Interacting Dark Energy [Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + many others…]

Idea: why not directly couple dark energy and dark matter?

$$\begin{aligned} \text{Ein eqn} &: \quad G_{\mu\nu} = 8\pi G T_{\mu\nu} \\ \text{General covariance} &: \quad \nabla_\mu G^\mu_\nu = 0 \rightarrow \nabla_\mu T^\mu_\nu = 0 \end{aligned}$$

$$T_{\mu\nu} = \sum_i T^{(i)}_{\mu\nu} \rightarrow \nabla_\mu T^{\mu\,(i)}_\nu = -\nabla_\mu T^{\mu\,(j)}_\nu \ \text{ is ok}$$

Couple dark energy and dark matter fluid in form:

$$\begin{aligned} \nabla_\mu T^{\mu\,(\phi)}_\nu &= \sqrt{\frac{2}{3}}\kappa\beta(\phi) T^{\alpha\,(m)}_\alpha \nabla_\nu \phi \\ \nabla_\mu T^{\mu\,(m)}_\nu &= -\sqrt{\frac{2}{3}}\kappa\beta(\phi) T^{\alpha\,(m)}_\alpha \nabla_\nu \phi \end{aligned}$$

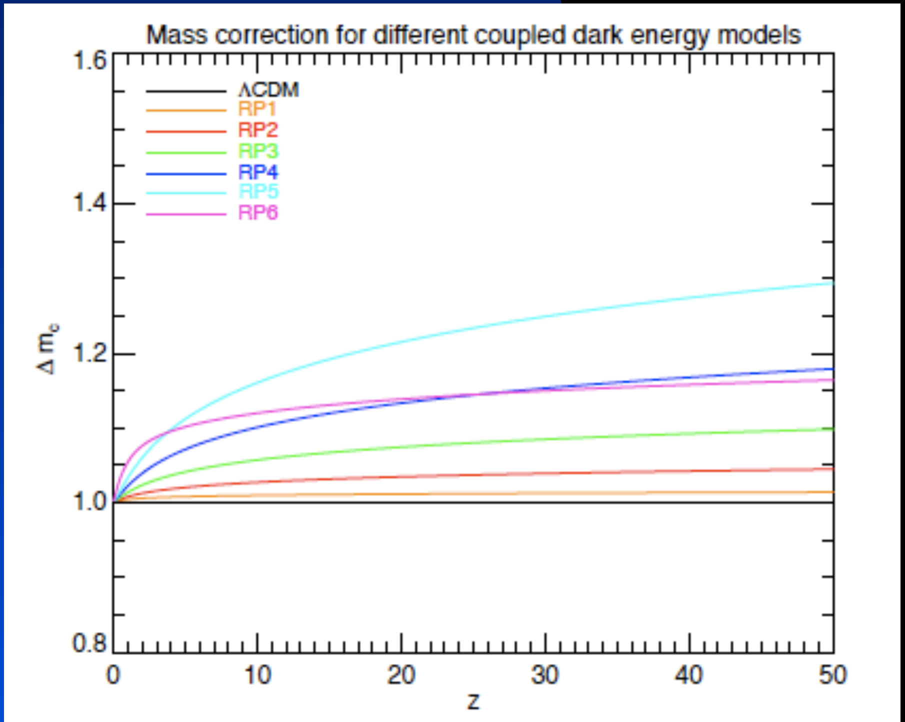
Evolution equations are then modified, H(a,β(ϕ)), and a variable dark matter mass emerges:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = \sqrt{\frac{2}{3}}\kappa\beta(\phi)\rho_m$$

$$\dot{\rho_m} + 3H\rho_m = -\sqrt{\frac{2}{3}}\kappa\beta(\phi)\dot{\phi}$$

$$\dot{\rho}_b + 3H\rho_b = 0$$

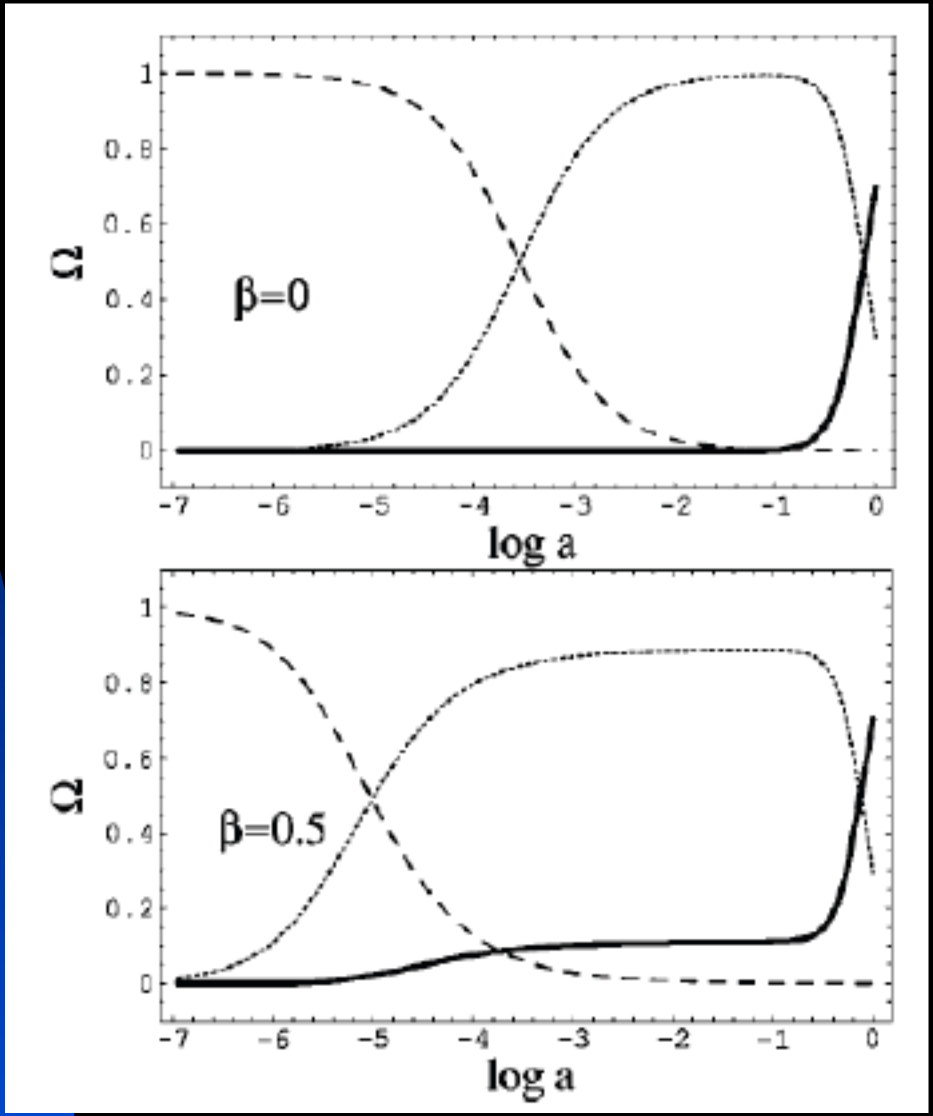
$$m(\phi) = m_0 \exp\left[\sqrt{\frac{2}{3}}\kappa\int_{\phi}^{\phi_0}\beta(\phi')d\phi'\right] = m_0 F_M(\phi)$$



Variation of dark matter mass:

Phase plane analysis leads to scaling solutions and fixed points:

For weak coupling $|\beta|<3/2$, find both late time accelerated DE attractor, and ϕ-MDE epoch early on



Perturbations in Interacting Dark Energy Models [Baldi et al (2008)]

Perturb everything linearly : Matter fluid example

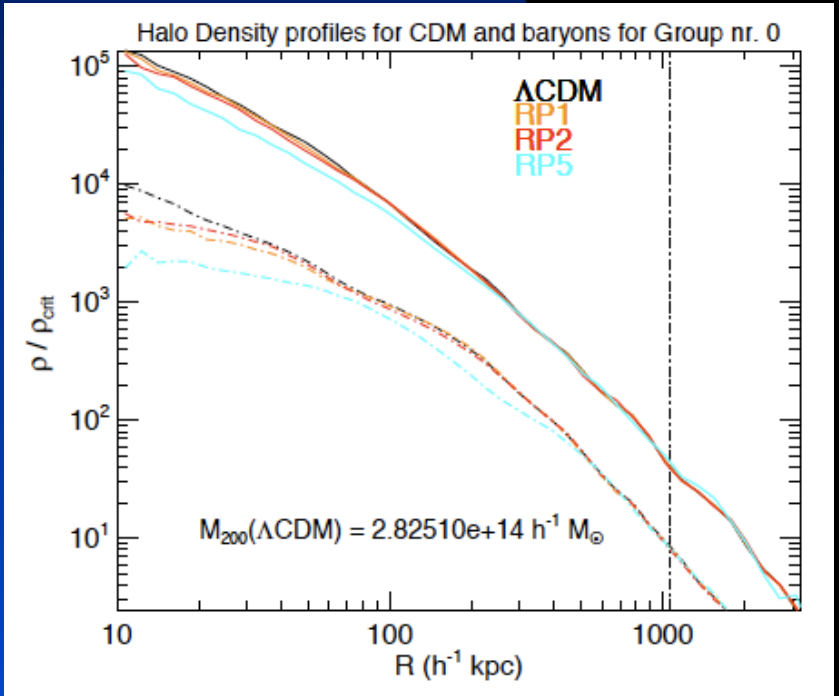
$$\ddot{\delta}_c + \left(2H - 2\beta\frac{\dot{\phi}}{M}\right)\dot{\delta}_c - \frac{3}{2}H^2[(1+2\beta^2)\Omega_c\delta_c + \Omega_b\delta_b] = 0$$

extra friction

modified grav interaction

vary DM particle mass

Include in simulations of structure formation : GADGET [Springel (2005)]



Density decreases as coupling β increases

Halo mass function modified.

Halos remain well fit by NFW profile.

Density decreases compared to ΛCDM as coupling β increases.

Scale dep bias develops from fifth force acting between CDM particles. enhanced as go from linear to smaller non-linear scales.

Still early days ..

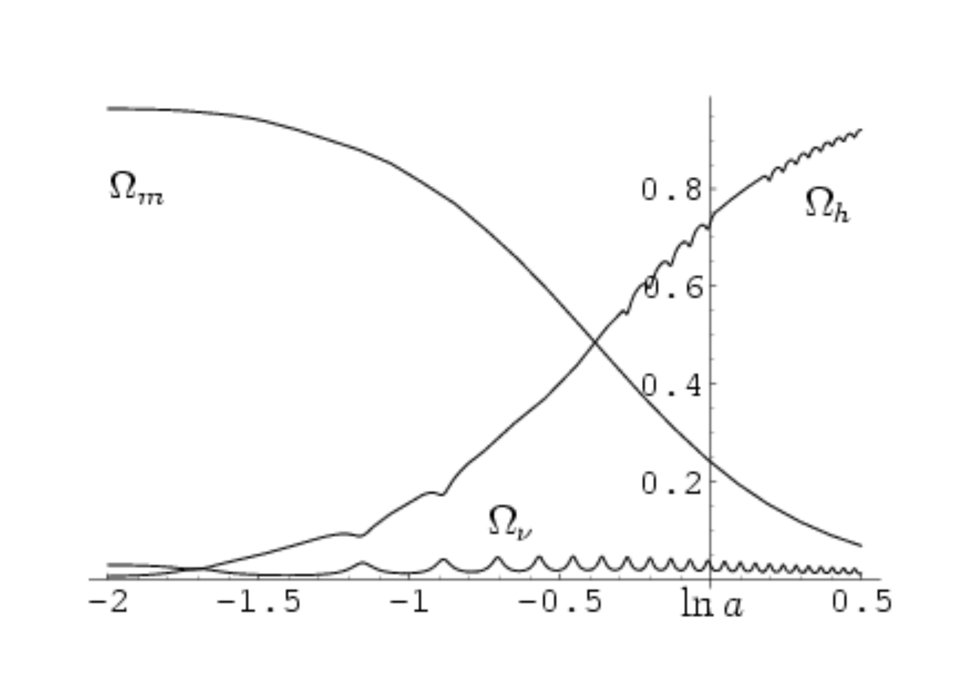
Including neutrinos -- 2 distinct DM families -- resolve coincidence problem [Amendola et al (2007)]

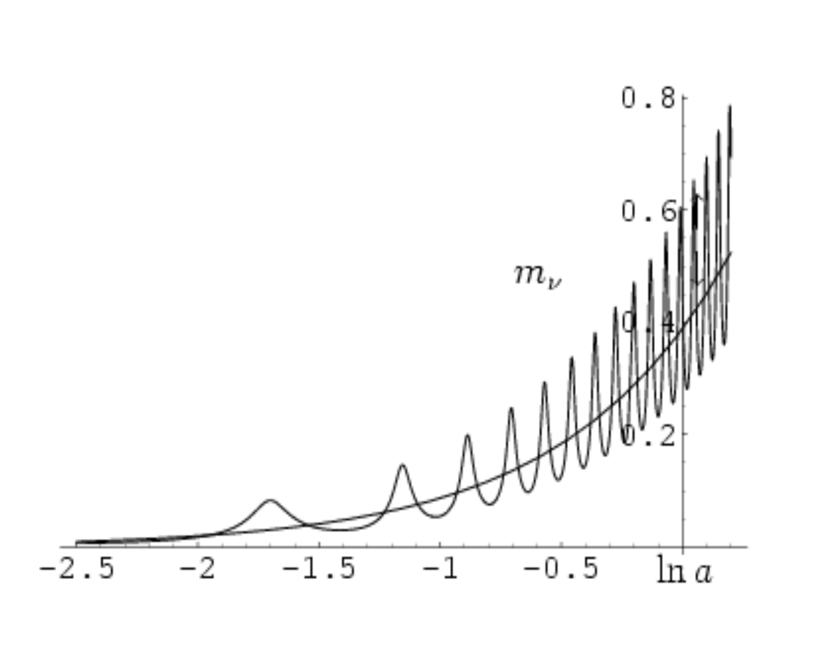
Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$





m_ν

Mass Varying Neutrino Models (MaVaNs). [Hung;Li et al; Fardon et al]

Coincidence ? $\rho_\Lambda \sim \Delta m_\nu^2(solar) \sim (10^{-3})^4 \mathrm{eV}^4$

Perhaps neutrinos coupled to dark energy with a mass depending on a scalar field -- acceleron

Field has instantaneous min which varies slowly as function of neutrino density. It can be heavy relative to Hubble rate (unlike standard Quintessence).

Eff pot for MaVaNs: $V = n_\nu m_\nu(\mathcal{A}) + V_0(\mathcal{A})$ with: $n_\nu = -\frac{\partial V_0}{\partial m_\nu}$

EOS for system (ignoring KE of acceleron): $w = \frac{p}{\rho} = -1 + \frac{n_\nu m_\nu}{V}$

$$w \sim -1 \quad \text{for} \quad n_\nu m_\nu \ll V_0$$

Many authors studied cosmology -- interesting model, example of coupled dark energy scenarios [Amendola; Brookfield et al 05 and 07]

Chaplygin gases -- acceleration by changing the equation of state of exotic background fluid rather than using a scalar field potential. [Kamenshchik, Moshella, Pasquier 2001]

$$p = -\frac{A}{\rho}$$

Sub in energy-momentum conservation

$$\rho = \sqrt{A + \frac{B}{a^6}}$$

Interpolates: dust dom -->De Sitter phase via stiff fluid

$$\rho = \sqrt{B}a^{-3} \qquad p = -\rho \qquad p = \rho$$

Representation in terms of generalised d-branes evolving in (d+1,1) dimensional spacetime [Bento et al, 2002]

Nice feature -- does not introduce new scalar field. Provides way of unifying dark matter and dark energy under one umbrella. (Note can write it as a potential if you want)

Need to understand ways of testing it observationally. Must link LSS and current acceleration.

Acc^n from new Gravitational Physics? [Starobinski 1980, Carroll et al 2003]

$$S = \frac{M_P^2}{2}\int d^4x\,\sqrt{-g}\left(R - \frac{\mu^4}{R}\right) + \int d^4x\,\sqrt{-g}\,\mathcal{L}_M$$

Modify Einstein

Const curv vac solutions:

$$\nabla_\mu R = 0, \rightarrow R = \pm\sqrt{3}\mu^2$$

de Sitter or Anti de Sitter

Transform to EH action:

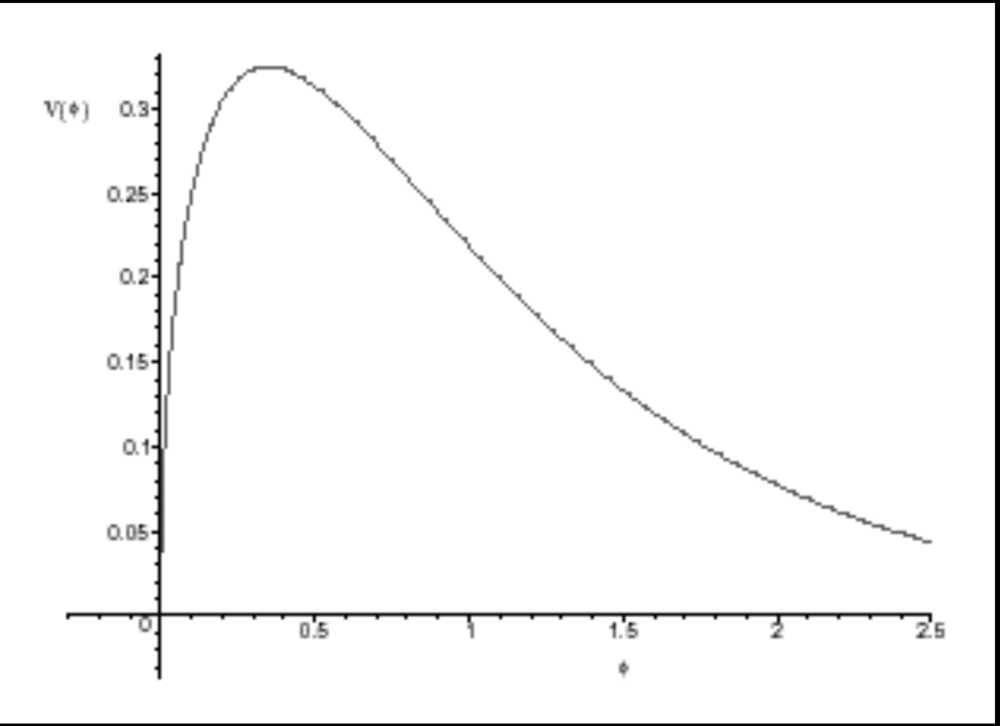
$$\tilde{g}_{\mu\nu} = p(\phi) g_{\mu\nu}\ , \quad p \equiv \exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right) \equiv 1 + \frac{\mu^4}{R^2}$$

Scalar field min coupled to gravity and non minimally coupled to matter fields with potential:

$$V(\phi) = \mu^2 M_P^2 \frac{\sqrt{p-1}}{p^2}$$

Cosmological solutions:

1. **Eternal de Sitter - ϕ just reaches V_{max} and stays there. Fine tuned and unstable.**
2. **Power law inflation -- ϕ overshoots V_{max}, universe asymptotes with w_{DE}=-2/3.**
3. **Future singularity-- ϕ doesn't reach V_{max}, and evolves back towards ϕ=0.**



1.Fine tuning needed so acceleration only recently: $m \sim 10^{-33}$eV

2. Also, not consistent with classic solar system tests of gravity.

3. Claim that such R^{-n} corrections fail to produce matter dom era [Amendola et al, 06]

But recent results based on singular perturbation theory suggests it is possible [Evans et al, 07]

Designer f (R) models [Hu and Sawicki (2007)]

Construct a model to satisfy observational requirements:

1.Mimic LCDM at high z as required by CMB

2. Accelerate univ at low z

3. Include enough dof to allow for variety of low z phenomena

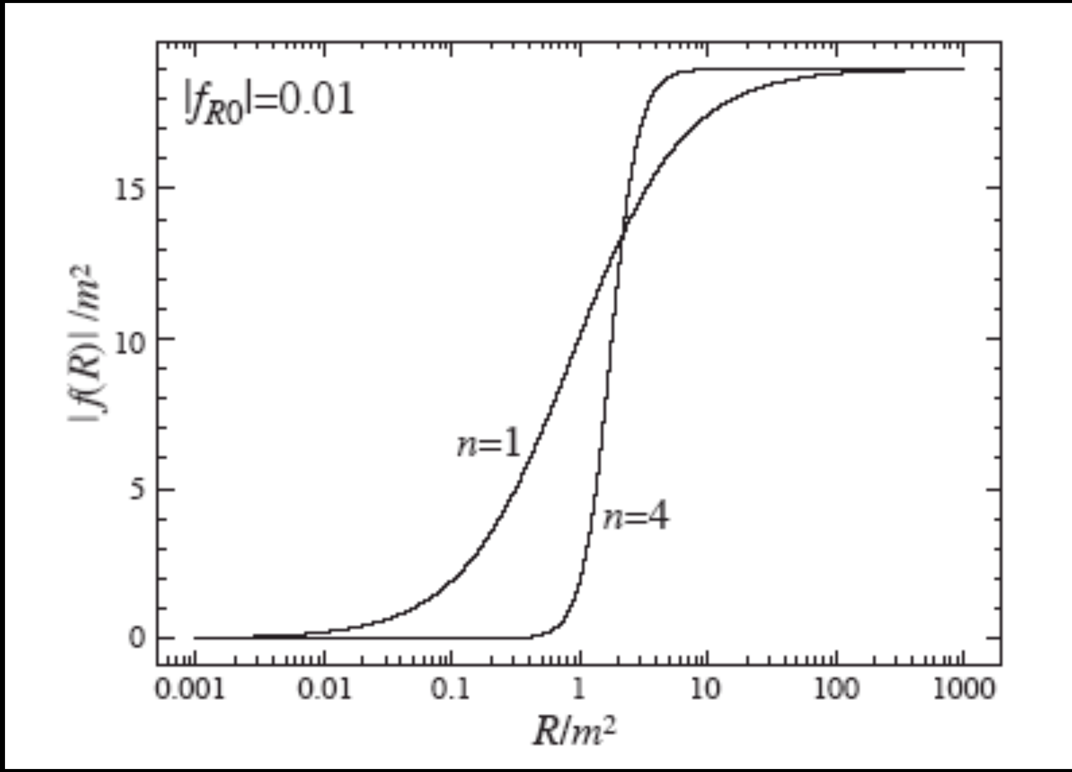
4. Include phenom of LCDM as limiting case.

5. Quantum corrections?

$$\lim_{R\to\infty} f(R) = \text{const.}\,,$$
$$\lim_{R\to 0} f(R) = 0\,,$$

$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1}\,,$$

$$f_{RR} \equiv \frac{\mathrm{d}^2 f(R)}{\mathrm{d}R^2} > 0$$



More general f (R) models [Gurovich & Starobinsky (79); Tkachev (92); Carloni et al (04,07); Amendola & Tsujikawa 08; Bean et al 07; Wu & Sawicki 07; Appleby & Battye (07) and (08); Starobinsky (07); Evans et al (07); Frolov (08)…]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R + f(R)}{2\kappa^2} + \mathcal{L}_\mathrm{m}\right]$$

No Λ

Usually f (R) struggles to satisfy both solar system bounds on deviations from GR and late time acceleration. It brings in extra light degree of freedom --> fifth force constraints.

Ans: Make scalar dof massive in high density solar vicinity and hidden from solar system tests by chameleon mechanism.

Requires form for f (R) where mass of scalar is large and positive at high curvature.

Issue over high freq oscillations in R and singularity in finite past.

In fact has to look like a standard cosmological constant [Song et al, Amendola et al]

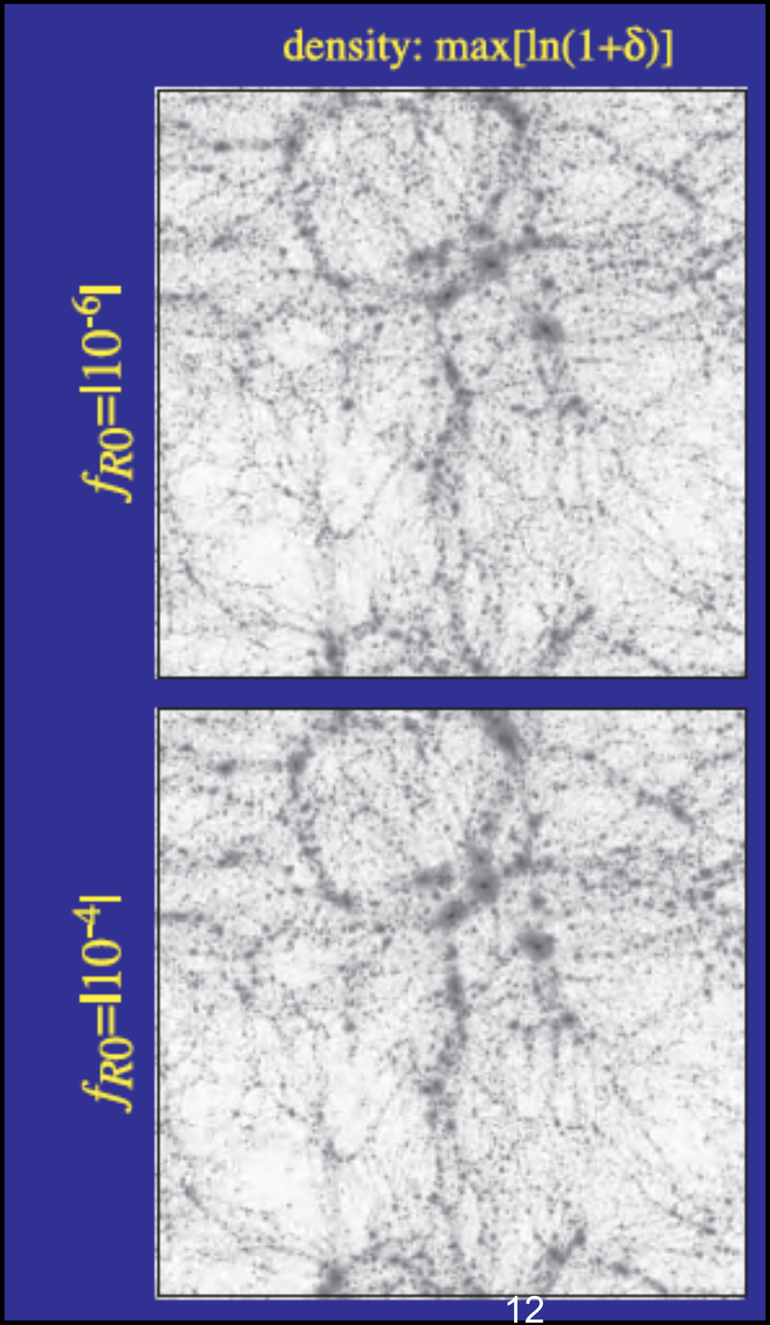
Non-linear evolution of f (R) models [Oyaizu, Lima and Hu (2008)]

Cosmological simulations of f(R) models. Extra scalar dof (df/dR) enhances force of gravity below the inverse mass of the scalar (d^2f/dR^2).

Simulation exhibits chameleon mechanism -> satisfy local constraints as the mass depends on the environment, in particular the depth of the local grav pot.

Find suppression of enhancement of power spectrum in non-linear regime but not at intermediate scales which are measureable.

For large bgd fields cmp to pot depth find enhanced forces and structure -- measurable?



Modifications of Friedmann equation in 4D:

Write: $$H^2 = \frac{8\pi}{3m_4^2}\rho L^2(\rho)$$

$L(\rho) = 1$ — Standard Friedmann

$$L(\rho) = \sqrt{1 + \frac{\rho}{2\sigma}};\ \ \sigma^{1/4} > 2.0MeV$$

Randall-Sundrum II: co-dimension one brane, embedded in 5D AdS space.

$$L(\rho) = \sqrt{1 - \frac{\rho}{2|\sigma|}};\quad \sigma < 0$$

Shtanov-Sahni: co-dimension one brane, negative tension embedded in 5D conformally flat Einstein space where signature of 5th dim is timelike

$$L(\rho) = \sqrt{1 + A\rho^n};\quad n < -1/3$$

Cardassian: only matter present --> late time acceleration. Freese & Lewis

$$L = \frac{1}{\sqrt{B\rho}}\left[\mp 1 + \sqrt{1 + B\rho}\right];B \equiv \frac{8\pi m_4^2}{3m_5^6}$$

Dvali-Gabadadze-Porrati: 3-brane embedded in flat 5D Minkowski with Ricci scalar term included in brane action. Bulk empty.

DGP model: $$H^2 \pm \frac{H}{r_0} = \frac{8\pi}{3m_4^2}\rho; \quad r_0 \equiv m_4^2/(2m_5^3)$$

Gravity 4D on short scales, but propagates into bulk on large scales. Induces corrections to Friedmann eqn, characterised by length r_0.

Two ways of embedding brane in bulk given by ±

- sign --> self accelerating phase (deS) for any decreasing energy density -- (w-->-1)

+ sign --> Minkowski phase. Brane extrinsically curved so that for $H \sim r_0^{-1}$ gravity screens the effects of the brane energy momentum

Consider our univ (brane) with homogeneous dust and lambda:

$$H^2 + \frac{H}{r_0} = \frac{8\pi}{3m_4^2}\rho_M(t) + \lambda$$

Infer effective dark energy :

$$\frac{8\pi}{3m_4^2}\rho_{DE}^{eff}(t) = \lambda - \frac{H}{r_0}$$

Lue & Starkman

H decreases with time, effective dark energy increases! For DE domination $w_{eff} < -1$ (mimics effect of phantom energy).

As universe evolves, screening term becomes weaker and eff dark energy density appears to increase

Degree of growth modulated by r_0. As $r_0 \to \infty$ recover standard ΛCDM.

For any cut off r_0, $w_{eff} \to -1$ with time and pure Λ cosmology recovered in future.

Possible concern over entering strong coupling regime for large distances.

Self acceleration branch contains ghost in spectrum for any value of brane tension -- instability

Charmousis et al 2006

Evolution of Fine Structure Constant

Olive and Pospelov;
Barrow et al; Avelino et al

Non-trivial coupling to emg:

$$L_m = -\frac{1}{4} B_F(\phi) F_{\mu\nu} F^{\mu\nu}$$

Bekenstein

Expand about current value of field:

$$B_F(\phi) = 1 + \zeta_F \phi + \frac{1}{2} \xi_F \phi^2$$

Eff fine structure const depends on value of field

$$\alpha(\phi) = \frac{e_0^2}{4\pi B_F(\phi)}$$

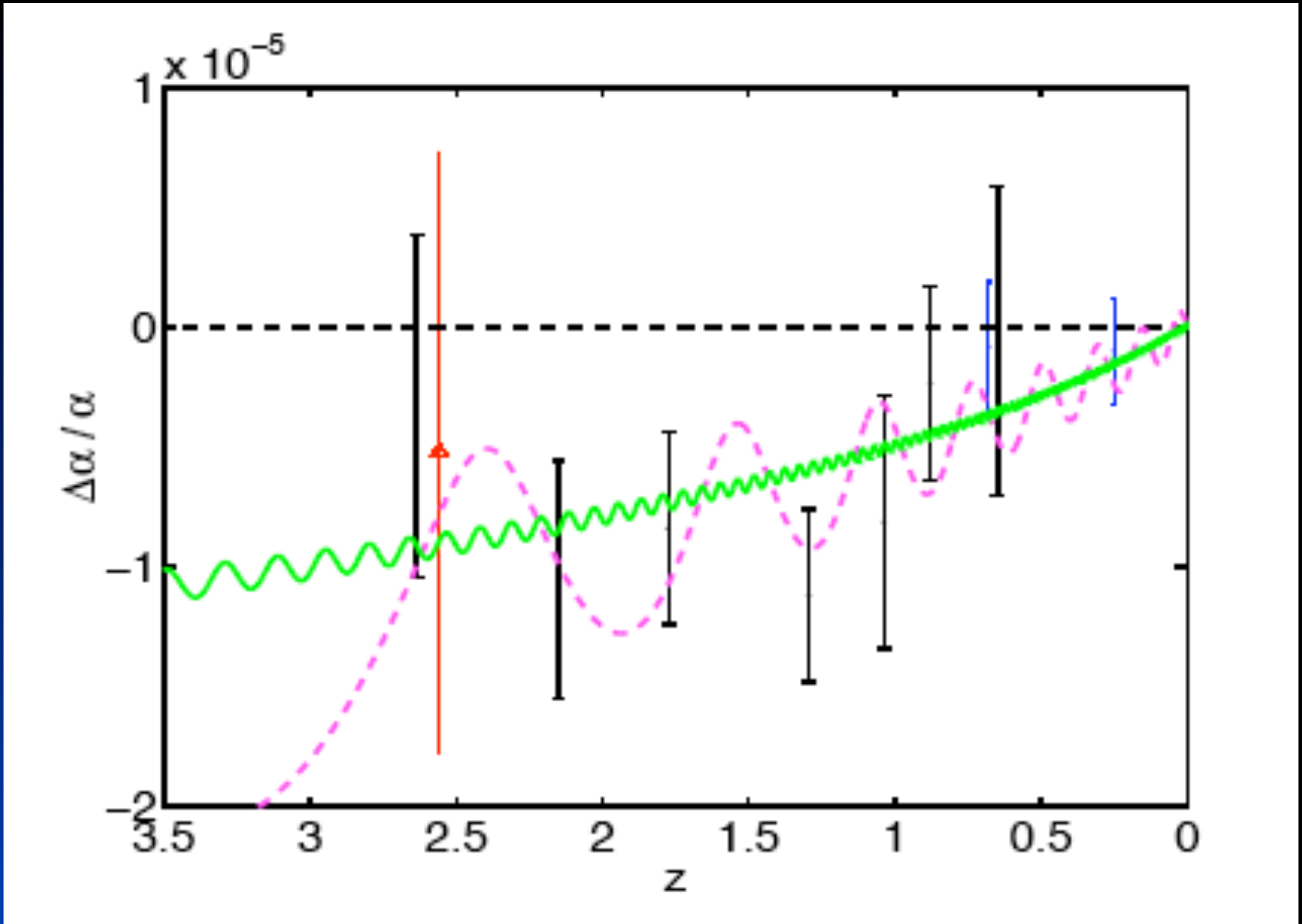
$$\frac{\Delta\alpha}{\alpha} = \zeta_F \phi + \frac{1}{2}(\xi_F - 2\zeta_F^2)\phi^2$$

Claim from analysing quasar absorption spectra:

$$\frac{\Delta\alpha}{\alpha}(z = 0.5 - 3.5) \approx 10^{-5}$$

Webb et al

$$V = V_0 e^{-\lambda \kappa \phi}$$



Nunes

$\lambda = 100$ – solid

$\lambda = 10$ – dashed

A way of constraining the eqn of state?

Evidence for dynamical dark energy ?

1. Precision CMB anisotropies – lots of models currently compatible.
2. Combined LSS , SN1a and CMB data – tend to give w_Q<-0.85 → best fit remains cosmological constant.
3. Look for more SN1a – SNAP will find over 2000 at large redshift – can then start to constrain eqn of state.
4. Constraining eqn of state with SZ cluster surveys – compute number of clusters for given set of cosm parameters.
5. Baryon Acoustic Oscillations in the LSS as a probe of dark energy.
6. Reconstruct eqn of state from observation – offers hope of method indep of potentials.
7. Look for evidence in variation of fine structure constant.
8. Using Gravitational lensing to constrain w --Dark Energy Survey
9. Sandage Loeb test -- measuring quasar spectra at different redshift between $2<z<5$. [Corasaniti et al 2007]

Dynamical evolution of w?

Weller and Albrecht; Kujat et al; Maor et al; Gerke and Efstathiou, Kratochvil et al; ...

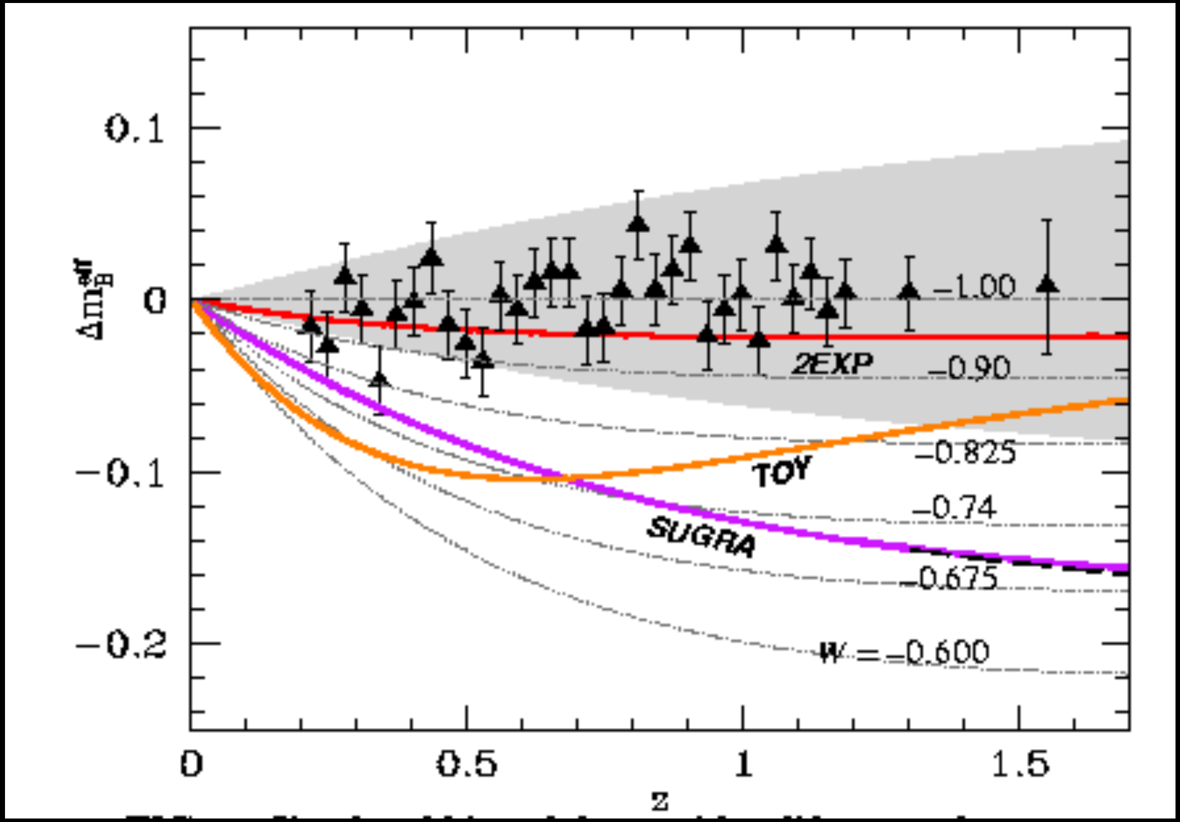
SNAP as a discriminator

Write:

$$w(z) = \sum_{i=0}^{N} w_i z^i$$

or:

$$w(z) = \sum_{i=0}^{N} w_i \ln(1+z)^i$$



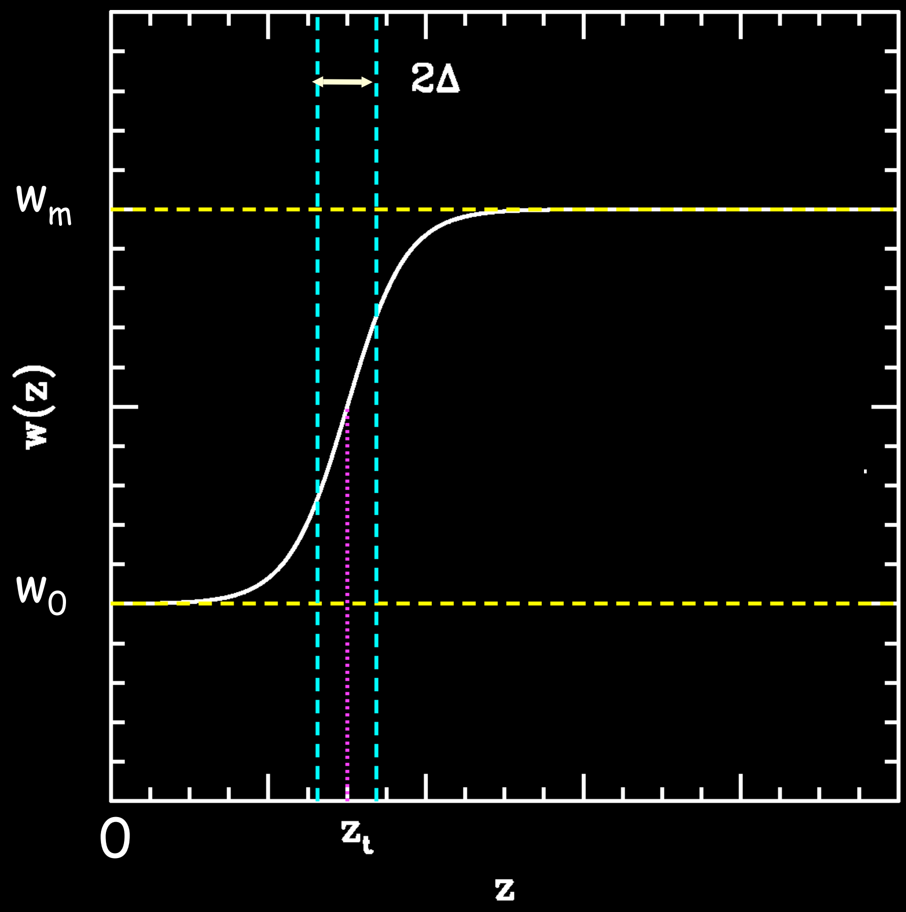
Evaluate magnitude difference for each model and compare with Monte Carlo simulated data sets.

Modelling quintessence

Impose an equation of state w(z) which captures the essential features of quintessence.

typical expectations:

- recent acceleration
  ➜ $w_0 < -1/3$
- avoid fine tuning the initial energy density ➜ $w_m > -1/3$
- there is a transition at a given redshift z_t with a given width Δ.
- Λ corresponds to $w_0 = -1$ and either $w_m = -1$ or $z_t >> 1$.



Strategy:

- compute predictions for many models with different parameters (ie H_0, w_0, w_m, n_s, t and the normalisation)
- compare with data sets (we use WMAP + SN-Ia)
- derive constraints on parameters (Markov-Chain Monte Carlo code with modified cmbfast)
- draw conclusions about the physical nature of the models.

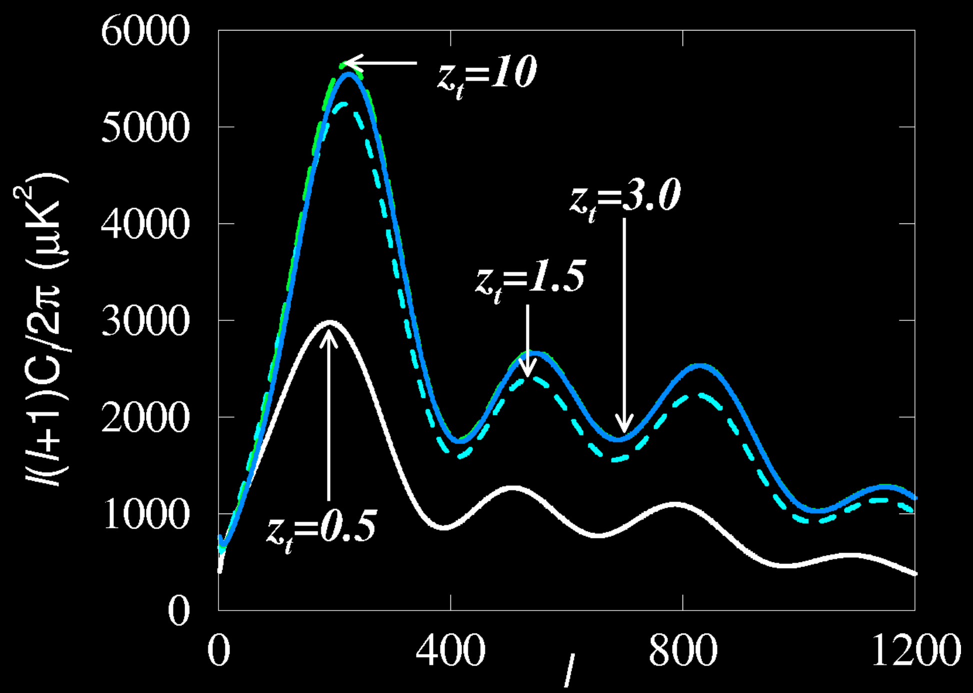
Kunz et al astro-ph/0307346; Corasaniti et al astro-ph/0406608

w(z) impact on the CMB through ISW

$$\left[\frac{\delta T(e)}{T}\right]_{SW} = \frac{1}{3}\Phi(ex_{ls}) + 2\int_{\tau_{ls}}^{\tau_0} \frac{\partial\Phi(ex,\tau)}{\partial\tau} d\tau$$

rapid transition :

- late onset of expansion changes ISW effect which acts at large l
- peak lower after COBE normalisation



- Cosmic variance makes the effect hard to observe, especially for models with slowly varying equation of state.
- A data set which connects large and small angular scales is crucial for a correct normalisation ➟ WMAP.

cosmological parameters --WMAP1

- limits slightly wider, but no clear difference
- **NO** new degeneracies!

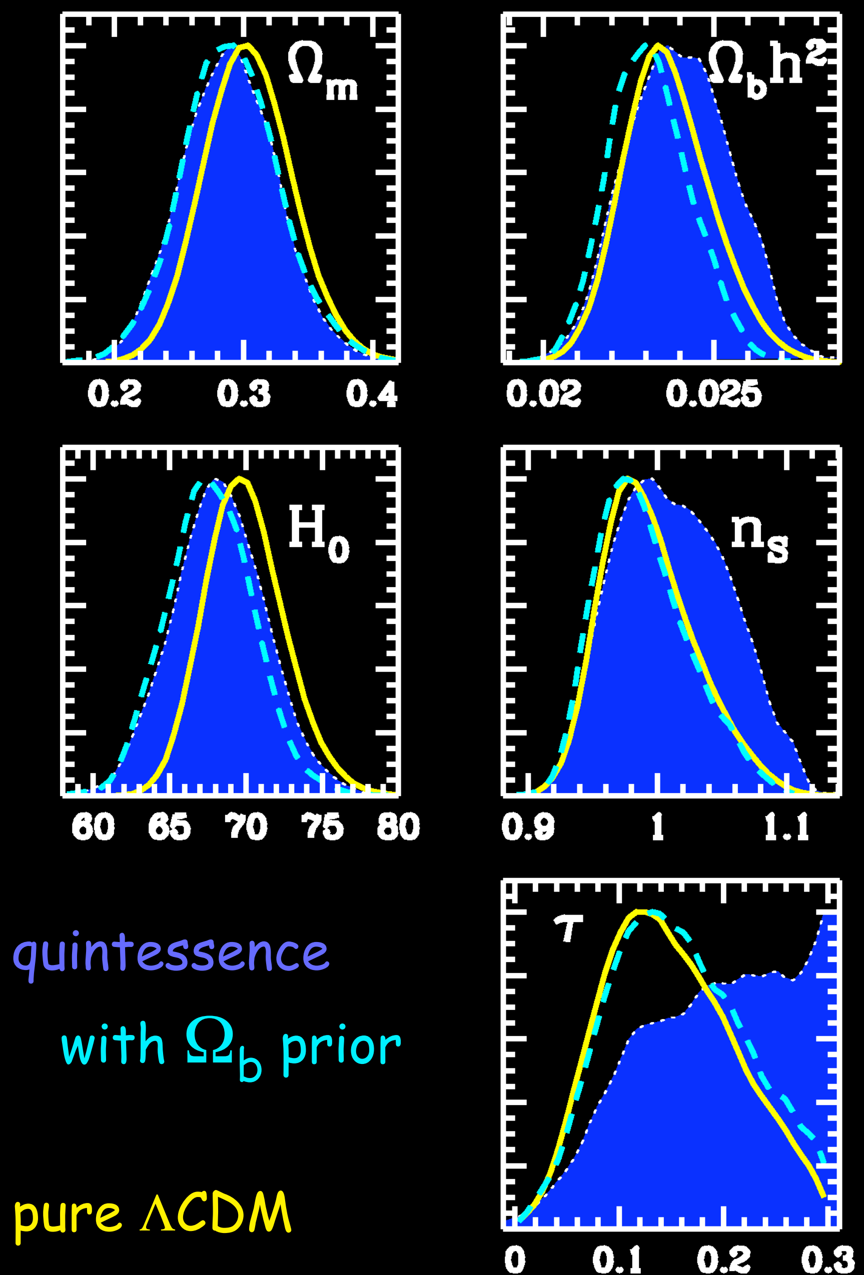
$\Omega_m = 0.29 \pm 0.04$

$\Omega_b h^2 = 0.0240 \pm 0.0015$

$H_0 = 68 \pm 3$

$n_s = 1.01 \pm 0.04$

$\tau = 0.19 \pm 0.07$



dark energy parameters

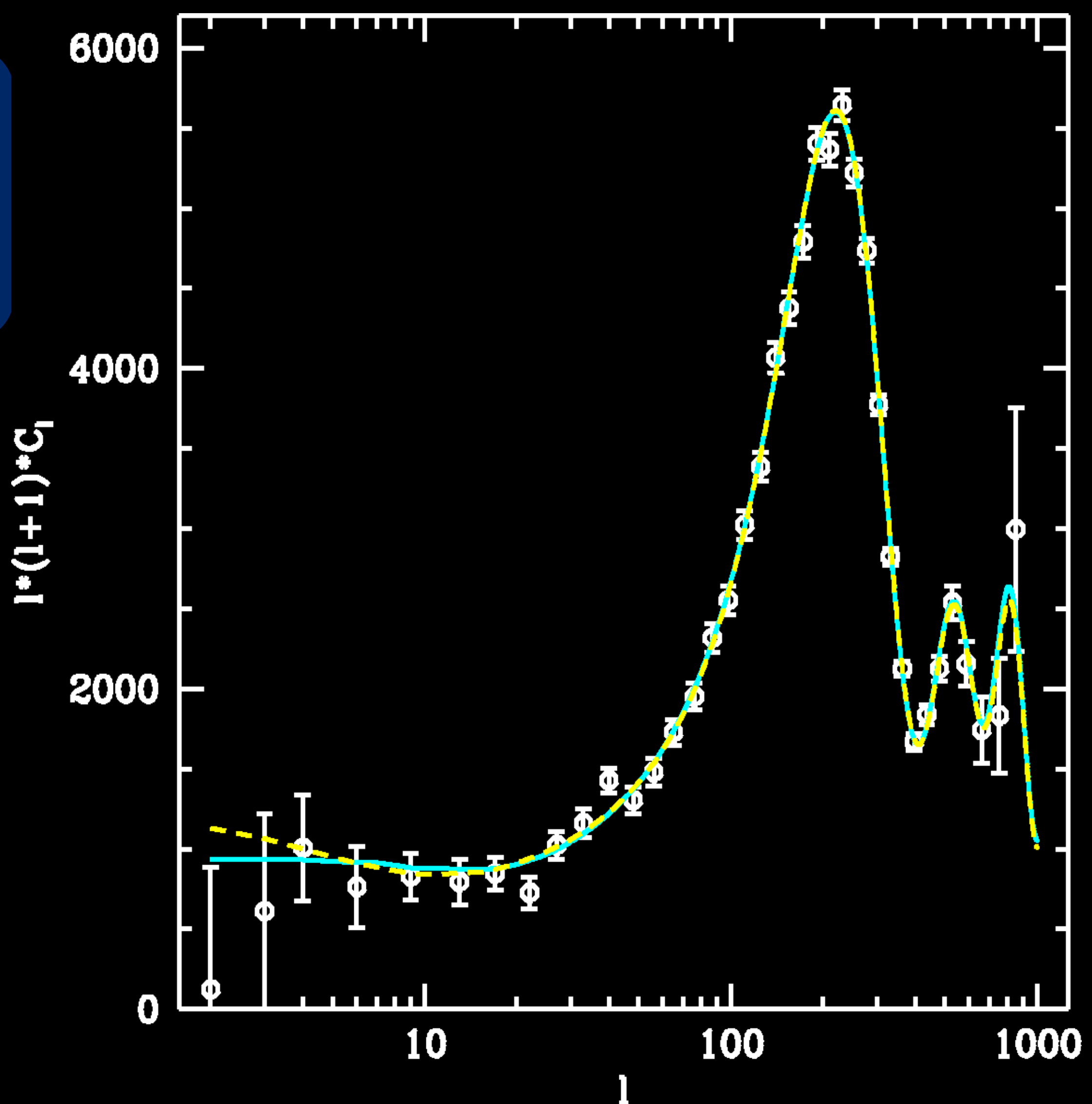
$w_0 < -0.80$ at 95% CL

$z_t > 0.6$ (fast transitions)

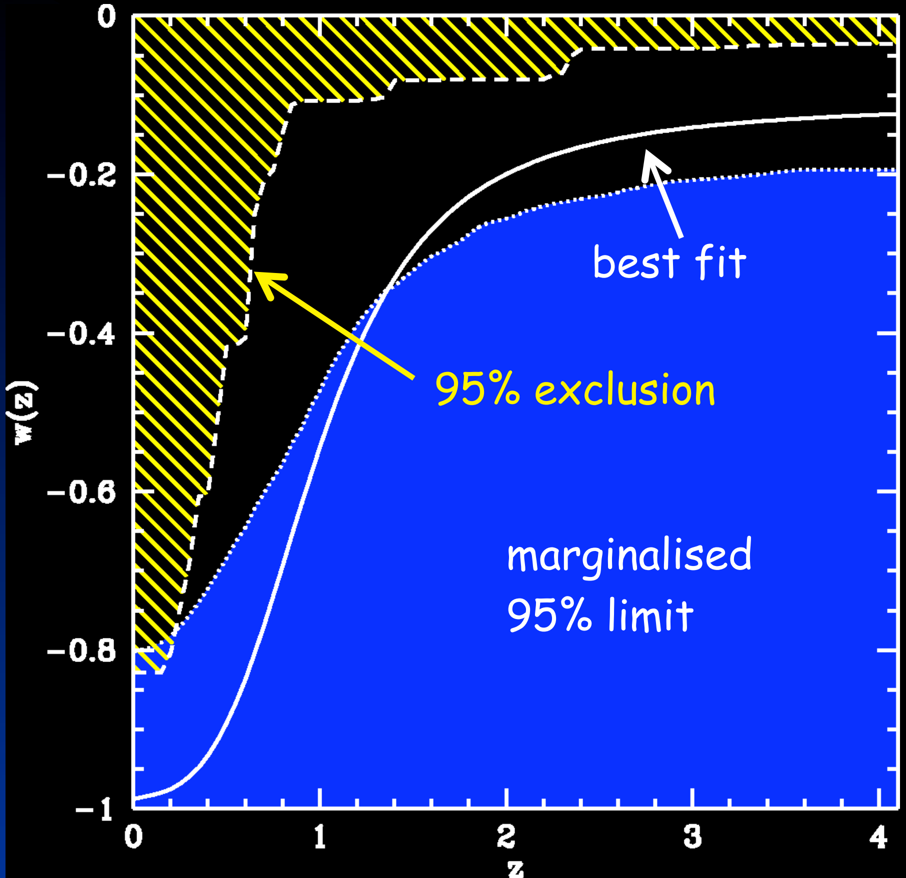
best-fit quintessence model:

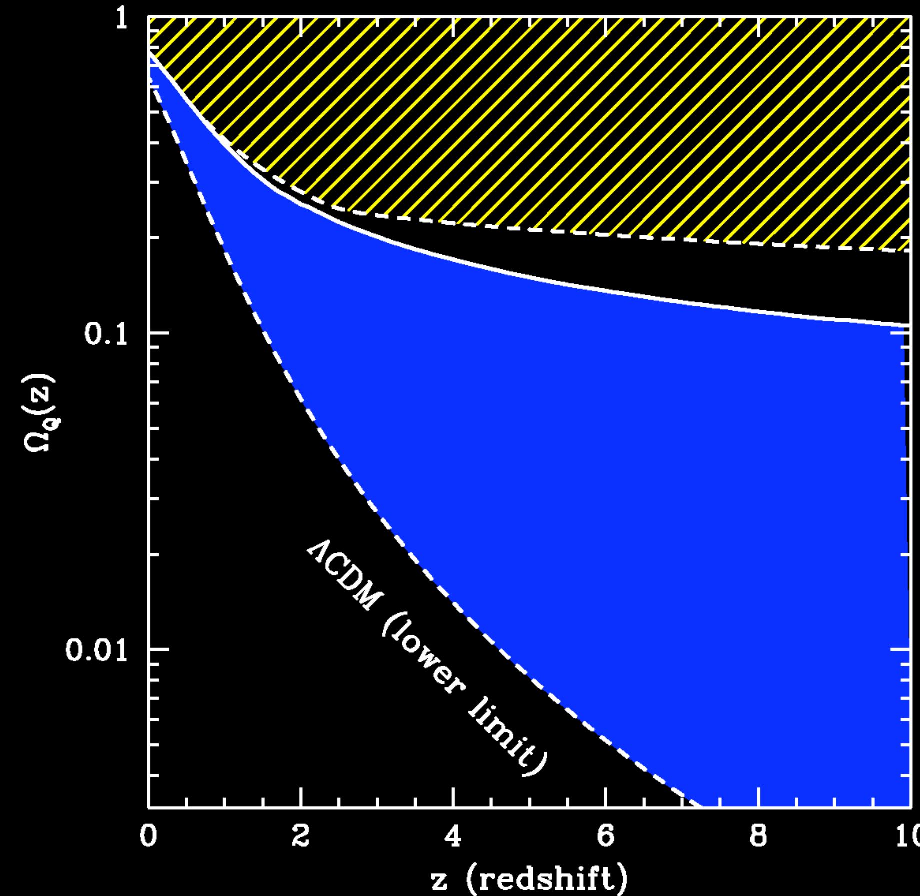
- $w_0 = -1$
- $w_m = -0.13$
- $a_t = 0.5$ ($z_t = 1$)
- effective $\chi^2 = 1603$

best ΛCDM : $\chi^2 = 1606$



time behaviour of the DE





- really strong constraints on w only for z < 0.2

Determining the best way to test for dark energy and parameterise the dark energy equation of state is a difficult task, not least given the number of approaches that exist to modelling it.

It deserves a lecture on its own, but Sabino wouldn't let me have a fifth lecture even though I pleaded with him.

Instead you will have to make do with the thorough review competed by Rocky and his colleagues making up the Dark Energy Task Force.

Albrecht et al : astro-ph/0609591

Then the findings on the search for the best figure of merit:

Albrecht et al: arXiv:0901.0721

Summary

•Observations transforming field, especially CMBR and LSS. -- everything consistent with a pure cosmological constant.

•Why is the universe inflating today?

•Is w=-1, the cosmological constant ? If not, then what value has it?

•Is w(z) -- dynamical?

•New Gravitational Physics -- perhaps modifying Friedmann equation on large scales?

•Lots of models of dark energy but may yet prove too difficult to separate one from another such as cosmological const – need to try though!

•Perhaps we will only be able to determine it from anthropic arguments and not from fundamental theory.

•or -- could we be wrong and we do not need a lambda term?