Pioneer anomaly and post-Einsteinian gravitation

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- arXiv:gr-qc/0410148 : Gravity tests in the solar system and the Pioneer anomaly
- arXiv:gr-qc/0501038 : Testing the Newton law at long distances
- arXiv:gr-qc/0502007 : Post-Einsteinian tests of linearized gravitation
- arXiv:gr-qc/0510068 : Post-Einsteinian tests of gravitation
- arXiv:gr-qc/0511020 : Gravity tests and the Pioneer anomaly

General Relativity: a metric theory of gravitation

- Gravitational field ≡ metric in Riemann space-time
  - ideal (atomic) clocks measure the proper time along their trajectory
  - freely falling probes (masses and light rays) follow geodesics

$$\mathrm{d}s^2 \equiv g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu$$

$$\delta\left[\int \mathrm{d}s\right] = 0$$

- One of the most accurately tested principles of physics

- Einstein-Hilbert equation
  - one curvature tensor has a null divergence (Bianchi identities) like the stress tensor (conservation laws)

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$D^\nu E_{\mu\nu} \equiv 0$$

$$D^\nu T_{\mu\nu} \equiv 0$$

  - in General Relativity, the two tensors are simply proportional to each other

$$E_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

- Equation tested by comparing predicted geodesics with observations/experiments

Parametrized post-Newtonian metrics

- Solution of GR in the solar system
  - with the Sun treated as a point-like motionless source
  - using spatially isotropic coordinates
  - with Newton potential

$$
\begin{aligned}
g_{00} &= \left(1 + 2\phi + 2\phi^2 + \ldots\right) \\
g_{jk} &= -\left(1 - 2\phi + \ldots\right)\delta_{jk}
\end{aligned}
$$

$$
\phi \equiv -\frac{G_N M}{rc^2} \quad , \quad |\phi| \ll 1
$$

- GR usually tested through its confrontation with the larger family of PPN metrics

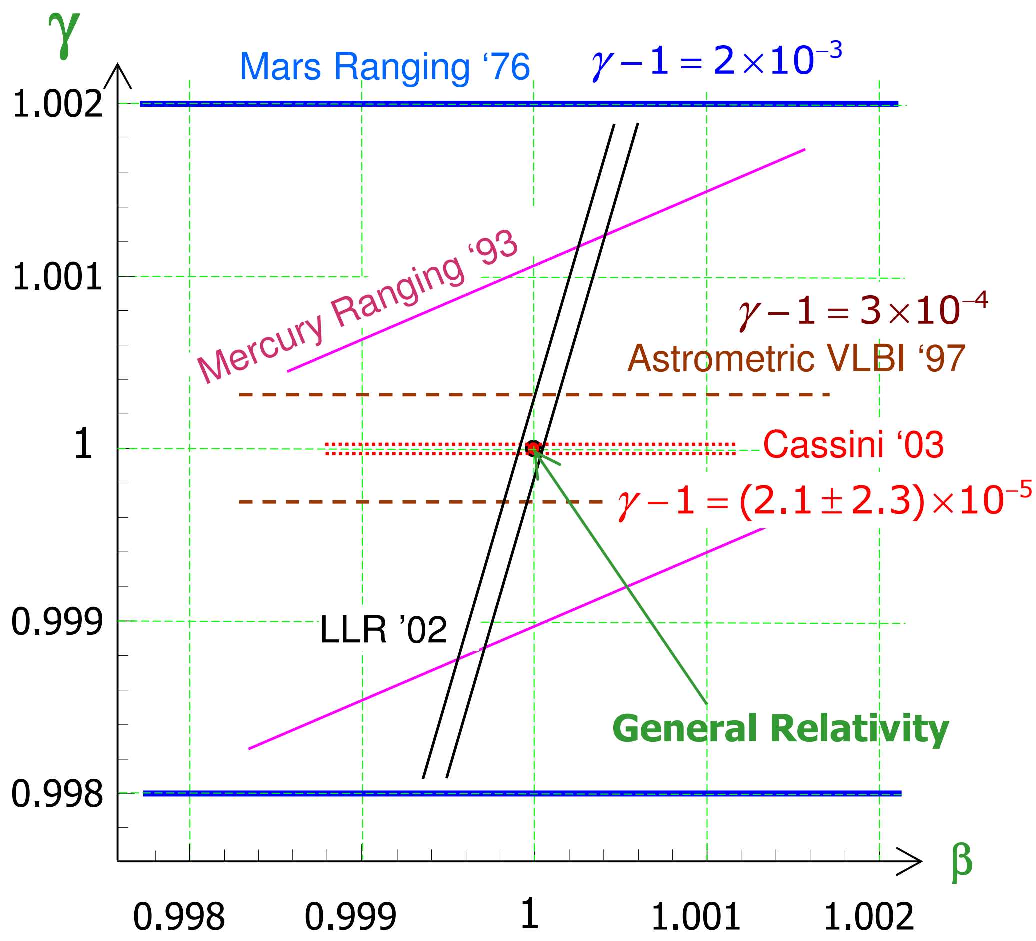
$$
\begin{aligned}
g_{00} &= \left(1 + 2\alpha\phi + 2\beta\phi^2 + \ldots\right) & \alpha = 1 \text{ fixes } G_N \\
g_{jk} &= -\left(1 - 2\gamma\phi + \ldots\right)\delta_{jk} & \beta = \gamma = 1 \text{ in GR}
\end{aligned}
$$

- Motions predicted as the geodesics of this metric
- Comparisons between observations and predictions expressed in terms of anomalies of the PPN parameters $\beta - 1\,,\ \gamma - 1$

Tests of General Relativity

Tests in the solar system confirm General Relativity

- Ranging on planets
- Astrometry and VLBI
- LLR = Lunar Laser Ranging (1969 – ongoing)
- Doppler velocimetry on artificial probes



S.G. Turyshev (JPL, NASA)

***Living Reviews in Relativity*, C.F. Will (2001)**

- All tests consistent with GR

$$|\gamma - 1| \lesssim 3 \times 10^{-5}$$

$$|\beta - 1| \lesssim 1 \times 10^{-4}$$

Tests of Newton law

The Newtonian dependence of Φ is very well tested

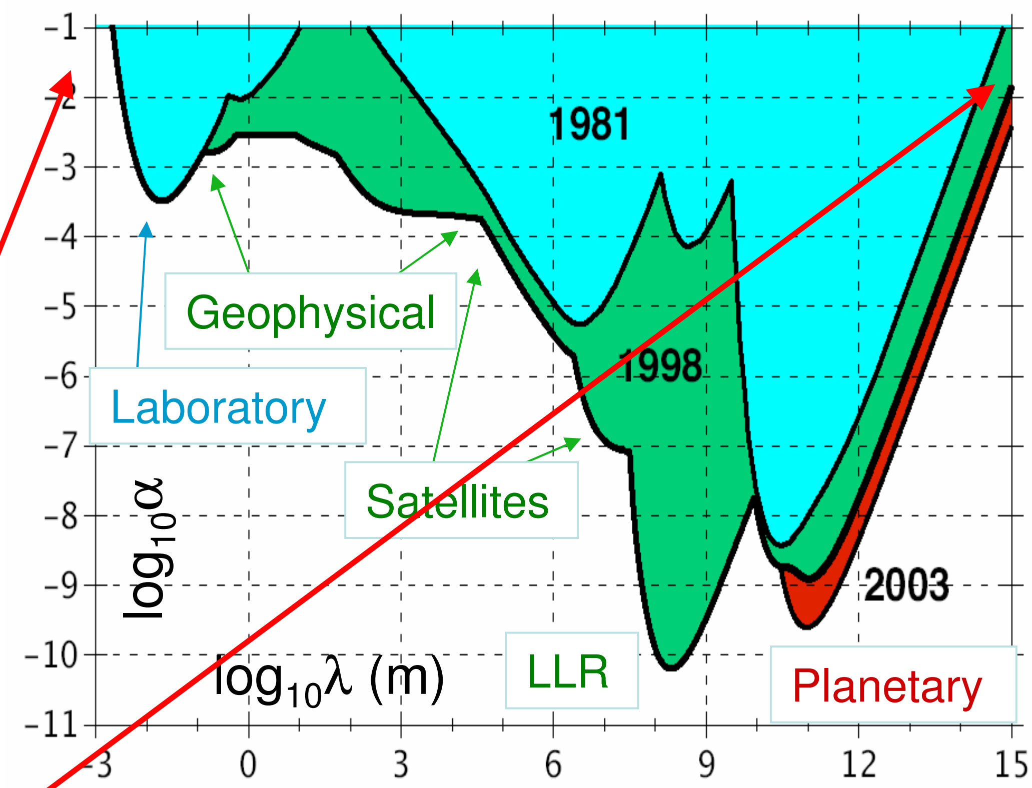
Search for a Yukawa correction

$$\Phi(r) = -\frac{GMm}{rc^2}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right)$$

Windows remain open for deviations

at short ranges $\lambda < 1\,\mathrm{mm}$

or long ranges $\lambda > 10^{16}\,\mathrm{m}$

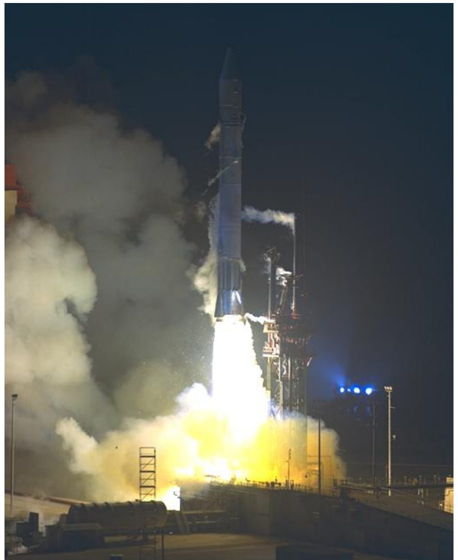


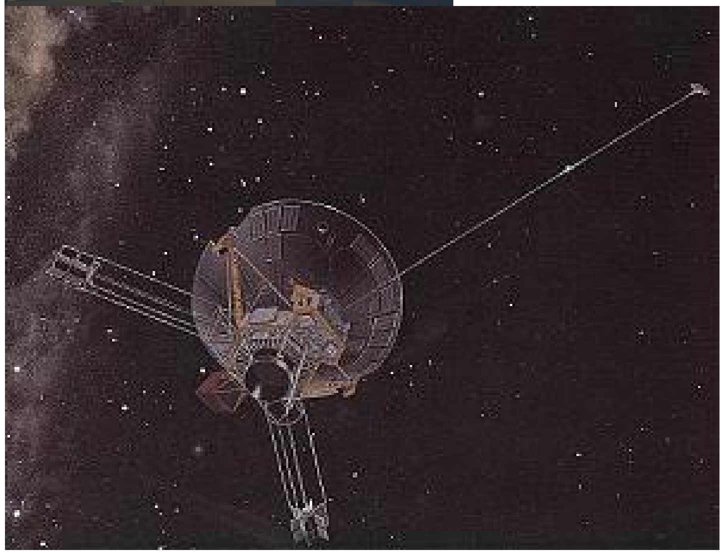
J. Coy, E. Fischbach, R. Hellings, C. Talmadge, and E. M. Standish (2003)

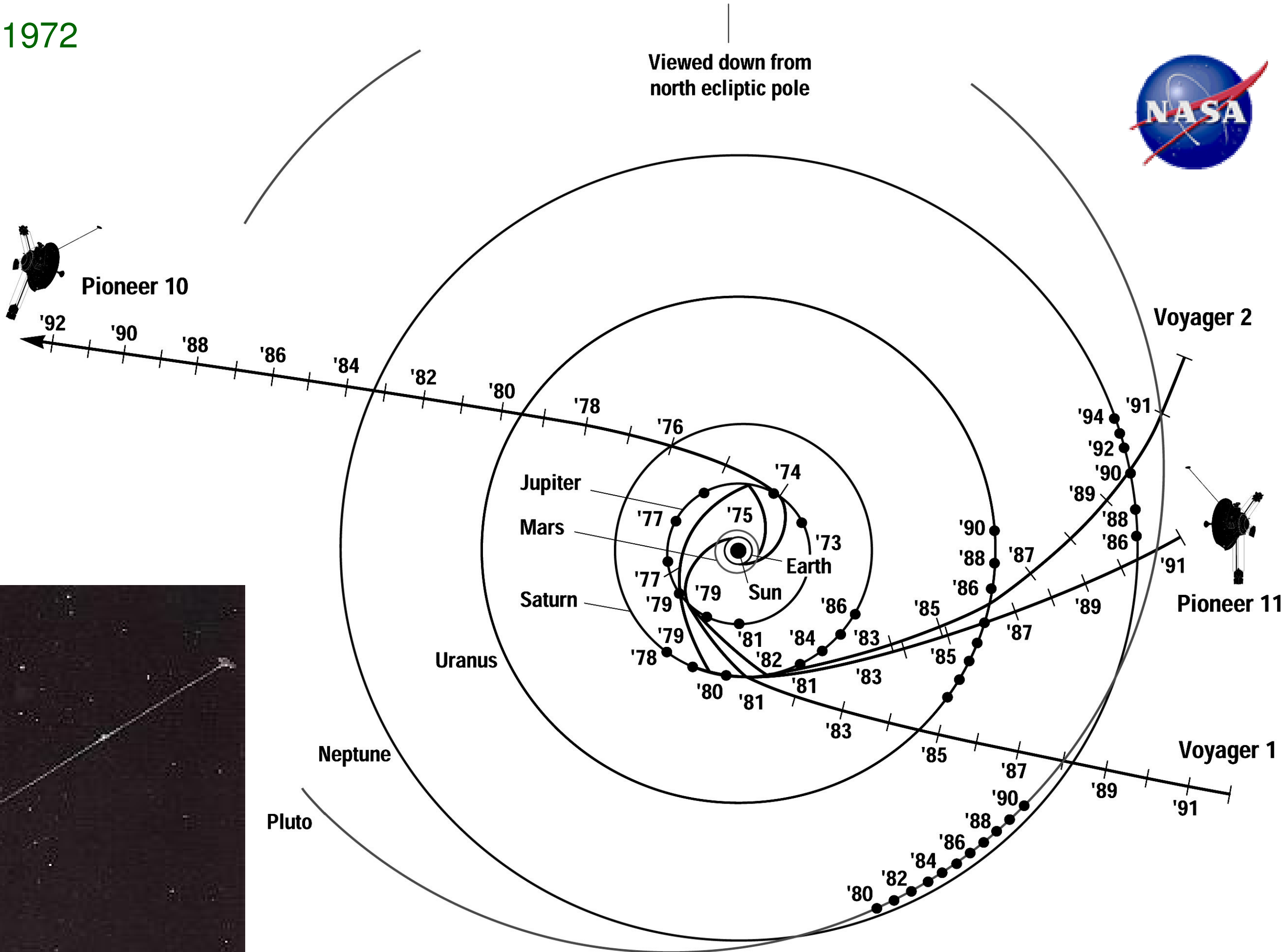
***The Search for Non-Newtonian Gravity*, E. Fischbach & C. Talmadge (1998)**

Pioneer « gravity test »

Pioneer 10 Launch: 2 March 1972







Trajectories of Pioneers:
Elliptical (bound) orbits before the last fly-by;
Hyperbolic (escape) orbits after the last fly-by.

Pioneer Doppler residuals

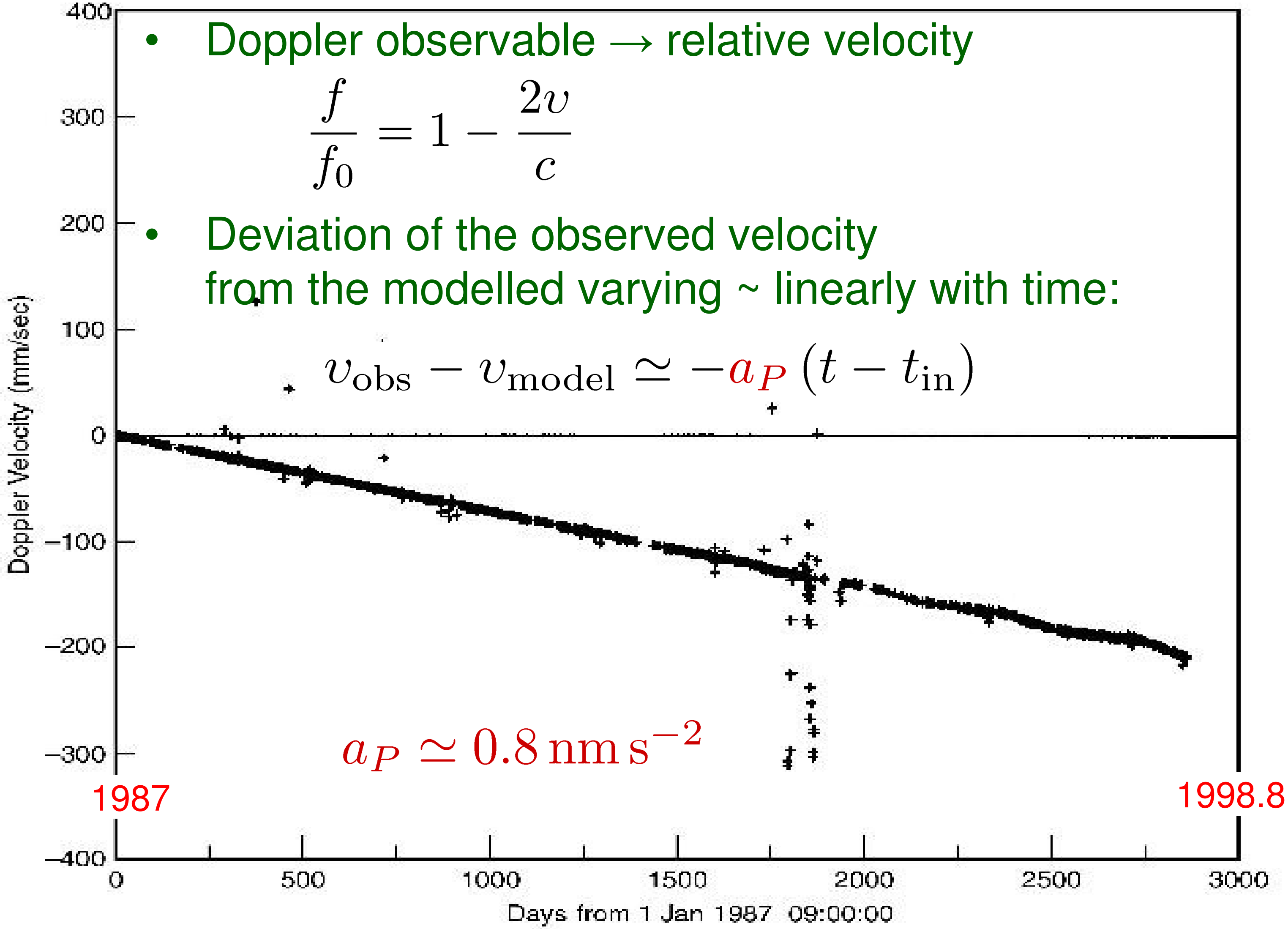
1 Hz is equivalent to 65 mm/s velocity

- Doppler observable → relative velocity

$$\frac{f}{f_0} = 1 - \frac{2v}{c}$$

- Deviation of the observed velocity from the modelled varying ~ linearly with time:

$$v_{\mathrm{obs}} - v_{\mathrm{model}} \simeq -a_P\,(t - t_{\mathrm{in}})$$



$$a_P \simeq 0.8\,\mathrm{nm\,s^{-2}}$$

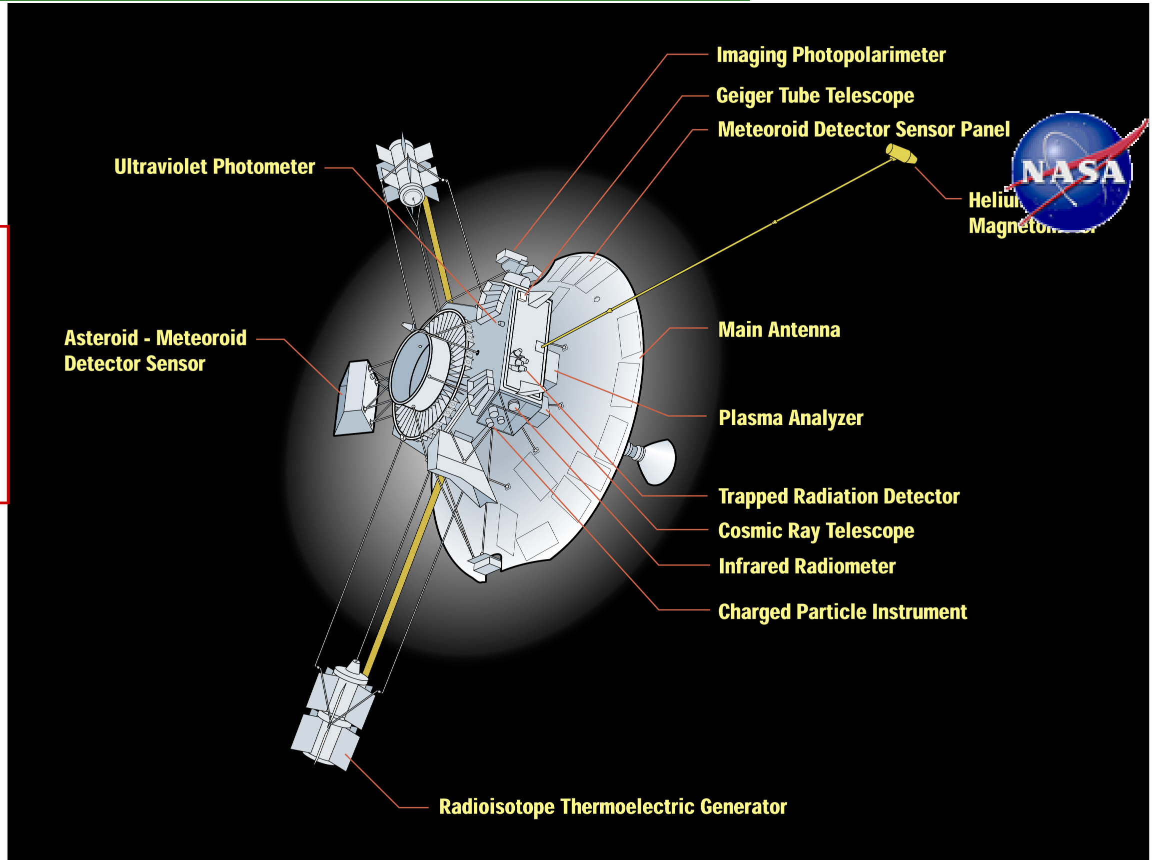
J. Anderson et al, Phys. Rev. D 65 (2002) 082004

Pioneer anomaly

The anomaly has been registered on the two deep space probes with the best navigation accuracy

The two probes were identical and had similar trajectories

- one experiment performed twice with the same result.



Artefact ?

- Satisfactory explanation has been looked for, not yet found.
- Data reanalysis soon performed ...

Is the Pioneer anomaly compatible with other gravity tests?

- The equivalence principle (EP) cannot be violated at the level of the Pioneer anomaly

$$a_N \sim 1\mu\mathrm{m\ s}^{-2} \quad , \quad a_P \sim 1\mathrm{nm\ s}^{-2}$$

→we preserve the geometrical interpretation of Einstein theory

✓gravitation described as a Riemannian metric theory

✓motions identified with geodesics

- But the Einstein-Hilbert equation can be modified

→modifications emerge naturally from radiative corrections to GR due to coupling between graviton and other fields

→gravitation coupling is modified and depends on scale

→this leads to modifications of the metric around a gravitational source and then of geodesic motions

→this entails a phenomenological framework larger than PPN

Metric extensions of GR: theoretical side

Two gravitation sectors

- Einstein curvature contains two independent components, one related to the trace (sector 1), the other one traceless (sector 0).
- Einstein-Hilbert relation is replaced by a general coupling involving two running constants. In the linearized theory (in Fourier space) and for a pointlike motionless stress tensor:

$$T_{\mu\nu} = \delta_{\mu 0}\delta_{\nu 0} T_{00}$$

$$\pi_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

$$E_{\mu\nu} = E^{(0)}_{\mu\nu} + E^{(1)}_{\mu\nu}$$

$$E^{(0)}_{\mu\nu} = \{\pi^0_\mu \pi^0_\nu - \frac{\pi_{\mu\nu}\pi^{00}}{3}\} \frac{8\pi \widetilde{G}^{(0)}}{c^4} T_{00} \qquad E^{(1)}_{\mu\nu} = \frac{\pi_{\mu\nu}\pi^{00}}{3} \frac{8\pi \widetilde{G}^{(1)}}{c^4} T_{00}$$

The solution remains in the vicinity of GR

$$\widetilde{G}^{(0)} = G_N + \delta\widetilde{G}^{(0)}$$
$$\widetilde{G}^{(1)} = G_N + \delta\widetilde{G}^{(1)}$$

$$E = [E]_{GR} + \delta E$$

- The Einstein curvature of GR vanishes outside gravitational sources:
  $[E]_{GR} = 0 \qquad \text{where} \qquad T \equiv 0$
- The Einstein curvature of a general metric extension contains two non vanishing components in empty space: $E = \delta E^{(0)} + \delta E^{(1)}$

Metric extensions of GR: phenomenological side

Two gravitation potentials

The general isotropic and stationary metric is written in terms of two potentials which parameterize the new phenomenological freedom:

$$g_{00} = [g_{00}]_{GR} + \delta g_{00}$$
$$g_{rr} = [g_{rr}]_{GR} + \delta g_{rr}$$

$$[g_{00}]_{GR} = 1 + 2\Phi = -\frac{1}{[g_{rr}]_{GR}}$$

$$\frac{\delta g_{00}}{[g_{00}]_{GR}} = 2\int \frac{\delta\Phi'_N + ([g_{00}]_{GR} - 1)\delta\Phi'_P}{[g_{00}]^2_{GR}} dr$$

$$\frac{\delta g_{rr}}{[g_{rr}]_{GR}} = \frac{2r(\delta\Phi_N - \delta\Phi_P)'}{[g_{00}]_{GR}}$$

$$\delta E^0_0 \equiv 2\Delta(\delta\Phi_N - \delta\Phi_P)$$

$$\delta E^r_r \equiv -\frac{2}{r}\delta\Phi'_P$$

The PPN metric is recovered as a particular case:

$$\delta\Phi_N = (\beta - 1)\Phi^2 + O(\Phi^3)$$
$$\delta\Phi_P = -(\gamma - 1)\Phi + O(\Phi^2)$$

The anomalous potentials $\delta\Phi_N\,,\ \delta\Phi_P$ promote Eddington parameters $\beta - 1\,,\ \gamma - 1$ to the status of space dependent functions

First sector: modified Newton potential

- The first anomalous potential $\delta\Phi_N$ corresponds to a modification of Newton law and is strongly constrained by planetary tests:
  - the third Kepler law is verified on the orbital period of Mars compared to the radius of its orbit (measured independently)
  - the perihelion precessions of the planets agree with GR

- Deviations $\delta\Phi_N$ needed to explain the Pioneer anomaly are too large to remain unnoticed on planetary tests, if they are assumed to have a Yukawa or linear form.
- Deviations could in principle appear only after Saturn:

  this possibility must be confronted to the ephemeris of outer planets (or other objects there)

  $\rightarrow$ Brownstein and Moffat, arxiv/gr-qc/0511026

Second sector: Pioneer-like anomalies

- The second potential $\delta\Phi_P$ corresponds to an Eddington parameter γ depending on the distance to the Sun: it modifies the propagation of electromagnetic signals and probe trajectories.
- a Pioneer-like anomaly results for probes with escape trajectories.
- We evaluate this effect by :
  - calculating the Doppler velocity taking into account the perturbation of light propagation to and from the probes as well as the perturbation of the motion of the probes
  - writing the derivative of this velocity as an acceleration $a \equiv \frac{dv}{dt}$
  - subtracting the result of the standard calculation for GR

$$\delta a \equiv a - [a]_{GR} \propto \delta\Phi_P$$

- The Pioneer anomaly may be used to determine the second potential in the outer part of the solar system

- The modification of GR producing the Pioneer anomaly should not spoil the agreement with other gravity tests,

Effects in the inner solar system

The two potentials affect the perihelion precessions of planets

$$\frac{\delta\Delta\varphi}{\pi} \simeq -\left(\delta g_{rr} - \frac{u\delta g_{00}''}{[g_{00}]_{GR}'} + \frac{e^2u^2}{4}\left(\delta g_{rr}'' - \frac{u\delta g_{00}^{(4)}}{2\,[g_{00}]_{GR}'}\right)\right) \quad u \equiv \frac{1}{r}$$

Planetary perihelion precessions may be used to obtain constraints on $\delta\Phi_P$ in the range $r \sim \mathrm{UA}$

The two potentials affect the propagation of electromagnetic waves

$$\delta\theta \simeq -\frac{G_N M}{c^2}\frac{\partial}{\partial r_0}\left(\delta\gamma(r_0)\ln\frac{4r_1r_2}{r_0^2}\right)$$

r_0 impact parameter

the deflection anomaly represents a spatial dependence of Eddington parameter

$$\delta\gamma(r_0) = -\frac{G_P}{G_N} + \frac{\zeta_P(r_0)r_0^2}{2G_N} \qquad r\delta\Phi_P(r) \equiv -\frac{G_P M}{c^2} + \frac{M}{c^2}r\zeta_P(r)$$

Eddington tests may be used to determine $\delta\Phi_P$ in the Sun vicinity

Conclusion and future tests

- A larger phenomenological framework
  - an extension of GR which preserves its geometrical interpretation but changes the coupling of stress tensor to gravity
  - an extension of PPN metric which promotes Eddington parameters to the status of scale-dependent functions

- Motions in the solar system must be reanalysed in the new framework, taking into account the two potentials
- Further tests
  - check predictions against recently recovered Pioneer data
    Recent Pioneer Data Recovery Effort turyshev@jpl.nasa.gov
  - search for Pioneer-related anomalies:
    - in the motions of planets or other objects, with dedicated probes
    - in Eddington / Shapiro tests
  - look for a scale dependence of Pioneer-related anomalies