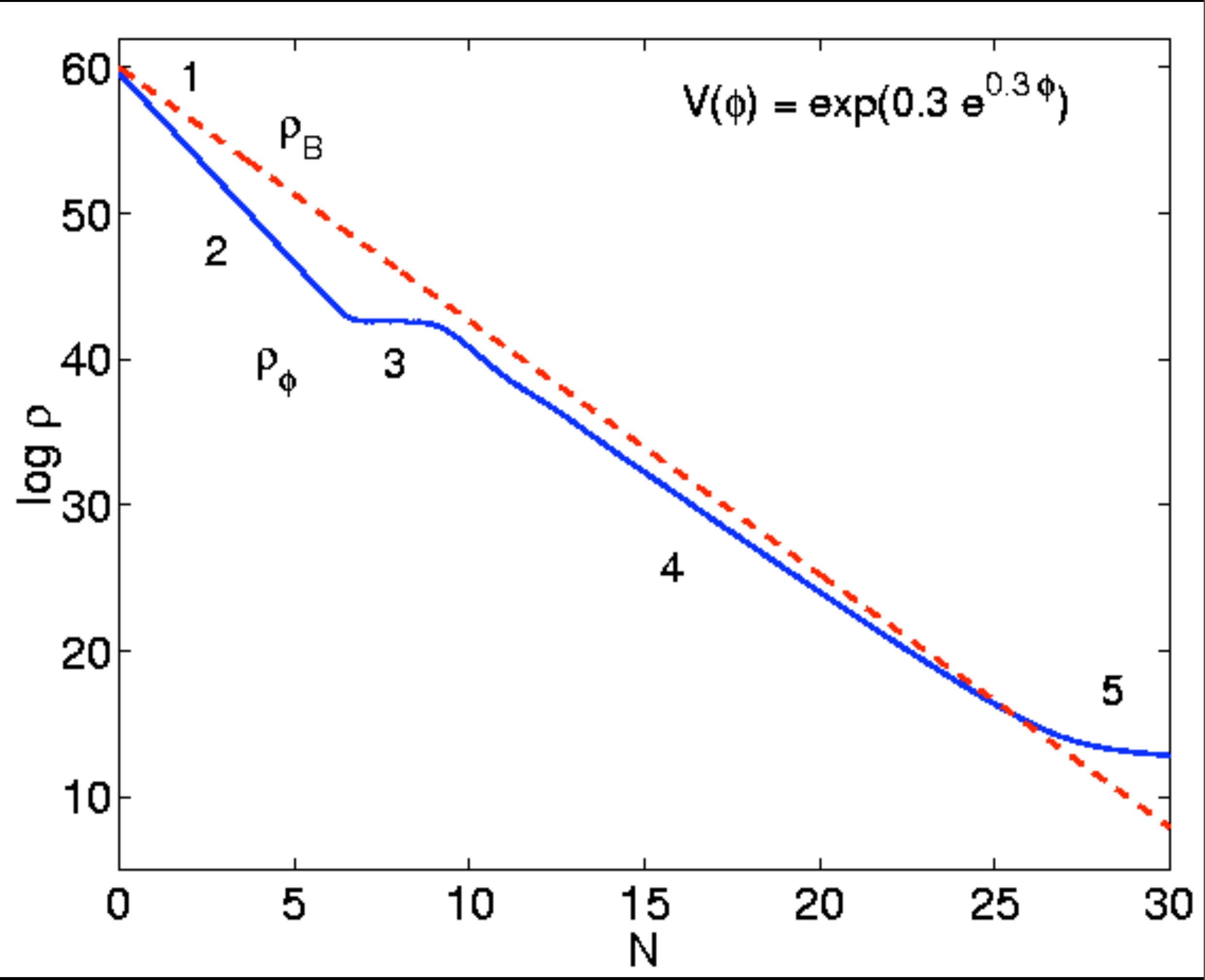
Slowly rolling scalar fields Quintessence - Generic behaviour

1. PE → KE
2. KE dom scalar field energy den.
3. Const field.
4. Attractor solution: almost const ratio KE/PE.
5. PE dom.



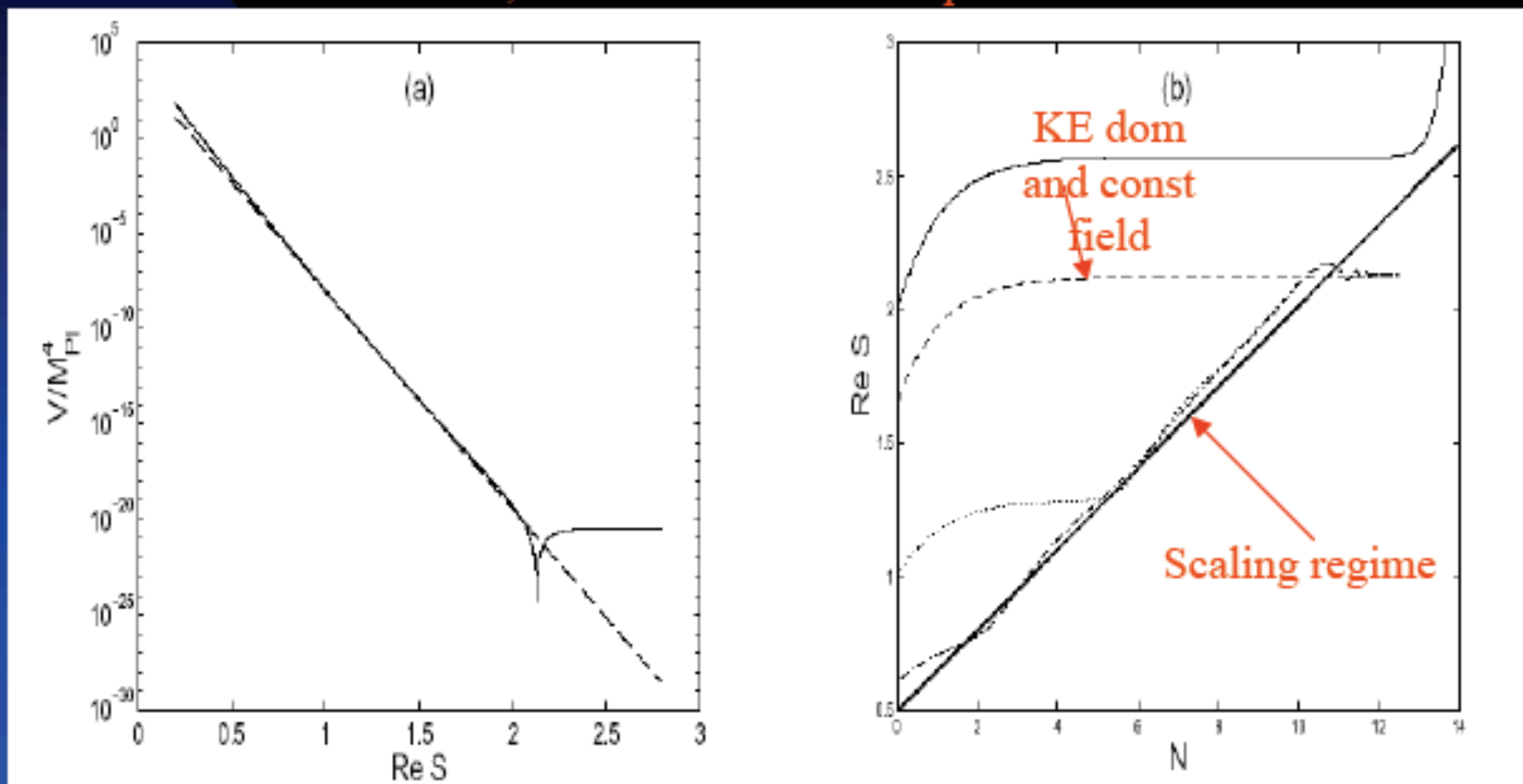
Nunes

Attractors make initial conditions less important

Useful way of stabilising moduli in string cosmology. Sources provide extra friction when potentials steep.

Barreiro, de Carlos and EC : hep/th-9805005

Brustein, Alwis and Martins : hep-th/0408160



Two condensate model with V~e^{-aReS} as approach minima

Barreiro et al : hep-th/0506045

Tracker solutions

Wetterich,

Peebles and Ratra,

Zlatev, Wang and Steinhardt

Scalar field: $\phi:\ \rho_\phi = \dfrac{\dot{\phi}^2}{2} + V(\phi)\ ;\ \ p_\phi = \dfrac{\dot{\phi}^2}{2} - V(\phi)$

EoM:

$$\dot{H} = -\frac{\kappa^2}{2}(\dot{\phi}^2 + \gamma\rho_B)$$

$$\dot{\rho}_B = -3\gamma H \rho_B$$

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV}{d\phi}$$

+ constraint:

$$H^2 = \frac{\kappa^2}{3}(\rho_\phi + \rho_B)$$

Intro: $x = \dfrac{\kappa\dot{\phi}}{\sqrt{6}H}$ $\quad y = \dfrac{\kappa\sqrt{V}}{\sqrt{3}H}$ $\quad \lambda \equiv \dfrac{-1}{\kappa V}\dfrac{dV}{d\phi}$ $\quad \Gamma - 1 \equiv \dfrac{d}{d\phi}\left(\dfrac{1}{\kappa\lambda}\right)$

Eff eqn of state: $\gamma_\phi = \dfrac{\dot{\phi}^2}{V + \dfrac{\dot{\phi}^2}{2}} = \dfrac{2x^2}{x^2 + y^2}$ $\Omega_\phi = \dfrac{\kappa^2 \rho_\phi}{3H^2} = x^2 + y^2$

Friedmann eqns and fluid eqns become:

$$x' = -3x + \lambda \sqrt{\frac{3}{2}} y^2 + \frac{3}{2} x \left[2x^2 + \gamma \left(1 - x^2 - y^2 \right) \right]$$

$$y' = -\lambda \sqrt{\frac{3}{2}} xy + \frac{3}{2} y \left[2x^2 + \gamma \left(1 - x^2 - y^2 \right) \right]$$

EC, Liddle and Wands

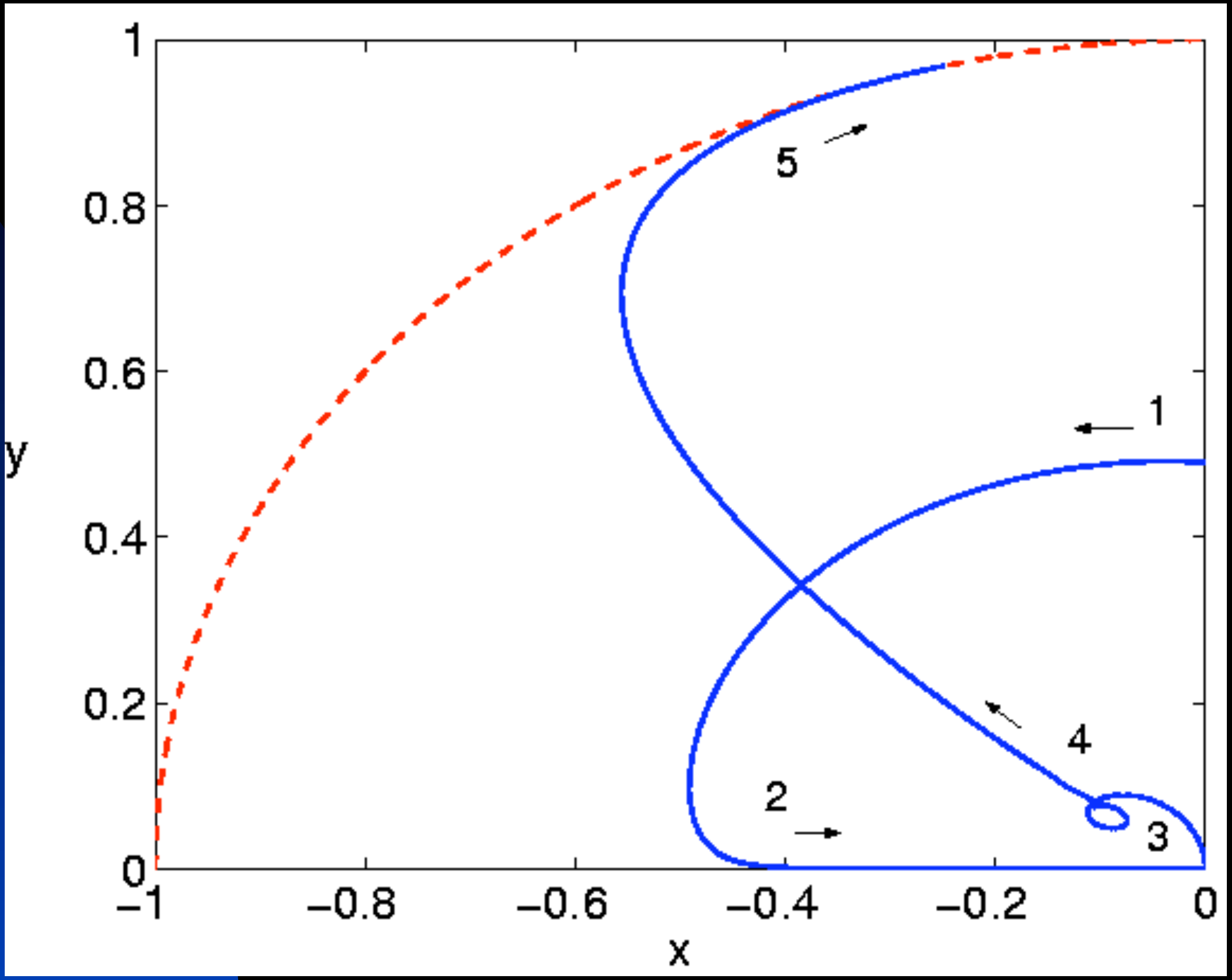
$$\lambda' = -\sqrt{6} \lambda^2 (\Gamma - 1)$$

$\dfrac{\kappa^2 \rho_\gamma}{3H^2} + x^2 + y^2 = 1$ where $' \equiv d / d(\ln a)$

Note: $0 \leq \gamma_\phi \leq 2 : 0 \leq \Omega_\phi \leq 1$

Phase Plane picture

Nunes



Typical example : Scaling solutions with exponential potentials. (EC, Liddle and Wands)

$$V(\phi) = V_0 e^{-\kappa\lambda\phi}$$

Scaling solutions: (x`=y`=0)

$$V = V_0 e^{-\lambda\kappa\phi}$$

No:	x_c	y_c	Existance	Stability	Ω_ϕ	γ_ϕ
1	0	0	$\forall\lambda,\gamma$	SP: $0<\gamma$ SN: $\gamma=0$	0	Undefined
2a	1	0	$\forall\lambda,\gamma$	UN: $\lambda<\sqrt{6}$ SP: $\lambda>\sqrt{6}$	1	2
2b	-1	0	$\forall\lambda,\gamma$	UN: $\lambda>-\sqrt{6}$ SP: $\lambda<-\sqrt{6}$	1	2
3	$\frac{\lambda}{\sqrt{6}}$	$\left(1-\frac{\lambda^2}{6}\right)^{1/2}$	$\lambda^2\le 6$	SP: $3\gamma<\lambda^2<6$ SN: $\lambda^2<3\gamma$	1	$\frac{\lambda^2}{3}$
4	$\left(\frac{3}{2}\right)^{1/2}\frac{\gamma}{\lambda}$	$\left[\frac{3(2-\gamma)\gamma}{2\lambda^2}\right]^{1/2}$	$\lambda^2\ge 3\gamma$	SN: $3\gamma<\lambda^2<\frac{24\gamma^2}{9\gamma-2}$ SS: $\lambda^2>\frac{24\gamma^2}{9\gamma-2}$	$\frac{3\gamma}{\lambda^2}$	γ

Late time attractor is scalar field dominated $\lambda^2 \le 6$

Field mimics background fluid. $\lambda^2 \ge 3\gamma$

Nucleosynthesis bound → $\lambda^2 > 20$

Useful classification scheme based on eom:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

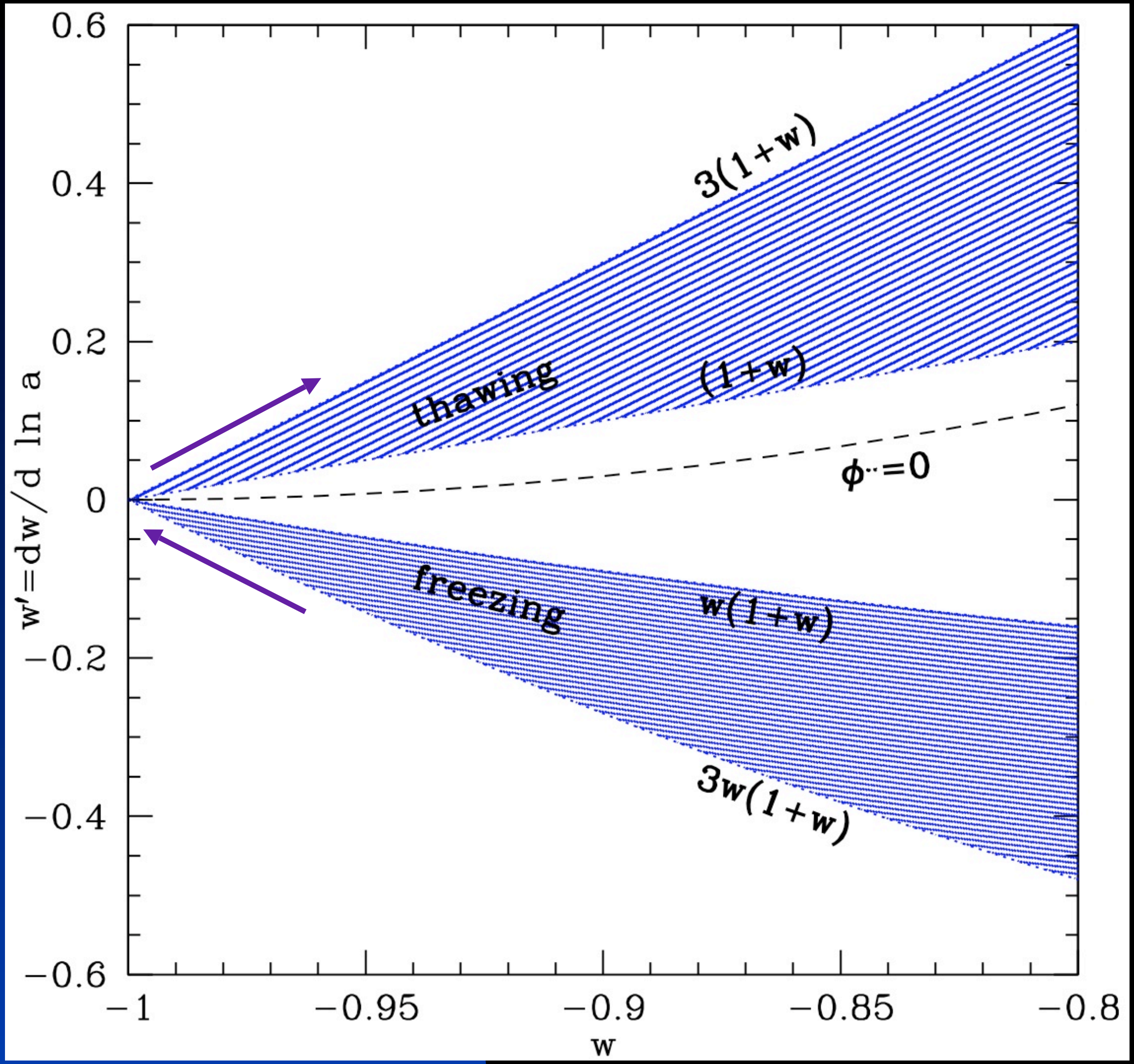
- Hubble friction slows field down
- Steepness of slope drives field

Depending which term dominates we characterise behaviour:

Freezers -- field rolls but decelerates as friction dominates

Thawers -- starts frozen by Hubble drag and then rolls.

Appear to be constrained to a narrow region of w-w' plane



$$w = \frac{\frac{\dot{\phi}^2}{2} - V(\phi)}{\frac{\dot{\phi}^2}{2} + V(\phi)}$$

Caldwell and Linder 2005

Original Quintessence model

Peebles and Ratra;

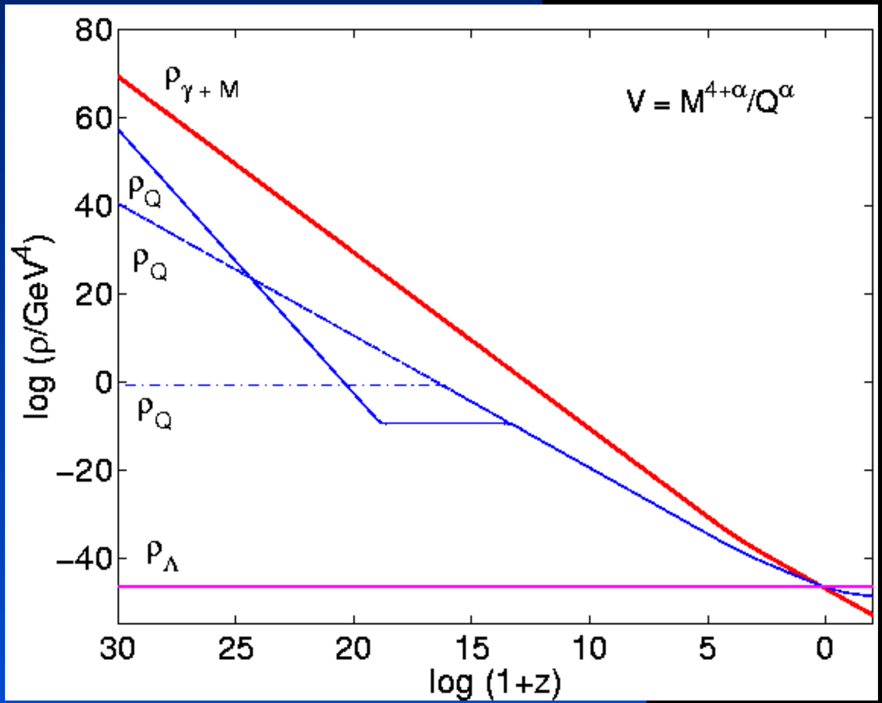
Zlatev, Wang and Steinhardt

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

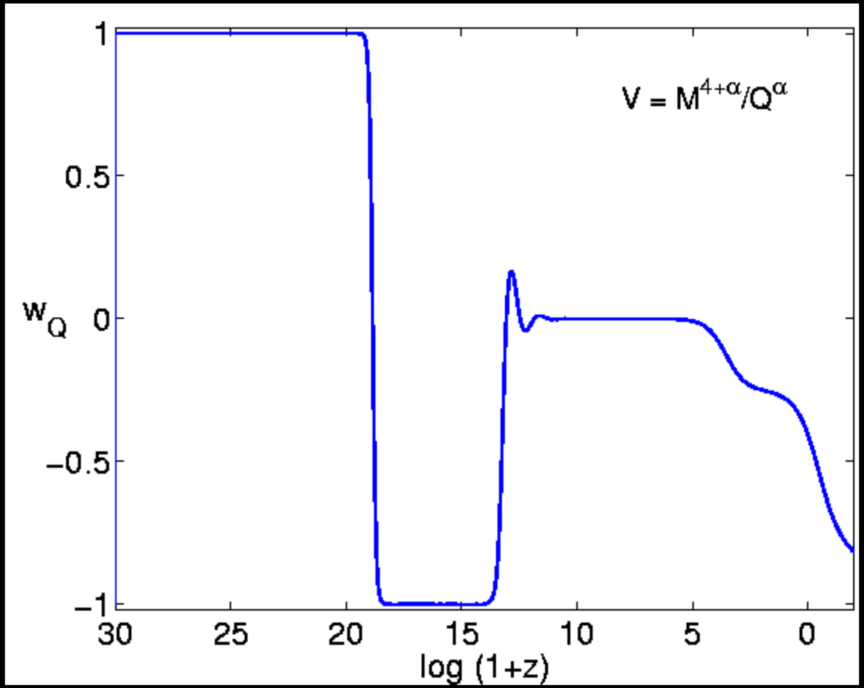
$$\lambda = \frac{-\alpha}{\kappa\phi}$$

$$\Gamma - 1 \equiv \frac{1}{\alpha}$$

Find: $\phi = \phi_i \left(\frac{a}{a_i}\right)^{\frac{3(1+w_B)}{2+\alpha}}$ and $w_\phi = \frac{\alpha w_B - 2}{2+\alpha}$



$\alpha = 6$



Fine Tuning in Quintessence

Need to match energy density in Quintessence field to current critical energy density.

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

$$\rho_{\Lambda} \leq \frac{H_0^2}{\kappa^2} \approx 10^{-47}\ \mathrm{GeV}^4$$

Find: $y_c^2 = \frac{\kappa^2 V}{3H^2} \propto \kappa^2 \phi^2$ so: $H^2 = \frac{V}{\phi^2} \propto \kappa^2 \rho_{\phi} \Rightarrow \phi_0 \approx M_{pl}$

Hence: $M = \left[\rho_{\phi}^{0}\, M_{pl}^{\alpha}\right]^{1/4+\alpha} \Rightarrow \alpha = 2; M = 1\,GeV$

A few models

1. Inverse polynomial – found in SUSY QCD - Binetruy

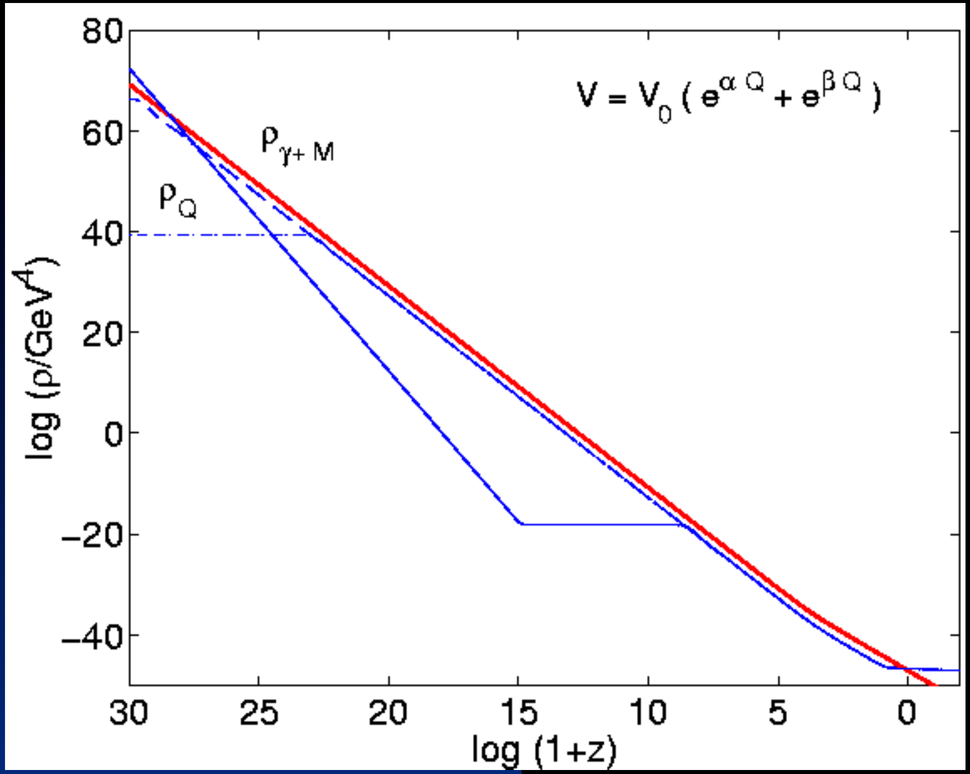
2. Multiple exponential potentials – SUGR and String compactification.

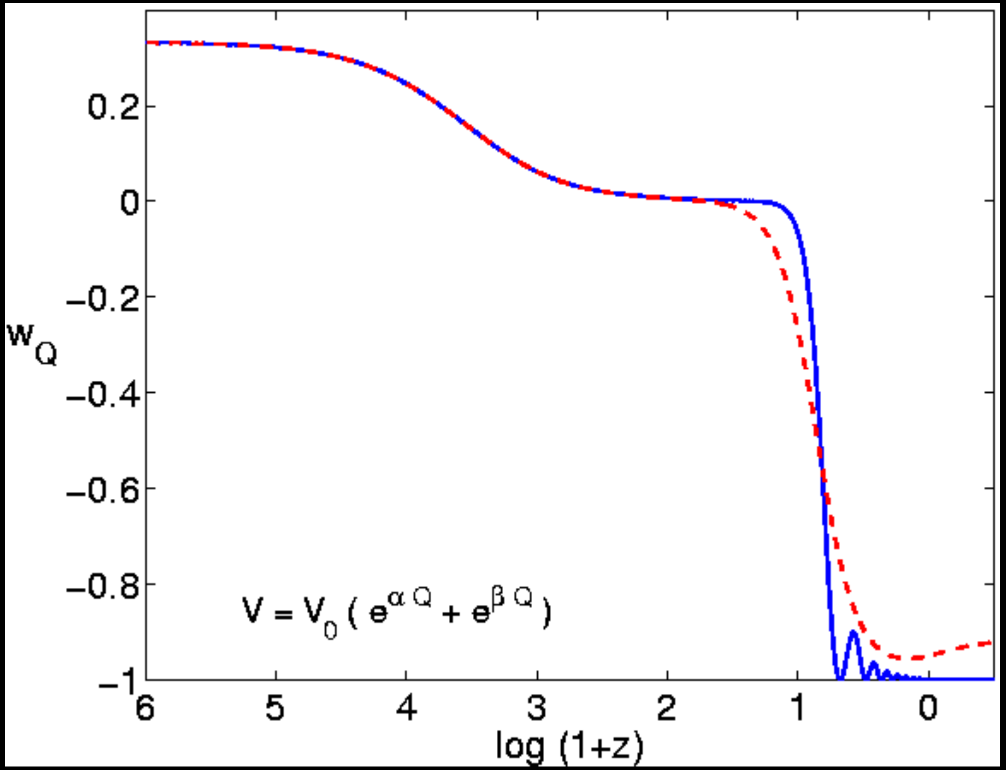
Barreiro, EC, Nunes

$$
\begin{aligned}
V(\phi) &= V_1 + V_2 \\
&= V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi}
\end{aligned}
$$

Enters two scaling regimes depends on lambda, one tracking radiation and matter, second one dominating at end. Must ensure do not violate nucleosynthesis constraints.

$\alpha = 20; \beta = 0.5$





Scaling for wide range of i.c.

Fine tuning:

$$V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$$

Mass:

$$m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$$

Fifth force !

3. Cosh potential model – Sahni and Wang

$$V(\phi) = V_0\left[\cosh(\alpha\kappa\phi) - 1\right]^n$$

$$V(\phi) \rightarrow \exp\left[n\alpha\kappa\phi\right] \;\; for \;\; |\alpha\kappa\phi| >> 1$$ Scales as rad and matter

$$V(\phi) \rightarrow \left[\alpha\kappa\phi\right]^n \;\; for \;\; |\alpha\kappa\phi| << 1$$ Oscillates about minima

Time ave eqn of state: $$\langle w_\phi \rangle = \frac{n-1}{n+1}$$

For n < 1/2, eqn of state less than – 1/3 and scalar field dominates at late times.

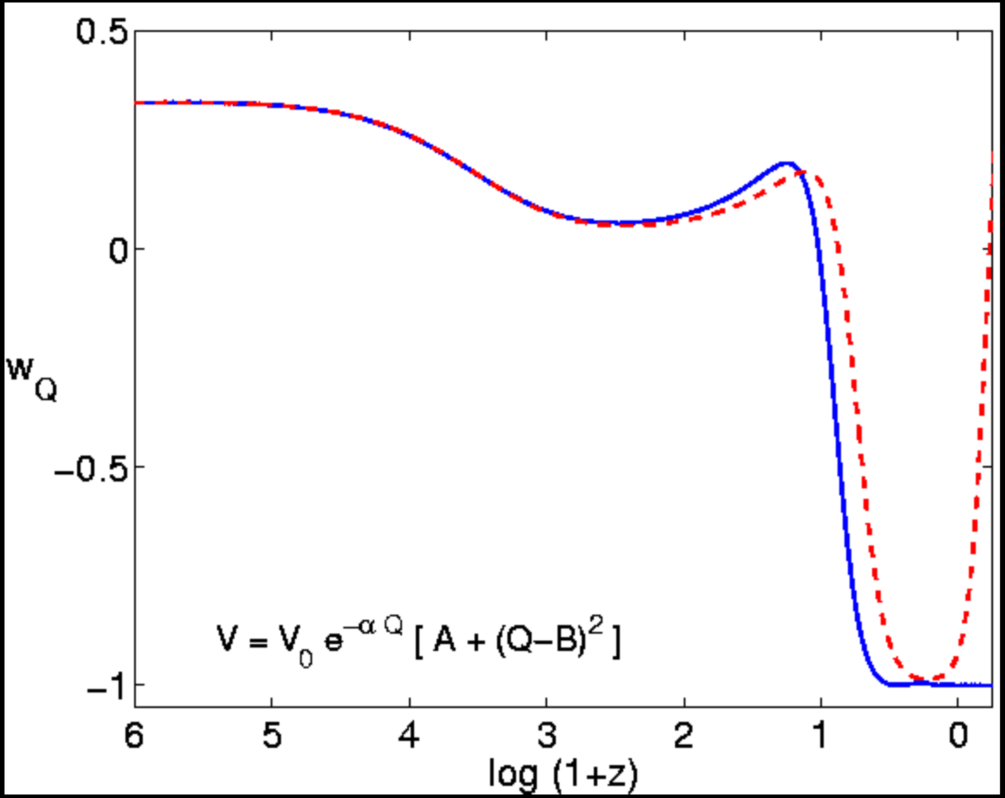
3. Albrecht-Skordis model – Albrecht and Skordis

$$V(\phi) = V_0\, e^{-\alpha\kappa\phi}\left[A + (\kappa\phi - B)^2\right]$$

-- Brane models

Early times: exp dominates and scales as rad or matter.

Field gets trapped in local minima and univ accelerates



Fine tuned as in previous cases.

4. Quintessential Inflation – Peebles and Vilenkin

Same field provides both initial inflaton and todays Quintessence – not tracker.

$$V(\phi) = \lambda(\phi^4 + M^4) \quad \text{for } \phi < 0$$
$$= \frac{\lambda M^4}{1 + (\phi/M)^\alpha} \quad \text{for } \phi \geq 0$$

Reheating at end of inflation from grav particle production

Avoids need for minima in inflaton potential Ford

$$\lambda = 10^{-14} : \Omega_{\phi_0} = 0.7 \Rightarrow \alpha = 4 ; M = 10^5 \text{ GeV},$$

Need to be careful do not overproduce grav waves.

Recent interesting proposal to link inflation, dark matter and dark energy through single mechanism Liddle

5. Supergravity inspired models – Brax and Martin; Choi; EC, Nunes, Rosati; …

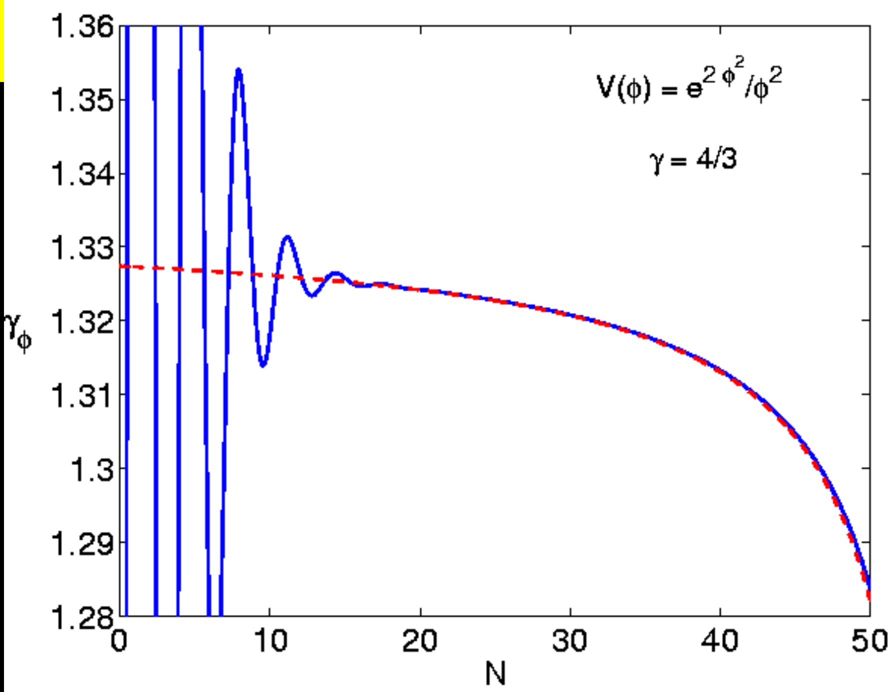
$$W = \Lambda^{3+\alpha}\phi^{-\alpha}$$

$$K = \phi\phi^{*}$$

$$\text{If} \quad \langle W \rangle = 0 \Rightarrow V(\phi) = \frac{\Lambda^{6+2\alpha}}{\phi^{2\alpha+2}} e^{\frac{\kappa^2}{2}\phi^2}$$

$$\alpha = 11 \Rightarrow w_{\phi_0} = -0.8$$

Issues over flatness of potential – Lyth and Kolda



Many more models!!

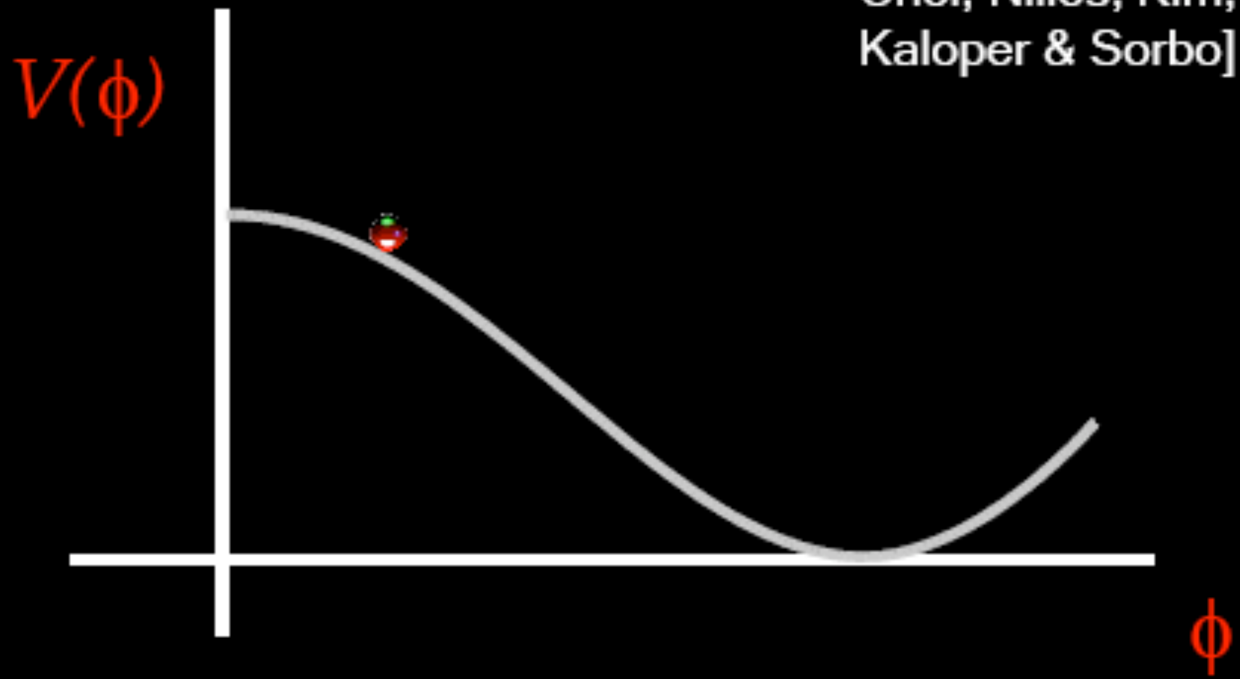
Particle physics inspired models?

Pseudo-Goldstone Bosons -- approx sym $\phi \rightarrow \phi$ + const.

Leads to naturally small masses, naturally small couplings

[Hill, Freiman, et al;
Choi; Nilles; Kim;
Kaloper & Sorbo]

Barbieri et al



$$V(\phi) = \lambda^4 (1 + \cos(\phi/F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy.

Strong CP problem intro axion : $m_a = \frac{\Lambda^2_{\rm QCD}}{F_a}; \; F_a \; - \; \text{decay constant}$

PQ axion ruled out but invisible axion still allowed:

$$10^9 \text{ GeV} \leq F_a \leq 10^{12} \text{ GeV}$$

Sun stability CDM constraint

String theory has lots of antisymmetric tensor fields in 10d, hence many light axion candidates.

Can have $F_a \sim 10^{17}$-10^{18} GeV

Quintessential axion -- dark energy candidate [Kim & Nilles].

Requires $F_a \sim 10^{18}$ GeV which can give:

$$E_{\rm vac} = (10^{-3} \text{ eV})^4 \rightarrow m_{\rm axion} \sim 10^{-33} \text{ eV}$$

Because axion is pseudoscalar -- mass is protected, hence avoids fifth force constraints

Quintessential Axion -- Kim and Nilles

Linear combination of two axions together through hidden sector supergravity breaking.

Light CDM axion (solve strong CP problem) with decay const through hidden sector squark condensation: $F_a \sim 10^{12}\mathrm{GeV}$

Quintaxion (dark energy) with decay const as expected for model independent axion of string theory: $F_q \sim 10^{18}\mathrm{GeV}$

Model works because of similarities in mass scales:

Scale of susy breaking and scale of QCD axion.

Scale of vacuum energy and mass of QCD axion. $\lambda^4 \sim (0.003\mathrm{eV})^4$

Potential for quintaxion remains very flat, because of smallness of hidden sector quark masses, ideal for Quintessence. Quintessence mass protected through existence of global symmetry associated with pseudo Nambu-Goldstone boson.

K-essence v Quintessence

K-essence -- scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model? -- Armendariz-Picon, Mukhanov, Steinhardt

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form.

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{16\pi G}R + K(\phi)\tilde{p}(X)\right]$$

$$K(\phi) > 0,\ X = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$

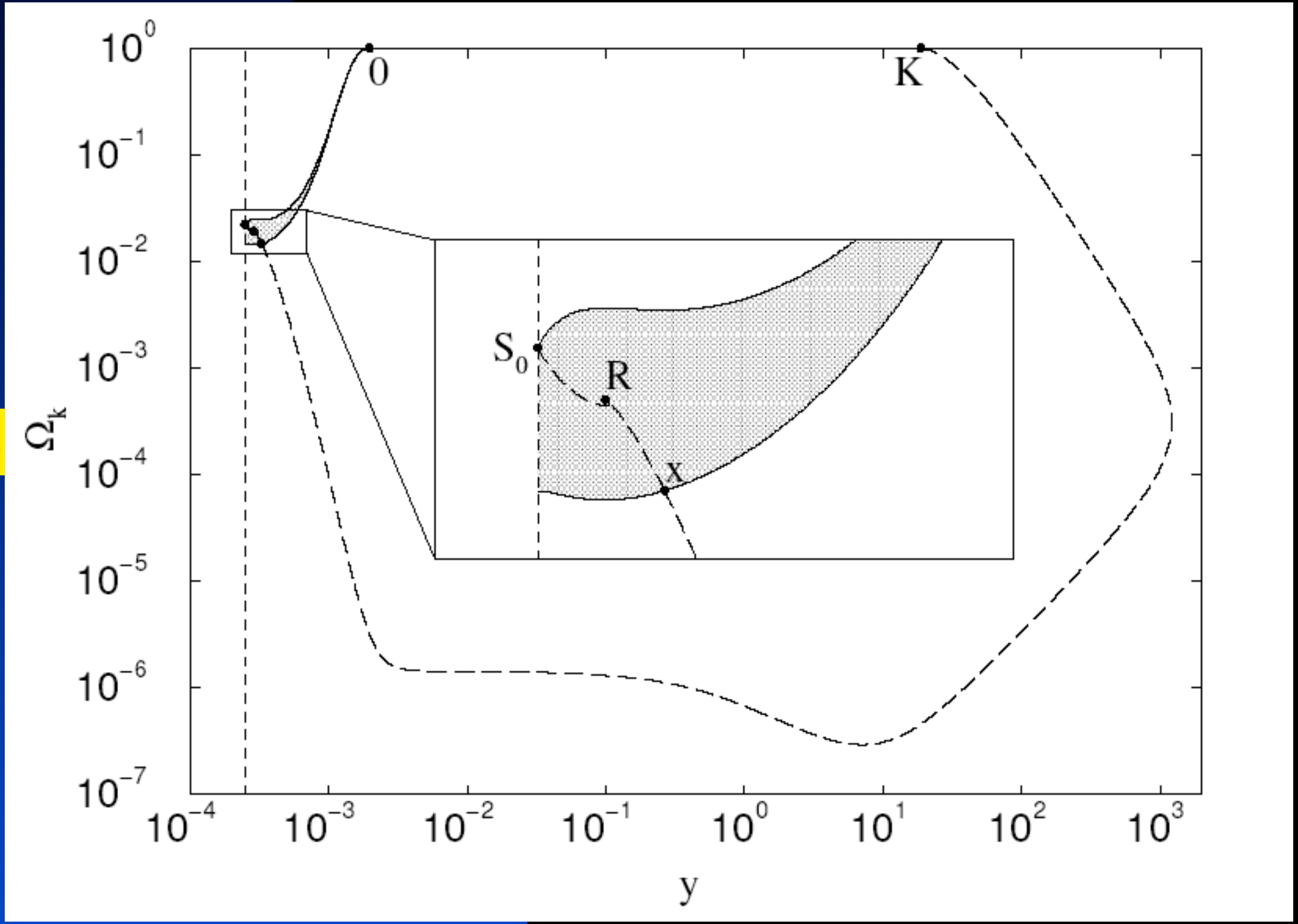
Eqn of state $w_k = \frac{\tilde{p}(X)}{\tilde{\varepsilon}(X)} = \frac{\tilde{p}(X)}{2X\tilde{p}'(X) - \tilde{p}(X)}$ can be < -1

However also requires similar level of find tuning as in Quintessence

Fine tuning in K-essence as well: -- Malquarti, EJC, Liddle

Not so clear that K-essence solves the coincidence problem. The basin of attraction into the regime of tracker solutions is small compared to those where it immediately goes into K-essence domination.

Ω_k



$y = 1/\sqrt{X}$

Shaded region is basin of attraction for stable tracker solution at point R. All other trajectories go to K-essence dom at point K.

Based on K-essence model astro-ph/0004134, Armendariz-Picon et al.

This could be related to speed of sound problem: -- Bonvin et al 2006

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{16\pi G}R + K(\phi)\tilde{p}(X)\right]$$

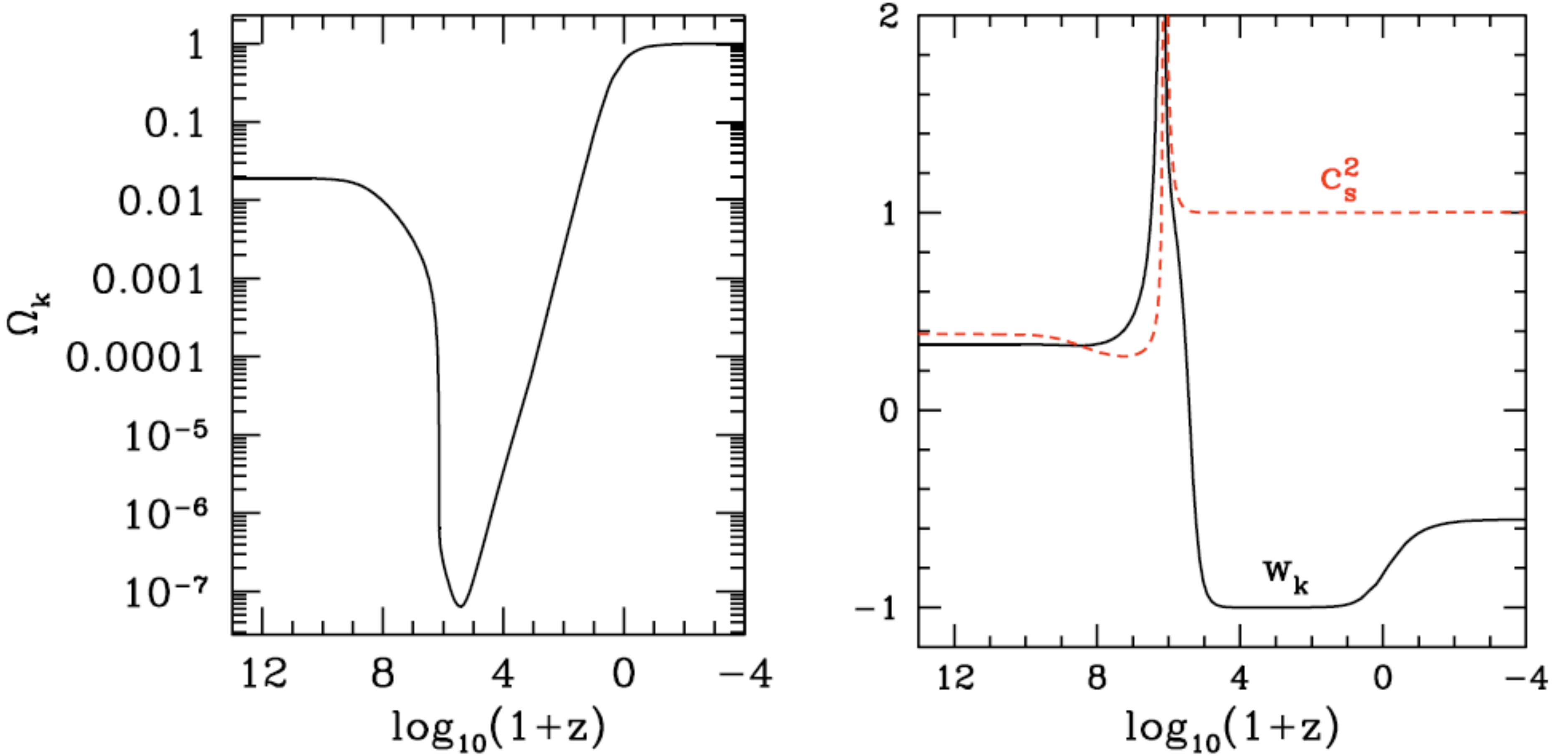
$$K(\phi) > 0,\ X = \tfrac{1}{2}\nabla_\mu\phi\nabla^\mu\phi \qquad w_k = \frac{\tilde{p}(X)}{\tilde{\varepsilon}(X)} = \frac{\tilde{p}(X)}{2X\tilde{p}'(X) - \tilde{p}(X)}$$

$$c_s^2 = \frac{P'}{2XP'' + P'}, \quad ' \equiv \frac{d}{dX}$$

Tracking solutions have radiation fixed point, $w_k = 1/3$, $\Omega_k << 1$ and k-essence fixed point $w_k < -1/3$, $\Omega_k \approx 1$.

As mentioned can have evolution from radiation fixed point to k-essence fixed point when univese becomes matter dominated.

But to do this universe passes through a phase where $c_s^2 > 1$ - requires super-luminal motion to solve coincidence problem.



Bonvin et al., 2006

Dark energy from Tachyon fields [Sen (2002), Garousi (2002), Gibbons (2002) …]

Introduced by Sen as a way of understanding the decay of D-branes, it has been noted that a rolling tachyon has an equation of state which varies between -1 and 0. Difficult to use it to have early Inflation but possible to have late time acceleration.

Tachyon on non BPS D3 brane:

$$S = -\int d^4x\, V(\phi)\sqrt{-\det(g_{ab} + \partial_a\phi\partial_b\phi)}$$

$$V(\phi) = \frac{V_0}{\cosh(\phi/\phi_0)}$$

Density and pressure and EOM:

$$\rho = -T_0^0 = \frac{V(\phi)}{\sqrt{1-\dot\phi^2}},$$

$$p = T_i^i = -V(\phi)\sqrt{1-\dot\phi^2}$$

$$H^2 = \frac{8\pi G V(\phi)}{3\sqrt{1-\dot\phi^2}},$$

$$\frac{\ddot\phi}{1-\dot\phi^2} + 3H\dot\phi + \frac{1}{V}\frac{dV}{d\phi} = 0$$

Accn: $$\frac{\ddot a}{a} = \frac{8\pi G V(\phi)}{3\sqrt{1-\dot\phi^2}}\left(1 - \frac{3}{2}\dot\phi^2\right)$$

Accn for: $\dot\phi^2 < 2/3$

Eqn of state: $$w_\phi = \frac{p}{\rho} = \dot\phi^2 - 1$$

Note, indep of steepness of potential, eos varies between 0 and -1

Phantom fields [Caldwell (2002) ...]

The data does not rule out w<-1. Can not accommodate in standard quintessence models but can by allowing negative kinetic energy for scalar field (amongst other approaches). Can arise from two time models in Type IIA strings, or low energy limit of F-theory in 12D Type IIB action.

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

leads to

$$w_\phi = \frac{p}{\rho} = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 - 2V(\phi)}$$

$$w_\phi < -1 \text{ for } \dot{\phi}^2/2 < V(\phi)$$

Super inflationary soln

$$a(t) = (t_s - t)^{\frac{2}{3(1+w)}}$$

$$H = \frac{n}{t_s - t}, \quad n = -\frac{2}{3(1+w)} > 0,$$

$$R = 6\left(2H^2 + \dot{H}\right) = \frac{6n(2n+1)}{(t_s - t)^2}.$$

Big Rip Singularity as $t \rightarrow t_s$

Depending on potential can avoid Big Rip but concerns over UV quantum instabilities. Vacuum unstable against production of ghosts and normal (+ve energy fields) [Carroll et al(2002), Cline et al (2004)]

But recent work suggests inclusion of higher order operators stabilise things on the w<-1 side [Creminelli et al (08)]

Chameleon fields [Khoury and Weltman (2003) …]

Key idea: in order to avoid fifth force type constraints on Quintessence models, have a situation where the mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

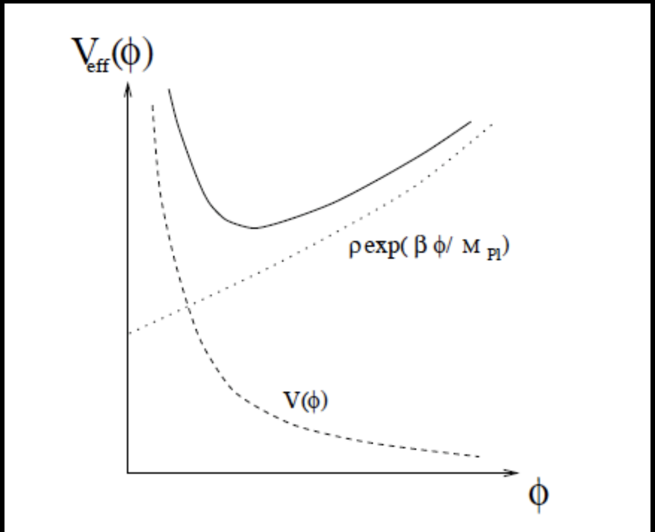
In that way can explain dark energy without violating solar system bounds.

Chameleons in a cosmological setting obtained for a wide range of potentials [Brax et al (04) …]

Proposed way of detecting chameleons through Casimir Force experiments because chameleon force between two nearby bodies is more like Casimir force than gravitational force [Brax et al (07) …]

$$V_{eff}(\phi) \equiv V(\phi) + \rho e^{\beta\phi/M_{Pl}}$$

mass of field depends on local matter density



Recent suggestion: [Gies, Mota and Shaw 2007]

Afterglow as a trace of chameleon field in optical expt. Vacuum interaction of a laser pulse with B field -> production and trapping of chameleons in vacuum chamber because their mass depends on ambient matter density. Magnetically induced re-conversion of trapped chameleons into photons creates afterglow over macroscopic timescales which can be searched for in current optical expts.

Hunting for chameleons in Axion Like Particle searches with GammeV experiment : [Weltman et al]

$$V_{\rm eff}(\phi,\vec{x}) = V(\phi) + e^{\beta_m\phi/M_{\rm Pl}}\rho_m(\vec{x}) + e^{\beta_\gamma\phi/M_{\rm Pl}}\rho_\gamma(\vec{x}),$$

Provides a method of detecting strongly coupled chameleons (coupled to photons -- $\beta_\gamma >> 1$

Mass Varying Neutrino Models (MaVaNs). [Hung;Li et al; Fardon et al]

Coincidence ? $\rho_\Lambda \sim \Delta m_\nu^2(solar) \sim (10^{-3})^4 \mathrm{eV}^4$

Perhaps neutrinos coupled to dark energy with a mass depending on a scalar field -- acceleron

Field has instantaneous min which varies slowly as function of neutrino density. It can be heavy relative to Hubble rate (unlike standard Quintessence).

Eff pot for MaVaNs: $V = n_\nu m_\nu(\mathcal{A}) + V_0(\mathcal{A})$ with: $n_\nu = -\frac{\partial V_0}{\partial m_\nu}$

EOS for system (ignoring KE of acceleron): $w = \frac{p}{\rho} = -1 + \frac{n_\nu m_\nu}{V}$

$$w \sim -1 \quad \text{for} \quad n_\nu m_\nu \ll V_0$$

Many authors studied cosmology -- interesting model, example of coupled dark energy scenarios [Amendola; Brookfield et al 05 and 07]

Chaplygin gases -- acceleration by changing the equation of state of exotic background fluid rather than using a scalar field potential. [Kamenshchik, Moshella, Pasquier 2001]

$$p = -\frac{A}{\rho}$$

Sub in energy-momentum conservation

$$\rho = \sqrt{A + \frac{B}{a^6}}$$

Interpolates: dust dom -->De Sitter phase via stiff fluid

$$\rho = \sqrt{B}a^{-3} \qquad p = -\rho \qquad p = \rho$$

Representation in terms of generalised d-branes evolving in (d +1,1) dimensional spacetime [Bento et al, 2002]

Nice feature -- does not introduce new scalar field. Provides way of unifying dark matter and dark energy under one umbrella. (Note can write it as a potential if you want)

Need to understand ways of testing it observationally. Must link LSS and current acceleration.