Local non-Gaussianity from inflation; beyond f_{NL}

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20/1/09

Empirical nG

* **any** features unlikely in Gaussian field

  e.g. lack large-scale correlations

  "axis of evil"

  hemispherical power asymmetry

  cold spot

  problem of a posteriori ~~sta~~ probabilities

Theory models of nG much more limited

* ~~k-inf~~ DBI / k-inflation

  causal sub-H scattering 

  "equilateral" $k \sim k_1 \sim k_2$

  suppressed in slow-roll inflation

* local evolution of initial Gaussian fields

  super-H evolution

  $$\Phi(x) = \Phi_1(x) + f_{NL}\left(\Phi_1^2(x) - \langle \Phi_1^2 \rangle\right)$$

  where $\Phi = +\frac{3}{5}\zeta$

  matter era / super H

  note sign ($\neq \Phi^{MFB}$)

  * observe (?) $0 \lesssim f_{NL} \lesssim 100$ *

  (see Ben Wandelt)

* others – topological defects?

Local evolution = "separate universes"

neglect spatial gradients + anisotropy

~~locally FRW~~

usually v. good approx. for $k \ll aH$

after H-exit
during + after inflation
→ "primordial era"

⇒ locally FRW

fully non-linear

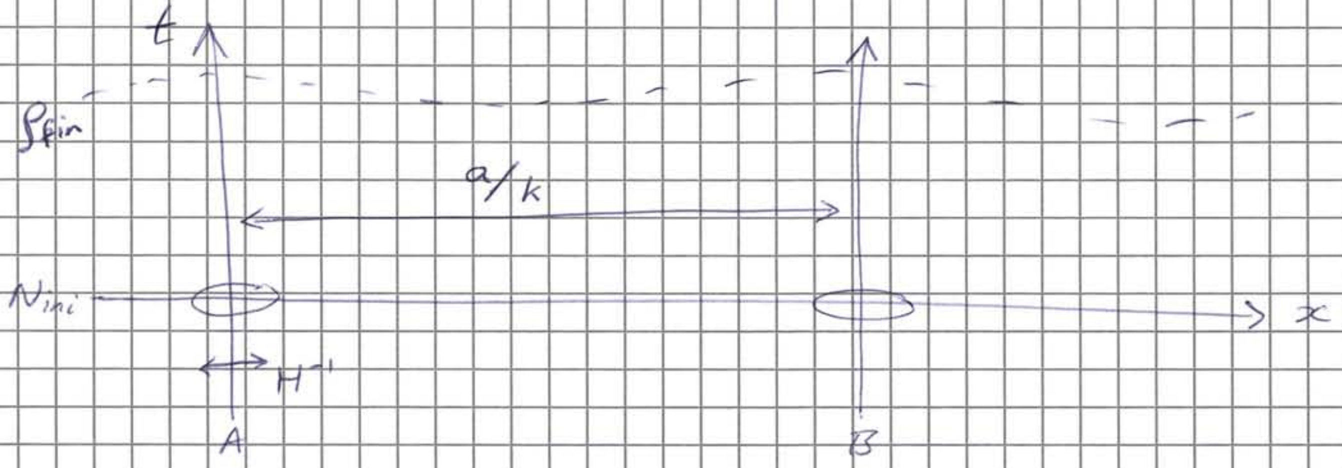
δN formalism identify non-linear $\zeta = \delta N$

RS, LMS, L+V

local expansion $N = \int_{N_{ini}}^{\rho_{fin}} H\, dt$

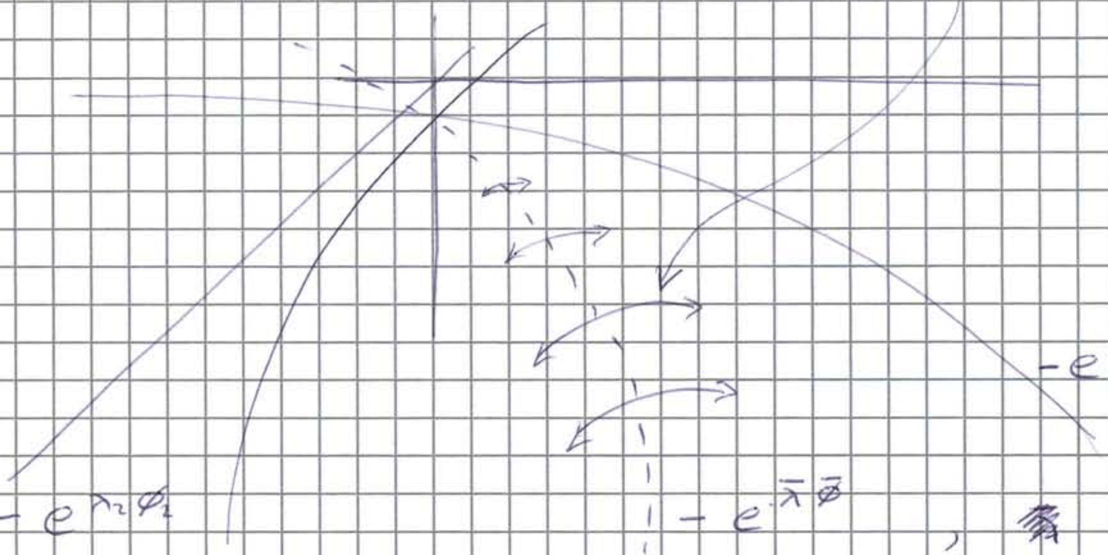
slow-roll inflation $N = N(\phi^A_{ini})$

where $\phi^A_{ini} = \phi^A \Big|_{N_{ini}}$ = unperturbed



Two main
Contenders for $f_{NL} \sim 100$

* new ekpyrotic, two fields $V(\phi^A) \propto -\exp(\lambda_A \phi^A)$
  $\lambda_A \gg 1$
  tachyonic growth of isocurvature fields



$-e^{\lambda_2 \phi_2}$ $\qquad -e^{\bar{\lambda}\bar{\phi}}$, $\qquad -e^{+\lambda_1 \phi_1}$

Koyama et al: $f_{NL} \simeq \frac{5}{12} \lambda_A^2$
0708.4321

c.f. tilt, $n-1 \simeq 4/\bar{\lambda}^2$, $\bar{\lambda} < \lambda_A \;\; \forall A$

More generally: f_{NL} large but model-dependent

* curvaton, late-decaying ~~modulus~~ scalar

$$\rho_\chi = \frac{1}{2} m^2 \cdot \chi^2$$, Gaussian $P(\delta\chi)$ from inflation

isocurvature $\rightarrow$ curvature

$$\zeta = r\,\zeta_\chi$$ where $r \sim \Omega_{\chi,\,decay}$

$$\zeta_\chi = \frac{1}{3}\frac{\delta\rho_\chi}{\rho_\chi}$$

$$\Rightarrow \zeta = \frac{r}{3}\left(\frac{2\chi\,\delta\chi + \delta\chi^2}{\chi^2}\right)$$

$$= \zeta_1 + \frac{3}{5} f_{NL}\, \zeta_1^2$$

where $\zeta_1 = \dfrac{2r}{3}\dfrac{\delta\chi}{\chi}$

$$f_{NL} = \frac{5}{4r} \leftarrow$$ note: scale & shape (k-independent)

$$f_{NL} \sim 100 \Rightarrow r \sim \Omega_{\chi,\,decay} \sim 0.01$$

Beyond f_{NL}?

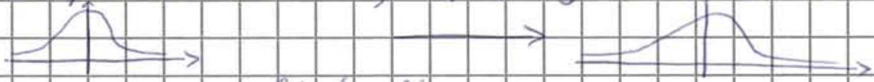
① Bispectrum $\longrightarrow$ higher order stats

(i) trispectrum, etc

(ii) full pdf

Gaussian $P(\phi)$, given $N(\phi) \longrightarrow$ nG $P(\delta N)$

δN is non-perturbative, f_{NL} just first term



② Adiabatic $\longrightarrow$ non-adiabatic

Kawasaki et al '08
Langlois, Vernizzi & DW '08

extend δN to multi-fluids

$$\zeta_\gamma = N_\gamma(\phi^A) - \cancel{N_\gamma(\phi_A)}\ \bar{N}$$

$$\zeta_m = N_m(\phi^A) - \bar{N}$$

$$\frac{1}{3} S_m = \zeta_m - \zeta_\gamma = N_m(\phi^A) - N_\gamma(\phi^A)$$

$$= \Delta N(\phi^A) = \Delta N(\delta s)$$

e.g. mixed inflaton - curvaton model

③ Multi-variate local nG

$\geqslant 2$ Gaussian fields

$$N(\phi^A) = \sum_A N_A\, \delta\varphi^A + \frac{1}{2}\sum N_{AB}\, \delta\varphi^A \delta\varphi^B + \dots$$

$$= N_\sigma\, \delta\sigma + N_s\, \delta s + \frac{1}{2} N_{\sigma\sigma}\, \delta\sigma^2 + \sum_s N_{\sigma s}\, \delta\sigma\, \delta s + \frac{1}{2} N_{ss}\, \delta s^2$$

adiabatic   entropy   slow-roll suppressed

δs independent of $\delta\sigma$

beware!

$$\text{effective } f_{NL} = \frac{\text{Bispectrum}}{(\text{Power spectrum})^2}$$

$\neq$ WMAP constraints

Conclusions

important to propose / evaluate theory models
for primordial nG
- more valuable than empirical tests alone

* f_{NL}, local function of single Gaussian
  excellent description of 3pt-fn
  for curvaton & ekpyrotic models

* can go beyond f_{NL}
  - higher order stats.
  - multi-variate distributions
  - non-adiabatic primordial pertbns

local evolution of Gaussian fields
well-motivated by slow-roll inflation