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REQUIREMENTS FOR MEASURING THE GRAVITATIONAL TIME DELAY BETWEEN DRAG-FREE SPACECRAFT

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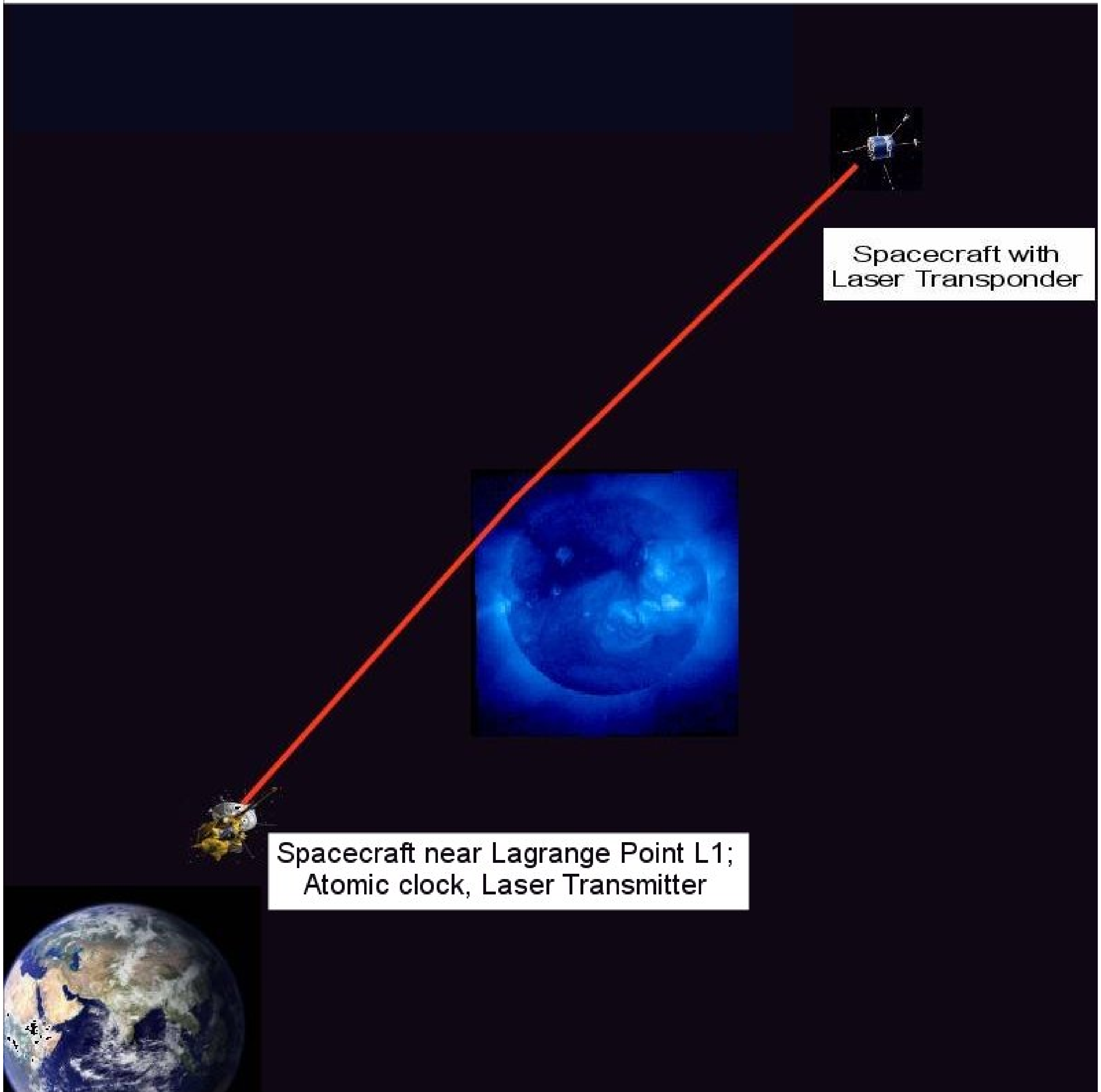
Gravitational Time Delay Measurement



Shapiro Gravitational Time Delay

1. The one-way extra time delay from a spacecraft to the point P nearest to the Sun along the line of sight, to first order, is:

$$\Delta\tau = \frac{GM}{C^3}\left\{(1+\gamma)\ln\left[\frac{r+\sqrt{r^2-b^2}}{b}\right]+\frac{\sqrt{r-b}}{\sqrt{r+b}}\right\}$$

2. Here: r = distance from spacecraft to P
   b = distance from P to the center of the Sun
   γ = PPN "space curvature" parameter
   M = mass of the Sun

3. For r_1 = 1 AU, r_2 = 1.2 AU, b = 1.4 solar radii, the total round-trip travel time is about 2200 s and the extra gravitational time delay is about 250 microseconds.

4. The present accuracy for γ is 2.3×10^{-5}, from the Cassini Mission measurement:
   B. Bertotti, L. Iess and P. Tortora, Nature **425**, 374-376 (2003).

Proposed Mission Design

1. Measurements from "near" spacecraft orbiting the L-1 point, 1.5×10^6 km from Earth.

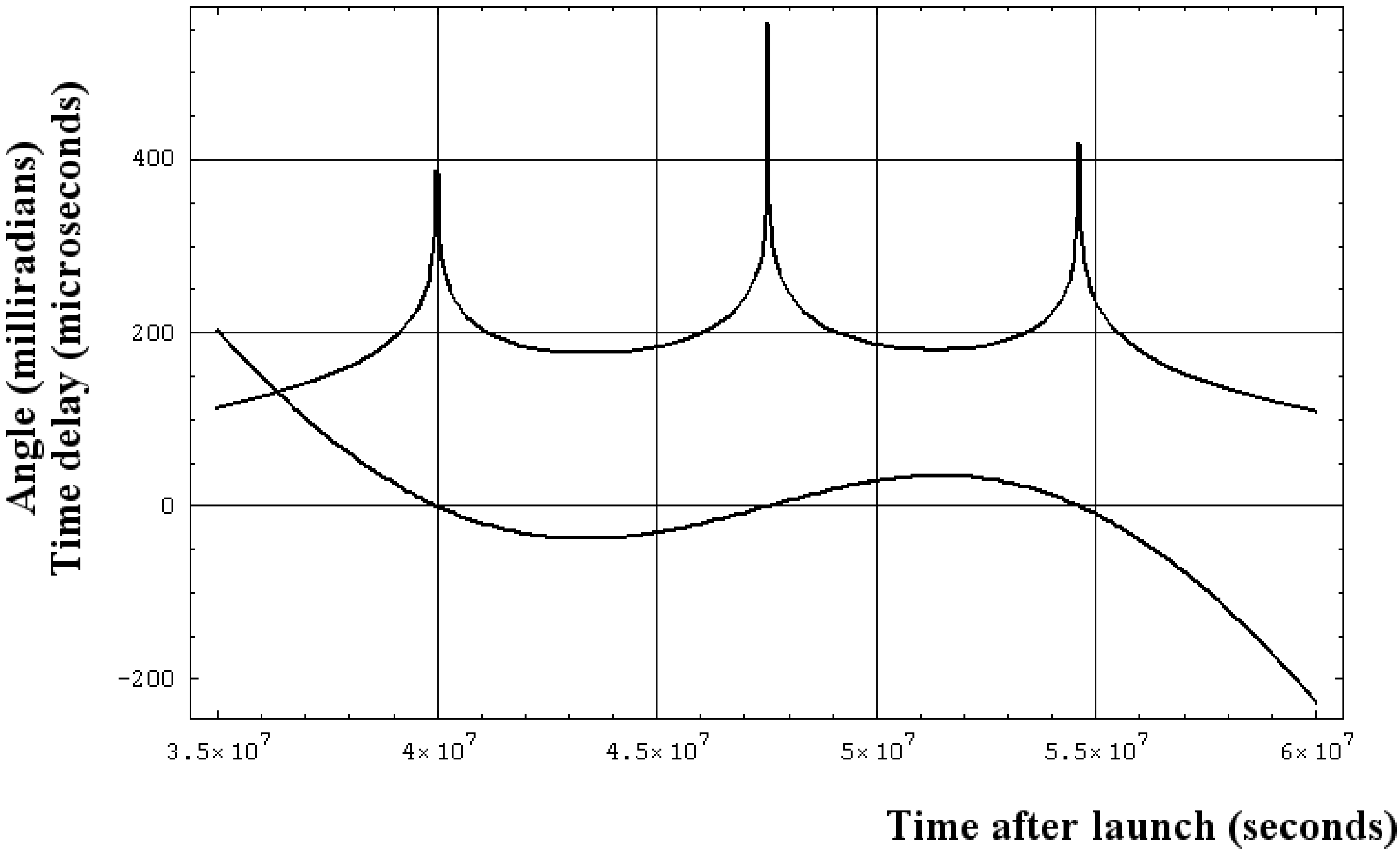
2. Two-way measurements to "far" spacecraft on same orbit as proposed for the LATOR mission: 1.5 year period.

3. Three conjunctions with the Sun: about 15, 18, and 21 months after launch.

4. Rate of change of b is 0.7 solar radii/day for 1st and 3rd conjunction and 0.2 solar radii/day for the 2nd conjunction.

5. Most accurate time delay measurements are over 20 day periods around 1st and 3rd conjunction.

6. Nearly continuous measurements for at least 40 days are required for accurate orbit determination.


Angle (milliradians)
Time delay (microseconds)
400
200
0
-200
3.5×10^7
4×10^7
4.5×10^7
5×10^7
5.5×10^7
6×10^7
Time after launch (seconds)

Required Technology Improvement

1. Spaceborne clock with $1 \times 10^{-13}/\sqrt{\text{Hz}}$ frequency stability down to 0.4 microhertz.

2. Drag-free spacecraft with less than 1.3×10^{-13} $\text{m/s}^2/\sqrt{\text{Hz}}$ spurious acceleration near 0.4 microhertz.

3. Round-trip transfer of timing information between distant spacecraft with 0.5 picosecond accuracy.

4. Accurate time delay measurements for lines of sight down to 0.1 degree from the limb of the Sun.

5. High accuracy orbit determination from spacecraft to spacecraft distance measurements plus ground tracking.

Spacecraft Clock Noise

1. $1 \times 10^{-13}/\sqrt{\text{Hz}}$ fractional frequency stability down to 0.4 microhertz is assumed.

2. For a single time delay measurement over about 2200 seconds, the clock jitter will limit the accuracy to about 3 ps. However, averaging over six hours will strongly reduce this timing error.

3. The main limitation is the variations in the clock frequency over the whole measurement time.

4. Little of the uncertainty in γ comes from clock noise at frequencies above 3 microhertz.

5. If accurate time transfer from the ground to the L-1 spacecraft can be added, it may be possible to reduce the error due to the L-1 clock.

Idealized Gravitational Time Delay Mission

1. Only white frequency noise in the L-1 clock is assumed.

2. The time delay is approximated by

$$g(t) = -B[\ \ln|Rt| - M\ ],\quad t_1<t<t_2.$$

3. $B = 3.8 \times 10^{-5}$ s
   $R = 0.7$ solar radii/day
   $M = \langle \ln|Rt| \rangle$
   $t_1 = 2$ days, $t_2 = 10$ days

4. $$(S/N)^2 = \frac{2}{n^2(f)} \int_0^{\infty} [g(f)]^2 df$$

5. Only about 5% of the integral of $[g(f)]^2$ comes from frequencies below 0.4 microhertz.

6. The uncertainty in γ from the assumed clock noise is 0.7×10^{-8}.

Drag-Free Spacecraft

1. The required performance builds on that planned for the LISA gravitational wave mission: $\sqrt{S_a} < a_{LISA}$ from 0.1 mHz to 1 Hz, with $a_{LISA} = 3 \times 10^{-15}$ m/s^2/$\sqrt{\text{Hz}}$. S_a is the power spectral density of the acceleration noise.

2. Requirements for extension from 0.1 to 0.01 mHz with $\sqrt{S_a} < a_{LISA} \times [0.1 \text{ mHz}/f]^{0.5}$ have been discussed: P. L. Bender, Proc. 6th Int. LISA Symp., June 2006, submitted.

3. Further extension to 4×10^{-7} Hz with $\sqrt{S_a} < a_{LISA} \times \sqrt{10} \times [0.01 \text{ mHz}/f]$ appears to be feasible.

4. The main challenge is the rapidly changing spacecraft temperature during the 20 day main observing period for the 1st and 3rd conjunction.

5. Without active temperature control, the temperature inside the far spacecraft would change by about 8 K during the 20 day period.

Round-Trip Transfer of Timing Information

1. The required accuracy is sub-picosecond, so timing of short laser pulses would be difficult.

2. An alternate approach is to use a cw laser, put 30 to 60 GHz sidetones on it, and use the phase of the beat frequency between the sidetones for the timing information.

3. For a 60 GHz beat frequency, with 16.7 ps period, a 5 deg phase error for a one-way measurement corresponds to a 0.23 ps timing error.

4. The main challenge is systematic measurement errors that drift with time. For the 1st and 3rd conjunction, the gravitational time delay increases by 6.4×10^{-5} s from 10 to 2 days before conjunction, and then decreases by the same amount afterwards.

5. A worst case drift of 0.23 ps in the round-trip timing error before conjunction and an opposite drift afterwards would give an error contribution of 0.7×10^{-8} for γ.

Signal Acquisition and Measurement Near the Sun

1. Two days from conjunction, the line of sight is 0.1 deg from the solar limb. Acquiring the signal at this time requires good knowledge of the spacecraft attitude.

2. Separate transmit and receive telescopes are planned, in order to reduce problems with scattered light from the transmitted beam.

3. Telescope diameters of 100 to 200 nm appear to be adequate with 0.5 W of laser power.

4. Doppler shifts of about 5 GHz for the carrier have to be allowed for by offsetting the local oscillator carrier by about this amount.

5. The Doppler shifts for the two sidebands will be the same to within about 2 MHz. Thus the beat of each received sideband with respect to its local oscillator sideband can be observed with the same photo-detector.

High-Accuracy Orbit Determination

1. The problem of determining the orbits for both spacecraft with sufficient accuracy has not yet been investigated.

2. Good performance of the drag-free system is required even down to 2×10^{-7} Hz.

3. Measurements of changes in the spacecraft-spacecraft distance will be supplemented by microwave tracking from Earth.

4. The main issue is errors that drift in one direction before conjunction and then in the opposite direction afterward.

5. The sensitivity to out-of-place errors in the orbits is a possible concern.

Conclusions

1. A mission to determine the gravitational time delay between drag-free spacecraft appears to be feasible.

2. However, the orbit determination part of the problem has not yet been investigated.

3. An error budget for determining γ might have equal entries of 1×10^{-8} each for the clock noise, the drag-free systems performance, the timing transfer process, and orbit determination.

4. Thus an accuracy of 2×10^{-8} for γ appears to be a reasonable goal for this kind of mission.