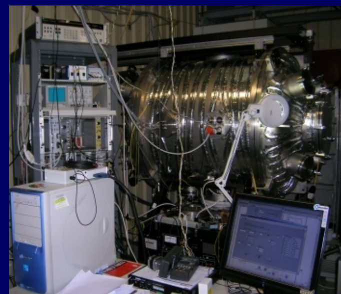
GG/GGG Webpage: http://eotvos.dm.unipi.it/nobili

GG space experiment to test the Equivalence Principle to 10^{-17}. Design, error budget and relevance of experimental results with GGG laboratory prototype

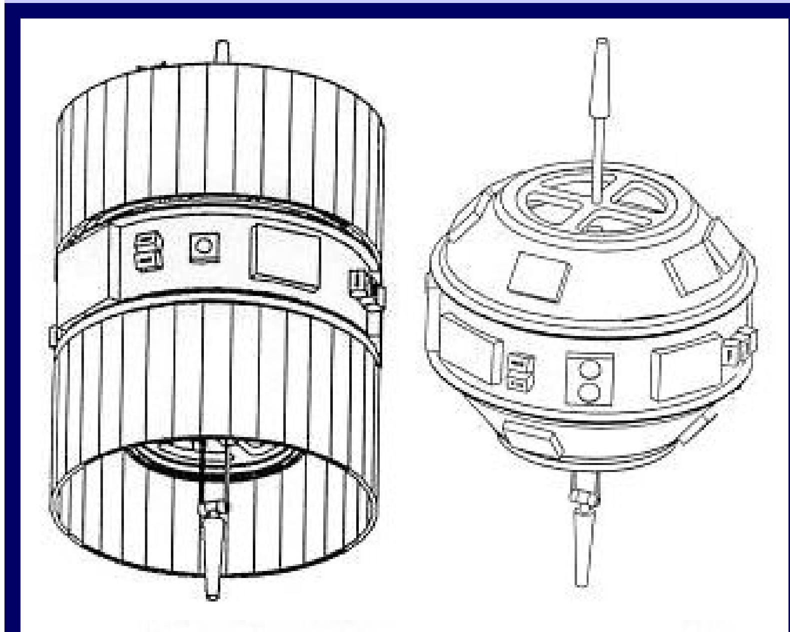


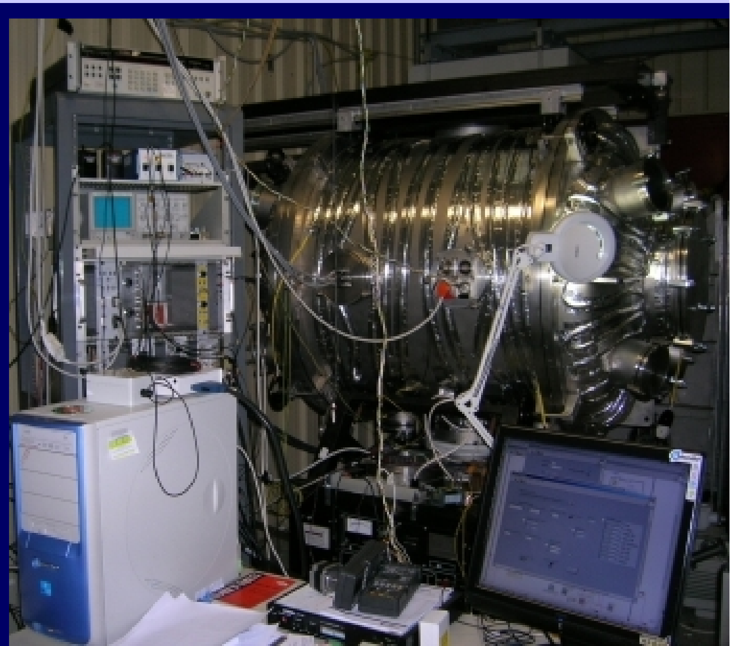
Anna Nobili, University of Pisa & INFN, for the GG collaboration

International Workshop "Advances in precision tests and experimental gravitation in space"

September 28-30 2006, Arcetri Italia

GG/GGG





GG satellite included in National Space Plan of Italian Space Agency (ASI) for the next 3 years

GGG lab prototype funded by INFN (Istituto Nazionale di Fisica Nucleare) + Indian collaboration

GG/GGG Webpage: http://eotvos.dm.unipi.it/nobili

Dynamical response of the “GGG” rotor to test the Equivalence Principle: theory, simulation and experiment.
Part I: The normal modes , Comandi et al. RSI, **77** 034501 (2006)
Part II: The rejection of common mode forces , Comandi et al. RSI, **77** 034502 (2006)

“Test of the Equivalence Principle with macroscopic bodies in rapid rotation: current sensitivity and relevance for a high accuray test in space” Nobili et al. to be submitetd to IJMPD

“Limitations to testing the Equivalence Principle with Satellite Laser Ranging” Nobili et al. to be submitetd to PRD

Guidelines for testing the equivalence principle in LEO

- Experimental consequence of EP is the UFF:

$$m_g = m_i \Rightarrow \eta \equiv \frac{\Delta a}{a} = 0$$

Δa relative (differential) acceleration between 2 bodies falling in the 1/r gravitational field of the Earth

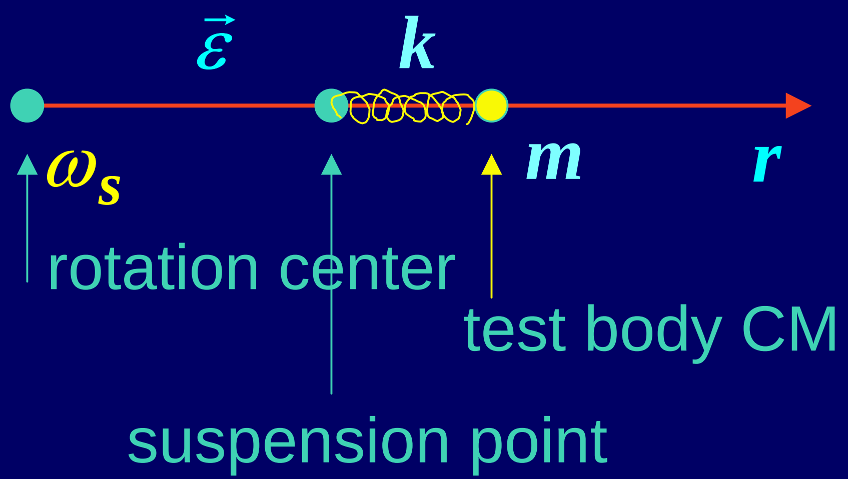
Each test body is in a 2-body motion around the Earth, but ONLY the effects of differential accelerations between them do matter to test UFF. If the two bodies are weakly coupled inside a spacecraft, these effects can be measured in situ far more accurately then by measuring their orbits from Earth

Δa very very small ⬇

- Coupling should be as weak as possible (to increase sensitivity to differential accelerations)
- Signal should be modulated at frequency as high as possible (to reduce “1/f” electronic noise)
- Test masses should (possibly…) be large (for low thermal noise even at room temperature)

There is no “free” test mass in these precision experiments…

Whatever the kind of suspension:



$\vec{\varepsilon}$ offset by construction

$$\omega_n = \sqrt{k/m} \quad , \quad \omega_s$$

$$\vec{r}_{eq} = \frac{1}{1-(\omega_s/\omega_n)^2} \cdot \vec{\varepsilon}$$

$\omega_s < \omega_n$ sub-critical rotation $r_{eq} > \varepsilon$

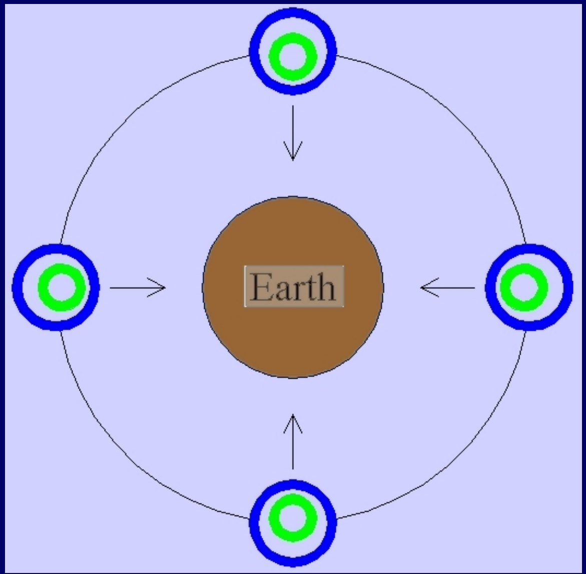
$\omega_s > \omega_n$ <u>super-critical rotation</u> $r_{eq} < \varepsilon$

$$\omega_s^2 \gg \omega_n^2 \quad , \quad \vec{r}_{eq} \simeq -\vec{\varepsilon}\,\frac{\omega_n^2}{\omega_s^2}$$

fast rotation and consequent self-centering require 2-D motion !!!!

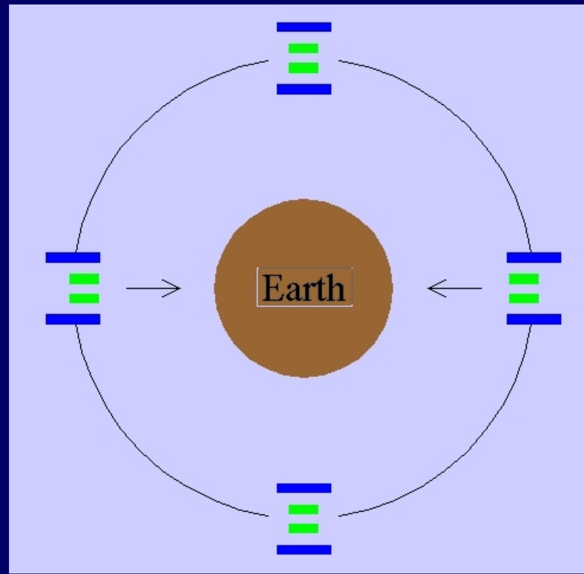
GG: the space experiment design

GG



2D
coupling of
test cylinders

STEP/μSCOPE



1D
coupling of
test cylinders

both must spin ⊥ to the orbit plane

EP signal of constant amplitude modulated by the rotating capacitors in between the test cylinders at

$$\omega_{spin} - \omega_{orb}$$

$$\Downarrow$$

$$\omega_{spin} - \omega_{orb} > \omega_{oscillator}$$

does not attenuate the signal

EP signal forces the oscillator at

$$\omega_{spin} - \omega_{orb}$$

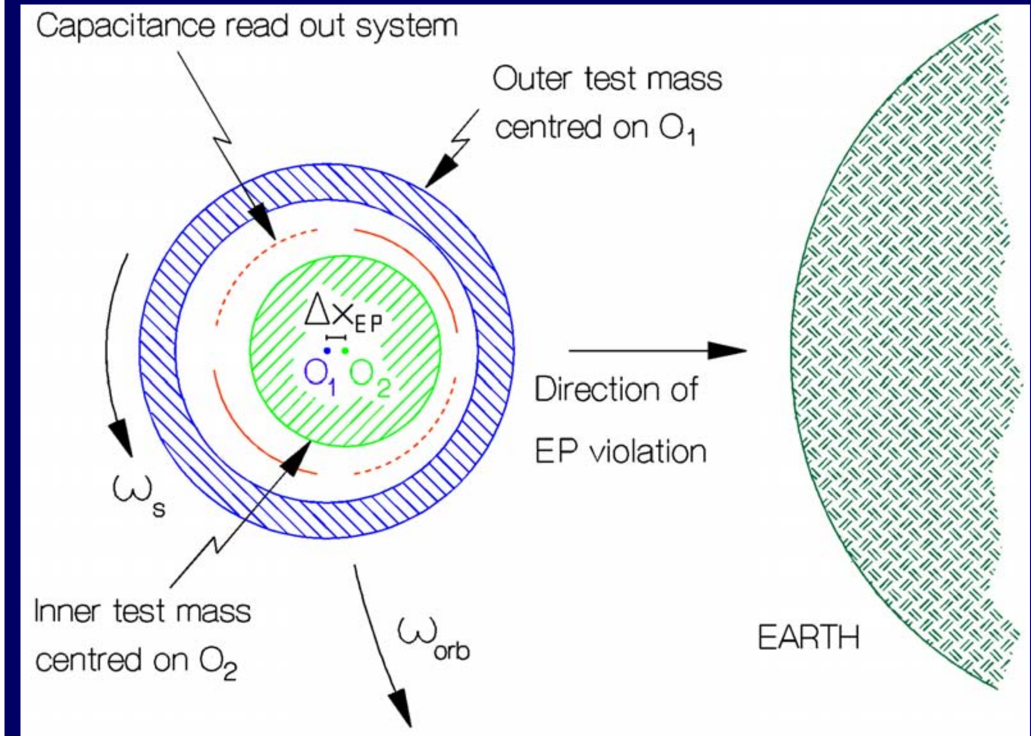
$$\Downarrow$$

$$\omega_{spin} - \omega_{orb} < \omega_{oscillator}$$

In ordernot to attenuate the forcing signal to be measured

GG: Signal modulation at <u>supercritical</u> spin frequency + passive stabilization of s/c attitude by 1-axis rotation

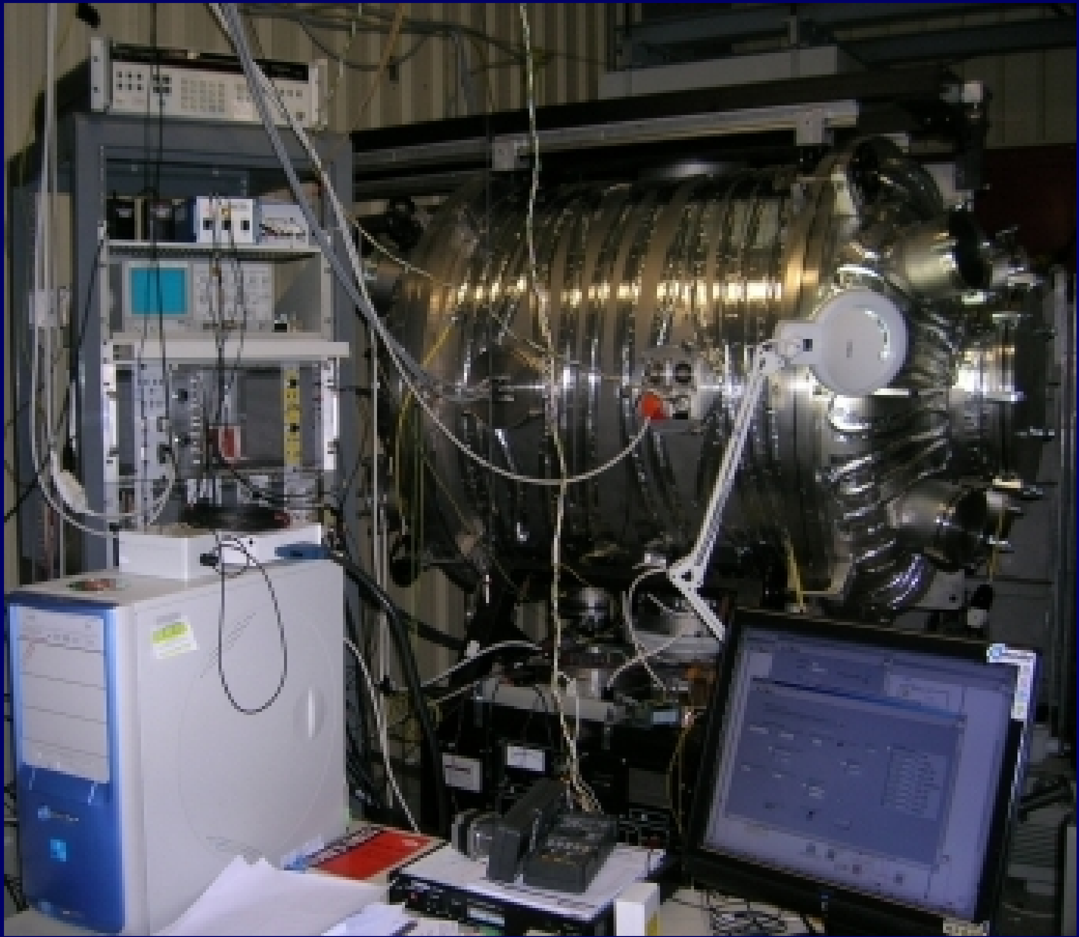
GG signal modulation concept

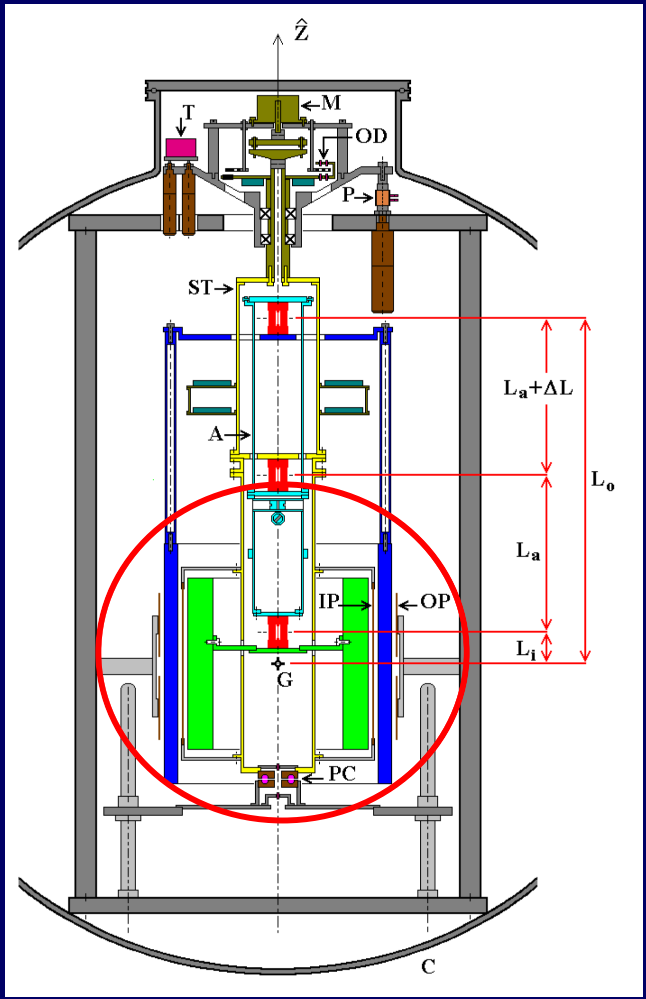


Section of the GG coaxial test cylinders and capacitance sensors in the plane perpendicular to the spin axis. They spin at angular velocity ω_s while orbiting around the Earth at angular velocity ω_{orb}. The capacitance plates of the read-out are shown in between the test bodies, in the case in which the centers of mass of the test bodies are displaced from one another by a vector due to an Equivalence Principle violation in the gravitational field of the Earth.

Under the (differential) effect of this new force the test masses, which are weakly coupled by mechanical suspensions, reach equilibrium at a displaced position where the new force is balanced by the weak restoring force of the suspension, while the bodies rotate independently around O_1 and O_2 respectively. The vector of this relative displacement has <u>constant amplitude</u> (for zero orbital eccentricity) and points to the center of the Earth. The signal is therefore <u>modulated</u> by the capacitors at their spinning frequency with respect to the center of the Earth.

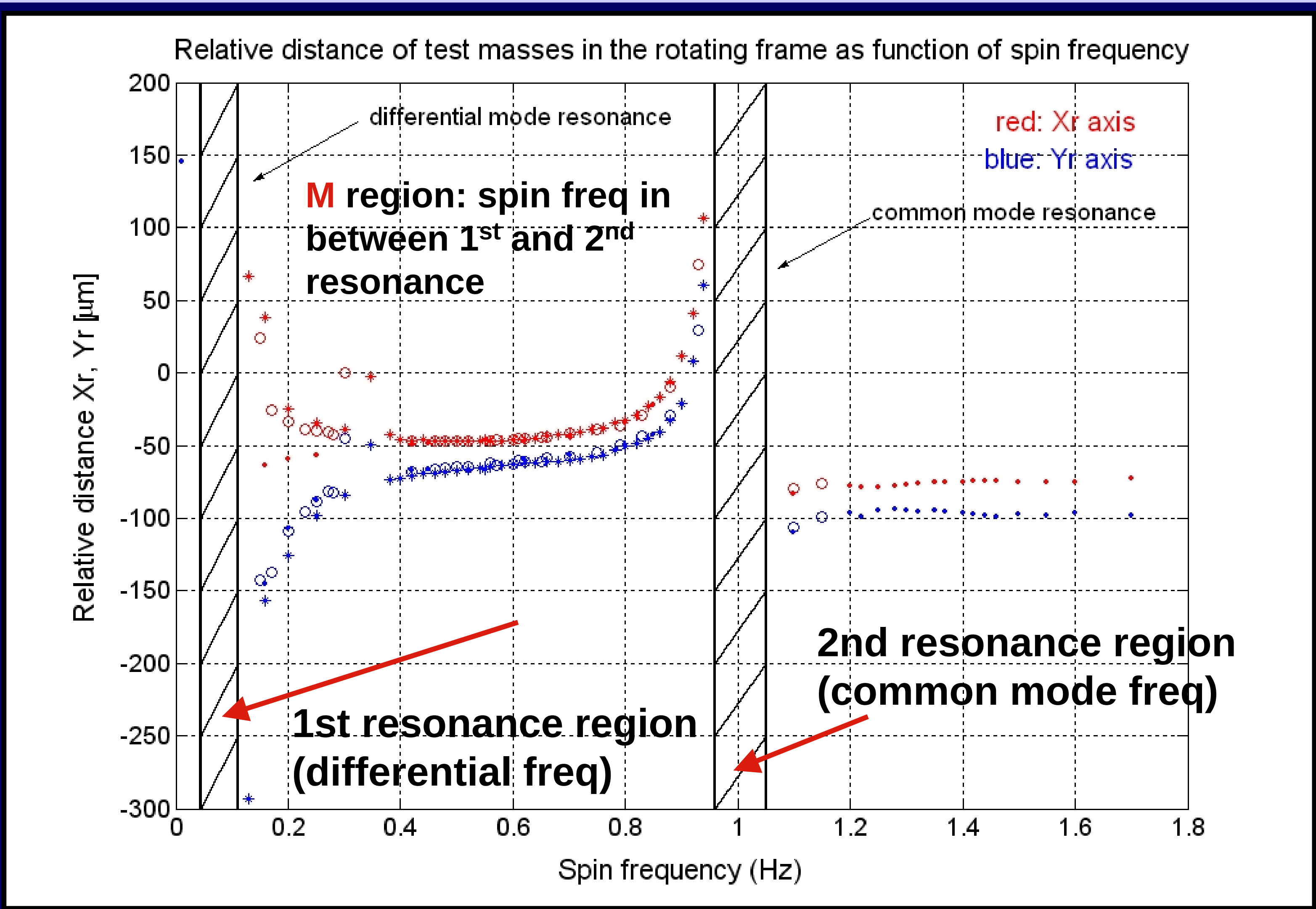
Experimental proof of test masses auto-centering in supercritical rotation (I)





Theory of rotation in supercritical regime (i.e. above natural frequencies) predicts auto-centering ➡ reduction of manufacturing and mounting errors of the <u>rotor</u>

Experimental proof of test masses auto-centering in supercritical rotation (I)

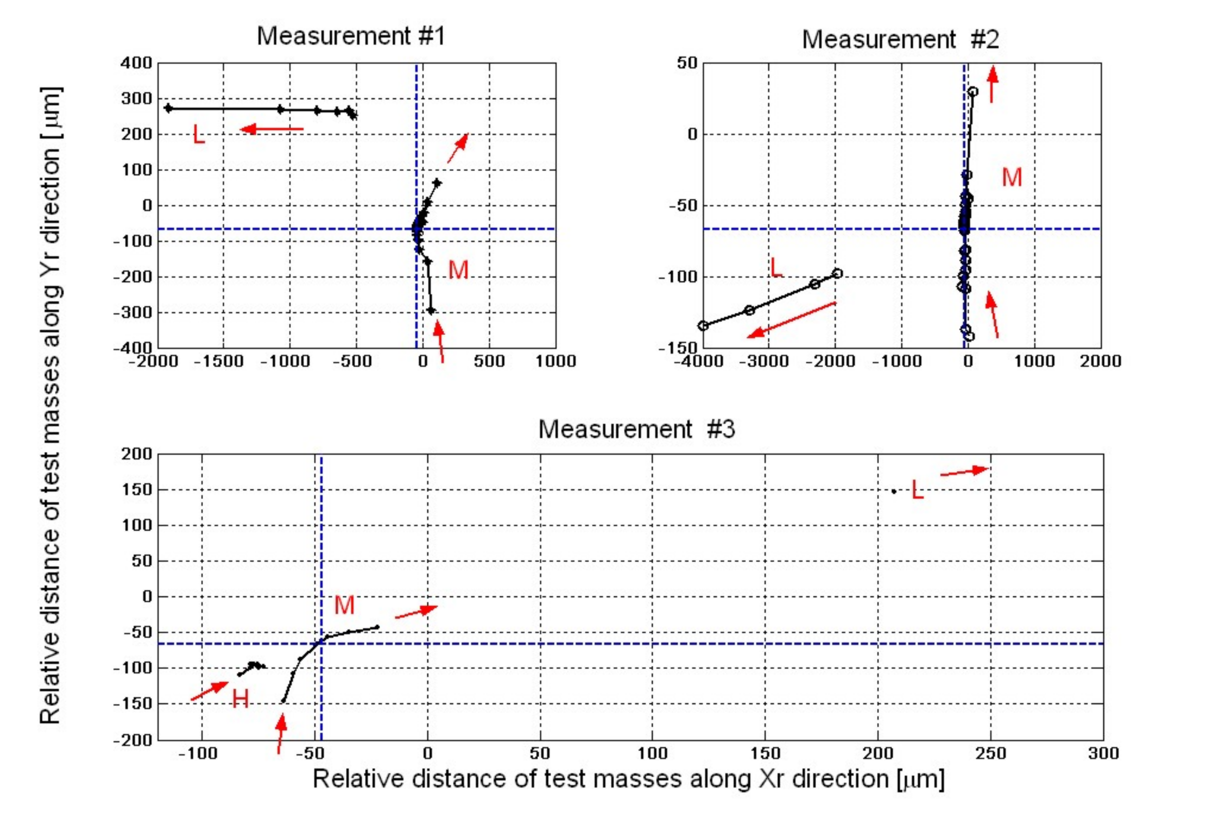


– Auto-centering never measured before for a multi-body supercritical rotor

– Allows unambiguous determination of the zero of capacitance the read-out

– Data from January to March 2006 runs (several hrs per data point….)

Experimental proof of test masses auto-centering in supercritical rotation (II)



Low spin freq: below 1st resonance

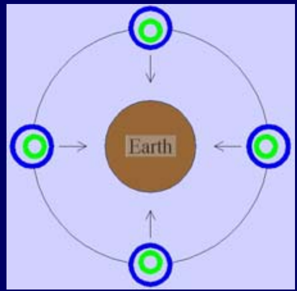
Medium spin freq: in between 1st and 2nd resonance

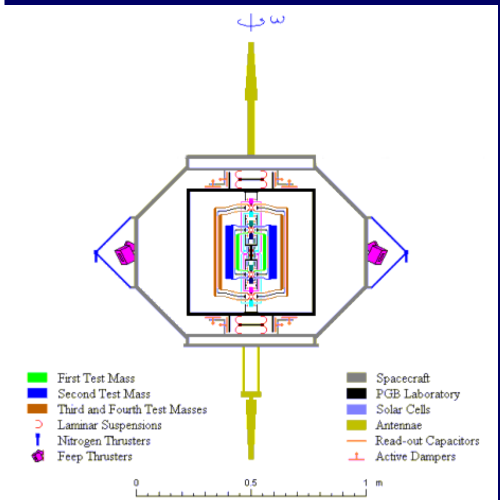
High spin: above 2nd resonance

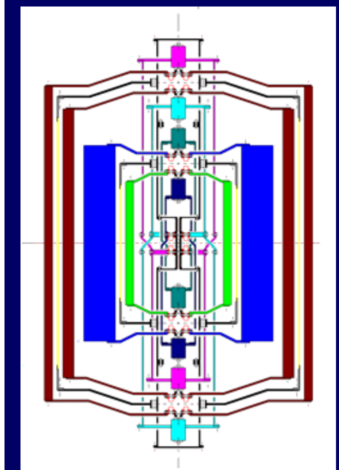
The red arrow shows the direction of increasing spin frequency

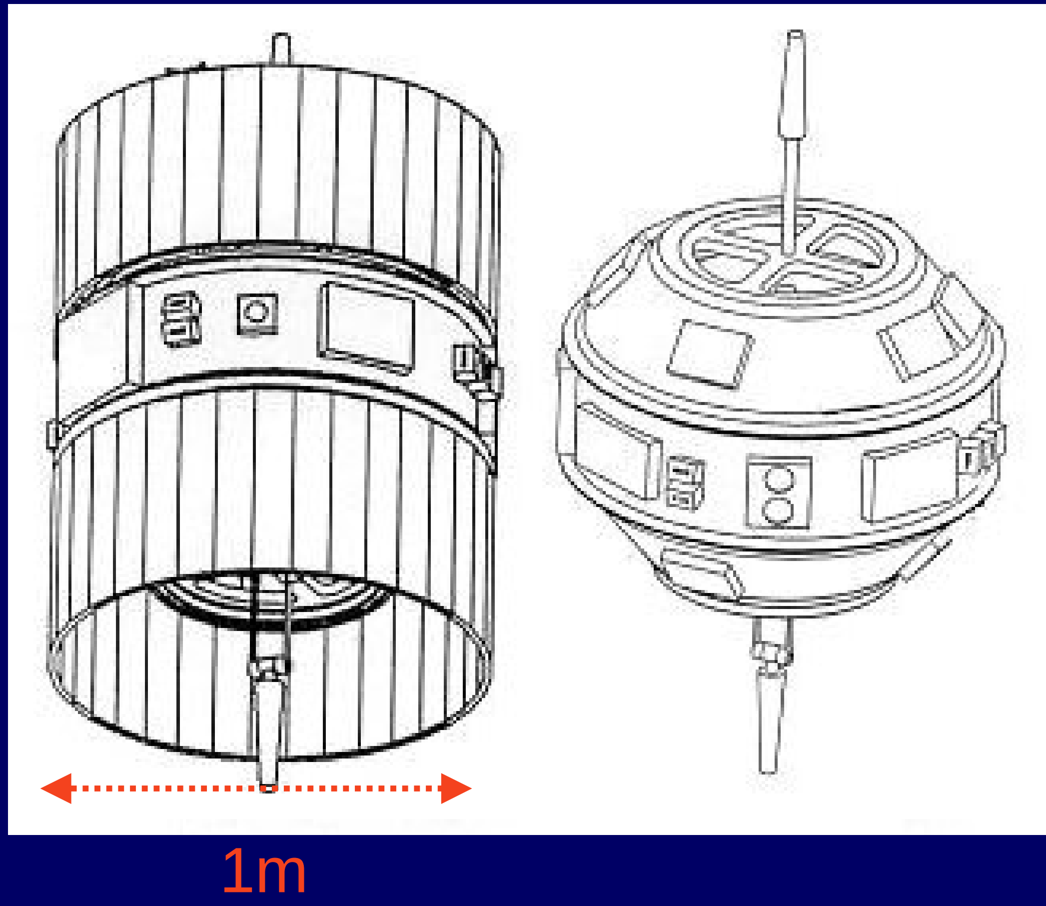
For a spin frequency in the region between the 1st and 2nd resonance (at "**M**edium" spin) there is the same position of relative equilibrium of the test cylinders (at the crossing of the blue dashed lines) independently of the initial conditions (Note: measurements #1, #2 and #3 start from different initial conditions). Only the laws of Physics (for given construction&mounting offset errors of the rotor) determine it. The test cylinders do not need to be centered; physics does it for us. The smaller the construction offsets, the better the centering achieved.

GG: configuration for equatorial orbit



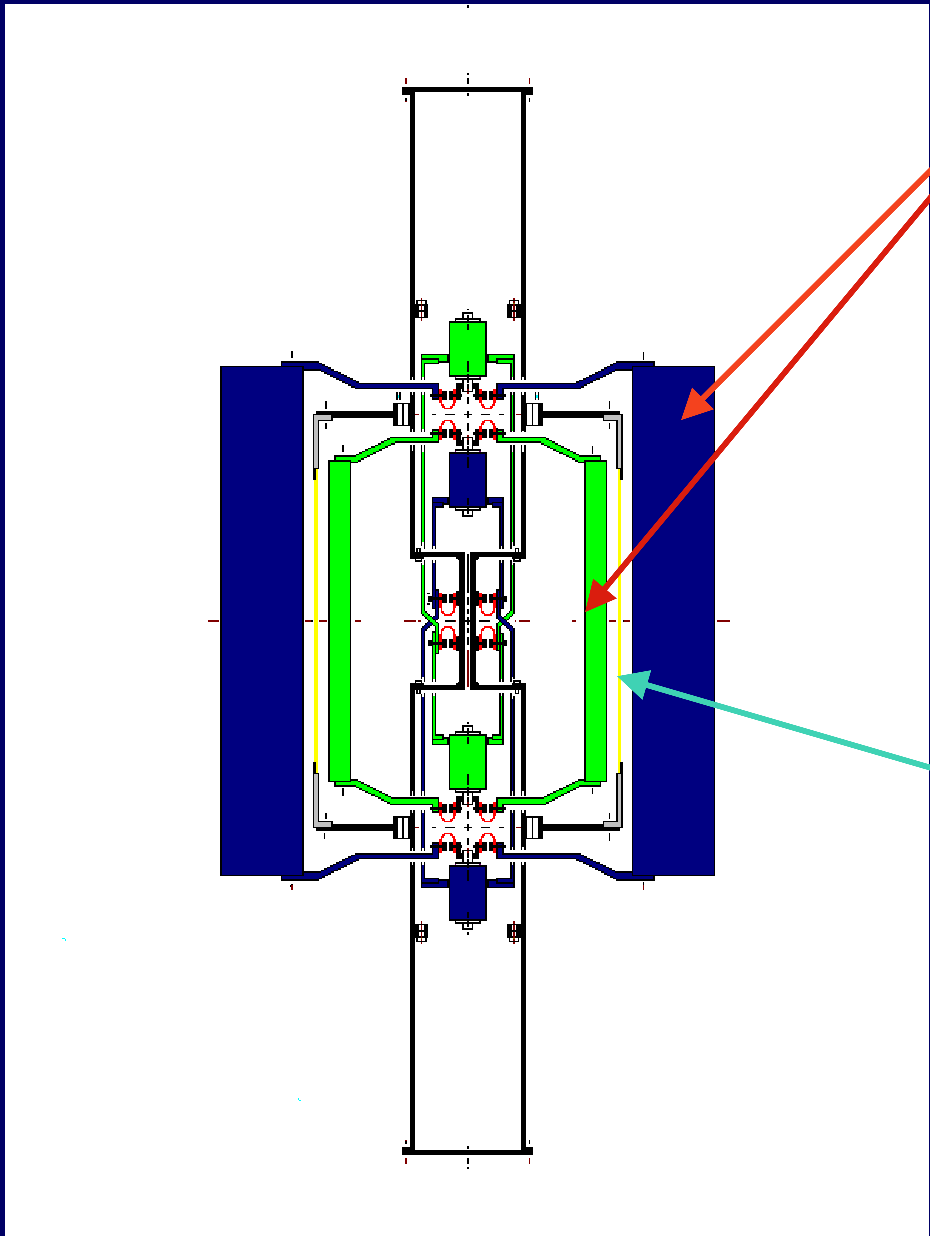






- 250 kg total mass
- passive 1-axis stabilization at 2 Hz
- room temperature (capacitance read out)
- partial along track drag compensation with electric thrusters
- VEGA launch from Kourou
- ground operantion from ASI station in Malindi, Kenia

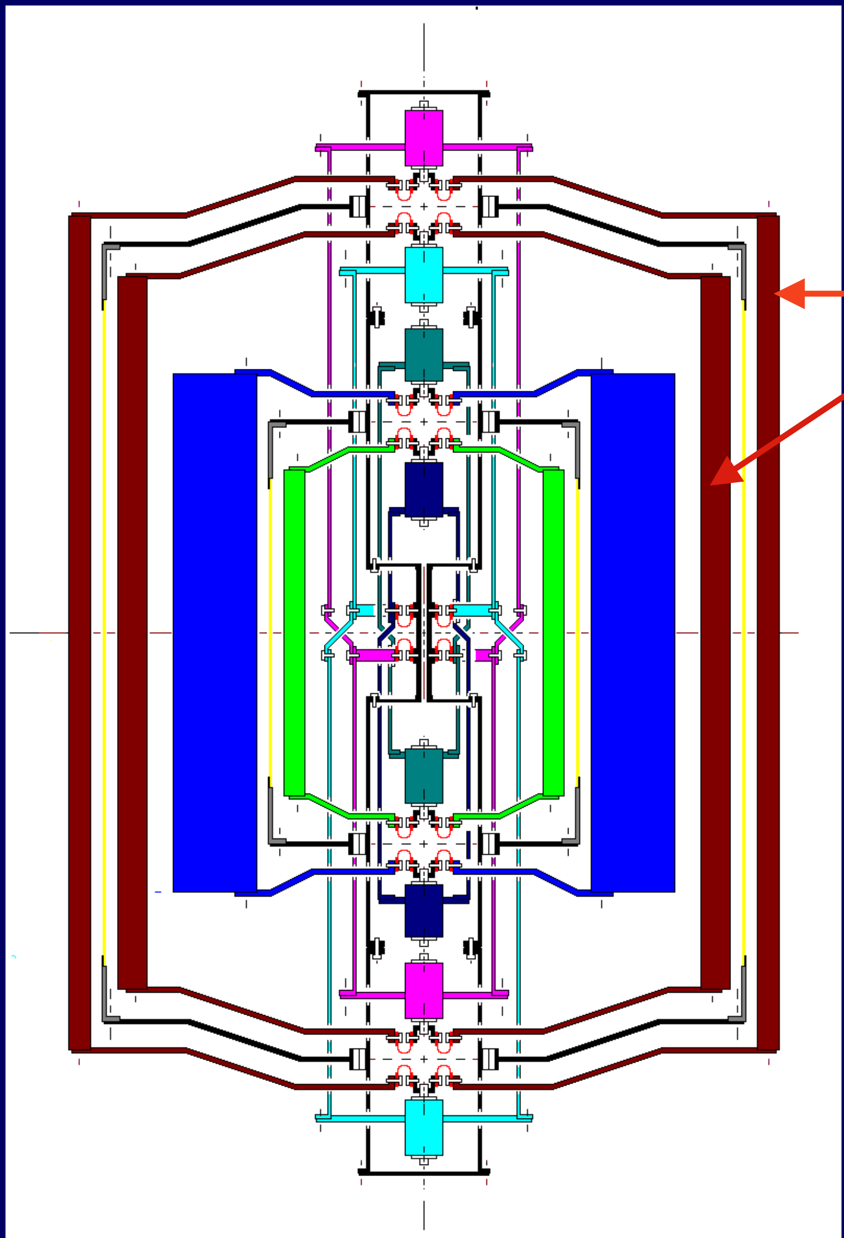
GG differential accelerometer for EP testing



Test masses of different composition (for EP testing)

– For CMR in the plane of sensitivity (⊥ to symmetry/spin axis):
test bodies coupled by suspensions
beam balance concept
– & coupled by read-out
1 single capacitance read out in between cylinders

GG accelerometers: section along the spin axis



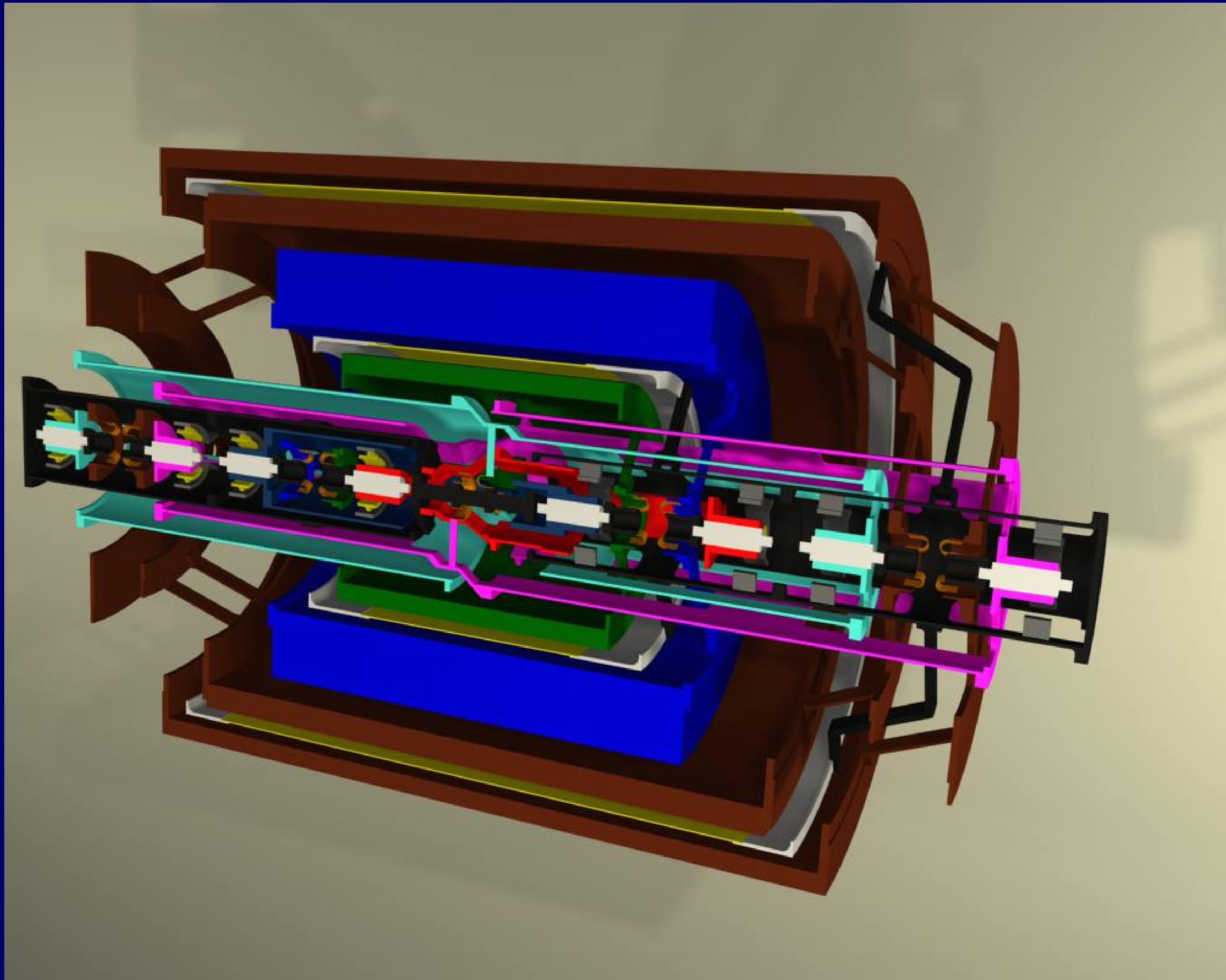
GG inner & outer accelerometer

– the outer one has equal composition test cylinders for systematic checks

– Accelerometers co-centered at center of mass of spacecraft for best symmetry and best checking of systematics…..

Beware… there is only 1 satellite center of mass!!!

GG accelerometers cutaway



GG error budget $\eta = 10^{-17}$ (I)

Effects indistinguishable from signal:

- Earth monopole coupling to different multipole moments of test cylinders in accelerometer: < 0.2
- Radiometer (not sensed by accelerometer with equal composition/density masses…): negligible in GG (PRD 2001; NA 2002)

Effects at same frequency as signal but different phase:

- Residual air drag (after FEEP compensation & CMR): <2.4 but with about 90 deg phase difference

GG error budget $\eta = 10^{-17}$ (II)

Effects at $\nu_{spin} - \nu_{orb}$:

- …..

Effects at $\nu_{spin} - \nu_{whirl}$ (differential whirl)

- …..

Effects at $2\nu_{spin}$:

- …..

DC or slowly varying effects (not an issue):

- Mass inhomogeneities (not moving)
- Parasitic capacitances (not moving)
- Patch effects (slowly moving)
- ……..

GG error budget $\eta = 10^{-17}$ (III)

Spin axis (axes):

- Dominant z moment of inertia, very high spin energy, essentially unaffected by any perturbing torque…(radiation pressure, "luni-solar" type precession…..)

- Self centering guaranteed by physical laws (given achieved offsets..)

- Locking-unlocking. Well defined procedure:

  - Hard locking (used only once at launch)
  - Mechanical stops
  - Inchworm and pressure sensors for "gentle unlocking" (first PGB all together; then one accelerometer masses at a time…)

GG error budget $\eta = 10^{-17}$ (IV)

Mechanical thermal noise:

$$a_{th} = \sqrt{\frac{4 K_B T \omega_{d.m.}}{mQ}} \cdot \frac{1}{\sqrt{T_{\text{int}}}}$$

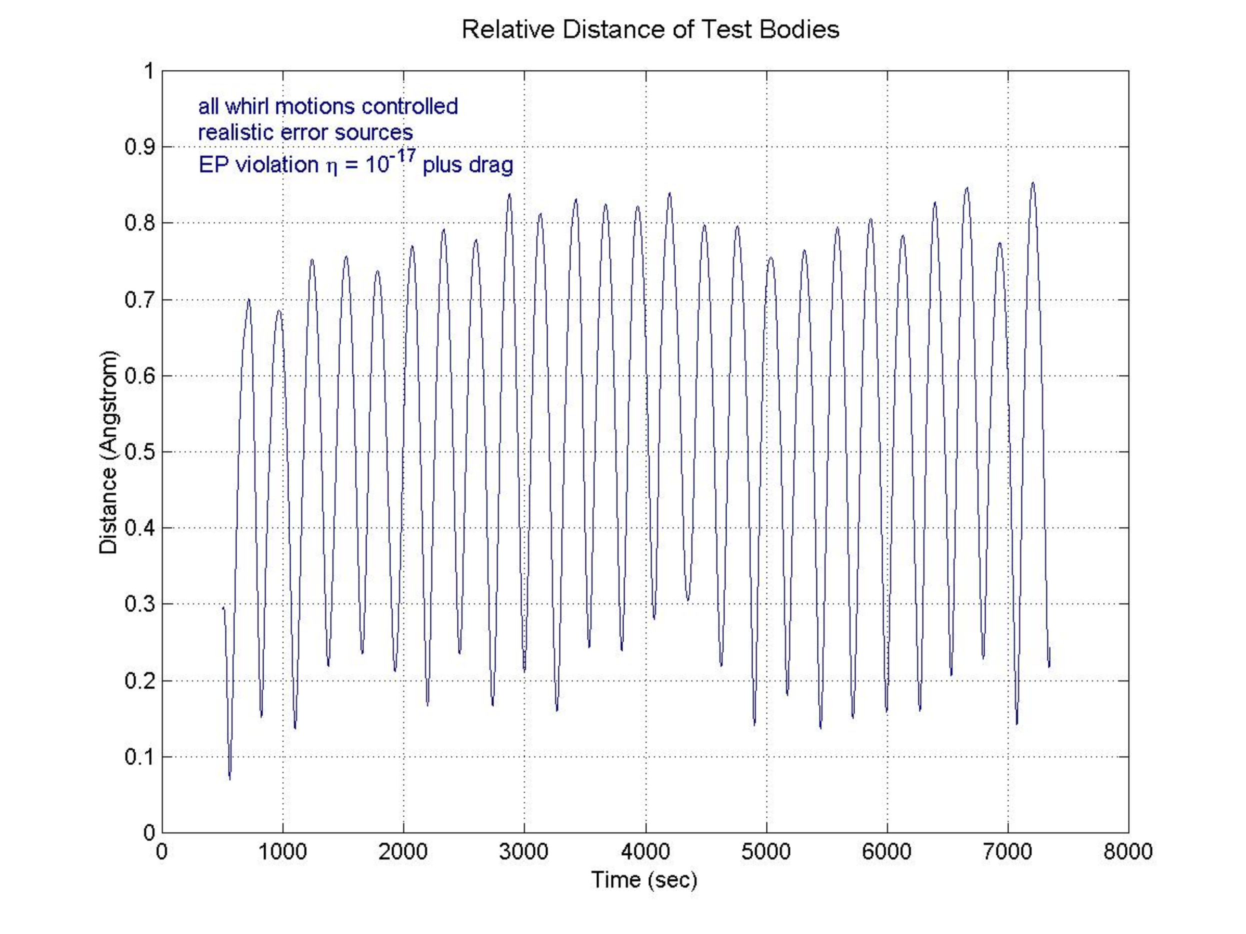
- with Q=20000, few days enough for SNR=2
- T/m : room temperature compensated by larger masses
- $\sqrt{2}$ gain in output data
- 2-yr mission duration certainly doable

plus, full scale ground prototype to learn from…..

GG Error Budget for EP testing to 10^{-17} (SI Units): close to solar maximum, maximum drag value along the orbit assumed

Acceleration (transverse plane) DUE TO:	Formula	Frequency (inertial frame) *(Hz)*	Frequency (detected by spinning sensors) *(Hz)*	Phase	Differential acceleration *(m/sec²)*	Differential displacement *(m)*
EP SIGNAL	$\frac{GM_{\oplus}}{a^2}\eta$	$\nu_{orb} \cong 1.75 \cdot 10^{-4}$	ν_{spin} w.r.t. Earth	Test body to center of Earth	$8.38 \cdot 10^{-17}$ $\eta = 10^{-17}$ *h=520 Km*	$6.3 \cdot 10^{-13}$ $\omega_{dm} \cong 1.15 \cdot 10^{-2}$ *545 sec* diff. period
AIR DRAG	$\frac{1}{2} C_D V_{sc}^2 \frac{A}{M} \rho_{atm}$	ν_{orb}	ν_{spin}	~ along track	$5.21 \cdot 10^{-17}$ AFTER: χFEEP $= \frac{1}{50000}$ χCMR $= \frac{1}{100000}$	$3.9 \cdot 10^{-13}$
SOLAR RADIATION PRESSURE	$\frac{A}{M}\frac{\Phi_{\Theta}}{c}$	$\nu_{orb} - \nu_{\Theta} \cong \nu_{orb}$	ν_{spin}	test body to center of Earth component	$9.57 \cdot 10^{-19}$ same χ_{FEEP}, χ_{CMR}	$7.2 \cdot 10^{-15}$
INFRARED RADIATION FROM EARTH	$\alpha_{\oplus} \frac{A}{M}\frac{\Phi_{\Theta}}{c}$	ν_{orb}	ν_{spin}	test body to center of Earth	$1.44 \cdot 10^{-18}$ same χ_{FEEP}, χ_{CMR}	$1.08 \cdot 10^{-15}$
EARTH COUPLING TO TEST BODIES QUADRUPOLE MOMENTS	$\frac{3}{8}\frac{GM_{\oplus}}{a^2}\frac{\Delta J}{J_x} \cdot \left(\frac{r_1^2 + r_2^2 + l^2/3}{a^2}\right)$	ν_{orb}	ν_{spin}	test body to center of Earth	$2.4 \cdot 10^{-19}$	$1.8 \cdot 10^{-15}$
MECHANICAL THERMAL NOISE	$\sqrt{\frac{4K_B T \omega_{dm}}{mQ}} \frac{1}{\sqrt{T_{int}}}$	$\nu_{d.m.}$	$\nu_{spin} \pm \nu_{d.m.}$	Random	$3.99 \cdot 10^{-17}$ $T_{int} \cong 7$ *days* $Q = 20000$	$3 \cdot 10^{-13}$
					TOTAL ERROR BUDGET	$3.59 \cdot 10^{-13}$

GG simulations during Phase A and advanced Phase A studies

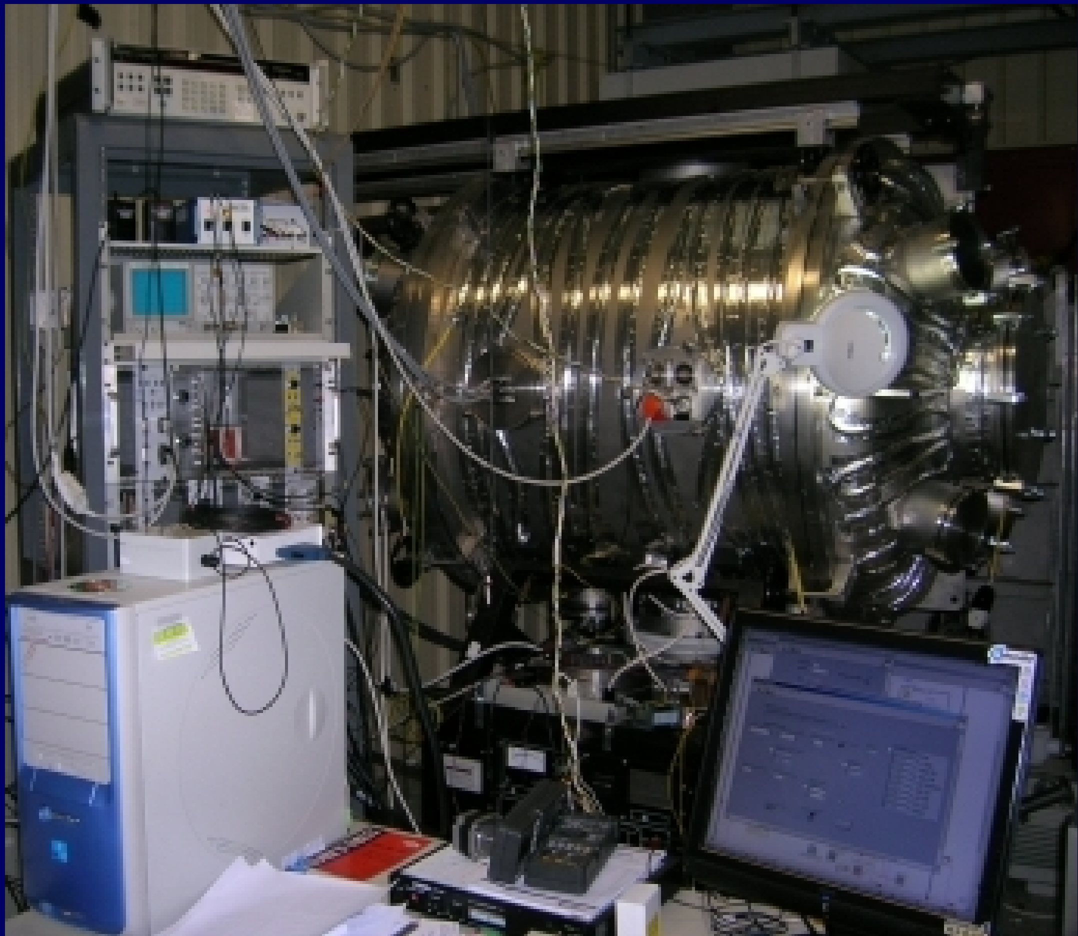


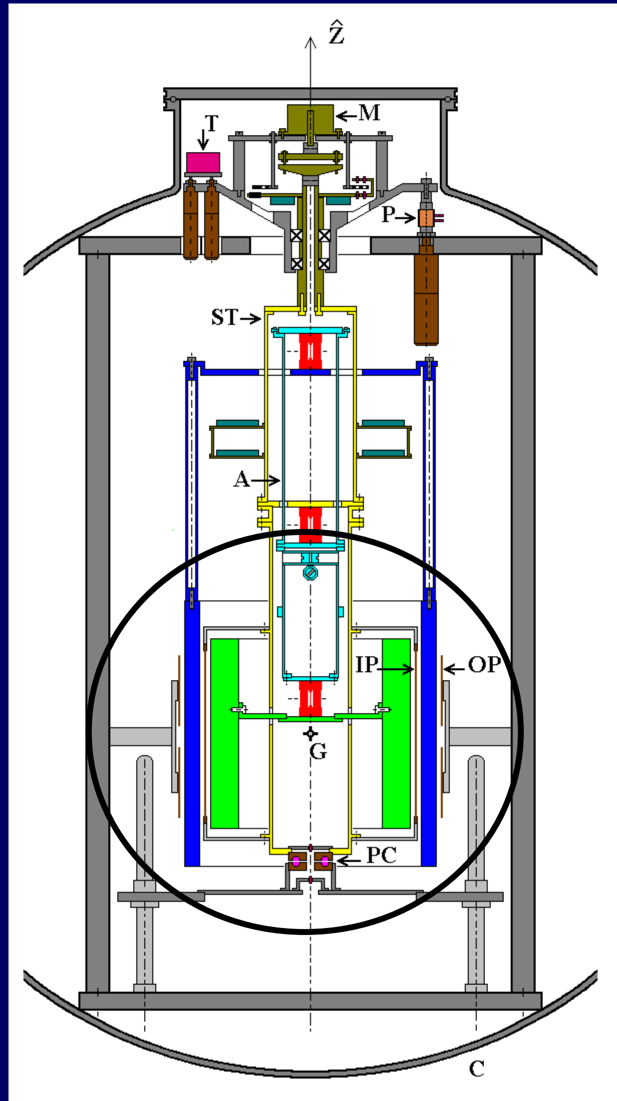
From GG Proposal to ESA, Jan 2000, p.16

http://eotvos.dm.unipi.it/nobili/ESA_F2&F3/gg.pdf

Realistic simulation of GG space experiment (errors according to requirements; see reference for details) showing the relative displacements of the test masses after whirl and drag control, with an applied "EP violation" signal to 10^{-17}. The applied EP signal could be recovered by separating it from residual whirl and drag, though they were both larger (see reference online to understand how…)

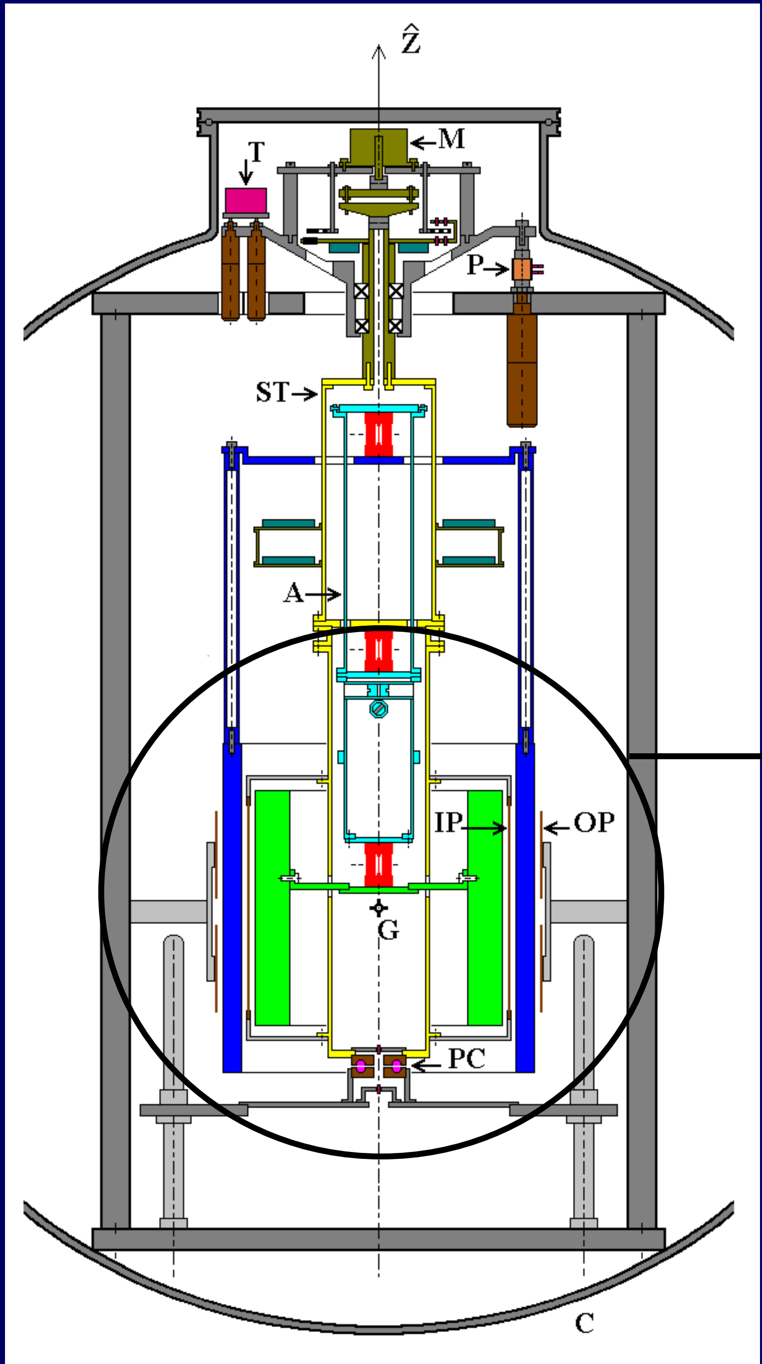
GGG prototype at INFN lab in San Piero a Grado (Pisa)

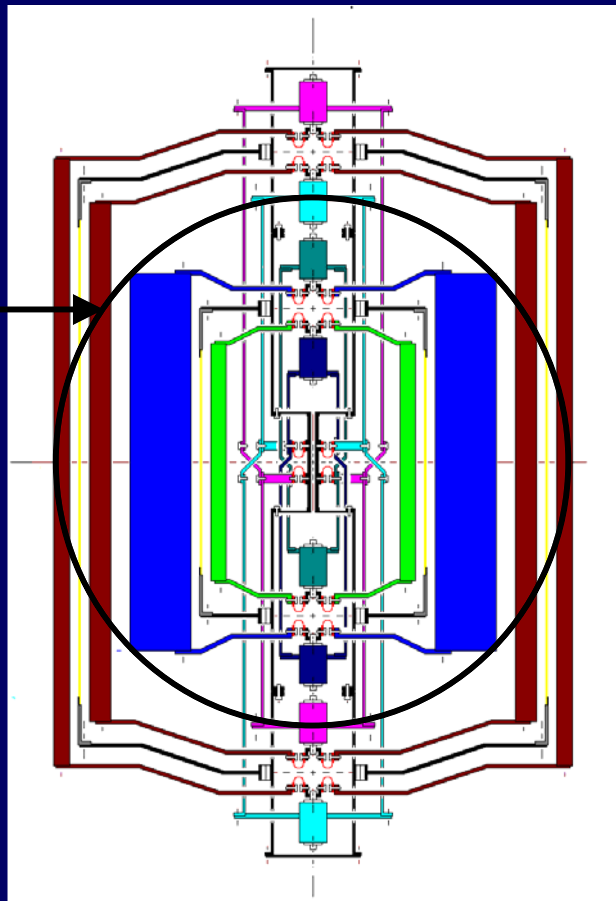




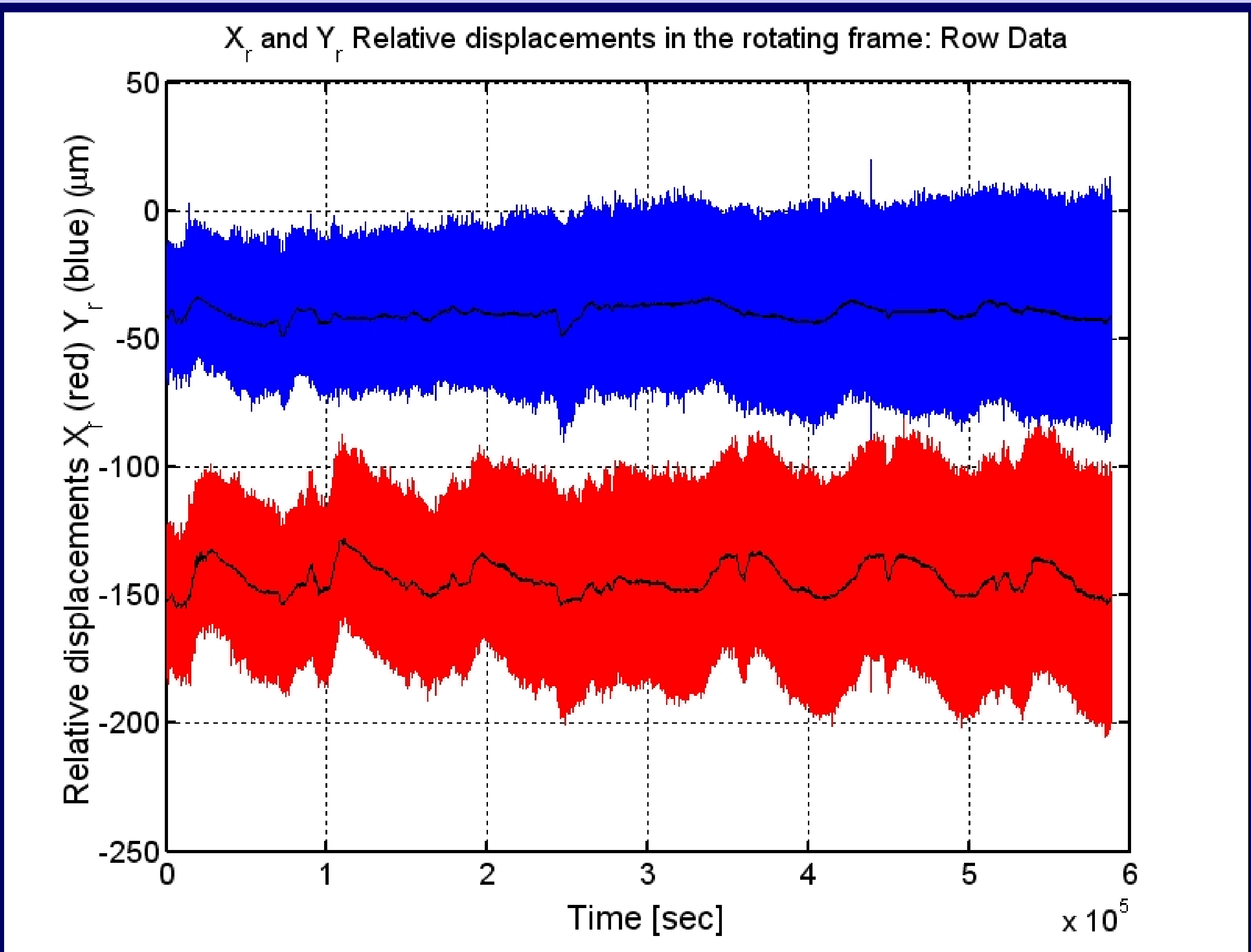
GGG vs GG design

Local gravity breaks the simmetry of the space accelerometer design…



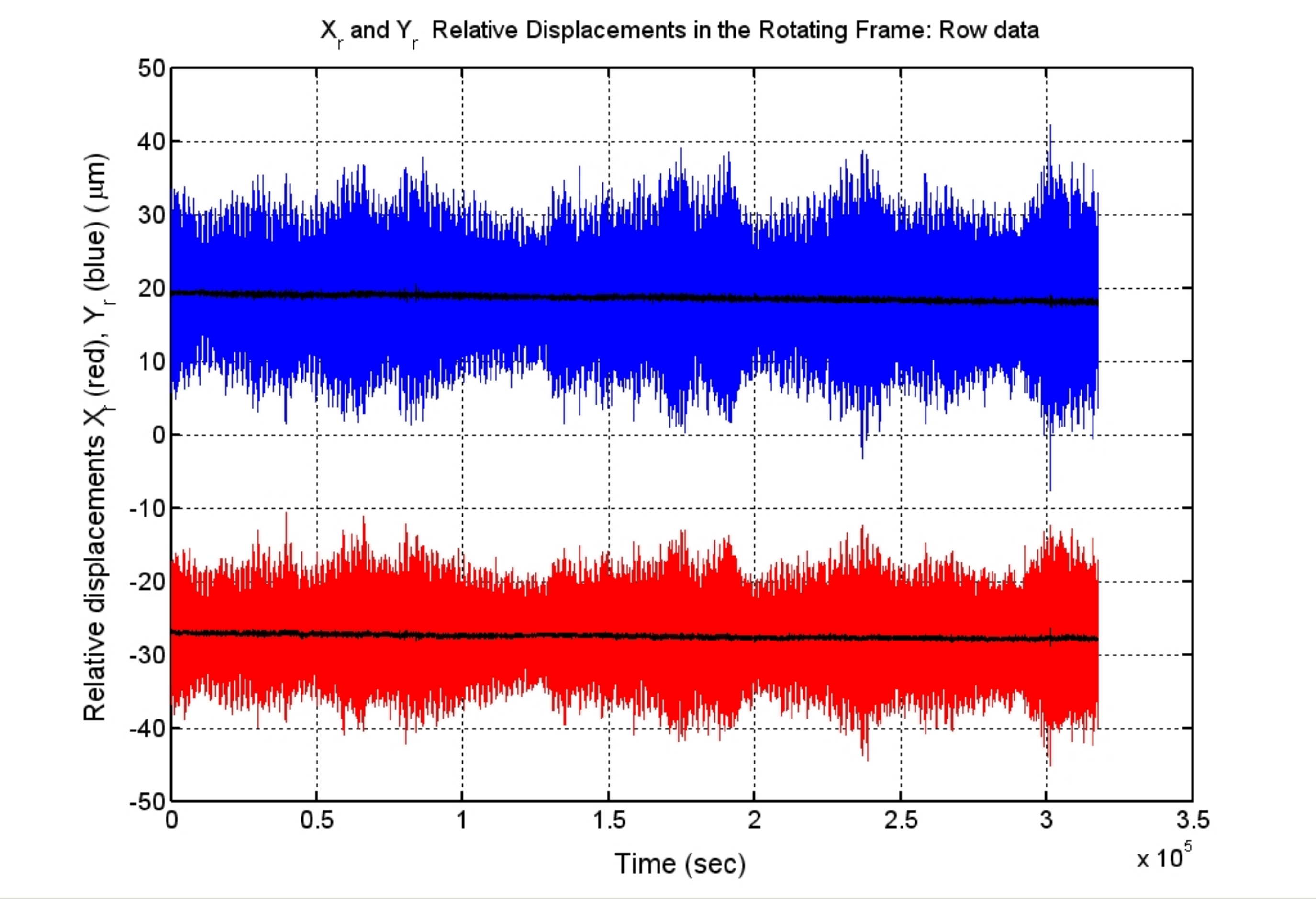


Relative displacements of rotating test cylinders: raw data (July 2005)



July 2005: several days of raw data as taken by the rotating capacitance bridges measuring the relative displacements of the test cylinders in the horizontal plane of the rotor. The black curves are the average of the same data over 5 periods of whirl; the center of the whirl orbit is the position of equilibrium of the test masses relative to each other which is affected by external forces, such as that of an Equivalence Principle violation

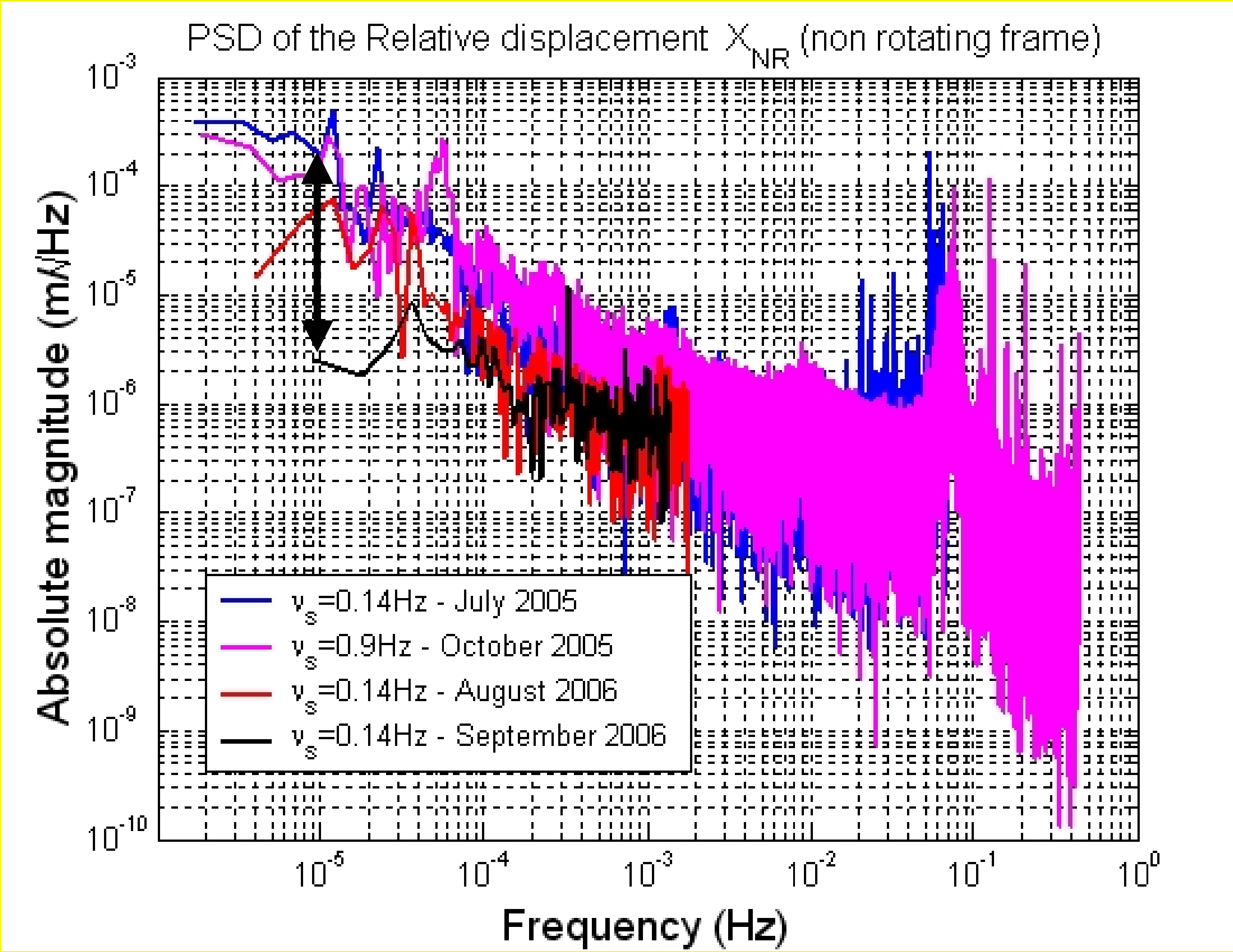
Relative displacements of rotating test cylinders: raw data (September 2006)



September 2006: same as in previous plot (see previous caption). The raw data have improved from 100 μm to 20 μm peak-to-peak relative displacements. More importantly, the average over 5 whirl periods (black curves), i.e. the position of relative equilibrium of the test cylinders, is now much more stable in time.

NOTE: The advantage of high frequency modulation (provided by rotation) will become apparent only after demodulation, i.e. after transformation to the non rotating horizontal plane of the laboratory

Relative displacements of rotating test cylinders (after demodulation): PSD



After demodulation (i.e. after transformation to the non rotating horizontal plane), the Power Spectral Density shows the improvement at low frequencies from 2005 to 2006. PSD allows us to compare the sensitivity achieved with runs of differemt duration. At the diurnal frequency, which is the frequency of an Equivalence Principle violation in the field of the Sun, the improvement since July 2005 is by a factor 100

Where do we stand (I)

@ 24 hr (10^{-5} Hz)

$$\Delta x_{PSD} \simeq 2\cdot 10^{-6}\, m/\sqrt{Hz} \qquad (T_{diff} \simeq 13.2\ s\ ,\ a_{\odot} \simeq 0.006\, m/s^{2})$$

In current 3-day measurement runs:

$$\Delta x_{3days} \simeq 2\cdot 10^{-6}\cdot\sqrt{2}/\sqrt{3\cdot 86400} \simeq 5.6\cdot 10^{-9}\ m$$

$$\Delta a_{3days} \simeq \Delta x_{3days}\cdot(2\pi/T_{diff}^{2}) \simeq 2\cdot 10^{-10}\ m/s^{2}$$

$$\eta_{\odot\_3days} \simeq \Delta a_{3days}/a_{\odot} \simeq 3\cdot 10^{-8}$$

In a 10-month run (possible; with current pumps vacuum is guaranteed for 11 months…)

$$\eta_{\odot\_300days} \simeq \Delta x_{300days}\cdot(2\pi/T_{diff}^{2})/a_{\odot} \simeq 3\cdot 10^{-9}$$

Work is in progress to improve:

– Read-out (absolute encoder) & coordinate transformation (demodulation) (single "good" clock thermally stabilized). Crucial, because advantage of high frequency modulation is wasted unless demodulation is performed correctly….

– Tilt control & thermal stability: control loop ok, but stability of Geomechanics tiltmeter not satisfactory… thermal stability can be improved

Where do we stand (II)

@ 5700 s ($1.75 \cdot 10^{-4}$ Hz)   (GG orbital frequency)

$$\Delta x_{PSD} \simeq 10^{-6}\, m/\sqrt{Hz} \qquad (T_{diff} \simeq 13.2\ s\ ,\ a_{\oplus GG} \simeq 8.4\, m/s^2)$$

In a 10-month run (11 months possible with GGG; 2 yr GG mission duration foreseen at Phase A studies):

$$\Delta x \simeq 10^{-6} \cdot \sqrt{2} / \sqrt{300 \cdot 86400} \simeq 2.8 \cdot 10^{-10}\ m$$

$$\eta_{\oplus} \simeq \Delta x \cdot (2\pi / T_{diff}^2) / a_{\oplus GG} \simeq 1.2 \cdot 10^{-12}$$

The GG target is $\eta_{GG} \simeq 10^{-17}$ and requires to detect a relative displacement of the test cylinders $\Delta x_{GG} \simeq 6.3 \cdot 10^{-13}\, m$ @ $1.75 \cdot 10^{-4}\ Hz$

The lab result is limited by motor/bearings + terrain tilt noise, which is absent in space $\Rightarrow$ lower noise level expected in GG… assume by factor 50…

This statement can be proved soon: run rotor with weaker coupling and show that the platform noise level does not increase….

Where do we stand (III)

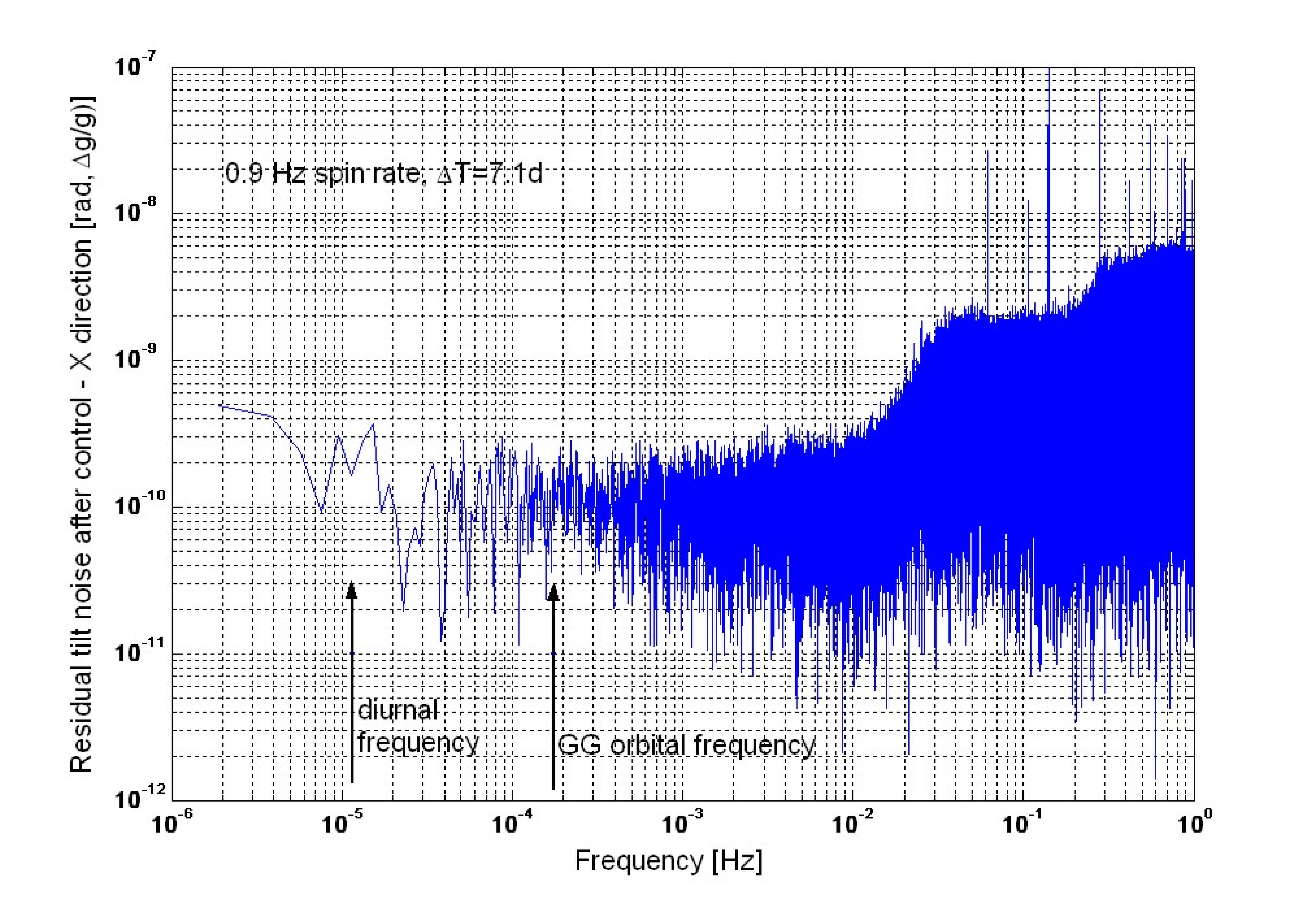
Since the coupling of the test cylinders in space is much weaker than at 1-g (because of weightlessness):

$$T_{diff}{}^{2} \propto 1/k$$

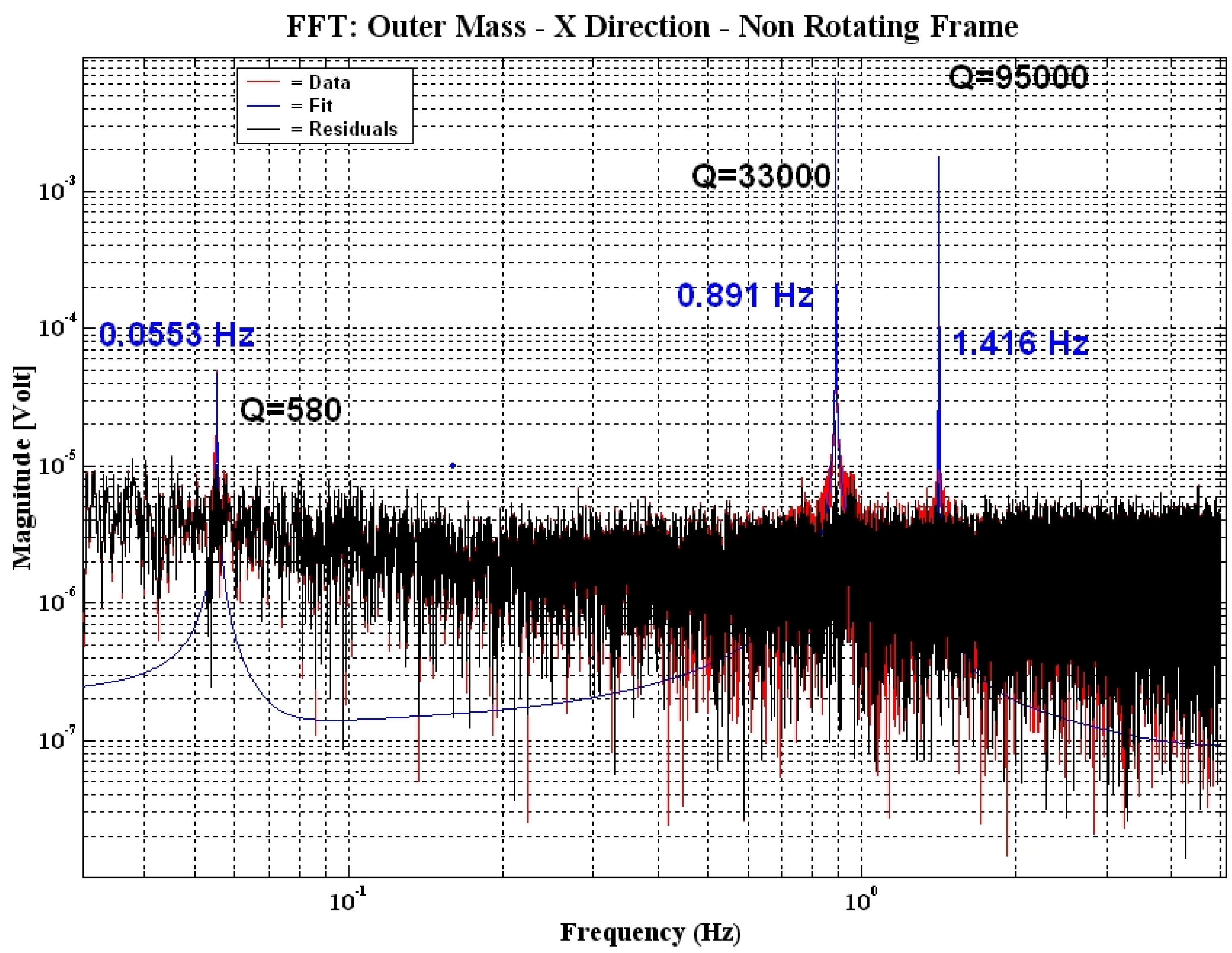
$$\left(\frac{T_{diff\_space}}{T_{diff\_GGG}}\right)^{2} \simeq \left(\frac{545\ s}{13.2\ s}\right)^{2} \simeq 1700$$

While the platform noise in space is lower (no motor/beraings, no terrain tilts: factor 50 assumed), the accelerometer in is sensitive to a signal of EP violation 1700 weaker than the current GGG, with an improvement over the current sensitivity by $8.5 \cdot 10^{4}$, bringing it to the level of the GG target sensitivity of 10^{-17}

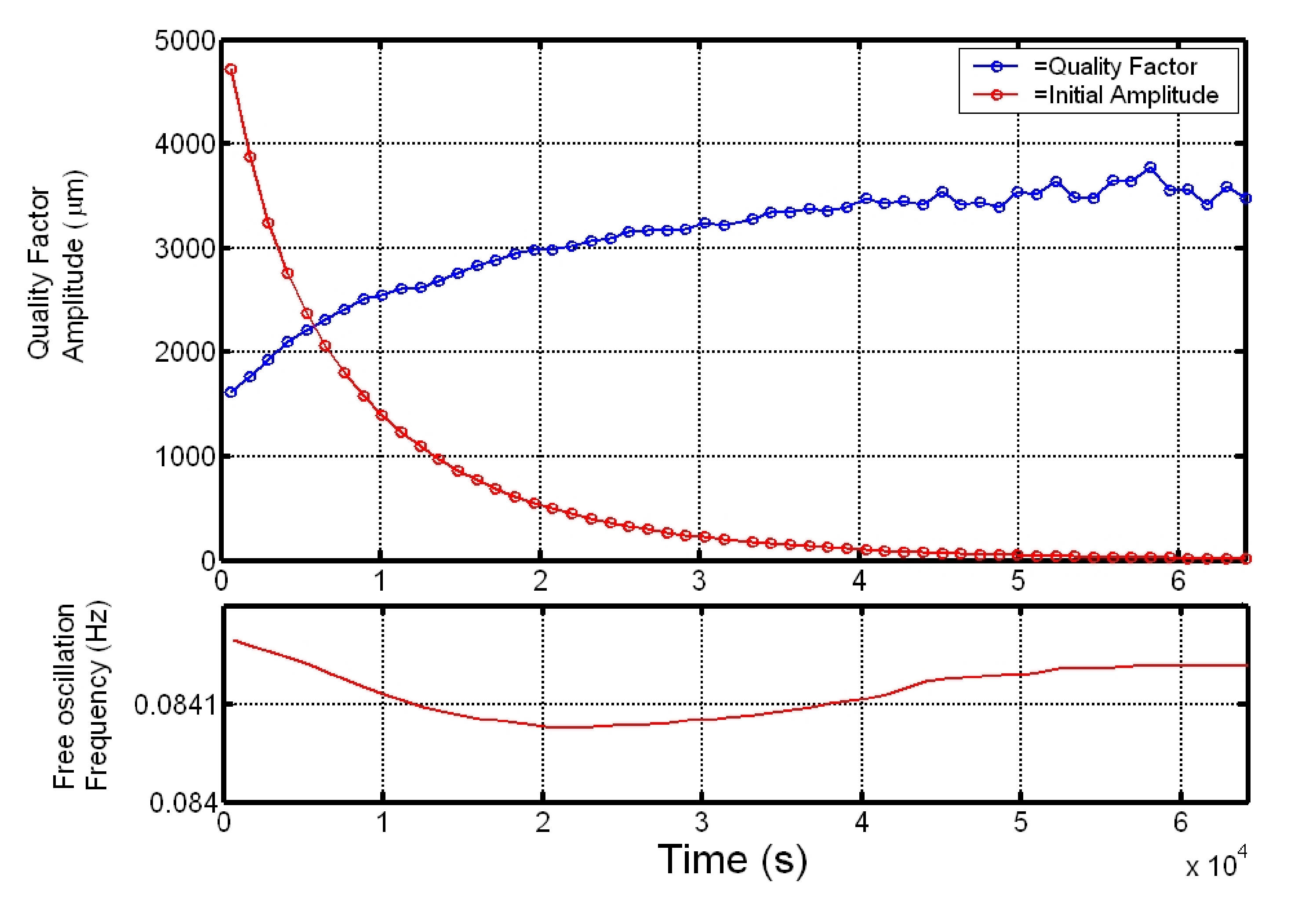
Terrain tilt control: check of control loop



Q measurements @ rotor natural frequencies (2003)

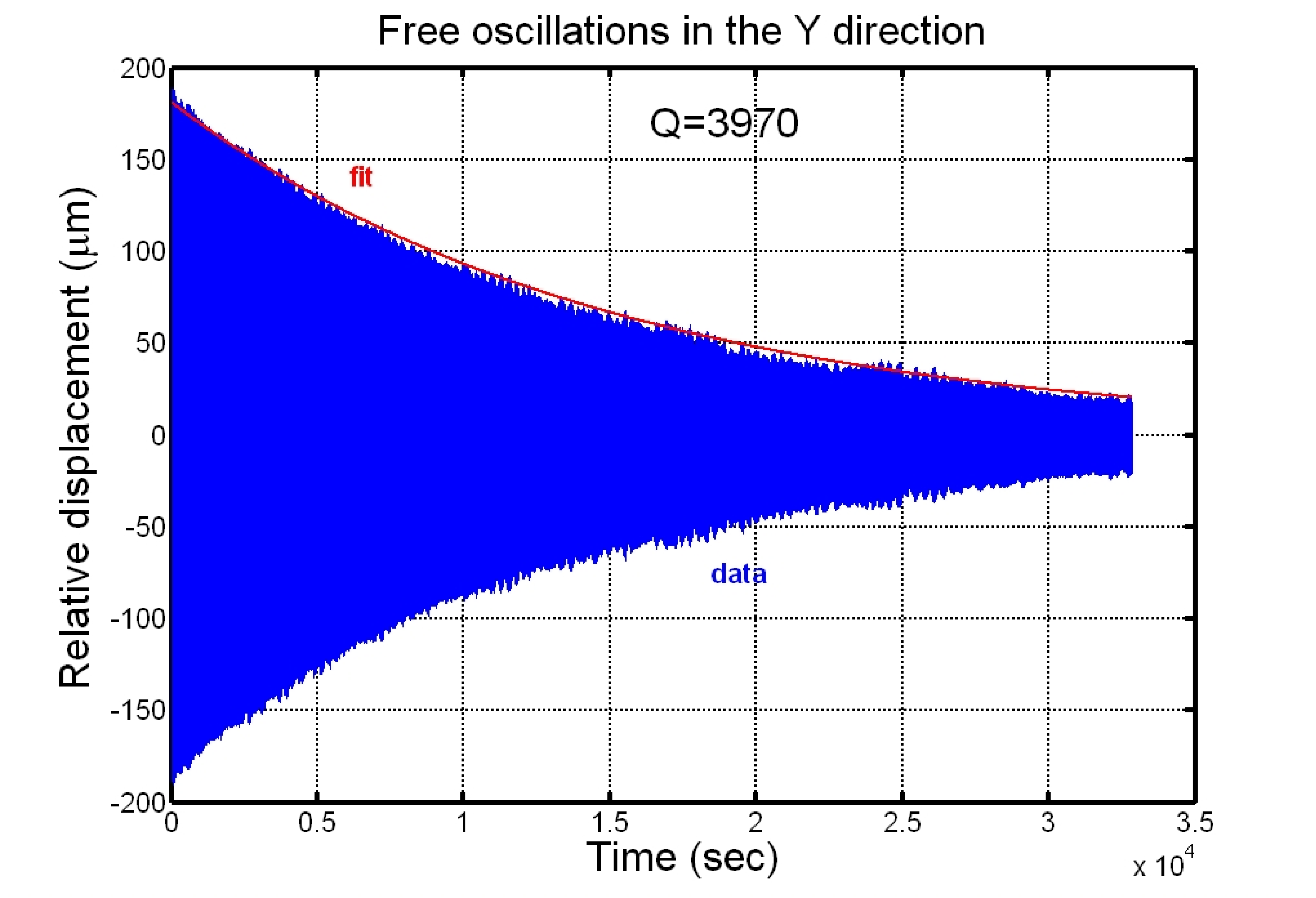


Q measurements @ rotor natural differential frequency (2006) (I)



Rotor (not spinning) excited at its natural differential frequency (as given in graph below): Q measured for decreasing initial oscillation amplitudes, starting from very large ones. It is found (see graph above) that Q increases as the initial oscillation amplitude decreases),,, see next slide for the best Q value measured

Q measurements @ rotor natural differential frequency (2006) (II)

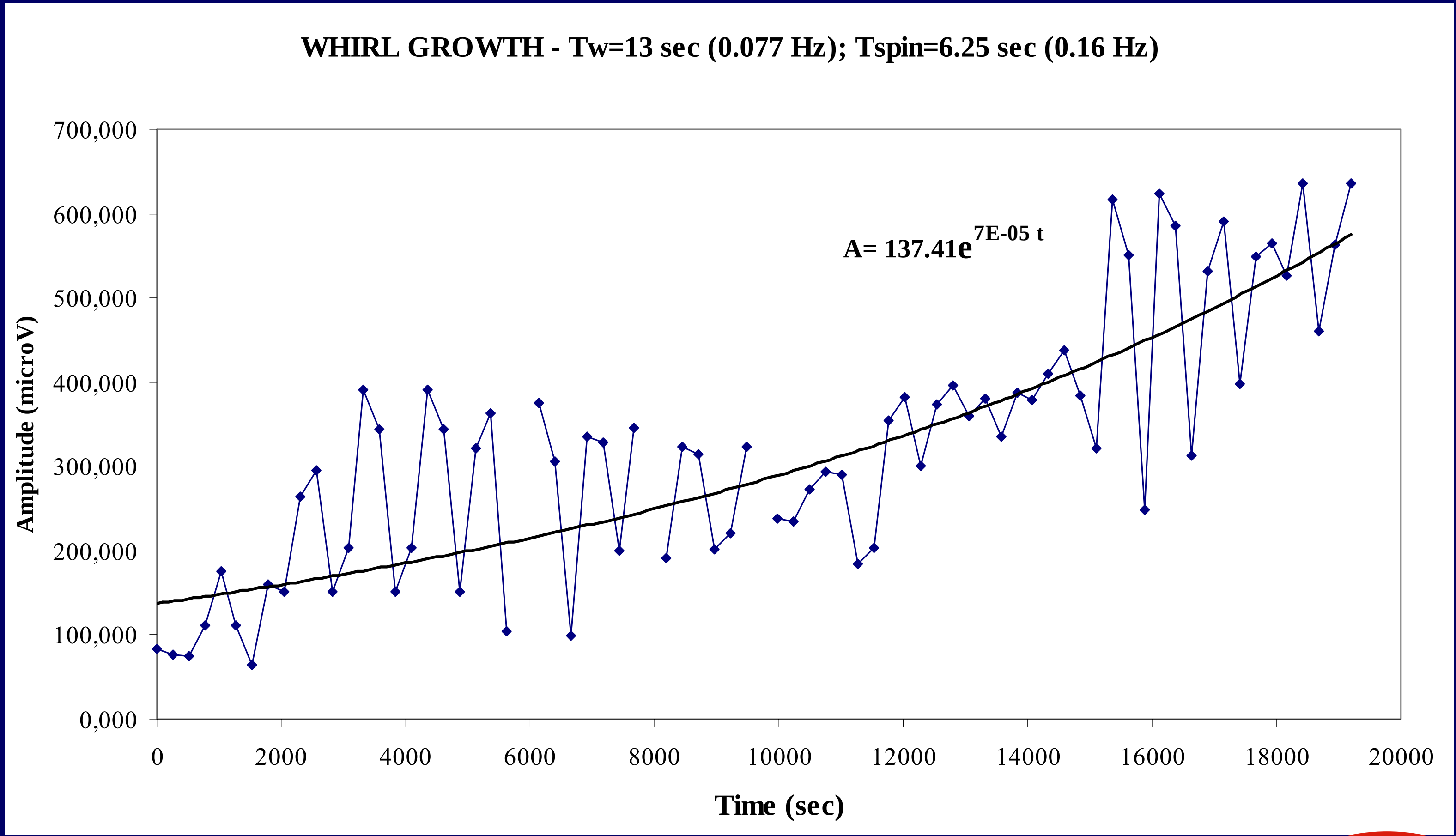


Rotor (not spinning) excited at its natural differential period of about 12 sec; for an initial amplitude of about 200 μm we measure Q=3970, which is a very good value for a complex cardanic suspension at this low frequency

Q measurements in supercritical rotation (2005) (I)

– Spin period 6.25 sec (0.16 Hz), whirl period 13 sec (0.0765 Hz), whirl control off

– From data of non rotating capacitors, which sense the motion fo the outer test cylinder

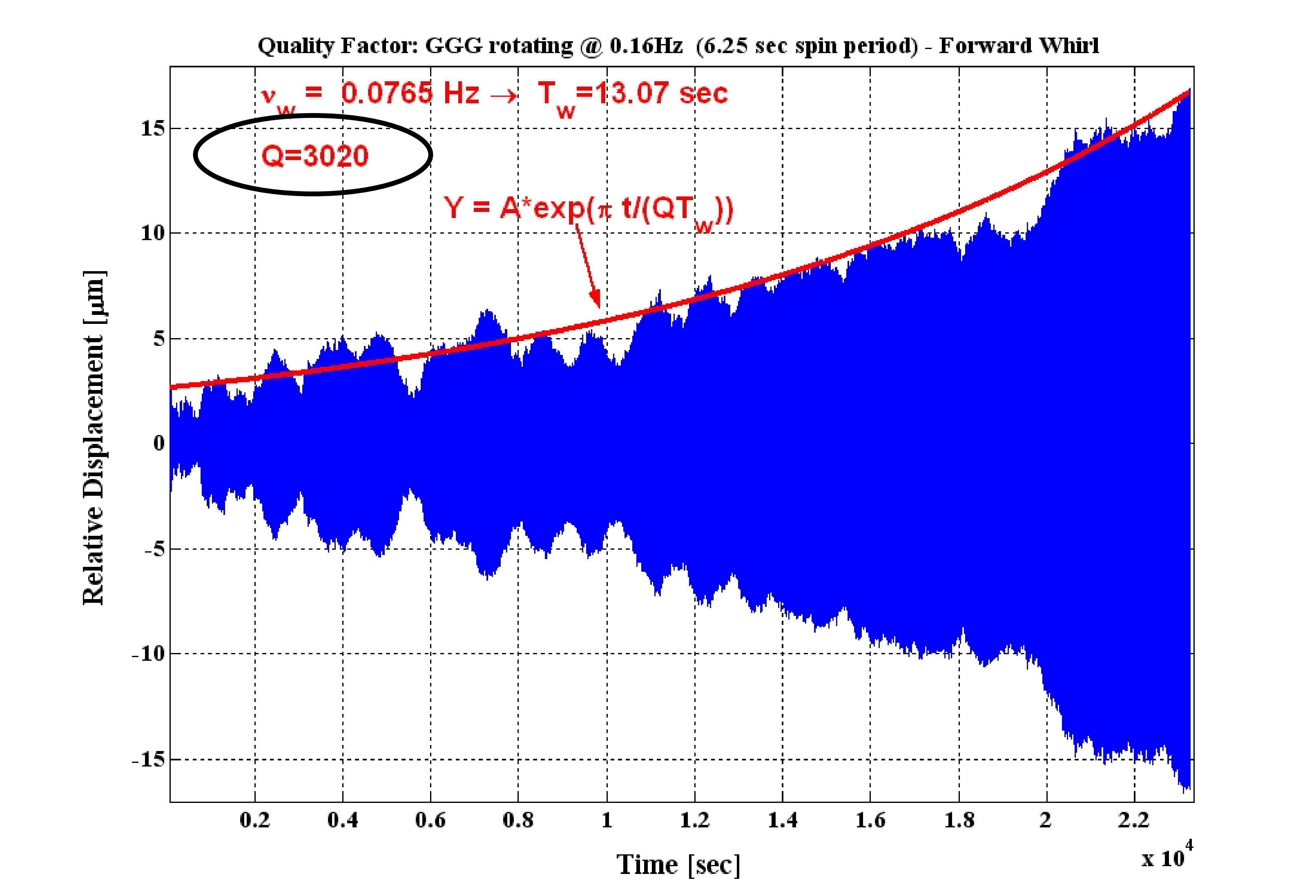


$$\frac{\pi}{-Q_{spin}} \frac{t}{T_w} = 7 \cdot 10^{-5}\, t \quad \Rightarrow \quad Q_{0.16\,Hz} = -3430$$

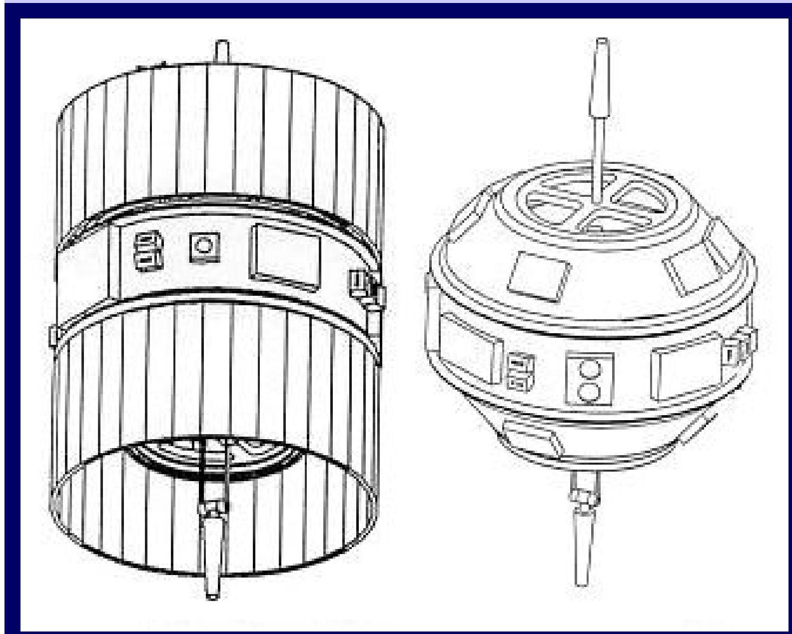
Q measurements in supercritical rotation (2005) (II)

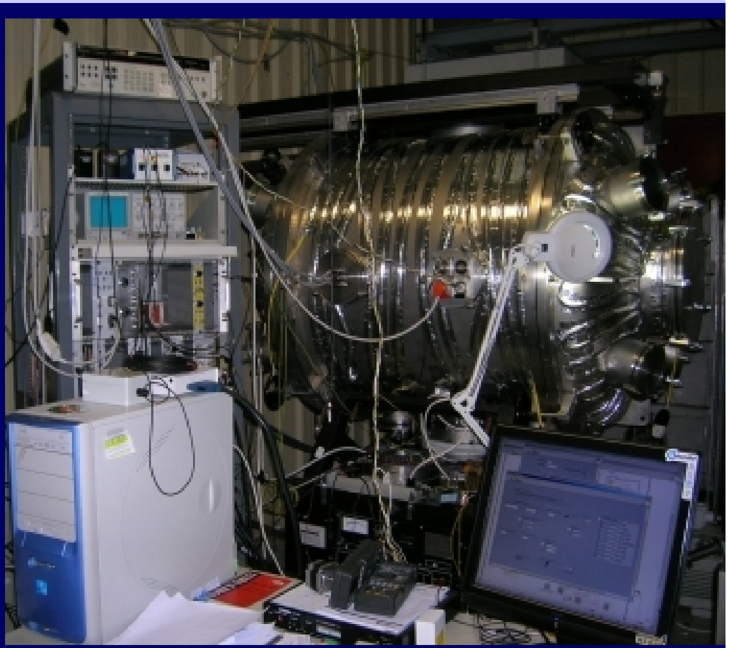
– Spin period 6.25 sec (0.16 Hz), whirl period 13 sec (0.0765 Hz), whirl control off

– From data of rotating capacitors in between the test cylinder



GG/GGG





GG satellite included in National Space Plan of Italian Space Agency (ASI) for the next 3 years

GGG lab prototype funded by INFN (Istituto Nazionale di Fisica Nucleare) + Indian collaboration

GG/GGG Webpage: http://eotvos.dm.unipi.it/nobili

Dynamical response of the "GGG" rotor to test the Equivalence Principle: theory, simulation and experiment.
Part I: The normal modes , Comandi et al. RSI, **77** 034501 (2006)
Part II: The rejection of common mode forces , Comandi et al. RSI, **77** 034502 (2006)

"Test of the Equivalence Principle with macroscopic bodies in rapid rotation: current sensitivity and relevance for a high accuray test in space" Nobili et al. to be submitted to IJMPD

"Limitations to testing the Equivalence Principle with Satellite Laser Ranging" Nobili et al. to be submitted to PRD