What I will assume in these 4 sessions is that everyone is familiar with the basic ideas of AdS/CFT.

(Basically that some backgrounds of String theory are dual to [capture the same information] as some (gauge) field theories.)

The point of these meetings is to discuss how one can learn about (particularly interesting) QFT using String theory.

This topic is VERY extense.

So, in these sessions I will focus on very specific aspects of this duality (but quite interesting ones!)

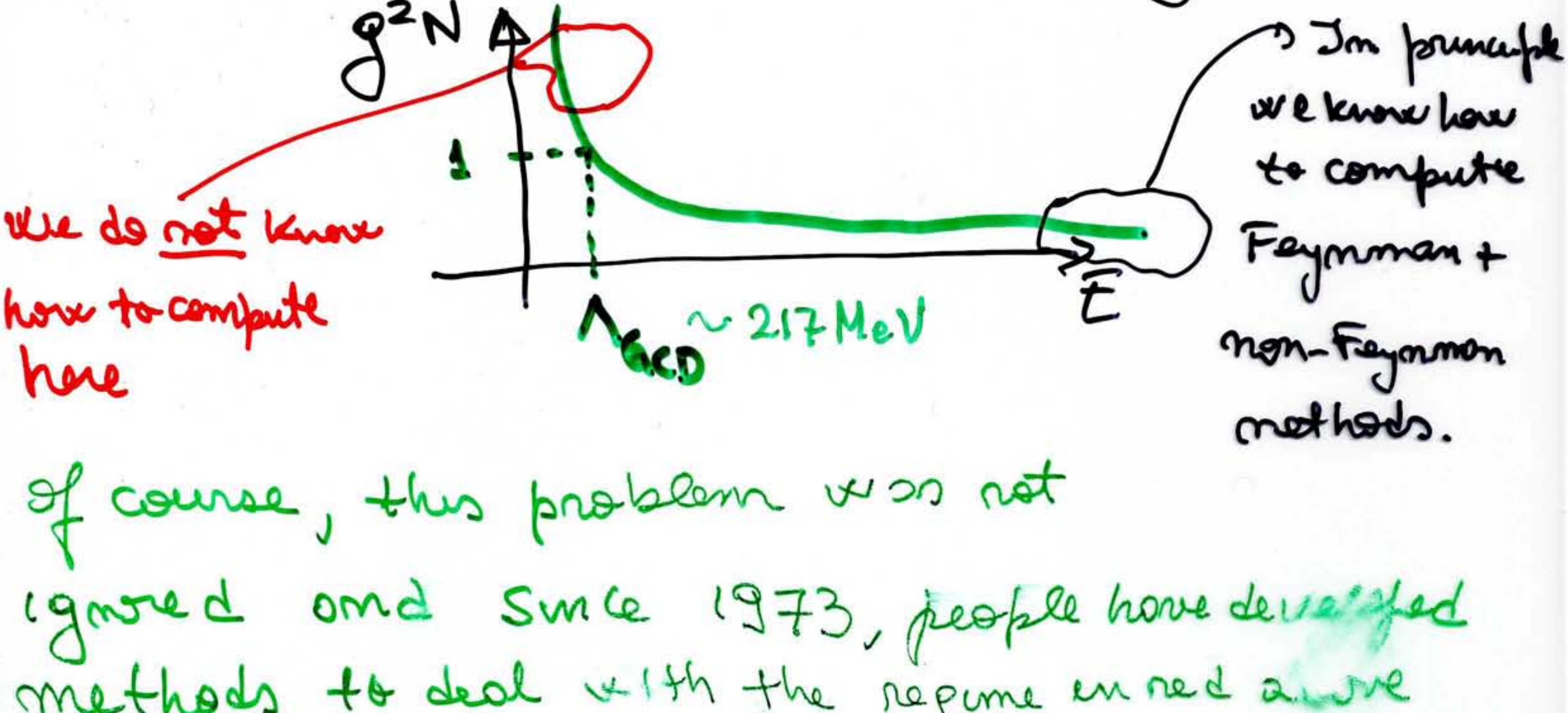
The problem of Physics behind these lectures is an old one:

[How to compute non-perturbative aspects of any given QFT (focus on gauge theories).]

The main Physics example to have in mind is "Nuclear Physics" (the "chemistry" of QCD)

$$\mathcal{L} = \mathrm{Tr}\left\{-\frac{1}{4}F_{\mu\nu}^2 + i\,\bar{\Psi}_k(\not{D} - m_k)\Psi_k\right\} \quad \begin{pmatrix} \text{group is } \overset{SU(N)}{SU(3)} \\ \text{and } k: 1, \dots, 6 \end{pmatrix}$$

We know that in this field theory



Of course, this problem was not ignored and since 1973, people have developed methods to deal with the regime in red above

These methods range from very good ones to very "effective" ones

- Lattice : the "correct way". Limitations that will be overcome with more computing power and new algorithms (prevents an analytic understanding)
- QFT elaborations : very nice, not so sucesfull

- Effective field theory : works by definition, but postpones the understanding of the connection between the effective Lagrangian and $\mathcal{L}_{QCD}$

In these lectures I will use a method inspired on String theory, that is the so called Maldacena Conjecture (or AdS/CFT and its extensions)

I will focus on uses of this conjecture to learn about phenomenologically interesting field theories

This topic is so extense that I will focus a little more. The basic problem I would like to address in these lectures is

"How to learn about non perturbative aspects of $\mathcal{N}=1$ SUSY gauge field theories in 3+1 dimensions"

(the small # of SUSY ⇝ good caricature of QCD among other 3+1 d interacting field theories)

Let me start by mentioning what I will not talk about. (even in this narrow topic there are different and very succesfull approaches)

I will not discuss:

Models starting with $\mathcal{N}=4$ ⇝ $\mathcal{N}=1$ [Girardello, Petrini, Porrati Zaffaroni; Polchinski + Strassler; Freedman, Gubser Pilch Warner + ...

Models based on cascading quivers [Klebanov + Tseytlin; Nekrasov + Strassler + Witten + Dymarsky + Seiberg + Murugan + Benini]

AdS/QCD [Erlich Katz, Son, Stephanov; Da Rold + Pomarol; ⋮

Models based on Black holes ⇝ Heavy Ion Physics [Policastro + Son + Starinets + ...]

But I will be quite pleased to discuss them outside the classes

So, what will I (try to) talk about?

. Models ~~of~~ using wrapped branes duals to

$\mathcal{N}=1$ SYM and $\mathcal{N}=1$ SQCD

and discuss in detail how can we learn about non perturbative aspects of these field theories looking at (computing with) String theory.

The topic may look narrow, but is actually quite extended, So, I will pick some particular aspects.

Of course, outside the lectures, we can discuss about everything I will not mention or other things.

$\boxed{\text{So, to summarize, the idea is to learn about } \mathcal{N}=1 \text{ SYM and } \mathcal{N}=1 \text{ SQCD from string theory. This is the topic of what follows}}$

Wrapped branes

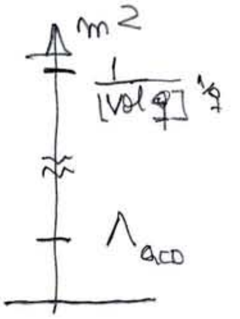
The idea is to construct a dual to a phenomenologically interesting field theory + <u>some UV completion</u> (that may have no relation with the theory itself)

One possibility is to take a Dp brane with $p>3$ and "carefully" wrap it over a q manifold (if the q-manifold is "small") $\rightsquigarrow$ we have an effective $p+1-q$ field theory. Two things should be clarified

① "carefully": so that this wrapping preserves some SUSY. (for example $\to$ $\boxed{\text{BPS eqs}}$)

② "small": compared with the inverse dynamical scale of the theory

the idea is:



$\rightsquigarrow$ so that the dynamics we would like to understand (non perturbative field theory is well separated from the UV completion)

How to do this actually? let us see different examples

D4 on S^1 $\rightsquigarrow$ Witten 1998 $\rightsquigarrow$ ~~[illegible]~~ wrapping is done with different boundary conditions for fermions and bosons. $\to$ a solution can be written that is non-SUSY.

D4 brane wrapping S^1 with anti-periodic boundary conditions

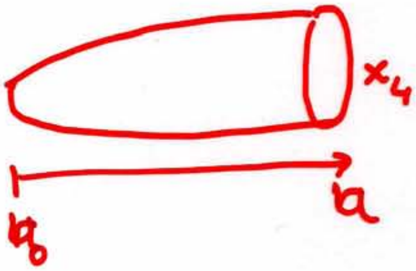
(Witten 1998).

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[-dt^2 + d\vec{x}_3^{\,2} + f(u)\,dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

$$F_4 = \frac{2\pi N_c}{\mathrm{Vol}\,S^4}\cdot \omega_{S^4} \quad ; \quad e^{2\phi} = g_s\left(\frac{u}{R}\right)^{3/2}$$

$$R = \pi g_s N_c \alpha'^{3/2} \quad ; \quad f(u) = 1 - \left(\frac{u_0}{u}\right)^3$$

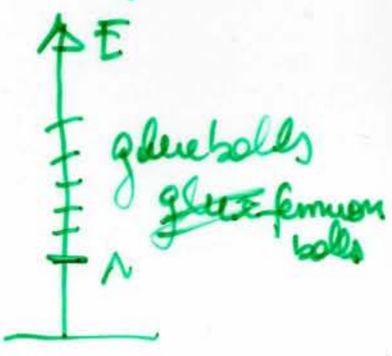
$\rightsquigarrow$ x_4, u have a cigar topology



—o—

This background may be thought of as an analytic continuation from a D4 brane black hole

the weakly coupled spectrum strongly coupled



T4.5

In this case a particular [illegible] A background exists

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[-dt^2 + d\vec{x}_3^{\,2} + f(u)\,dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4\right]$$

$$F_4 = 2\pi N \ \mathrm{vol}\,\Omega_4$$

$$e^{2\phi} = g_s^2 \left(\frac{u}{R}\right)^{3/2}$$

$$R = \pi g_s N_c (\alpha')^{3/2} \qquad ; \ f(u) = 1 - \left(\frac{u_0}{u}\right)^3$$

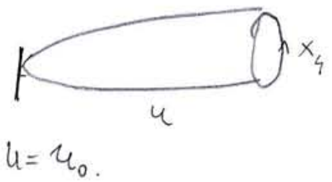
Ths can be thought of as taking the D4 black hole and

continuing $x_4 \to i t$

$t \to i x_4$

Transparencies

The sub-manifold $x_4 - u$ mo a cgr



$$g_5^2 = (2\pi)^2 g_s (\alpha')^{1/2}$$

$$g_4^2 = \frac{g_5^2}{2\pi R}$$

$$M_{\text{glueballs}} = \frac{1}{R}$$

$$T_{QCD} = \frac{1}{2\pi\alpha'}\sqrt{g_{xx}\,g_{tt}}\,\Big|_{u=u_0} = \frac{1}{2\pi\alpha'}\left(\frac{u_0}{R}\right)^{3/2}$$

When wrapped D6 branes on a S^3 one obtains a metric that when lifted to 11 dimensional supergravity reads:

$$ds^2 = dx_{1,3}^2 + \alpha^2 dr^2 + \gamma^2 \tilde{\omega}_a^2 + \beta^2 \left(\omega^a - \frac{1}{2}\tilde{\omega}^a\right)^2$$

$$\alpha^2 = \frac{1}{1 - a^3/r^3} \quad ; \quad \beta^2 = \frac{r^2}{9}\left(1 - a^3/r^3\right) \ ; \quad \gamma^2 = r^2/12$$

$$\tilde{\omega}_1 = \cos\tilde{\psi}\, d\tilde{\theta} + \sin\tilde{\psi}\sin\tilde{\theta}\, d\tilde{\varphi} \qquad \omega_1 = \cos\psi\, d\theta + \sin\psi\sin\theta\, d\varphi$$

$$\tilde{\omega}_2 = -\sin\psi\, d\tilde{\theta} + \cos\tilde{\psi}\sin\tilde{\theta}\, d\tilde{\varphi} \qquad \omega_2 = -\sin\psi\, d\theta + \cos\psi\sin\theta\, d\varphi$$

$$\tilde{\omega}_3 = d\tilde{\psi} + \cos\tilde{\theta}\, d\tilde{\varphi} \qquad \omega_3 = d\psi + \cos\theta\, d\varphi$$

In IIA this looks like a background with

$g_{\mu\nu}$, Φ, F_2

$$ds^2 = e^{4/3\phi}(dx_{11} + A)^2 + e^{-2/3\phi} ds_{10}^2$$

One can see that (a few) aspects of $\mathcal{N}=1$ SYM are captured by the background above

Another possibility is to wrap a D6 brane on a 3 cycle

$g_{\mu\nu}$, Φ, F_2. When lifted to 11 dimensions these solutions

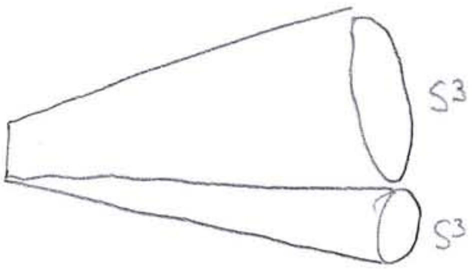
look like M theory on a G_2 manifold. The first solution I know of

was written by Bryant + Salamon and used by Atiyah, Maldacena, Vafa

$$ds^2 = dx_{1,3}^2 + \alpha^2 dr^2 + \gamma^2 \tilde{\omega}_a^2 + \beta^2 (\omega^a - \tfrac{1}{2}\tilde{\omega}^a)^2$$

$$\alpha^2 = \frac{1}{1 - \frac{a^3}{r^3}} \quad ; \quad \beta^2 = \frac{r^2}{9}\left(1 - \frac{a^3}{r^3}\right) \quad ; \quad \gamma^2 = \frac{r^2}{12}$$

when reduced to type IIA

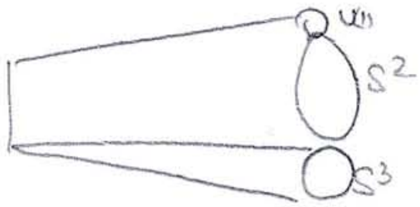


one must pick an invariant direction to do the reduction, the size of this direction is the value of the dilaton

$$ds_{11}^2 = e^{\frac{4}{3}\phi}(dx_{11} + A)^2 + e^{-2\phi/3} ds_{10}^2$$

$\boxed{\text{minimal twisting gauge}}$

So, a problem of interest in 2002 was to find G_2 manifolds with a stabilized $U(1)$, that is:



this problem was solved by

BGGG

02, 03

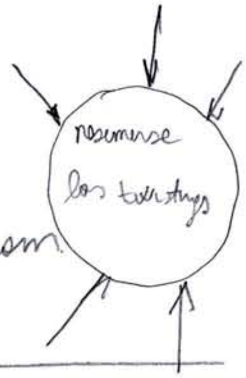
CGPL.

Another possibility that was explored was ~~to~~ wrap a D5 brane on a small 2-cycle

There are two different ways of doing this wrapping.

① Preserving $\mathcal{N}=1$ in 3+1 ⟶ I will focus on this.

② Preserving $\mathcal{N}=2$ in 3+1 . Bigazzi, Cotrone, Zaffaroni
· Gauntlett, Kim, Martelli, Waldram



We now focus on $\mathcal{N}=1$ case. Let us first see the solution for a D5 on S^2

It is ~~good~~ ~~to~~ gain some intuition by starting with a flat D5 brane

$$ds^2_{\text{str}} = \alpha' e^{\frac{\phi}{2}} \left[\frac{u\, dx^2_{1,5}}{\sqrt{g^2_{YM} N}} + \sqrt{g^2 N}\, \frac{du^2}{u} + \sqrt{g^2 N}\, u \; d\Omega_3 \right]$$

we took the decoupling limit

$$u = \frac{r}{\alpha'} = \text{fixed} \qquad \alpha' \to 0$$

$$g^2_{YM} = g_s \alpha' = \text{fixed}$$

$$e^{\phi} = \frac{g^2_{YM}\, u}{(2\pi)^{3/2} \sqrt{g^2_{YM} N}}$$

$$F_3 = -\frac{N}{4}\, \omega_1 \wedge \omega_2 \wedge \omega_3$$

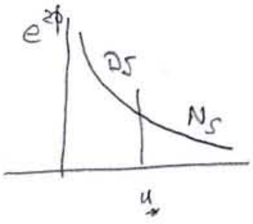
$$\left\{ \begin{array}{l} \omega_1 = \cos\psi\, d\tilde\theta + \sin\psi \sin\tilde\theta\, d\tilde\varphi \\ \omega_2 = -\sin\psi\, d\tilde\theta + \cos\psi \sin\tilde\theta\, d\tilde\varphi \\ \omega_3 = d\psi + \cos\tilde\theta\, d\tilde\varphi \end{array} \right\} \longrightarrow d\Omega_3 = \frac{1}{4}\left(\omega_1^2 + \omega_2^2 + \omega_3^2\right)$$

Nota

En realidad esta solución se debe pensar como el IR de una NS_5 brane

$$ds^2_{\text{str}} = dx^2_{1,5} + \tilde\alpha' \left(N \frac{du^2}{u^2} + N d\Omega_3 \right)$$

$$e^{2\phi} = \frac{N}{g^2_{YM}} \left(\frac{1}{u^2} \right) \qquad \tilde\alpha' = g_s \alpha' = \text{fixed}$$



$$H_3 = -\frac{N}{4}\, \omega_1 \wedge \omega_2 \wedge \omega_3 .$$

one can check that the solution above preserves $\mathcal{N}=1$ (4 SUSY's)

$$\left.\begin{array}{l}\delta\psi_\mu = 0\\ \delta\lambda = 0\end{array}\right. \rightsquigarrow \quad \varepsilon: \qquad \Gamma_{x_0x_1x_2x_3}\Gamma_{\theta\varphi}\,\eta = +\eta$$

$$\Gamma_{\theta\varphi}\,\eta = \Gamma_{\tilde\theta\tilde\varphi}\,\eta$$

$$\eta = i\eta^*$$

$$\varepsilon = e^{\phi/8}\, e^{\frac{\alpha}{2}\Gamma_\theta\Gamma_{\tilde\theta}}\,\eta$$

$$\boxed{\cos\alpha = \coth 2r - \frac{2r}{\sinh^2(2r)}}$$

$$\left.\right\} \quad \boxed{\Gamma_{x_0x_1x_2x_3}\left(\cos\alpha\,\Gamma_{12} + \sin\alpha\,\Gamma_1\Gamma_{\hat 2}\right)\varepsilon = \varepsilon}$$

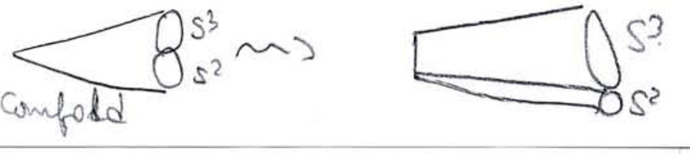
Let me comment something. If one considers the leading asymptotics

$$\left.\begin{array}{l} e^{2\phi} \sim e^{2\phi_0}\dfrac{e^{2r}}{4\sqrt{r}} \\ e^{2h} \sim r \\ a \sim 0 \end{array}\right\} \quad ds^2_E = e^{\phi/2}\Big[dx^2_{1,3} + \alpha' g_s N\Big\{dr^2 + r(d\theta^2 + \sin^2\theta\, d\varphi^2) + \frac{1}{4}(d\tilde\theta^2 + \sin^2\tilde\theta\, d\tilde\varphi^2) + \frac{1}{4}(d\psi + \cos\theta\, d\varphi + \cos\tilde\theta\, d\tilde\varphi)^2\Big\}\Big]$$

$$F_3 = -\frac{N}{4}\Big[-\omega_1\wedge\omega_2\wedge\omega_3 + \sin\theta\, d\theta\wedge d\varphi\wedge\omega_3\Big]$$

this is also a solution. It is singular at $r \to 0$ $\quad R \sim \frac{1}{r}$ (badly singular)

So, basically what the function $a(r)$ is doing is "resolving" the singular behaviour $\rightsquigarrow$ introducing the deformed conifold



So, the solution on which the D5 wraps S^2 has to have a "topology" of the form.

$$ds^2 \sim e^{f_1} dx_{1,3}^2 + e^{f_2} d\Omega_2 + e^{f_3} d\Omega_3$$

but there will be some "complications." (to preserve SUSY)

<u>In reality it is</u> (coordinates $\{x^4, r, \theta, \varphi, \tilde{\theta}, \tilde{\varphi}, \psi\}$)

$$ds^2_E = e^{\frac{\phi}{2}} \left[dx_{1,3}^2 + \alpha' g_s N \left\{ dr^2 + e^{2h}(d\theta^2 + \sin^2\theta\, d\varphi^2) + \frac{1}{4}(\tilde{\omega}_1 + a\, d\theta)^2 + \frac{1}{4}(\tilde{\omega}_2 - a \sin\theta\, d\varphi)^2 + \frac{1}{4}(\tilde{\omega}_3 + \cos\theta\, d\varphi)^2 \right\} \right].$$

$$F_3 = + \frac{N_c}{4}\Big[-(\omega_1 + a\, d\theta)\wedge(\omega_2 - a \sin\theta\, d\varphi)\wedge(\omega_3 + \cos\theta\, d\varphi) + a'\, dr \wedge (\tilde{\omega}_1 \wedge d\theta + \sin\theta\, d\varphi \wedge \tilde{\omega}_2) + (1-a^2) \sin\theta\, d\theta \wedge d\varphi \wedge (\omega_3 + \cos\theta\, d\varphi)\Big].$$

$\phi(r)$

<u>where the functions</u>

$$a(r) = \frac{2r}{\sinh 2r} \quad ; \quad e^{2h} = r \coth 2r - \frac{r^2}{\sinh^2(2r)} - \frac{1}{4}$$

$$e^{2\phi} = e^{2\phi_0} \frac{\sinh 2r}{2 e^{h(r)}}$$

The geometry is such that when $r \to 0$

$$ds^2 \sim dx_{1,3}^2 + \underbrace{dr^2 + r^2 d\Omega^2 + \frac{1}{4}(\tilde{\omega}_1 + d\theta)^2 + \frac{1}{4}(\tilde{\omega}_2 - \sin\theta\, d\varphi)^2 + \frac{1}{4}(\tilde{\omega}_3 + \cos\theta\, d\varphi)^2}_{\text{Deformed conifold}}$$

more geometry Pof Tatfor

More general solutions   hep-th/0602027

Casero, Nuñez, Paredes

one can propose a metric, F_3 ; ϕ

$$ds^2 = e^{2f(r)}\Big\{ dx_{1,3}^2 + dr^2 + e^{2h}(d\theta^2 + \sin^2\theta\, d\varphi^2) +$$

$$\frac{e^{2g}}{4}\Big[(\tilde{\omega}_1 + a\, d\theta)^2 + (\tilde{\omega}_2 - a\sin\theta\, d\varphi)^2\Big] + \frac{e^{2k}}{4}\Big[\omega_3 + \cos\theta\, d\varphi\Big]^2\Big\};$$

$$F_3 = \frac{N_c}{4}\Big[-(\tilde{\omega}_1 + b\, d\theta)\wedge(\tilde{\omega}_2 - b\sin\theta\, d\varphi)\wedge(\tilde{\omega}_3 + \cos\theta\, d\varphi) +$$

$$b'\, dr \wedge(\tilde{\omega}_1 \wedge d\theta + \sin\theta\, d\varphi\wedge\tilde{\omega}_2) + (1-b^2)\sin\theta\, d\theta\wedge d\varphi\wedge\tilde{\omega}_3\Big]$$

$\phi(r)$

write the BPS eqs (check that they solve the Einstein eqs) and find

$b(r) = \dfrac{2r}{\sinh 2r}$; $\phi(r) = f(r)$ and a one parameter family of solutions arise.

$$\begin{cases} a \sim 1 + \mu\rho^2 \\ e^{2k} \sim N_c\left(\dfrac{4}{6+3\mu}\right) - N_c\dfrac{(20+36\mu+9\mu^2)}{15(2+\mu)}\rho^2 \end{cases}$$

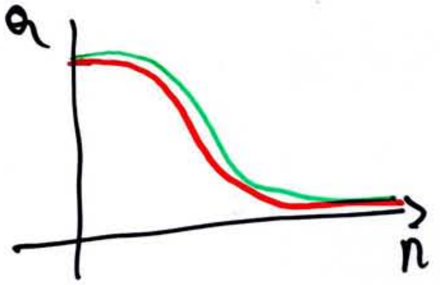
$\mu \in [-2, -\tfrac{2}{3}]$

$$e^{2g} \sim N_c \frac{4}{6+3\mu} \quad ; \quad e^{2\phi} \sim e^{2\phi_0}(1 + (\dots)\rho^2)$$

$$e^{2h} \sim N_c \frac{4\rho^2}{6+3\mu}$$

and plotting these functions

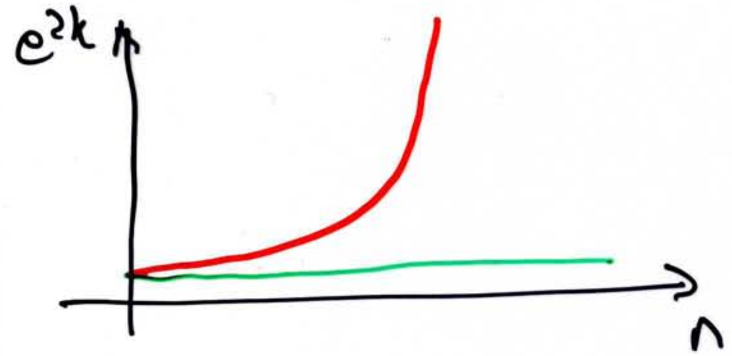
Plot of the deformed functions

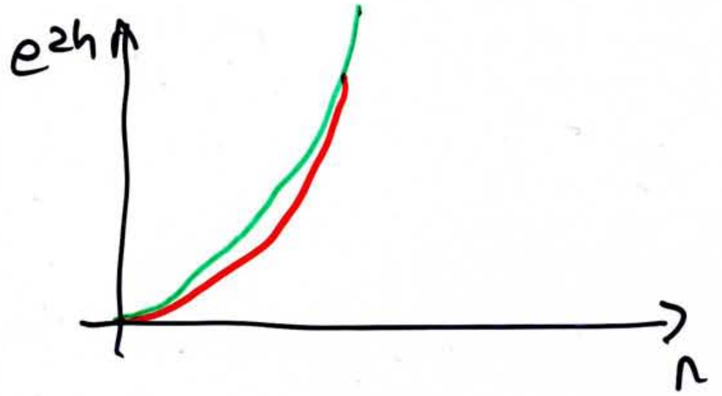


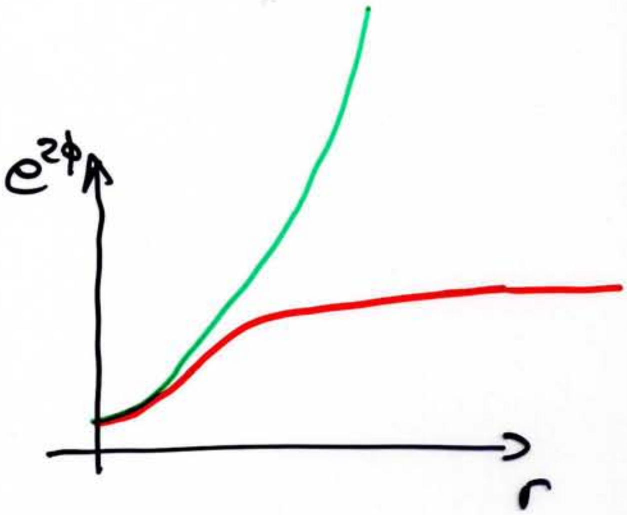
$\mu \in [-2, -2/3]$

exact solution

$\begin{bmatrix} \text{Mink}_3 \times \text{def conifold} \\ N_c = 0 \end{bmatrix}$







— exact solution $\mu = -2/3$

— new deformed solution

<u>Notice</u> the stabilization of the dilaton!

These new solutions present <u>μ-dependent</u> gauge theory dual quantities w/ quite interesting structure

See hep-th/0602027 (Section 3)

There are many checks that this solution captures very nicely non-perturbative aspects of $\mathcal{N}=1$ SYM:

Confinement of quarks. Maldacena, Nuñez

Screening of monopoles.

$U(1)_R$ symmetry breaking / chiral symmetry breaking Hartnoll Portugues Gursoy

Extension to SQCD Casero, Nuñez, Paredes.

Instantons Maldacena Nuñez, ...

Dipole deformation and dynamics of KK modes gursoy Nuñez

gluino condensate. Petrini, Zaffaroni; Aharony Loewy Sonnenschein

Domain walls Cotrone, Merlatti

Beta function. di Vecchia, Lerda, Merlatti; Bertolini-Merlatti, ...

Correlators and holography Berg, Haack, Mück,

strings tensions. Herzog Klebanov, Hartnoll Portugues

Domain walls. Maldacena, Nuñez, Sonnenschein Loewy.

Finite temperature; viscosity. Son, Starinets Kovtun Buchel Liu

Addition of flavors; SQCD. quenched; mesonic dynamics.
Nuñez Paredes Ramallo

glueballs; non susy deformations. Caceres, Nuñez; Pons Talavera.

Baryonic vertex Sonnenschein, Loewy; Ramallo, Arean Aharony Loewy Sonnenschein

Veneziano Yankielowicz - Superpotential Mück Evans Petrini Zaffaroni

Non-commutative version Mateos, Pons, Talavera

Broadening of flux tubes Sonnenschein, Loewy

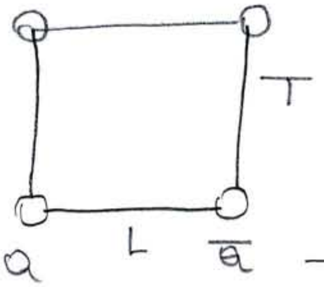
9

Tests of MN (D5 on S^2) (All these tests are not influenced by the KK modes) (*)

The first thing we should do is to see if the dual field theory confines. For this one would like to compute the

$\langle$Wilson loop$\rangle$

$$\langle W \rangle = \langle e^{\oint A} \rangle$$



$\rightsquigarrow$ action = in euclidean $S = E \cdot T$

$\rightarrow$ ~~quarks~~ non dynamical quarks

if $E \sim \sigma L +$ constant

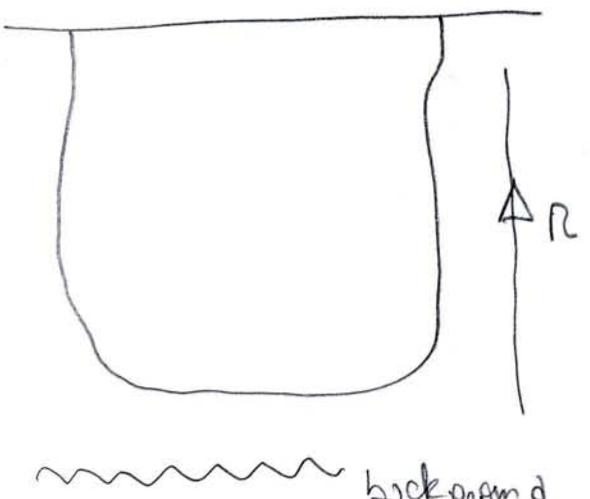
$\Downarrow$

confinement. $(\forall L)$

how to do this?

Maldacena / Rey + ~~Y~~ Yee $\rightsquigarrow$ prescription $\langle W \rangle = e^{-S_{NG}}$

for a string hanging from a D3 at ∞ (mass $\to \infty$ of the Q, $\bar{Q}$)



$t = \tau \qquad \tau \in [0, \infty]$

$X = \sigma \qquad \sigma \in [0, L]$

$R = R(\sigma)$

background

Fortunately the Wilson loop was analyzed for a general background by Kinar - Schreiber - Sonnenschein (1998) who proved a theorem

$$ds^2 = -g_{tt}\, dt^2 + g_{xx}\, dx^2 + g_{rr}\, dr^2 + \text{Internal part}$$

$$ds^2_{\substack{\text{ind}\\ F_1}} = -g_{tt}\, d\tau^2 + \left(g_{xx} + g_{rr}\, r'^2\right) d\sigma^2$$

$$S_{NG} = \frac{1}{2\pi\alpha'}\int d\sigma\, d\tau\, \sqrt{g_{tt}\,(g_{xx}) + g_{tt}\, g_{rr}\, r'^2} = \frac{T}{2\pi\alpha'}\int d\sigma \sqrt{F^2 + G^2 r'^2}$$

$\rightsquigarrow$ eq of motion $\frac{d}{d\sigma}\frac{\partial \mathcal{L}}{\partial r'} = \frac{\partial \mathcal{L}}{\partial r}$ $\rightsquigarrow$ complicated but use Energy $= p\dot{q} - L$ is conserved

$$p = \frac{G^2 r'^2}{\left[F^2 + G^2 r'^2\right]^{1/2}} \longrightarrow \boxed{E = \frac{-F^2}{\sqrt{F^2 + G^2 r'^2}} = -C}$$

C ↓ $F(r_0)$ $\rightsquigarrow$ turning point

$$\frac{G\, dr}{F\sqrt{F^2(r) - F^2_{(0)}}} = d\sigma$$

$$L = \int_0^L d\sigma = \int_{r_0}^{\infty} \frac{G}{F}\frac{dr}{\sqrt{F^2 - F_{(0)}^2}}$$

heavier others

$$E = F_{(0)}\, L + 2\int_{r_0}^{\infty} \frac{G}{F}\left(\sqrt{F^2 - F_{(0)}^2} - F\right) dr - 2\int_0^{r_0} G\, dr$$

where the infinite mass of the quark $m_q = \int_0^{\infty} G\, dr = \infty$ was substracted

So, they proved that

- $F(r)$ is analytic near the IR

$$F = F(0) + a_k r^k$$

$$k > 0$$
$$a_k > 0$$

- G is smooth $r \in [0, \infty]$

$$g \sim b_j r^j \qquad j > -1$$

- $F, G > 0$ on $(0, \infty)$

- $F' > 0$ on $[0, \infty]$

- $\int \frac{G}{F^2} < \infty$

$$\Rightarrow \quad E = F(0) \cdot L + \begin{cases} O\left[(\log L)^{\beta} e^{-\alpha L}\right] \\ O\left[2k - d\, L^{-n}\right] \end{cases}$$

$\rightsquigarrow$ ~~if~~ $F(0) \neq 0 \rightsquigarrow$ confinement

if $F(0) = 0 \rightsquigarrow \quad E = -\frac{1}{L^{\alpha}}$

Kinon, Schreiber and Sonnenschein showed that

$F(n)$ is analitic

$F \quad F(0) + a_k n^k$

$k>0$
$a_k>0$

G is smooth

$g \sim b_j n^j$ $\quad j>-1$

$F, G > 0$ in $[0,\infty]$

$F' > 0$ in $[0,\infty]$

$\int \frac{G}{F^2} < \infty$

$$E = F(0)\, L + \begin{cases} \Theta\left[\log L^{p} e^{-\alpha L}\right] \\ \Theta\left[2k - d\, L^{-\mu}\right] \end{cases}$$

$\rightsquigarrow$ if $F(0) \neq 0 \rightsquigarrow$ confinement

if $F(0) = 0 \rightsquigarrow E \sim \frac{1}{L^{\alpha}}$

$F^2 = e^{\phi} = -g_{tt}\, g_{xx}$

$G^2 = \alpha'_{EN}\, e^{\phi} = -g_{tt}\, g_{rr}$

In our background

all the Hypotheses are satisfied

$E_{Q\bar{Q}} = e^{2\phi_0} \cdot L$ + corrections $\rightsquigarrow$ Wilson's confinement

$L = \displaystyle\int_0^\infty \frac{dr}{\sqrt{e^{\phi} - e^{\phi_0}}} \longrightarrow \infty$ so the pair is separated without problem

---

Usual ideas about confinement $\rightsquigarrow$ Quarks are ~~condensed~~ confined

$\downarrow$

monopoles are screened

how to compute screening of monopoles $\rightsquigarrow$ compute the t'Hooft loop $\rightsquigarrow$ Wilson loop for magnetic charges.

In this case the "string" is simulated by a D3 brane wrapping the cycle $\theta = \tilde{\theta}$, $\varphi = 2\pi - \tilde{\varphi}$, $\psi = \pi$

Screening of monopoles

The t' Hooft loop is computed using a D3 brane wrapped on a particular 2 cycle

The effective string computes the magnetic version of the Wilson loop.

$$\left.\begin{array}{l}\theta = \tilde{\theta}\ , \ \varphi = 2\pi - \tilde{\varphi}\ , \ \psi = \pi \\ t = \tau\ ;\ x = \sigma\ ,\ r = r(\sigma)\end{array}\right\}$$ is the ~~cycle~~ configuration for the D3 brane

$$ds^2_{ind} = e^{\phi}\left[-dt^2 + d\cancel{x}^2 + r'^2 d\sigma^2 + \left(e^{2h} + \frac{(a-1)^2}{4}\right)(\cancel{\ } d\theta^2 + \sin^2\theta\, d\varphi^2)\right]$$

$$\int\sqrt{\det g_{ind}} = e^{-\phi}\int d\tau\, d\sigma \cdot \left(\int d\theta\, d\varphi \sin\theta \left[e^{2h} + \frac{(a-1)^2}{4}\right]\right)\sqrt{-g_{tt}g_{xx} - g_{tt}g_{rr}r'^2}$$

$\Rightarrow$ effective string

$$S = \underbrace{\left[e^{-\phi}e^{2\phi}\left(e^{2h} + \frac{(a-1)^2}{4}\right)4\pi\right]_{r\to 0}}_{T_{eff} = 0} \cdot \int d\tau\, d\sigma \sqrt{-g_{tt}g_{xx} - g_{rr}g_{tt}r'^2}$$

$\rightsquigarrow$ so the monopoles are free $\rightsquigarrow$ Screening of monopoles

The Beta function (di Vecchia-Lerda-Merlatti, Bertolini-Merlatti)

one possible way of presenting this

The idea is to read the gauge coupling from a "brane probe"

For this, we wrap a D5 probe on a particular cycle defined by

$\theta = \tilde{\theta}$ $\quad x^\mu \quad$ ρ : fixed

$\varphi = 2\pi - \tilde{\varphi}$

$\psi = (2n+1)\pi$

How to define the gauge coupling?

we have a 6d theory on ND5 field

$g^2_{YM_6} N = g_s \alpha' N \, . \, [g^2_{YM_6}] = L^2$

$$\boxed{g^2_{YM_4} = \frac{g^2_{YM_6}}{\mathrm{Vol}\,\Sigma_2}} \quad ?$$

The idea is that the action on this probe

$$S = -T_5 \int d^5x \, e^{-\phi} \sqrt{\det(g + F_2)} + T_5 \int C_2 \wedge F_2 \wedge F_2 .$$

~~[illegible]~~ the idea is that expanding this around the background

$$S \sim \underbrace{O}_{\sim \frac{1}{g^2_{YM}}} \int d^4x \, F_{\mu\nu}^2 + \underbrace{O}_{\sim \theta .} \int d^4x \, F \wedge F$$

~~$S = T_5 \int d^5x \, e^{-\phi} \sqrt{\det g}$~~

what each coefficient is?

Compute

$g_{ab} =$

$$\omega_1 \big|_{cycle} = \cos\psi \, d\tilde{\theta} + \sin\psi \sin\tilde{\theta} \, d\tilde{\varphi} \to (-1)^n \, d\theta$$

$$\omega_2 \big|_{cycle} = -\sin\psi \, d\theta + \cos\psi \sin\theta \, d\varphi \to +1 \sin\theta \, d\varphi$$

$$\omega_3 \big|_{cycle} = d\psi + \cos\theta \, d\varphi - \cos\theta \, d\varphi \to 0 .$$

$$ds^2_{\substack{ind \\ string}} = e^{\phi} \left\{ dx^2_{1,3} + \alpha' g_s N_c \left[e^{2h} + \frac{(a-1)^2}{4} \right] \left(d\theta^2 + \sin^2\theta \, d\varphi^2 \right) \right\}$$

$$g_{ab} + 2\pi\alpha' F_{ab} = \begin{array}{c} \\ t \\ x_1 \\ x_2 \\ x_3 \\ \theta \\ \varphi \end{array} \begin{array}{c} \begin{array}{cccccc} t & x_1 & x_2 & x_3 & \theta & \varphi \end{array} \\ \left[\begin{array}{cccccc} -e^{\phi} & F_{tx_1} & F_{tx_2} & F_{tx_3} & & \\ -F_{tx_1} & e^{\phi} & F_{x_1x_2} & F_{x_1x_3} & & \\ -F_{tx_2} & F_{x_2x_1} & e^{\phi} & F_{x_2x_3} & & \\ -F_{tx_3} & F_{x_3x_1} & F_{x_3x_2} & e^{\phi} & & \\ & & & & \alpha' g_s N \left(e^{2h} + \frac{(a-1)^2}{4}\right) e^{\phi} & \\ & & & & & e^{\phi} \alpha' g_s N \left(e^{2h} + \frac{(a-1)^2}{4}\right) \sin^2\theta \end{array} \right] \end{array}$$

$$\Rightarrow \sqrt{\det(g_{ab} + 2\pi\alpha' F_{ab})} = \alpha' g_s N \left(e^{2h} + \frac{(a-1)^2}{4}\right) \sin\theta \, . \, e^{3\phi} \left[-\eta_{ab} \times e^{-2\phi} F^2_{\mu\nu} \right]$$

So from here $e^{\phi}\sqrt{\det(g+F)}$

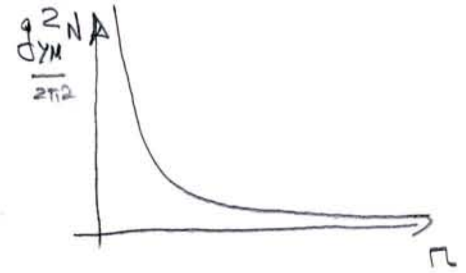
$$S_{D5} = -\underbrace{\boxed{\left(T_{D5}\cdot \alpha' g_s N \left(e^{2h}+\frac{(a-1)^2}{4}\right)\int d\Omega_2 \sin\theta\right)(2\pi\alpha')^2}}_{\frac{1}{4g^2_{YM\,4D}}}\int d^4x \overset{\text{flat space}}{F_{\mu\nu}^2}$$

$$T_{D5} = \frac{1}{(2\pi)^5 g_s \alpha'^3}$$

$$\frac{1}{(2\pi)^5 g_s\alpha'^3}\cdot \alpha'^3 g_s N \cdot 4\pi^2 \cdot 4\pi = \frac{1\cdot 16\pi^3}{(2\pi)^5} N$$

$$\frac{1}{2\pi^2}$$

$$\boxed{\frac{2\pi^2}{g^2_{YM}N} = \left[e^{2h}+\frac{(a-1)^2}{4}\right]} \qquad \boxed{e^{2h}+\frac{(a-1)^2}{4} = r\,\mathrm{tgh}\, r}$$



Now, can we compute the running of this coupling?

for this we need to propose some relation between

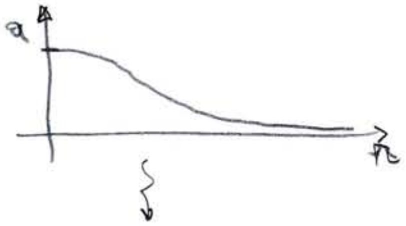
$r \longleftrightarrow$ Energy.

Catrone, Bigazzi and Zaffaroni found that the gaugino condensate

$\langle\lambda\lambda\rangle$ was related with the function $a(r)$.

This is VERY natural (though computationally involved)

Since



this is what a lattice person wants to see for $\langle\lambda\lambda\rangle$ in a simulation

$$a = \frac{2r}{\sinh 2r}$$

So $\boxed{a(r) \Leftrightarrow \langle\lambda\lambda\rangle = \left(\frac{\Lambda}{\mu}\right)^3}$

$$\boxed{a = \left(\frac{\Lambda}{\mu}\right)^3}$$

Now, the point is to compute

$$\frac{d\, g_{YM}}{d \log \mu/\Lambda} = \frac{d\, g_{YM}}{-\frac{1}{3}\, d \log a}$$

with

$$\left.\begin{aligned} a &= \frac{2r}{\sinh 2r} \\ \frac{2\pi^2}{g_{YM}^2 N} &= r \tgh r \end{aligned}\right\}$$ This gives a complicated expression.

$$-3 \frac{d \sqrt{\frac{2\pi^2}{N r \tgh r}}}{d \log\left(\frac{2r}{\sinh 2r}\right)} = f(r)$$

We gain some understanding by looking at this expression for $r \to \infty$

using that for large r $\quad a \sim 4r e^{-2r} = \left(\frac{\Lambda}{\mu}\right)^3$

$$\beta \sim \frac{3}{2}\left(1 + 4 e^{-2r}\right) =$$

$$\boxed{\beta_{\frac{8\pi^2}{g^2 N}} = -\frac{3 g_{YM}^3 N}{4\pi}\left(\frac{1}{1 - \frac{g_{YM}^2 N}{8\pi^2}}\right)}$$ $\rightsquigarrow \left\{\begin{array}{l} NSVZ \\ \text{Jones} \end{array}\right.$

- Criticisms
- Computation done at $r \to \infty$ $\rightsquigarrow$ effect of the KK modes?
  they decouple from the computation $\rightsquigarrow$ unchanged on S^3 $V(S^1 \times V(?))$
- $r \to \infty$ is the only SUSY point
  may be the formula is more general
- Solutions with stabilized dilaton give the same result in an approximate way.

The spectrum of this model

One starts by considering the theory on a D5 brane.
(5+1)d field theory with 16 SUSY's. The global symmetry is

$$SO(1,5) \times SO(4) \equiv SU(4) \times SU(2)_A \times SU(2)_B$$

The fields on the D5 brane A_M, $4 \times \phi$; $\lambda, \bar{\lambda}$ (Weyl)
Transform as

	$SU(4)$	$\times\ SU(2)_A$	$\times\ SU(2)_B$
A_M	6	1	1
ϕ_i	1	2	2
λ	4	1	2
$\bar{\lambda}$	$\bar{4}$	2	1

Then, we wrap the D5 on S^2 $\rightsquigarrow$ $SO(1,5) \to SO(1,3) \times \underbrace{SO(2)}_{(4,5)}$
$\equiv SO(4) \times SO(2) \equiv SU(2)_L \times SU(2)_R \times U(1)_{45}$
$\left(A_M \to A_\mu \,;\, m_\pm = A_4 \pm i A_5\right)$; $\left(\lambda \to \lambda_\alpha + \psi_{\dot{\alpha}} \,;\, \bar{\lambda} \to \bar{\lambda}_{\dot{\alpha}} + \bar{\psi}_{\dot{\alpha}}\right)$

	$SU(2)_L$	$\times\ SU(2)_R$	$\times\ U(1)_{45}$	$\times\ SU(2)_A$	$\times\ SU(2)_B$
A_μ	2	2	0	1	1
m_i	1	1	±2	1	1
ϕ_i	1	1	0	2	2
λ	2	1	+1	2	1
ψ	2	1	−1	1	2
$\bar{\lambda}$	1	2	−1	1	2
$\bar{\psi}$	1	2	+1	2	1

Now, the twisting comes. In order we SUSY we must mix $U(1)_{45}$ with $SU(2)_A$ or $SO(2)_B$ (a U(1) inside them)

	$U(1)_A$	$U(1)_T$
A_μ	0	0
$m_\pm$	0	± 2
ρ	± 1	± 1
λ_α	± 1	$\begin{pmatrix}2\\0\end{pmatrix}$
$\lambda_{\dot\alpha}$	± 1	$\begin{pmatrix}0\\-2\end{pmatrix}$
ψ_α	0	-1
$\psi_{\dot\alpha}$	0	$+1$

$U(1)_T = \mathrm{diag}(U(1)_A, U(1)_{45})$

$Q_T = Q_{45} + Q_A$

So, now, we compactify with $U(1)_T$ (instead of $U(1)_{45}$)

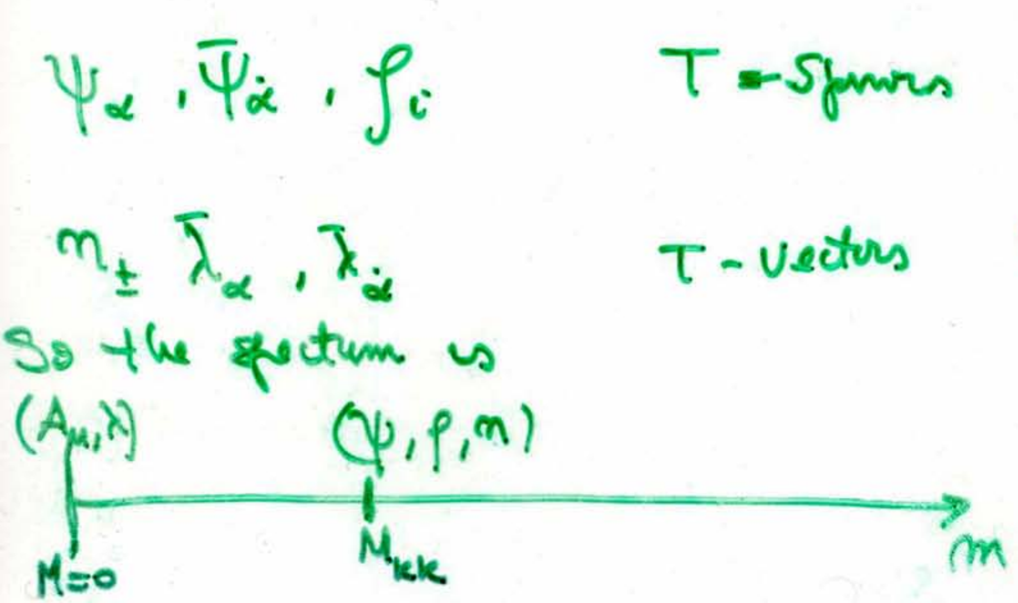
The T-spin of each particle

$A_\mu, \lambda_\alpha, \bar\lambda_{\dot\alpha}$ T-scalars $Q_T = 0$

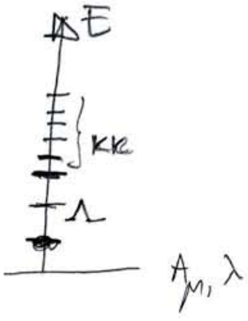
$\psi_\alpha, \bar\psi_{\dot\alpha}, \rho_i$ T-spinors $Q_T = 1$

$m_\pm$ $\bar\lambda_\alpha, \lambda_{\dot\alpha}$ T-vectors $Q_T = 2$

So the spectrum is



Now, one has a field theory whose spectrum at weak coupling is



one would like $\frac{\Lambda}{m_{KK}} \to 0$

but one actually gets $\underline{\Lambda \sim m_{KK}}$

Thus "degeneracy" between the strong coupling scale and the UV completion afflicts <u>all</u> supergravity duals. It is an effect of the supergravity approximation $\boxed{\text{(into M theory for few } N_{D6}\text{)}}$

. Now one may wonder, why so many tests are passed nicely ~~while~~ when the effect of the KK modes could be noticeable?

The reason why so many tests are succesfull would only be the fact for some reason, the "effect cancel". But how to distinguish for which quantities the effect of the UV is absent?

Gursoy and I (2005) have proposed a way of doing it

It basically consist on starting from a background that is related to the ~~original~~ original one by TST transformation.

<u>Some background</u>

Suppose that one takes $N=$ SYM $(A_\mu,\ 3\ \text{complex}\ \phi_i\ ;\ \lambda \times 4)$

$$\mathcal{L} = \text{kinetic} + \int d^2\theta\ W \quad ; \quad W = \Phi_1 \Phi_2 \Phi_3 - \Phi_1 \Phi_3 \Phi_2$$

there is a theory call $N=1\ \beta$ deformed with $W = e^{i\beta}\phi_1\phi_2\phi_3 - e^{-i\beta}\phi_1\phi_3\phi_2$

this deformation is $\longrightarrow$ marginal $\quad N=1_\beta$ is conformal

The background dual to $N=1_\beta$ is constructed by a TST around two $U(1)\times U(1)$ of S^5

Basically what is going on is that for fields

Charged under these $U(1)$'s the product of fields is replaced by a $*$-product $\quad \phi_1 * \phi_2 * \phi_3$

So, we used this to construct a new metric after transforming under $U(1)\times U(1)$ $\xrightarrow[\text{which}]{\text{under}}$ the KK modes are charged

All the details to check when the KK modes do (not) influence a computation one in hep-th/0505100 by Umut Gürsoy and me.

We first write the full metric in the following appropriate form for the $SL(3,R)$ rotation in which the torus and the transverse parts have been separated as follows:

$$
\begin{aligned}
& ds^2_{string} = e^{\phi}[dx^2_{1,3} + \alpha' g_s N dr^2] + D_1 d\psi^2 + D_2 d\theta^2 + D_3 d\tilde{\theta}^2 + E_1 d\theta d\tilde{\theta} + \\
& E_2 d\theta d\psi + E_3 d\tilde{\theta} d\psi + \frac{F}{\sqrt{\Delta}}[d\varphi + (\alpha_1 - C\beta_1)d\theta + (\alpha_2 - C\beta_2)d\tilde{\theta} + (\alpha_3 - C\beta_3)d\psi - Cd\tilde{\varphi}]^2 + \\
& F\sqrt{\Delta}(d\tilde{\varphi} + \beta_1 d\theta + \beta_2 d\tilde{\theta} + \beta_3 d\psi)^2
\end{aligned} \tag{1.1}
$$

To simplify the notation let us define,

$$
f = 4e^{2h}\sin^2\theta + \cos^2\theta + a^2\sin^2\theta, \quad g = a\sin\theta\sin\tilde{\theta}\cos\psi - \cos\theta\cos\tilde{\theta}. \tag{1.2}
$$

Then, various functions in (1.1) are given as,

$$
\begin{aligned}
& F = \frac{\alpha' g_s N e^{\phi}}{4}\sqrt{f-g^2}, \quad \Delta = \frac{f-g^2}{f^2}, \quad C = \frac{g}{f}, \\
& \beta_1 = \frac{f}{f-g^2} a\sin\psi\sin\tilde{\theta}, \quad \beta_2 = \frac{g}{f-g^2} a\sin\psi\sin\theta, \quad \beta_3 = \frac{f\cos\tilde{\theta} + g\cos\theta}{f-g^2} \\
& \alpha_1 = \frac{a\sin\tilde{\theta}\sin\psi g}{f-g^2}, \quad \alpha_2 = \frac{a\sin\theta\sin\psi}{f-g^2}, \quad \alpha_3 = \frac{\cos\theta + g\cos\tilde{\theta}}{f-g^2} \\
& D_1 = \frac{\alpha' g_s N e^{\phi}}{4(f-g^2)}(f\sin^2\tilde{\theta} - g^2 - \cos^2\theta - 2g\cos\theta\cos\tilde{\theta}), \\
& D_2 = \frac{\alpha' g_s N e^{\phi}}{4}(a^2 + 4e^{2h} - \frac{f}{f-g^2} a^2\sin^2\psi\sin^2\tilde{\theta}) \\
& D_3 = \frac{\alpha' g_s N e^{\phi}}{4}(1 - \frac{a^2\sin^2\theta\sin^2\psi}{f-g^2}), \quad E_1 = \frac{a}{2}\alpha' g_s N e^{\phi}(\cos\psi - \frac{g}{f-g^2} a\sin^2\psi\sin\theta\sin\tilde{\theta}) \\
& E_2 = -\frac{\alpha' g_s N e^{\phi} a\sin\psi\sin\tilde{\theta}}{2}\frac{(f\cos\tilde{\theta} + g\cos\theta)}{f-g^2}, \quad E_3 = -\frac{\alpha' g_s N e^{\phi}}{2}\frac{a\sin\theta\sin\psi}{f-g^2}(\cos\theta + g\cos\tilde{\theta}).
\end{aligned} \tag{1.3}
$$

Now, let us focus on the RR two form. It is useful to define four one forms as $\mathcal{A}^{(i)}, \mathcal{C}^{(i)}, \ i = 1,2$

$$
\mathcal{A}^{(1)} = \alpha_1 d\theta + \alpha_2 d\tilde{\theta} + \alpha_3 d\psi, \quad \mathcal{A}^{(2)} = \beta_1 d\theta + \beta_1 d\tilde{\theta} + \beta_3 d\psi, \tag{1.4}
$$

and

$$
\mathcal{C}^{(1)} = C^{(1)}_{\theta} d\theta + C^{(1)}_{\tilde{\theta}} d\tilde{\theta} + C^{(1)}_{\psi} d\psi, \quad \mathcal{C}^{(2)} = C^{(2)}_{\theta} d\theta + C^{(2)}_{\tilde{\theta}} d\tilde{\theta} + C^{(2)}_{\psi} d\psi, \tag{1.5}
$$

and the two form $\tilde{c}_{\mu\nu} dx^{\mu} \wedge dx^{\nu}$, with components $\tilde{c}_{\theta\tilde{\theta}}, \tilde{c}_{\tilde{\theta}\psi}$, and all others being zero.

more details

In order to reproduce the RR two-form to the following form,

$$C^{(2)} = C_{12} D\varphi \wedge D\tilde{\varphi} - \mathcal{C}^{(1)} \wedge D\varphi - \mathcal{C}^{(2)} \wedge D\tilde{\varphi} - \frac{1}{2}(A^{(1)} \wedge \mathcal{C}^{(1)} + A^{(2)} \wedge \mathcal{C}^{(2)} - \tilde{c}); \quad (1.6)$$

the quantities defined above are determined as,

$$\begin{aligned}
C_{12} &= \frac{1}{4}(a(r)\sin\theta\sin\tilde{\theta}\cos\psi - \cos\theta\cos\tilde{\theta}),\\
\mathcal{C}^{(1)} &= (-\frac{\psi}{4}\sin\theta - C_{12}\beta_1)d\theta - (\frac{1}{4}a(r)\sin\theta\sin\psi + C_{12}\beta_2)d\tilde{\theta} - \beta_3 C_{12} d\psi,\\
\mathcal{C}^{(2)} &= (\frac{1}{4}a(r)\sin\tilde{\theta}\sin\psi + C_{12}\alpha_1)d\theta + (\frac{\psi}{4}\sin\tilde{\theta} - C_{12}\alpha_2)d\tilde{\theta} + \alpha_3 C_{12} d\psi,\\
\tilde{c} &= \tilde{c}_{\theta\tilde{\theta}} d\theta \wedge d\tilde{\theta} + \tilde{c}_{\theta\psi} d\theta \wedge d\psi + \tilde{c}_{\tilde{\theta}\psi} d\tilde{\theta} \wedge d\psi
\end{aligned} \quad (1.7)$$

with

$$\begin{aligned}
\tilde{c}_{\theta\tilde{\theta}} &= \alpha_1\alpha_2 + \beta_1\beta_2 + C_{12}(\alpha_1\beta_2 - \alpha_1\beta_2) - \frac{1}{4}\psi\sin\tilde{\theta}\beta_1 + \frac{1}{4}a\sin\theta\sin\psi\alpha_1\\
&-\frac{1}{4}a\cos\psi + \frac{1}{4}a\beta_2\sin\psi\sin\tilde{\theta},\\
\tilde{c}_{\theta\psi} &= \alpha_1\alpha_3 + \beta_1\beta_3 - C_{12}(\alpha_1\beta_3 - \alpha_3\beta_1) - \frac{1}{4}\psi\sin\theta\alpha_3 + \frac{1}{4}a\sin\psi\sin\tilde{\theta}\beta_3,\\
\tilde{c}_{\tilde{\theta}\psi} &= \alpha_2\alpha_3 + \beta_2\beta_3 - C_{12}(\alpha_2\beta_3 - \alpha_3\beta_2) + \frac{1}{4}\psi\sin\tilde{\theta}\beta_3 - \frac{1}{4}a\sin\psi\sin\theta\alpha_3.
\end{aligned} \quad (1.8)$$

So, after the $SL(3,R)$ rotation is done, we will end with a metric that reads

$$\begin{aligned}
(ds^2_{string})' &= (\frac{e^{2(\phi'-\phi)}F}{F'})^{1/3}\big(e^{\phi}[dx^2_{1,3} + \alpha' g_s N dr^2] + D_1 d\psi^2 + D_2 d\theta^2 + D_3 d\tilde{\theta}^2 + E_1 d\theta d\tilde{\theta} +\\
&E_2 d\theta d\psi + E_3 d\tilde{\theta} d\psi\big) + \frac{F'}{\sqrt{\Delta}}[d\varphi + (\alpha_1 - C\beta_1)d\theta + (\alpha_2 - C\beta_2)d\tilde{\theta} + (\alpha_3 - C\beta_3)d\psi - C d\tilde{\varphi}]^2\\
&+F'\sqrt{\Delta}(d\tilde{\varphi} + \beta_1 d\theta + \beta_2 d\tilde{\theta} + \beta_3 d\psi)^2
\end{aligned} \quad (1.9)$$

↳ using this metric and the RR fields above one can disentangle when/how the KK modes influence a computations

Many checks for this in hep-th/0505100.