

# Is Physics in the Solar System really understood?

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*MPI Solar System Research, Katlenburg-Lindau, Germany*

**Slava G. Turyshev, John D. Anderson, Jim K. Campbell**

*Jet Propulsion Laboratory, California Institute of Technology, USA,*

**and the**

**Pioneer Anomaly Explorer Science Team**

# The Pioneer Anomaly Explorer Mission Science Team

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# Background

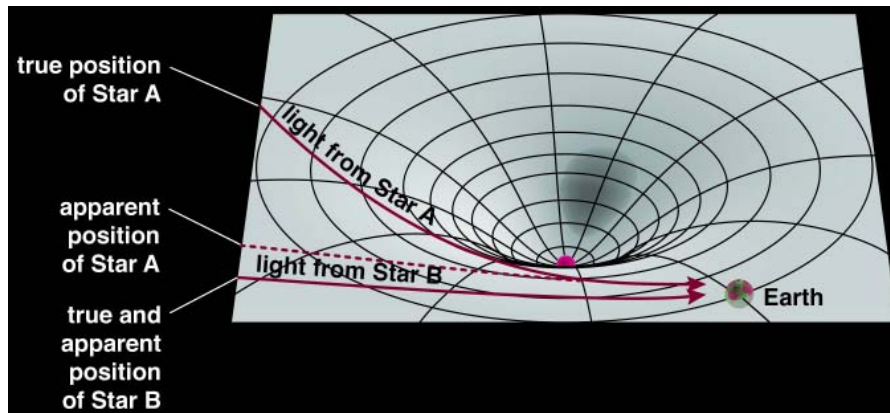
- Many aspects of General Relativity are well tested and confirmed:

## Foundations:

- Universality of Free Fall
- Local Lorentz Invariance
- Universality of Gravitational Redshift

## Predictions:

- Solar System Effects
  - Perihelion shift
  - Gravitational Redshift
  - Light deflection
  - Time delay
  - Gravitomagnetic effects
- Strong field observations
  - Binary systems
  - Black holes
- Gravitational waves
- Cosmology



# Experimental confirmation

## ■ Tests within PPN frame

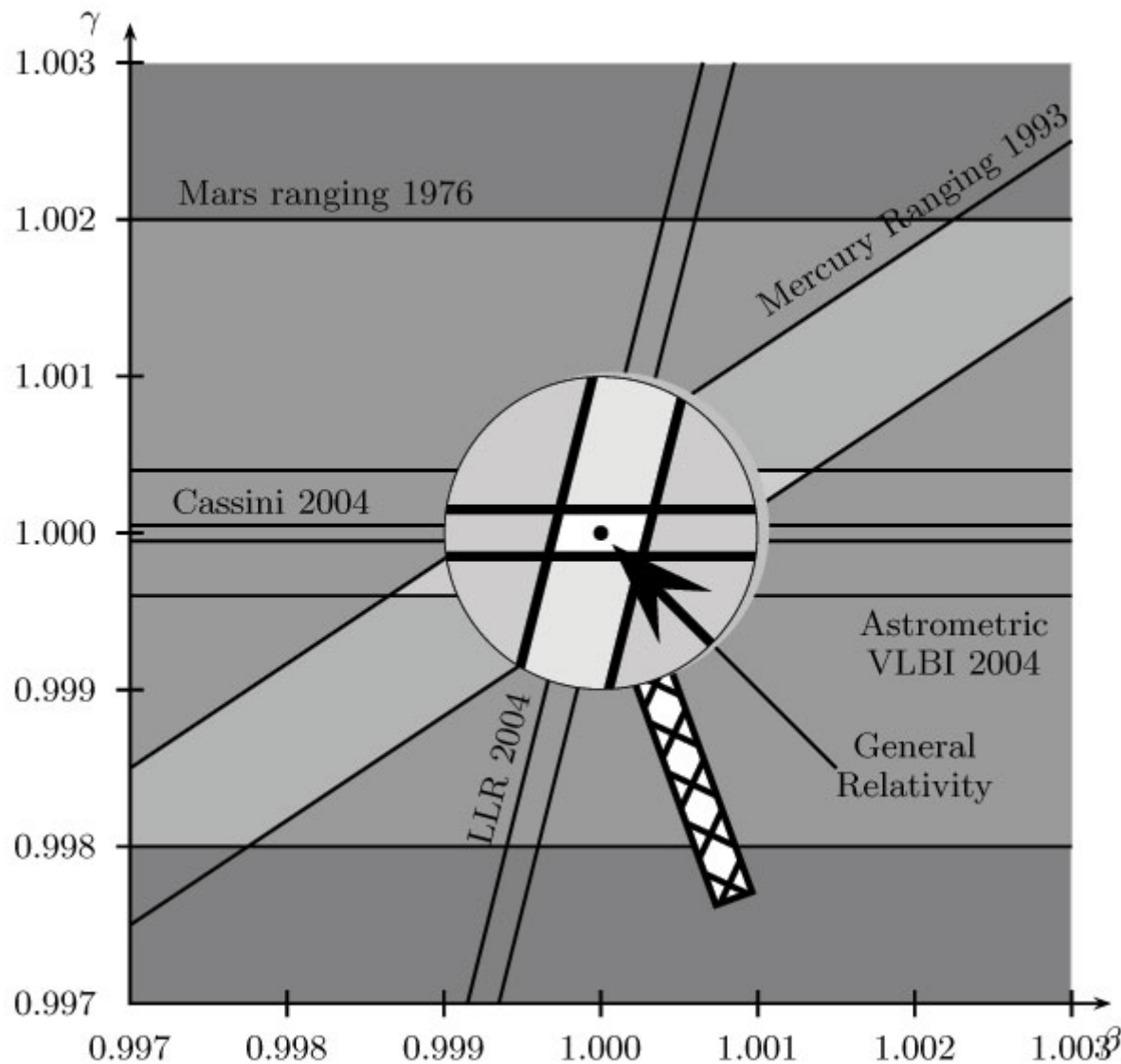
$$g_{00} = -1 + 2\alpha \frac{U}{c^2} - 2\beta \frac{U^2}{c^4}$$

$$g_{0i} = 4\mu \frac{(\vec{J} \times \vec{r})_i}{c^3 r^3}$$

$$g_{ij} = (1 + 2\gamma) \frac{U}{c^2}$$

perihelion shift	astronomical observations	$ \frac{2}{3}(\alpha + \gamma) - \frac{1}{3}\beta - 1  \leq 10^{-4}$
light deflection	Very Long Baseline Interference	$ \gamma - 1  \leq 10^{-4}$
time delay	Cassini S/C	$ \gamma - 1  \leq 2 \cdot 10^{-5}$
gravitational redshift	Gravity Probe A	$ \alpha - 1  \leq 1.4 \cdot 10^{-4}$
Lense-Thirring effect	LAGEOS satellites	$\leq 0.1$
Schiff effect	Gravity Probe B	$\leq 5 \cdot 10^{-3}$ not yet confirmed)

# Space experiments for Eddington parameters



*Turyshchev et al., 2006*

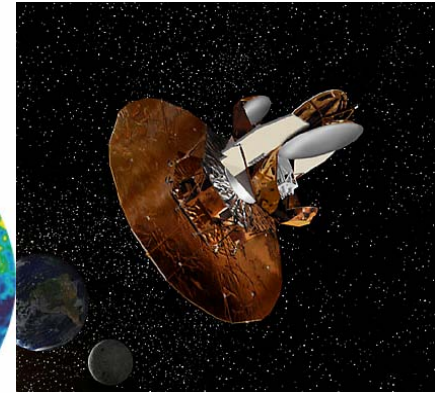
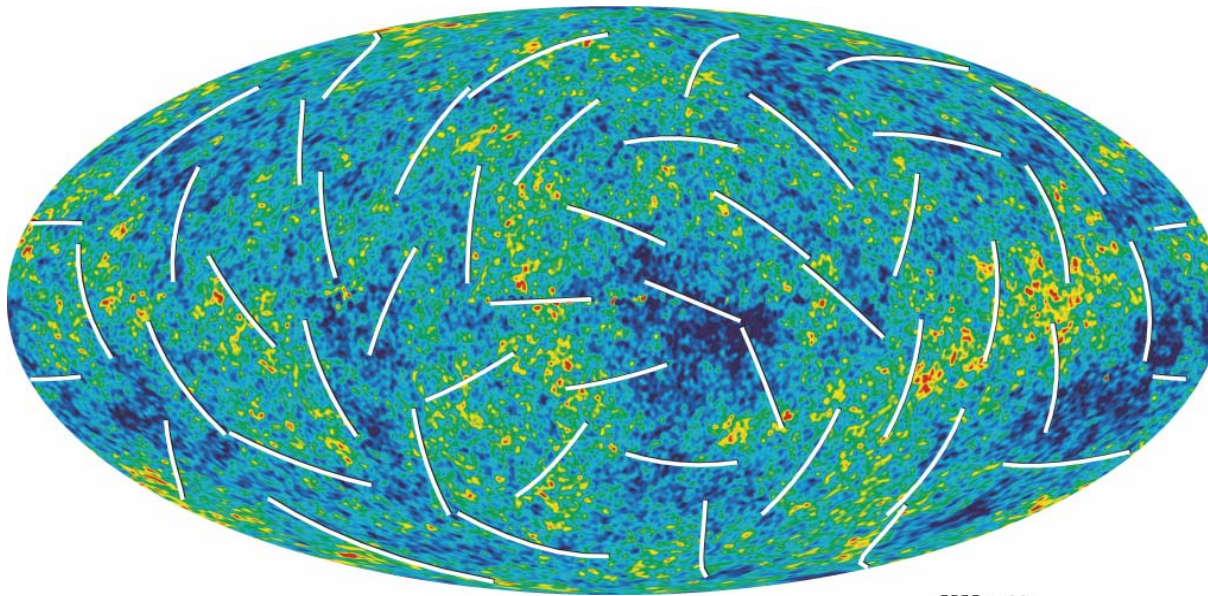
# Open questions and observed phenomena

- Unexplained phenomena within GR

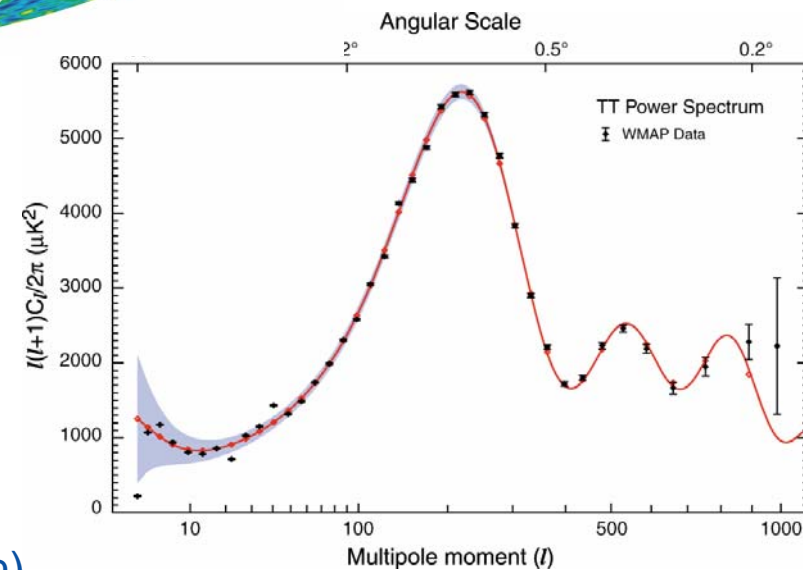




# W-MAP and the Cosmic Questions



The anisotropy of the cosmic microwave background  
measured by WMAP (3 year result 2006)  
*WMAP Science Team 2006*



- confirms cosmological model / inflation)
- dark energy /dark matter
- only 5 % of the Universe consists of „ordinary“ matter ??

# Open questions and observed phenomena

## ■ Unexplained phenomena within GR

### – Dark Matter (*Zwicky* 1933):

to describe galactic rotation curves, gravitational lensing effects and early structure formation in cosmological models

### – Dark Energy (*Turner* 1999):

to describe the accelerated expansion of the universe seen from supernovae observations and CMB anisotropy measurements



# Increase of the Astronomical Unit

## ■ Observation:

- *Krasinsky and Blumberg (2005)*:  $15 \pm 4$  m / 100 a
- *Pitjeva (in Standish (2005))*:  $7 \pm 1$  m / 100 a

## ■ Remarks and questions:

- $dG/dt \neq 0$  excluded by Lunar Laser Ranging
- Mass loss of Sun causes only 1 m / 100 a
- Influence by cosmic expansion many orders of magnitude too small
- Increase of solar wind plasma on long time scales ?
- Drift of clocks  $t \rightarrow t + \alpha t^2$  with  $\alpha \approx 3 \cdot 10^{-20} \text{ s}^{-1}$  ?

# Open questions and observed phenomena

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to describe galactic rotation curves, gravitational lensing effects and early structure formation in cosmological models
- **Dark Energy** (*Turner* 1999):  
to describe the accelerated expansion of the universe seen from supernovae observations and CMB anisotropy measurements
- **Increase of the Astronomical Unit** (*Pitjeva* 2005, *Krasinski* 2005):  
length scale related to the earth-sun distance increases by  $7 \pm 1$  m per 100 years (confirmed by astronomical observations); solar mass loss only explains ca. 1 m per century

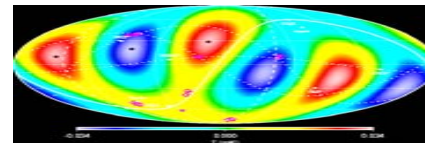
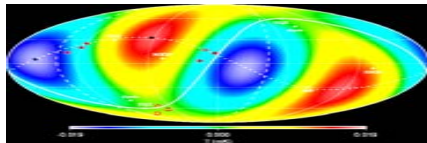
# Quadrupole / octopole anomaly

## ■ Observation:

- Anomalous behaviour of low  $\ell$  contributions to CMB quadrupole and octopole aligned to  $> 99.87\%$
- Quadrupole and octopole aligned to ecliptic to  $> 99\%$
- No correlation with the galactic plane  
(*Oliveira et al (2004)*, *Schwarz et al (2005)*)

## ■ Remarks and questions:

- Influence of solar system on CMB observations ?
- Systematics ?



# Open questions and motivation

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- **Quadrupole/Octopole Anomaly** (*Tegmark et al.* 2005, *Schwarz et al.* 2005):  
quadrupole and octopole of CMB are correlated with solar system eclipse

04)

# Open questions and motivation

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- **Quadrupole/Octopole Anomaly** (*Tegmark et al.* 2005, *Schwarz et al.* 2005):  
quadrupole and octopole of CMB are correlated with solar system eclipse
- **Pioneer Anomaly** (*Anderson et al.* 1998,2002/04)

# Pioneer Anomaly

- Pioneers 10/11: most precisely navigated deep space satellites (Jet Propulsion Lab., Pasadena CA)
- Observation of a small, anomalous, blue-shifted Doppler frequency drift (*Anderson et al.* 1998, 2002), uniformly changing with the rate of

$$\dot{f}_p = (5.99 \pm 0.01) \cdot 10^{-9} \text{ Hz / s}$$

- Drift can be interpreted as a sunward constant acceleration of

$$a_p = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m/s}^2$$

- This interpretation has become known as the Pioneer Anomaly:
  - Constant acceleration of the spacecraft **toward** the Sun
  - Analysed with data (1987–1998) for heliocentric distances 20 - 70 AU
  - Anomaly occurs when satellites have set to hyperbolic (escape) orbits
  - No real indication of how far out the anomaly goes.
  - Temporal and spatial variations are less than 3%



Viewed down from north ecliptic pole

Sun

Earth

Mars

Jupiter

Saturn

Uranus

Neptune

Pluto

Voyager 1

Voyager 2

'88

'86

'84

'82

'80

'78

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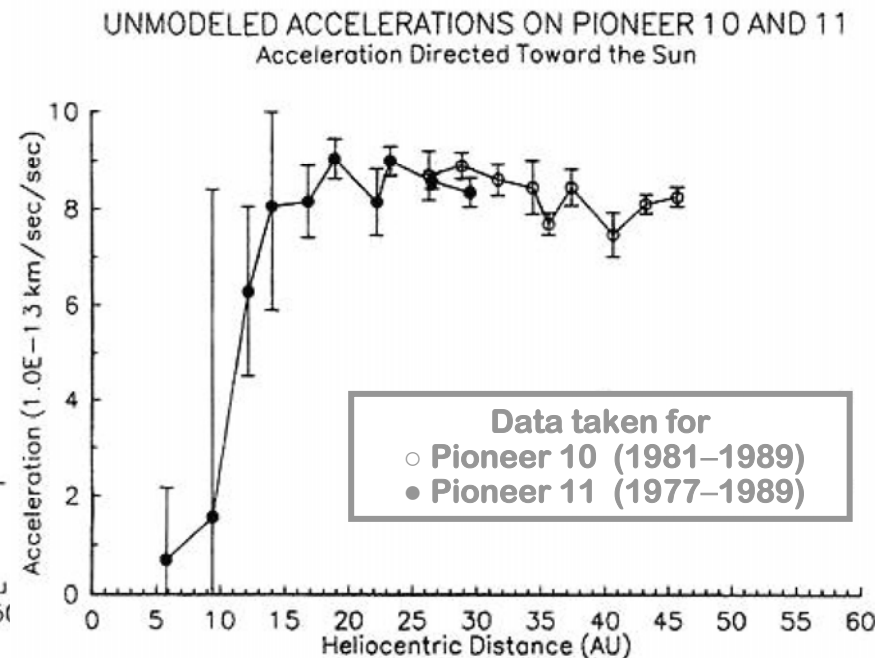
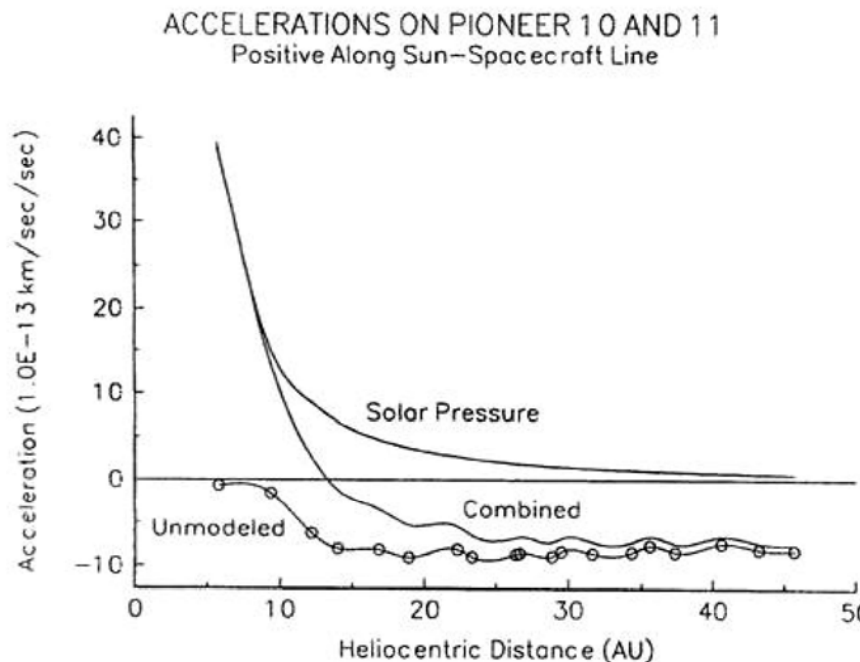
'92

'94

- Elliptical (bound) orbits **before** last fly-by
- Hyperbolic (escape) orbits **after** last fly-by

# Detection of the Anomaly

- Search for unmodeled accelerations with Pioneers started in 1979:
  - Motivation: search for Planet X – initiated when Pioneer 10 was at 20 AU;
  - The solar-radiation pressure **away** from the Sun became  $< 5 \times 10^{-10} \text{ m/s}^2$
- Original detection of the anomaly by JPL orbit determination in 1980:
  - The analysis found the biggest systematic error in the acceleration residuals is a constant bias  $a_p \sim (8 \pm 3) \times 10^{-10} \text{ m/s}^2$  directed **towards** the Sun



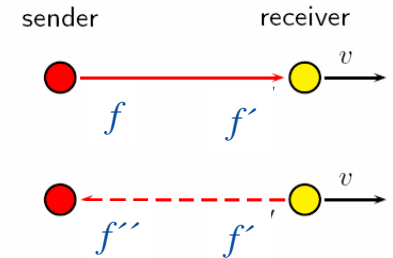
# Observed Anomalous Doppler Drift

frequency received at S/C:

$$f' = \frac{1}{\sqrt{1 - v^2/c^2}} \left( 1 - \frac{v}{c} \right) \cdot f$$

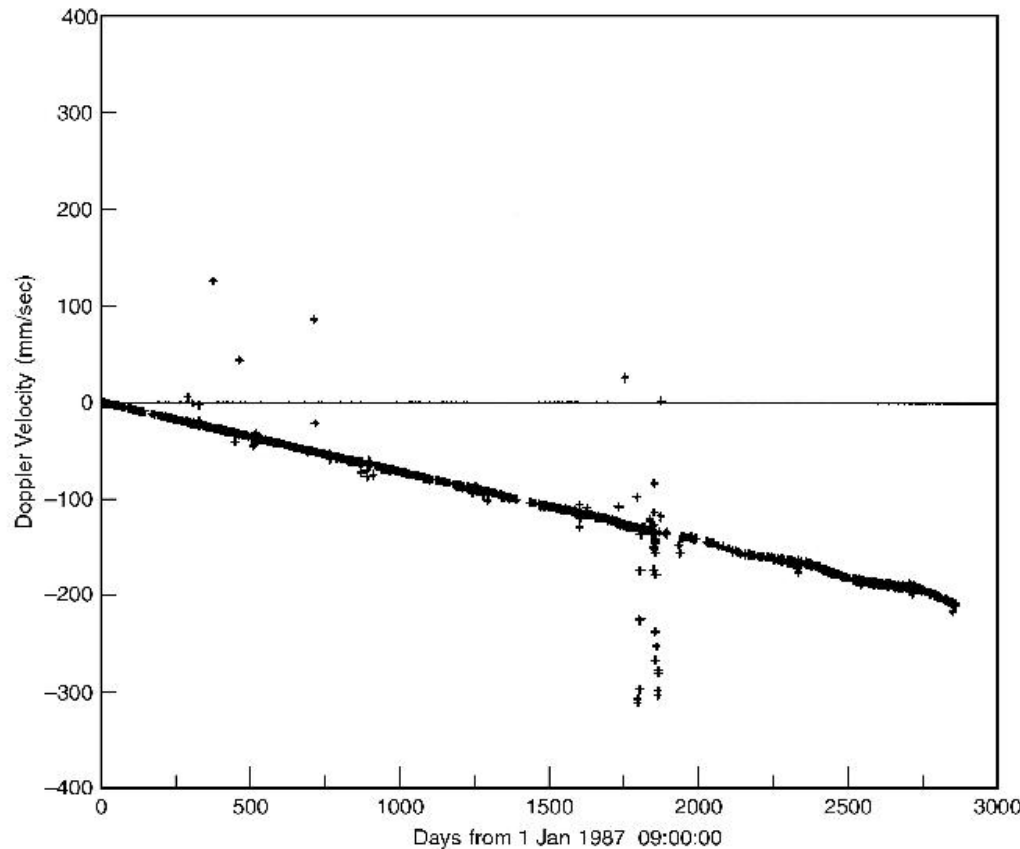
frequency sent back and  
received on earth:  
(neglecting the transponder shift)

$$f'' = \frac{1}{\sqrt{1 - v^2/c^2}} \left( 1 - \frac{v}{c} \right) \cdot f'$$



→  $\frac{f'' - f}{f} = -2 \frac{v/c}{1 + v/c} \approx -2 \frac{v}{c}$

$$v_{\text{observed}} - v_{\text{modelled}} = -a_p t$$



The two-way Doppler residuals for  
Pioneer 10 vs time  
[1 Hz is equal to 65 mm/s range  
change per second].

*Anderson et al.*

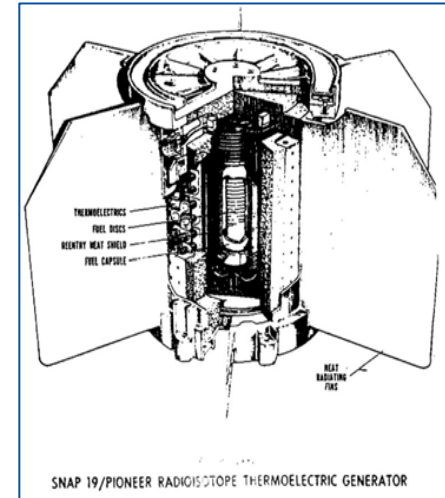
# Sources of External Systematic Error

error budget constituents	bias [ $10^{-10}\text{m/s}^2$ ]	uncertainty [ $10^{-10}\text{ m/s}^2$ ]
<b>sources of extrenal systematics</b>		
solar radiation pressure		$\pm 0.001$
→ sol. rad. press. from mass uncertainties	+ 0.03	$\pm 0.01$
solar wind		$\pm 0.00001$
solar corona effects		$\pm 0.02$
Lorentz force (em-effects)		$\pm 0.0001$
Kuiper belt's gravity		$\pm 0.03$
earth rotation		$\pm 0.001$
mechanical / phase stability of DSN antenna		$\pm 0.001$
clock effects on phase stability		$\pm 0.001$
DSN station location		$\pm 0.00001$
tropospheric and ionospheric effects		$\pm 0.001$
<b>computational systematics</b>		
numerical stability of least-square estimations		$\pm 0.02$
accuracy of consistency / model tests		$\pm 0.13$
→ mismodelling of manoeuvres		$\pm 0.01$
→ mismodelling of solar corona		$\pm 0.02$
annual / diurnal terms		$\pm 0.32$

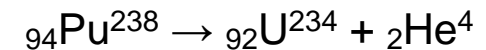
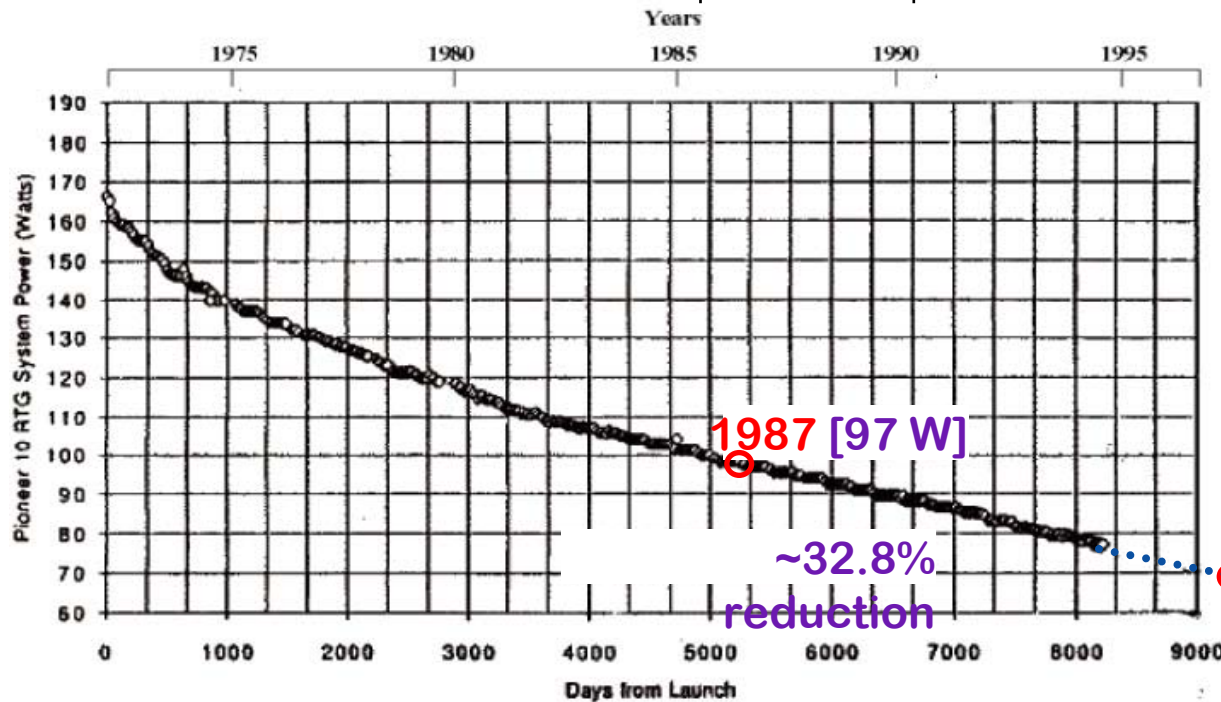
# On-board Systematics: Power and Heat

## error budget constituents

	bias [ $10^{-10}$ m/s <sup>2</sup> ]	uncertainty [ $10^{-10}$ m/s <sup>2</sup> ]
radio beam reaction force	+ 1.10	± 0.11
thermal and propulsion effects from RTGs		
→ RTG heat reflected off the S/C	-0.55	± 0.55
→ differential emissivity of the RTGs		± 0.85
→ non-isotropic radiative cooling of S/C		± 0.16
→ expelled He produced within the RTGs	+0.15	± 0.16
mass expulsion / gas leakage		± 0.56
variation between S/C determinations	+0.17	± 0.17



Radioisotope Thermoelectrical Generator (SNAP-19)



half life: 87.74 years

# A Drag Through Dust ?

## ■ Interplanetary Medium

- is a thinly scattered matter (neutral Hydrogen, microscopic particles) with two main contributions, IPD and ISD:
- Interplanetary Dust (IPD), modelled:

$$\rho_{IPD} \leq 10^{-24} \text{ g/cm}^3$$


- Interstellar Dust (ISD), measured on Ulysses S/C:

$$\rho_{ISD} \leq 10^{-26} \text{ g/cm}^3$$

## ■ Drag on a spacecraft is given

by:

$$a_{drag} = -c_s \frac{\rho(r) v_s A_s}{m_s}$$

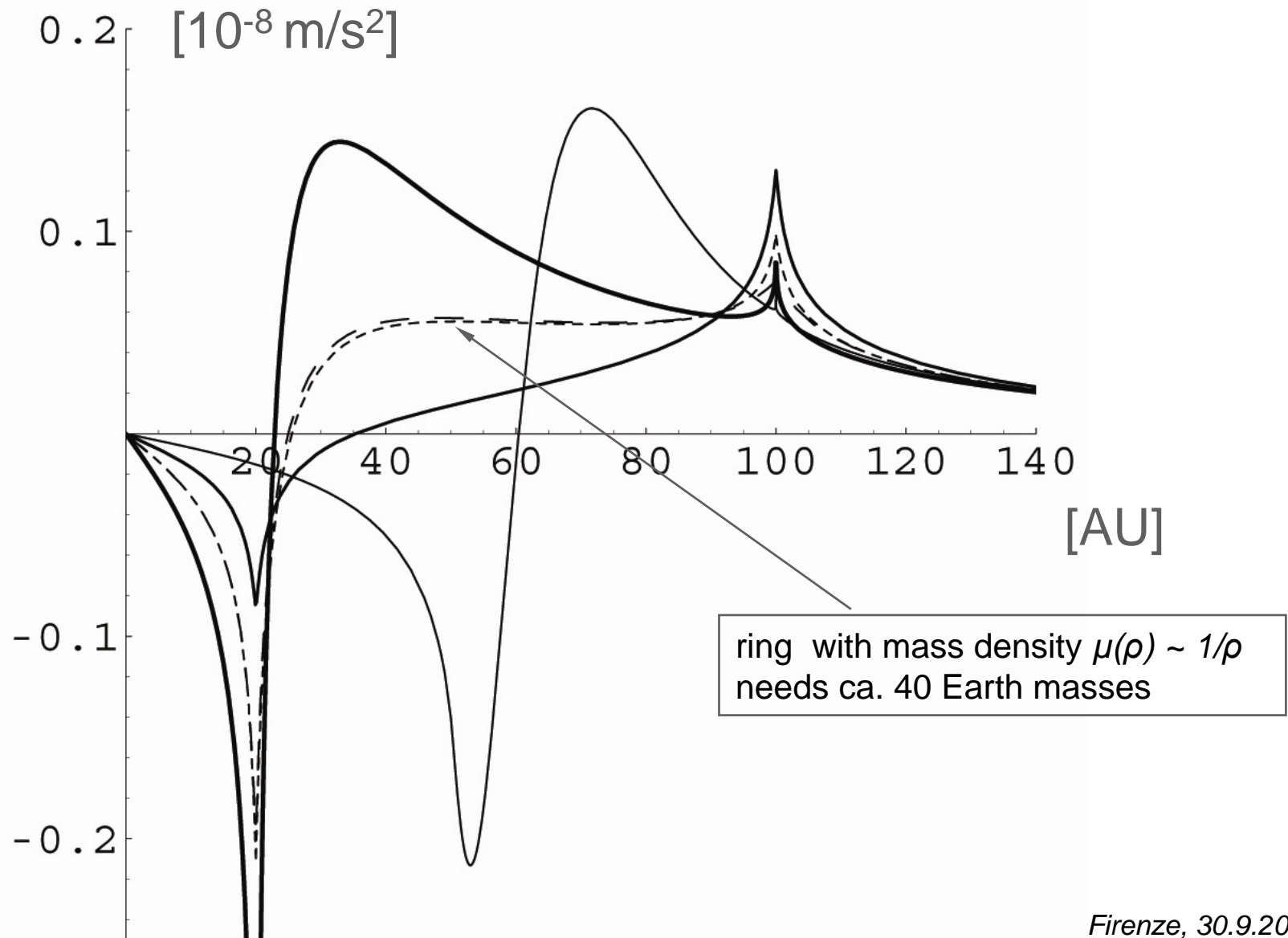
 The Pioneer Anomaly (between 20 and 70 AU) could only be explained with an axially-symmetric dust distribution with a constant uniform density of

$$\rho(r) \leq \rho_0 = 3 \cdot 10^{-19} \text{ g/cm}^3 \approx 300.000 (\rho_{IPD} + \rho_{ISD})$$



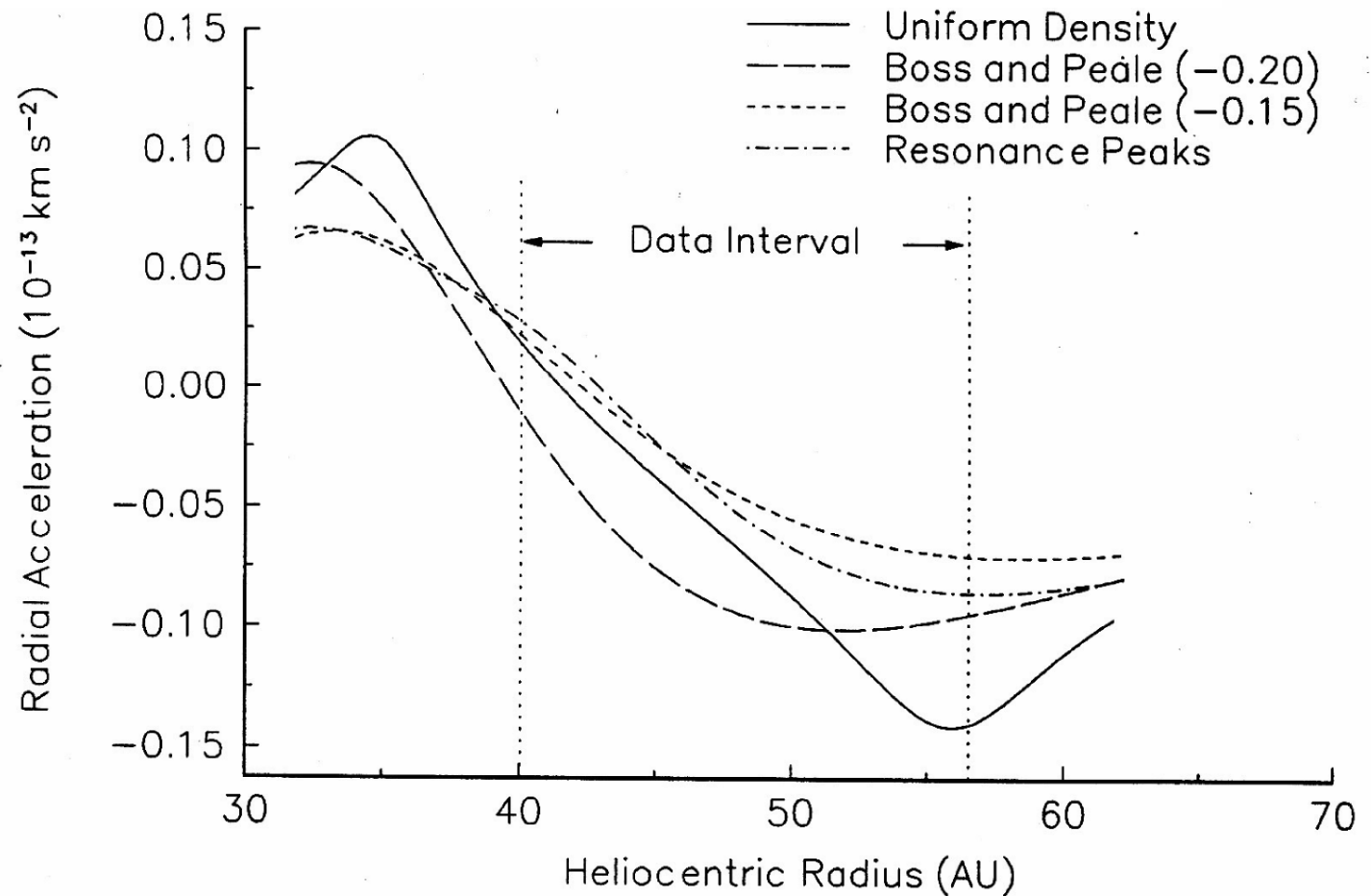
# Unexpected masses in the solar system (2)

- acceleration vs. distance for different mass density distribution  
(Nieto, 2005)



# Unexpected Masses in the Kuiper Belt

Models for 1 Earth mass in the Kuiper belt



# Suggested Explanations: New Physics

- Observation [REDACTED] stimulated some suggestions:
  - Gravity of the solar system is not static w.r.t. the cosmic expansion
  - Cosmological models with a time-varying gravit. constant  $G(t)$
- Scalar-field approaches:
  - Long-range scalar field, with oscillatory decline in  $a_p$ ,  $d \geq 100$  AU
  - Flavor oscillations of neutrinos in the Brans-Dicke theory of gravity may produce a quantum mechanical phase shift of neutrinos
  - A theory of conformal gravity with dynamical mass generation
- Drift of clocks
  - $t \rightarrow t + \alpha \ell^2$  with  $\alpha \approx 3 \cdot 10^{-19} \text{ s}^{-1}$ ?

# Doppler Tracking in the expanding universe

## ■ Cosmology

### – Dynamics in curved space-time

light rays:  $g(l, l) = 0, \quad g(k, k) = 0, \quad D_l l = 0$

point particles:  $g(v, v) = 1, \quad D_v v = 0$

---

### – Measured quantities

Observer 4-velocity  $u$  with  $g(u, u) = 1$

frequency:  $\nu_u = k(u)$

velocity:  $g(u, v) = 1 / \sqrt{1 - V^2 / c^2}$

---

### – Einstein-deSitter universe

metric:  $ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2)$

Hubble constant:  $H = \dot{R} / R$

deceleration parameter:  $q = -\frac{1}{H^2} \frac{\ddot{R}}{\dot{R}}$

# Doppler Tracking in the expanding universe

## ■ Redshifts

### – Conserved quantities

light

$$\nu_u(t)R(t) = \text{const.}$$

→ Hubble red shift

$$\nu_u(t) = \frac{R(t_0)}{R(t)} \nu_u(t_0) \approx (1 - H(t - t_0)) \nu(t_0)$$

massive particles

$$R^2(t) \frac{dr(s)}{ds} = \text{const.} \quad \Leftrightarrow \quad R(t) \frac{1}{\sqrt{1 - V^2(t)}} V(t) = \text{const.}$$

→ slow down at small velocities:

$$R(t)V(t) = \text{const.} \quad \Rightarrow \quad V(t_2) = \frac{R(t_1)}{R(t_2)} V(t_1) = (1 - H(t_2 - t_1)) V(t_1)$$

# Doppler Tracking in the expanding universe

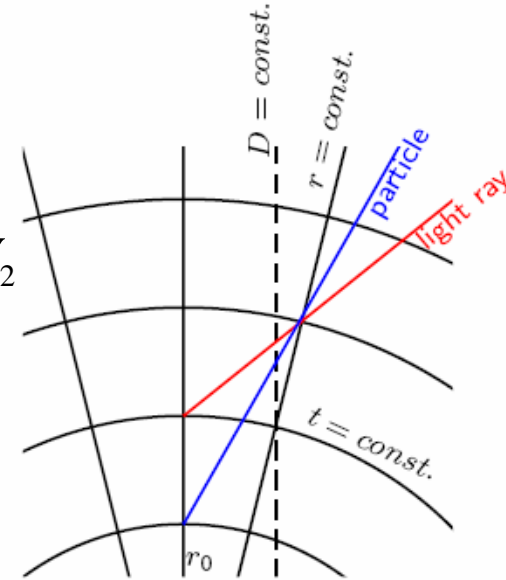
## ■ Kinematics

- Distance measured by time-of-flight of light rays

$$D = R(r_2 - r_1) \quad \text{with} \quad \frac{dD}{dt} = \dot{R}(r_2 - r_1) + R\dot{r}_2 = HD + V_2$$

- Trajectory at constant distance to observer has local velocity

$$0 = \dot{D} = HD + V_2 \quad \Rightarrow \quad V_2 = -HD$$



- Observer at rest in cosmic substrate
- Pioneer S/C move on geodesics and become slowed down
- Cosmic redshift of frequency
- Resulting Doppler effect (velocity of points of constant distance wrt cosmic substrate)

$$\nu_2(t_2) = (1 - H(t_2 - t_1) - V_2^{tot}) \nu_0(t_1) \quad \text{and} \quad V_2^{tot}(t_2) = H(t_2 - t_1) - V_2^{tot} H(t_2 - t_1)$$

- Red shift and Doppler effect from the velocity induced by constant distance cancel
- Only the satellite's slow down is left over.

$$a = HV = \frac{V}{c} cH \ll cH$$



# Yukawa modification? (1)

- Ansatz

(Lämmerzahl, 2005)

$$V(r) = G \frac{M_{Sun}}{r} \left( 1 + \alpha \cdot e^{-r/\lambda} \right)$$

- with Taylor extension and

$G_0 = (1 + \alpha)G$  as observed grav. constant for  $r \rightarrow \infty$

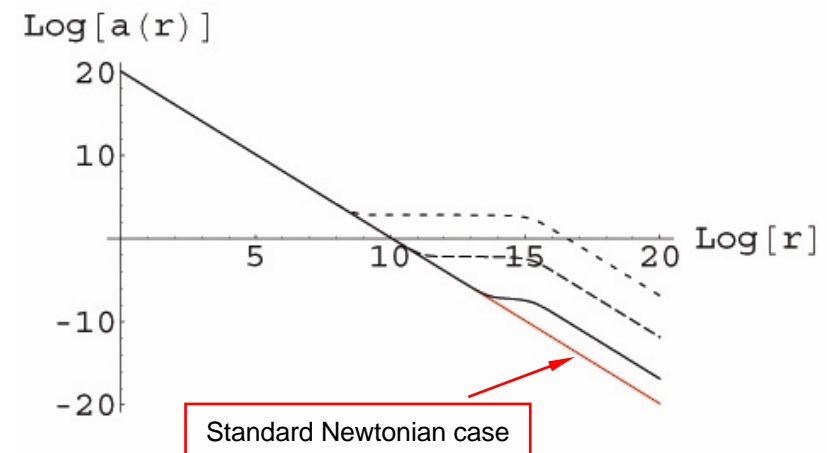
$$a(r) = -G_0 \frac{M_{Sun}}{r^2} + \text{[red circle]} - \frac{\alpha}{1 + \alpha} G_0 \frac{M_{Sun}}{3\lambda^2} \frac{r}{\lambda} + \dots$$

$= a_p$  (anomalous Pioneer acceleration)

- Next order term smaller by  $2/3(r/\lambda) \leq 0.06$  (could account for small decrease observed during missions)

- Strong  $\alpha (\approx 1)$

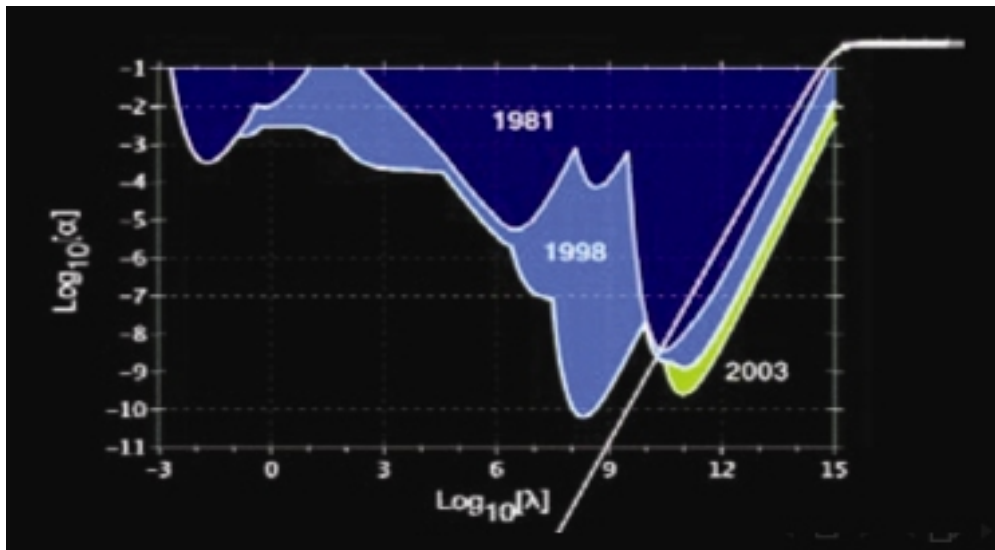
→ long range coupling  
→ acceleration plateau  
between ca. 1 – 100 AU.



# Yukawa modification? (2)

$$a_p = \frac{\alpha}{1 + \alpha} G_0 \frac{M_{Sun}}{2\lambda^2} \Rightarrow \alpha = \frac{2\lambda^2 a_p}{G_0 M_{Sun} - 2\lambda^2 a_p}$$

$$\Rightarrow \lambda \geq \sqrt{\frac{G_0 M_{Sun}}{2a_p}} = 2.8 \cdot 10^{14} \text{ m}$$



## A viable model?

### ■ Pioneer anomaly

$$\log_{10}(\lambda) > 16, \alpha + 1 \leq 10^{-5}$$

### ■ Galactic rotation curves

$$\log_{10}(\lambda) > 16, \alpha + 1 \leq 10^{-1}$$

### ■ Local strength: modification by „Yukawa in Yukawa“

$\log_{10}(\lambda) > 16$  for  $\log_{10}|\alpha| = 1$  compatible with present experimental results in the solar system (including planetary orbits)

(Lämmerzahl, 2005)

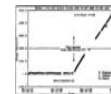
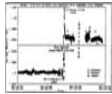
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- **Quadrupole/Octopole Anomaly** (*Tegmark et al.* 2005, *Schwarz et al.* 2005):  
quadrupole and octopole of CMB are correlated with solar system eclipse
- **Pioneer Anomaly** (*Anderson et al.* 1998/2002)
- **Fly-by Anomalies** (*Antresian and Guinn* 1998, *Anderson and Williams* 2001, *Morley* 2005, *Campbell* 2006)  
confirmed for 3 satellites  
satellite trajectory velocities are too high by some [mm/s] after planetary fly-bys / require non-conservative gravitational potential

# Fly-by Anomaly

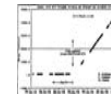
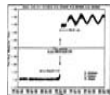
- 2-way S-band Doppler and range residuals during fly-bys at Earth show an exit (asymptotic) velocity greater than expected reported by several authors (*Antreasian & Guinn 1998, Anderson & Williams 2001, Morley 2005, Preuss 2006*)



Galileo

2-way Doppler  
S-band residuals

range residuals



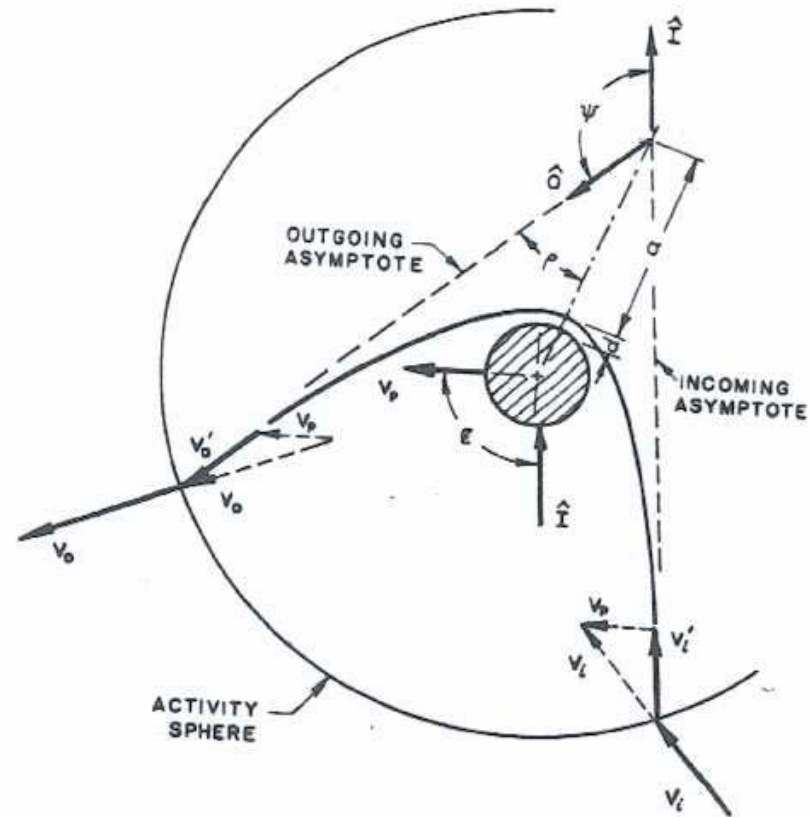
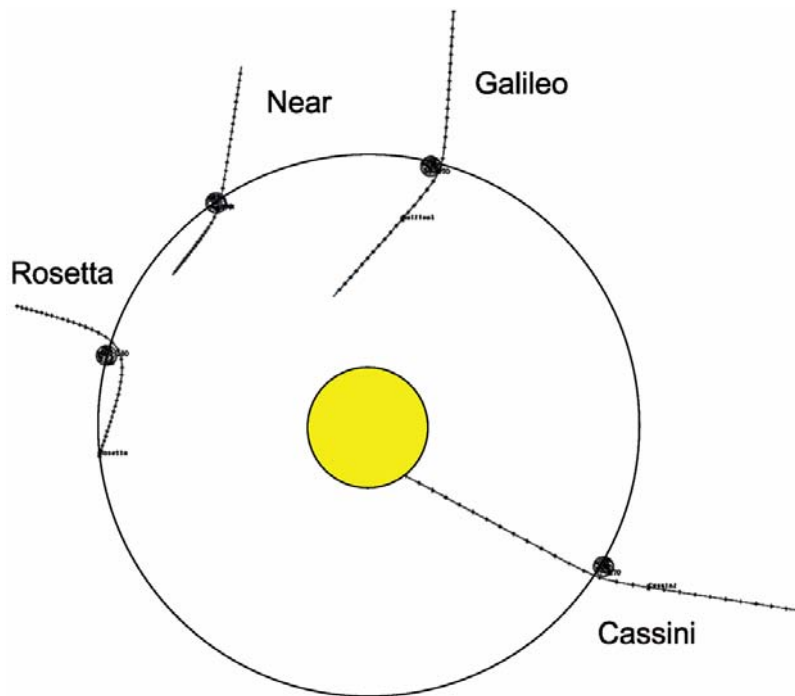
NEAR

2-way Doppler  
S-band residuals

range residuals

# Fly-by / gravity assist maneuver

$$\begin{aligned}\Delta E_{kin} / m_{sat} &= (v_0^2 - v_i^2) / 2 \\ &= v_{earth} (v'_0 - v'_i)\end{aligned}$$



Anderson, Campbell, Nieto, 2006

# Earth fly-by's analyzed

	Galileo (1 <sup>st</sup> fly-by)	NEAR	Cassini	Rosetta	Messenger
$v_{\infty}$ [km/s]	8.949	6.851	16.01	3.863	4.056
$v_F$ [km/s]	13.738	12.739	19.03	10.517	10.389
$h$ [km]	956	532	1,172	1,954	2,336
$\varepsilon$	2.47	1.81	5.86	1.31	1.13
$\Theta$ [°]	47.67	66.92	19.66	99.396	94.7
$i$ [°]	142.9	108.0	25.4	144.9	133.1
Fly-by	8.12.1990	23.1.1998	18.8.1999	4.3.2005	2.8.2005
$\Delta v_{\infty}$ [mm/s]	$3.92 \pm 0.08$	$13.46 \pm 0.13$		$1.82 \pm 0.05$	
$\Delta v_F$ [mm/s]	$2.56 \pm 0.05$	$7.24 \pm 0.07$	$ -0.2 $ (?)	$0.67 \pm 0.02$	$O(0)$

- to be implemented: Hayabusa fly by 05/2004 ( $h = 3,725$  km)
- Cassini data not reliable due to perigee manoeuver
- 2<sup>nd</sup> Galileo fly by too deep / large atmospheric influence

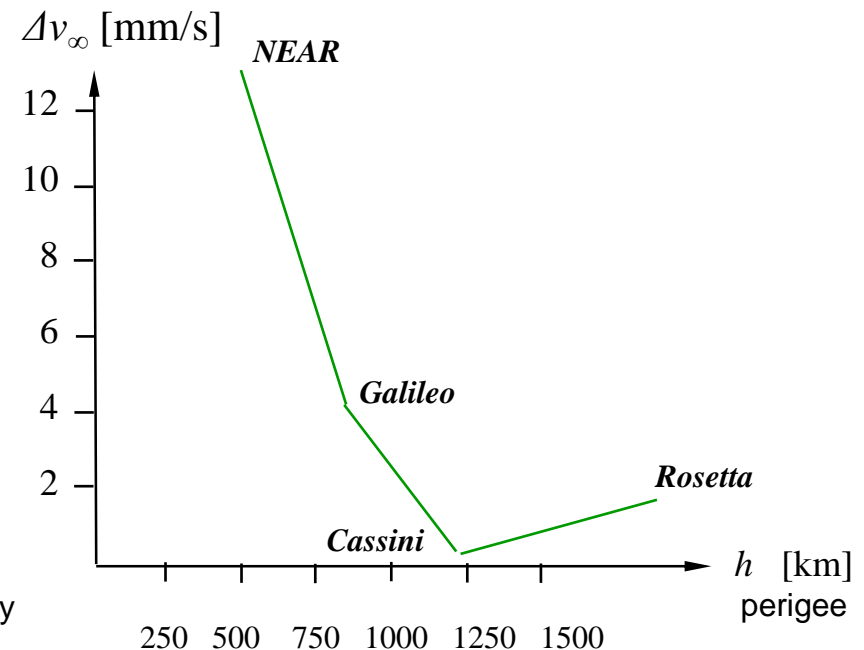
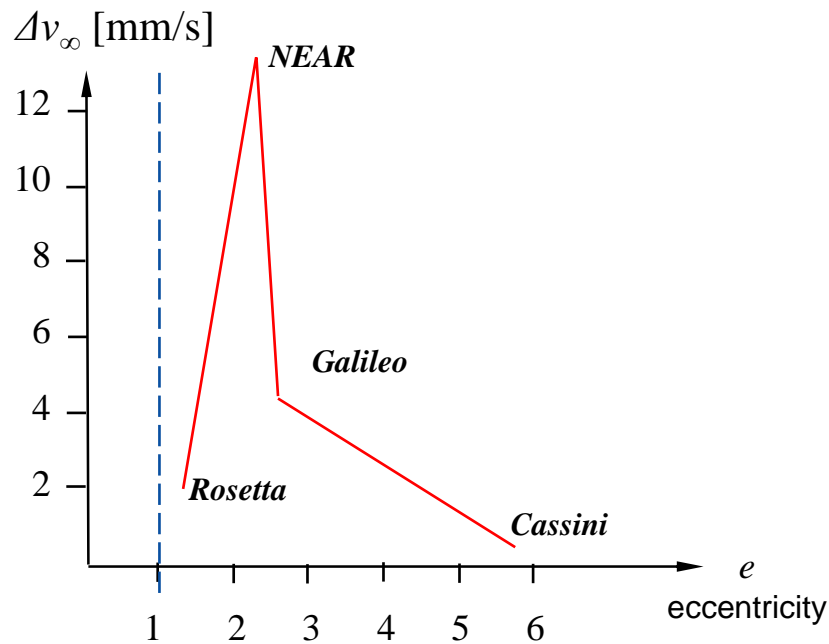


# Error analysis

error budget constituents	bias [ $10^{-5} \text{ m/s}^2$ ]
Atmospheric drag	- 0.0001
Ocean tides	$\pm 0.1$
Solid earth tides	$\ll  0.15 $
S/C charging (modeled / analyzed for LISA; for charging $Q < 10^{-7} \text{ C}$ )	$\pm 0.0001$
Magnetic moments ( $< 2 \cdot 10^{-7} \text{ G/m}$ )	$\pm 10^{-10}$
Earth albedo (1 t S/C)	$\pm 0.00024$
Solar wind	$\pm 0.0003$
Relativistic corrections $U \cdot v^2 / c^2 \approx 10^{-20}$	not affecting
Spin rotation coupling (coupling of the helicity of radio waves with S/C spin and Earth rotation ( only effective for 2-way Doppler ranging)	not affecting

# Phenomenological observations

- $\Delta v$  decreases with increasing eccentricity and perigee height
- $\Delta v$  disappears at  $e = 1$  (as expected for bound orbits)



$$a_{Fly-by} = 10^{-4} \text{ m/s}^2$$

$$\frac{a_{Fly-by}}{a_{Newton}} \approx 10^{-5} \approx \frac{a_{PA}}{a_{Newton}}$$

# General modification of particle motion

## ■ Universality of Free Fall

$$0 = v^\nu \partial_\mu v^\nu + H^\mu(x, v) = v^\nu \partial_\mu v^\nu + \left\{ \begin{smallmatrix} \mu \\ \rho\sigma \end{smallmatrix} \right\} v^\rho v^\sigma + \text{non-metric}$$

## ■ Non-relativistic approximation / expansion for small velocities

$$\frac{d^2 x^i}{dt^2} = - \left( \left\{ \begin{smallmatrix} i \\ \mu\nu \end{smallmatrix} \right\} - \left\{ \begin{smallmatrix} 0 \\ \mu\nu \end{smallmatrix} \right\} \frac{dx^i}{dt} \right) \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} + \left( \frac{dt}{ds} \right)^{-2} \left( \gamma^i(v, x) - \frac{dx^i}{dt} \gamma^0(v, x) \right)$$

$$\approx \text{Newton} + \text{Lense-Thirring} + \dot{x}^2 \partial_i V + \dot{x}^i \dot{V} + Y^i + Y_j^i \dot{x}^j + Y_{jk}^i x^j x^k + \dots$$

with:

$$Y^i = A_{11} \frac{GM}{r^2} \frac{r^i}{r}$$

$$Y_j^i = A_{21} \frac{GM}{r^2} \frac{r^i r^j}{r^2} + A_{22} \frac{GM}{r^2} \delta_j^i$$

$$Y_{jk}^i = A_{31} \frac{GM}{r^2} \frac{r^i r^j r^k}{r^3} + A_{32} \frac{GM}{r^2} \frac{r^i}{r} \delta_{jk} + A_{33} \frac{GM}{r^2} \frac{r^j}{r} \delta_k^i$$

# General modification of particle motion

## ■ Acceleration terms obtained:

$$\ddot{x}_{(1)}^i = A_{11} \frac{GM}{r^2} \frac{r^i}{r} \quad (\text{Newton})$$

$$\ddot{x}_{(2)}^i = \underbrace{\left( \text{blue oval} \right) \frac{GM}{r^2} \frac{\dot{r}_{11}^i}{c}}_{\substack{\text{chosen} = 0 \\ \text{0 at perigee} \\ \text{0 for } r \rightarrow \infty}} + \underbrace{\left( \text{red oval} \right)}_{\substack{\text{Maximum at perigee} \\ \text{applies to fly-bys}}} \approx 10^{-4} \text{ m/s}^2$$

$$\ddot{x}_{(3)}^i = A_{31} \frac{GM}{r^2 c^2} \frac{\dot{r}^i (r \cdot \dot{r})^2}{r^3} + A_{32} \frac{GM}{r^2 c^2} \frac{\dot{r}^i}{r} \dot{r}^2 + A_{33} \frac{GM}{r^2 c^2} \frac{\dot{r}^i}{r} (r \cdot \dot{r}) \ll 10^{-4} \text{ m/s}^2$$

## ■ Remarks:

- Energy not conserved
- Universality of Free Fall still valid
- Gravity cannot be transformed away (contradicts the Einstein elevator)
- $r$  – dependence in all terms (cannot explain Pioneer anomaly)
- Unstable bound orbits: How can dynamical equations distinguish between bound and escape orbits?
- Hyperbolic orbits are rare natural orbits.

# Clocks to explore the anomalies

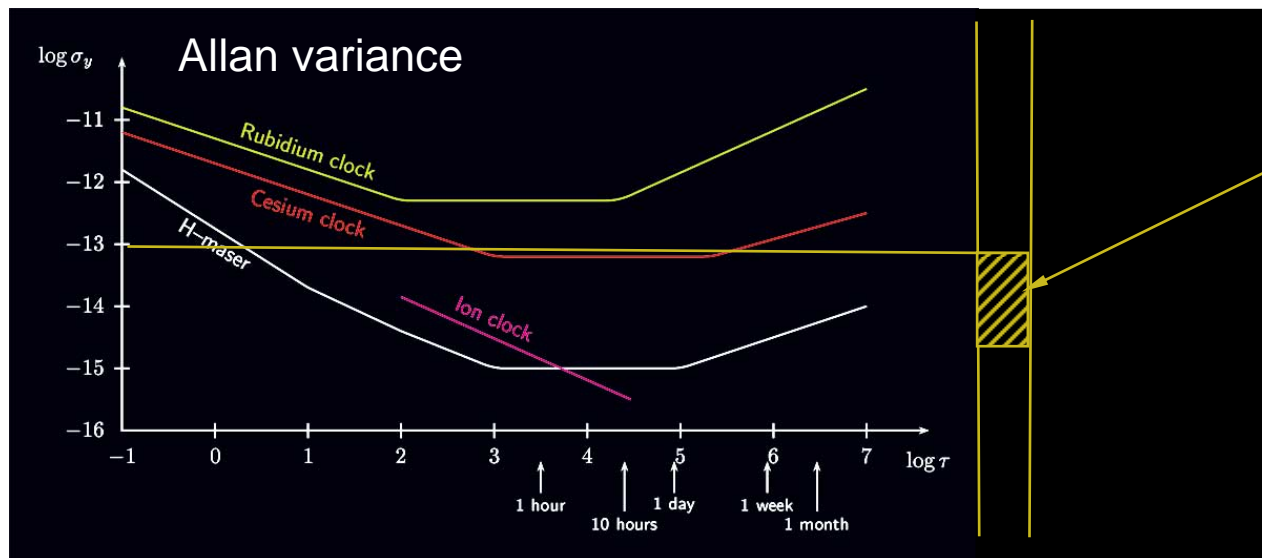
## ■ Redundant measurements

- Measuring acceleration of S/C on geodesic via ranging and Doppler tracking
- Measuring redshift of clocks on-board S/C

for Pioneer Anomaly

$$\frac{\Delta\nu}{\nu} = \frac{1}{c^2} \int_{20AU}^{90AU} a_{PA} dx \approx 10^{-13}$$

- Clock exploration does not depend on geodesic motion, independent from acceleration
- Clock exploration is cumulative
- Clocks automatically isolate the pure gravity sector
- Clocks represent an absolute DC-accelerometer



DSGP requirement

**Challenge:**  
*long term stability*

# Conclusion and final remarks

## ■ Unexplained phenomena

- Dark matter (does it affect solar system physics?)
- Dark energy
- Increase of AU
- Quadrupole / Octopole anomaly
- Pioneer Anomaly
- Fly-by anomalies

$$a_{Fly-by} = 10^{-4} \text{ m/s}^2 \qquad \frac{a_{Fly-by}}{a_{Newton}} \approx 10^{-5} \approx \frac{a_{PA}}{a_{Newton}}$$

## ■ It's worth to discuss the anomalies

- Try to find systematics
- Try to find conventional explanations
- Try to find relations between anomalies (Anomalies most probably are not isolated phenomena.)
- Are there similar effects in other gravitating systems?
- What's about hyperbolic orbits?

## ■ Observation of future fly-bys of satellites

- Rosetta Mars fly-by 02 / 2007 (orbital height: ca. 250 km)
- Rosetta Earth fly-by 11 / 2007 (orbital height: ca. 5,000 km)
- Rosetta Earth fly-by 11 / 2009 (orbital height: ca. 2,500 km)
- New Horizon Jupiter fly-by in 2008 ?

## ■ Use clocks