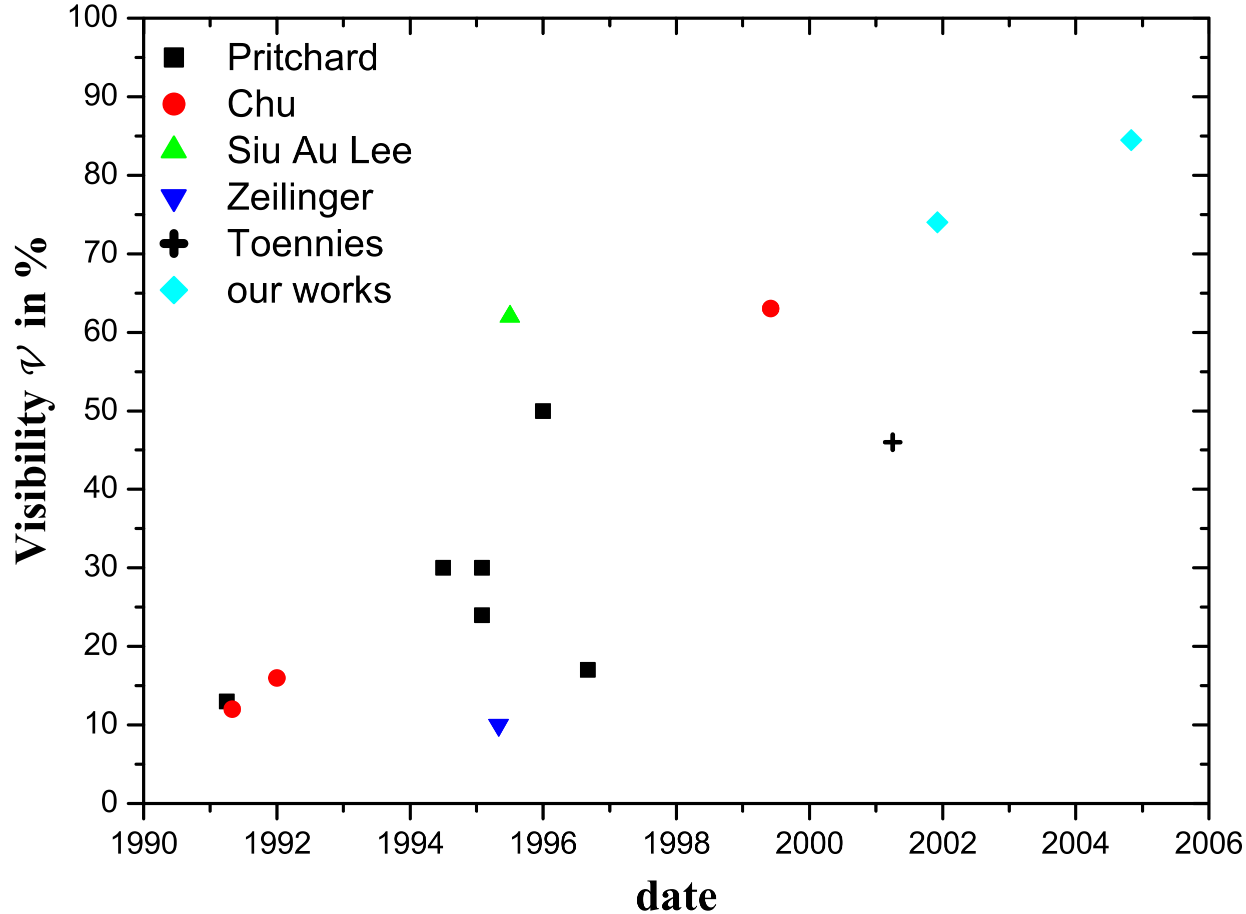
first flexion resonance

calculated Sagnac only
phase noise spectrum
$|\phi_{Sagnac}(\nu)|^2$ in rad^2/Hz

approximate spectrum of the
seismic noise of the support
$|x_\varepsilon(\nu)|^2$ (in 10^{-10} m^2/Hz)

calculated phase noise $\langle\Delta\phi_1^{\,2}\rangle = 0.16$ rad^2
(measured value from visibility data $\langle\Delta\phi_1^{\,2}\rangle = 0.286 \pm 0.008$ rad^2)

Fringe visibility in Mach-Zehnder atom interferometers as a function of publication date



Conclusion

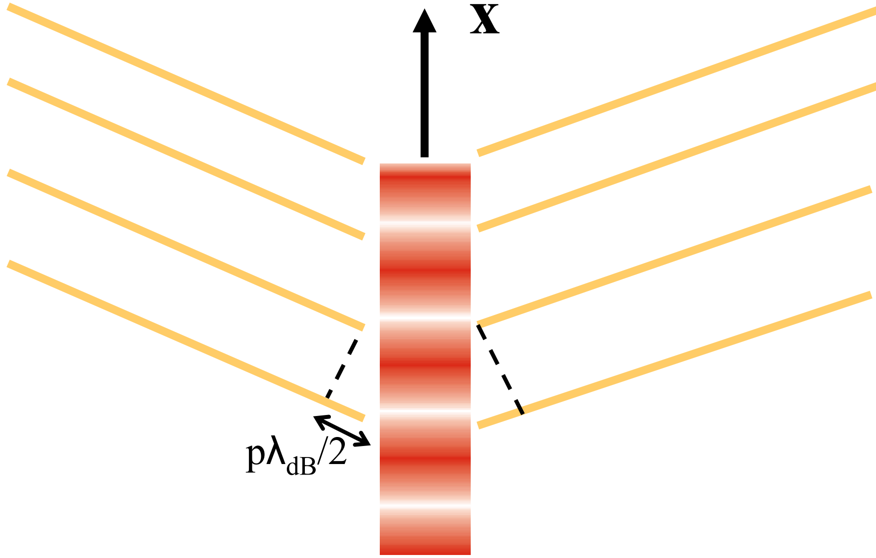
- the existence of an important phase noise due to vibrations in our atom interferometer.
- a large reduction of the fringe visibility.
- With a very stiff rail, the dominant noise term is due to Sagnac effect. Need for a better rail suspension, with low resonance frequencies.

- With a reduced phase noise, atom interference fringes with a high visibility should be observed:

a) with higher diffraction orders p $\rightarrow$ larger separation of the atomic beams

b) with slower atomic beams $\rightarrow$ the time of flight $T=L/u$ increases when the velocity u decreases (Sagnac phase term $\propto T$ and acceleration phase term $\propto T^2$).

All my thanks!

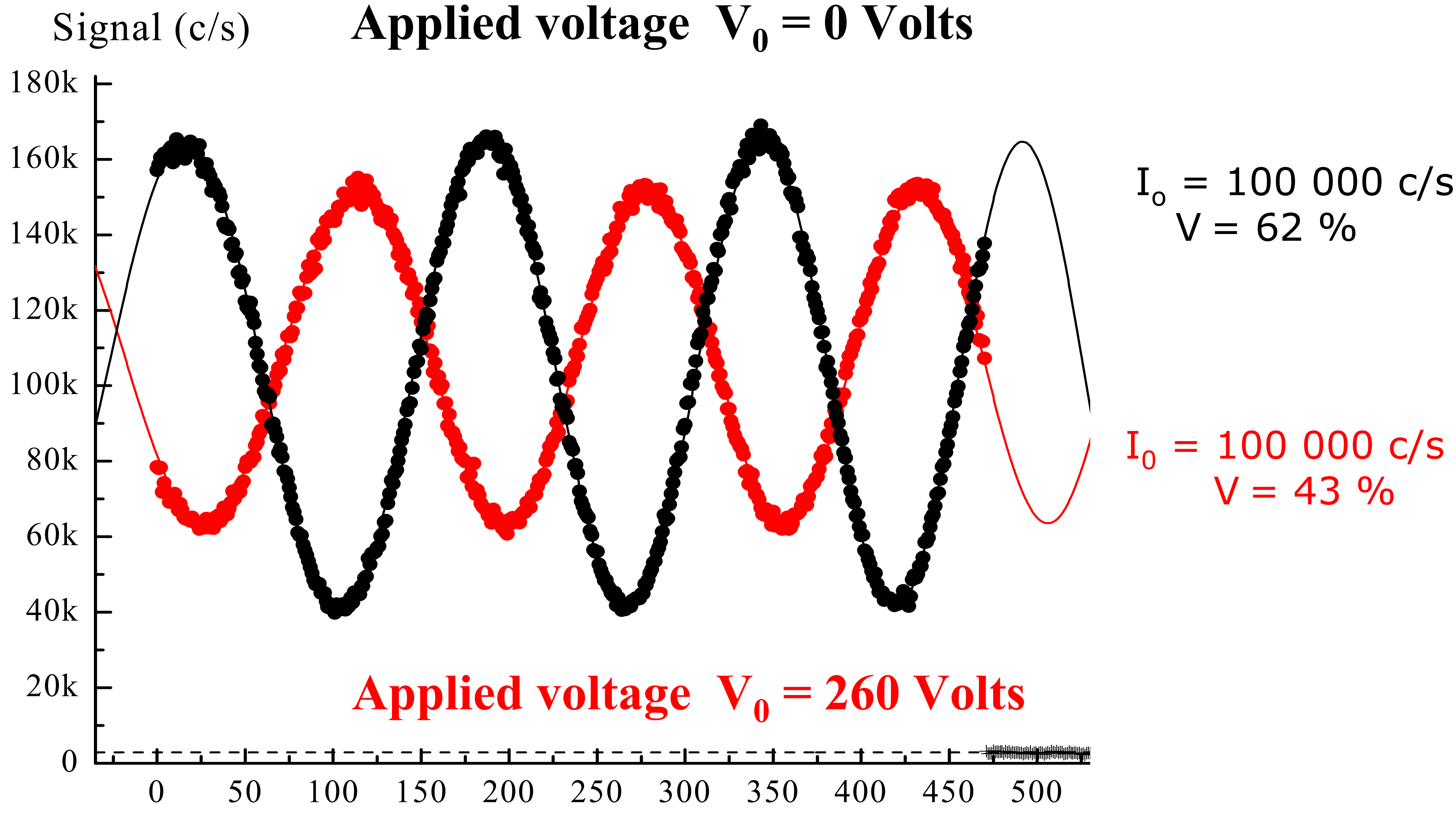


$$\phi = p\, k_G\, (x_1 + x_3 - 2x_2)$$

Main advantage: this non dispersive phase is useful to observe interference fringes

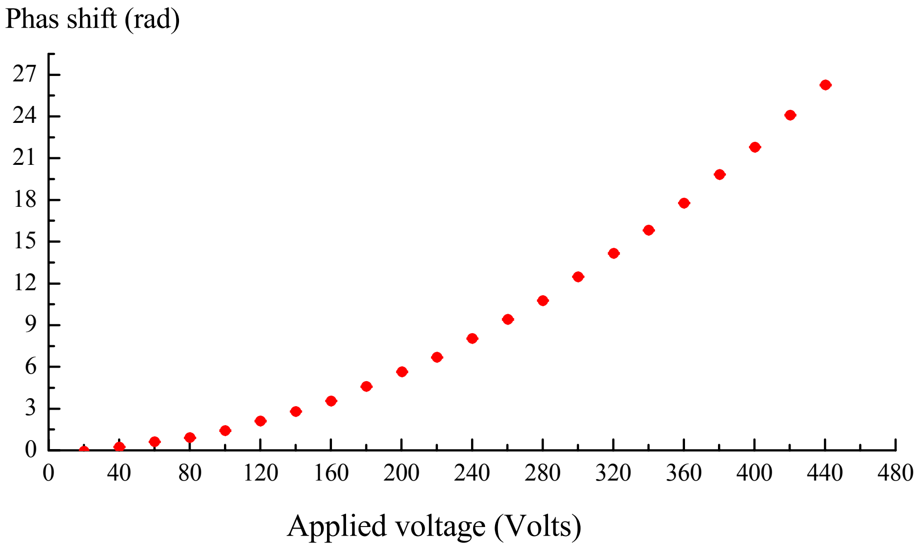
Main problem: a high stability of the grating positions is needed (for example: in our experiment, a 1 radians phase shift corresponds to a variation of x_1 or x_3 of 53 nm)

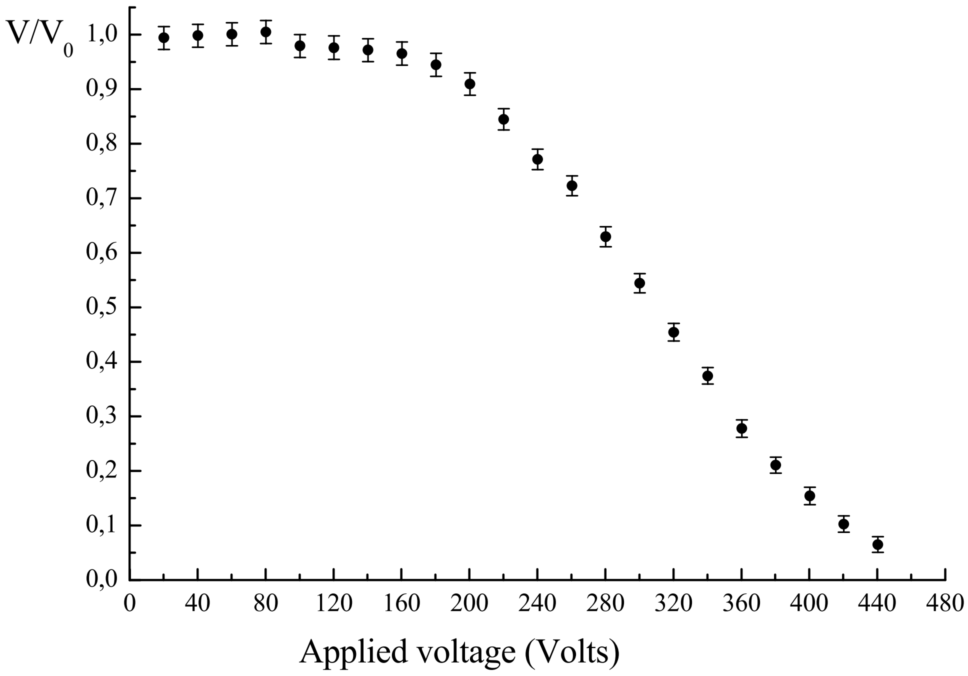
Phase shift induced by the electric field

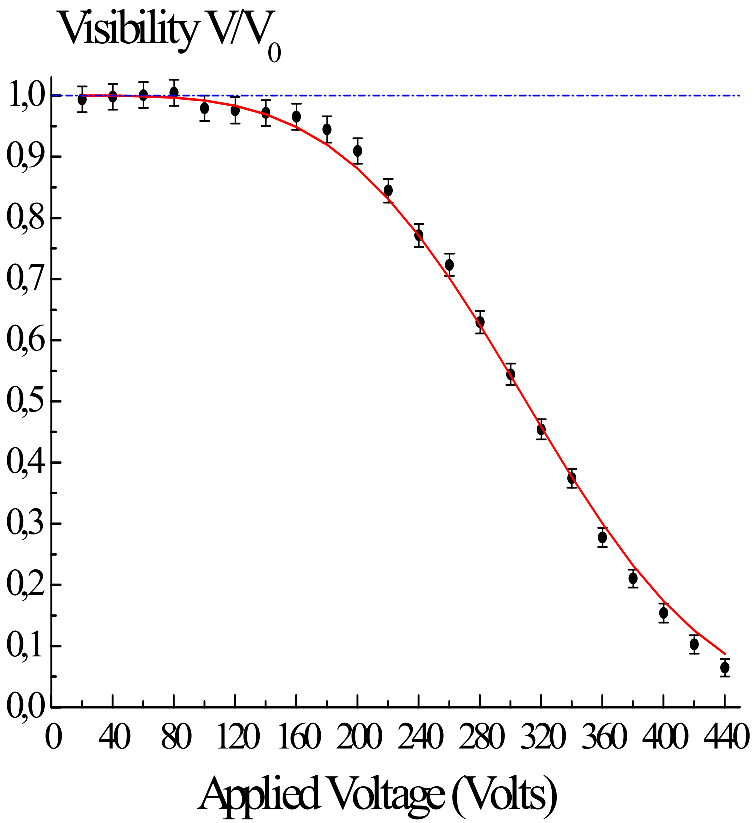


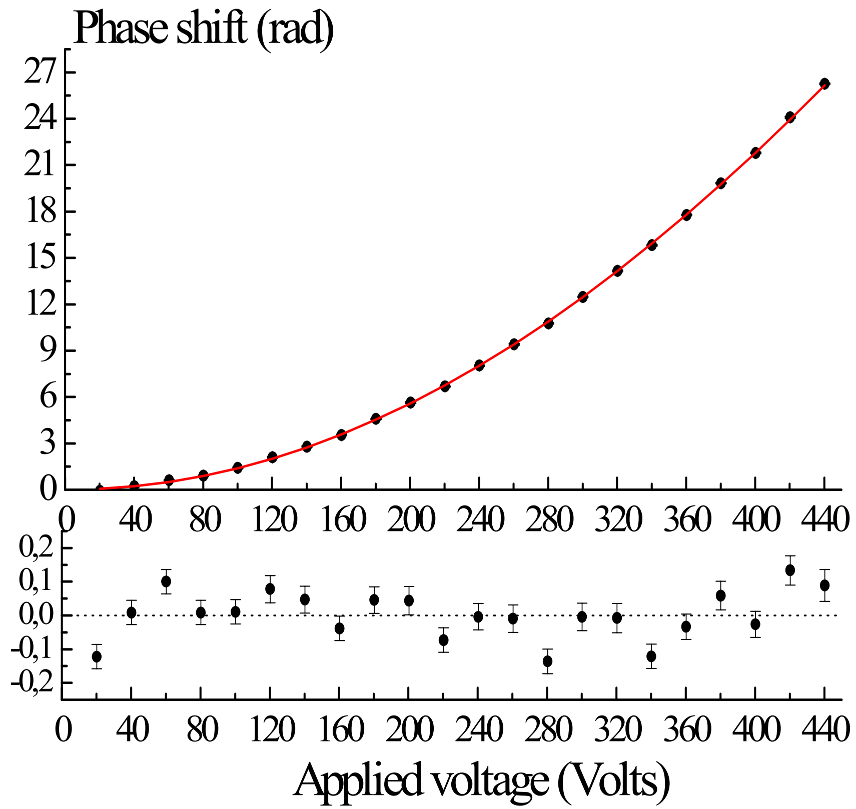
counting time 0.36 s per data point

Phase shift and visibility reduction due to the electric field









$$\begin{cases} \Phi_m/V_0^2 = (1{,}3870 \pm 0{,}0010) \times 10^{-4}\ \mathrm{rad.V^{-2}} \\ S_{//} = 8{,}00 \pm 0{,}06 \end{cases}$$

Lithium electric polarizability values

Experiment or calculation	**Result** 10^{-30} m^3	**Result atomic units**
B. Bederson et al. (experiment 1974)	24.3 ± 0.5	163.98 ± 3.4
Our experiment (2005)	24.33 ± 0.16	164.2 ± 1.1
Kassimi and Thakkar, Hartree-Fock calculation (1994)*		169.946
Kassimi and Thakkar, extrapolated value from MP2,MP3 et MP4 calculations (1994)*		164.2 ± 0.1
Drake et al., Hylleraas calculation (1996) #		164.111 ± 0.002

* Phys.Rev. A **50**, 2948 (1994) # Phys.Rev. A **54**, 2824 (1996)