Constraining the physics of (slow-roll) inflation

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based on work with

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Galileo Galilei Institute 2009

Overview

I. Some basics of inflation

II. Present status:

What have we learnt about slow-roll inflation?

III. Outlook

What can we learn from future observations?

IV. Beyond the simplest scenario

I. Inflation

Inflation

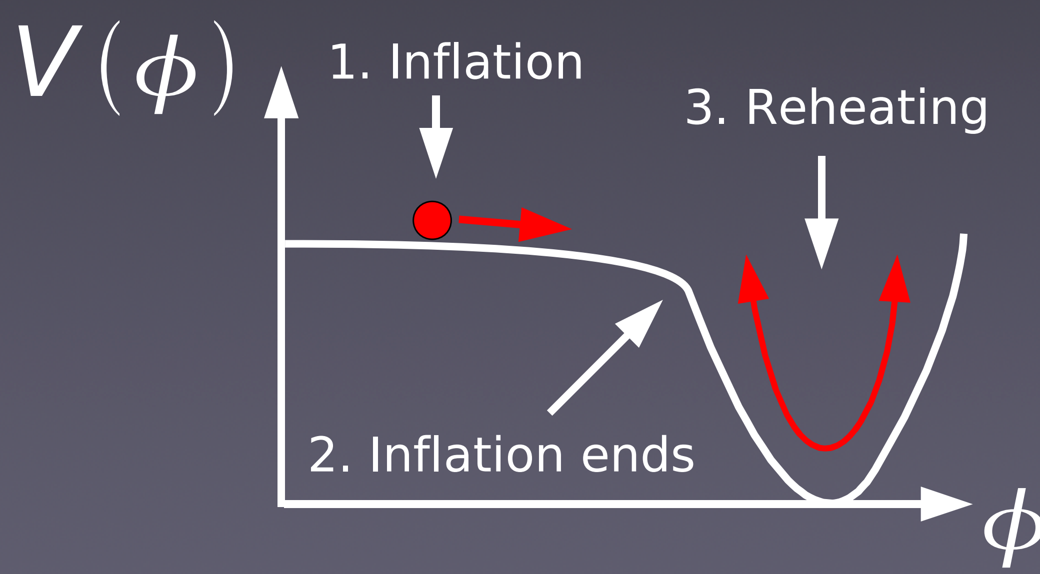
- Accelerated growth of scale factor

$$\ddot{a}(t) > 0$$

- Simplest feasible realisation:

Canonical scalar field (inflaton) ϕ

with $V(\phi) \gg \dot{\phi}^2/2$



Inflation at zeroth order

Friedmann equation $$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Klein-Gordon equation $$\ddot{\phi} + 3H\dot{\phi} - \frac{dV}{d\phi} = 0$$

⇨ $a(t) \sim \exp(H_{\text{inf}} t)$

Scale of inflation: $H_{\text{inf}} \sim$ const.

Pre-(post-?!)dictions of inflation

- Generic
  - Isotropy
  - Homogeneity

    } on large scales
  - Absence of relics (monopoles, etc.)
  - Spatial flatness
  - Perturbations from quantum fluctuations
- Model-dependent
  - Statistical properties of fluctuations

Inflation

- Very likely to have happened...

  ... but how exactly?

Models of Inflation

old, new, pre-owned,
chaotic, quixotic, ergodic,
ekpyrotic, autoerotic,
faith-based, free-based,
D-term, F-term, summer-term,
brane, braneless, brainless,
supersymmetric, supercilious,
natural, supernatural, *au natural,*
hybrid, low-bred, white-bread,
one-field, two-field, left-field,
eternal, internal, infernal,
self-reproducing, self-promoting,
dilaton, dilettante, …….

[shamelessly stolen from Rocky Kolb]

Inflation

- Very likely to have happened...

  ... but how exactly?

⇨ information from statistical properties of perturbations

Inflationary perturbations

Quantum fluctuations of inflaton field

Einstein's equations

Fluctuations of the metric

→ two (non-decaying) types of perturbations:

(and their corresponding gauge-invariant quantities)

- Scalar (curvature) perturbations (→ u_S)
- Tensor perturbations (gravity waves) (→ u_T)

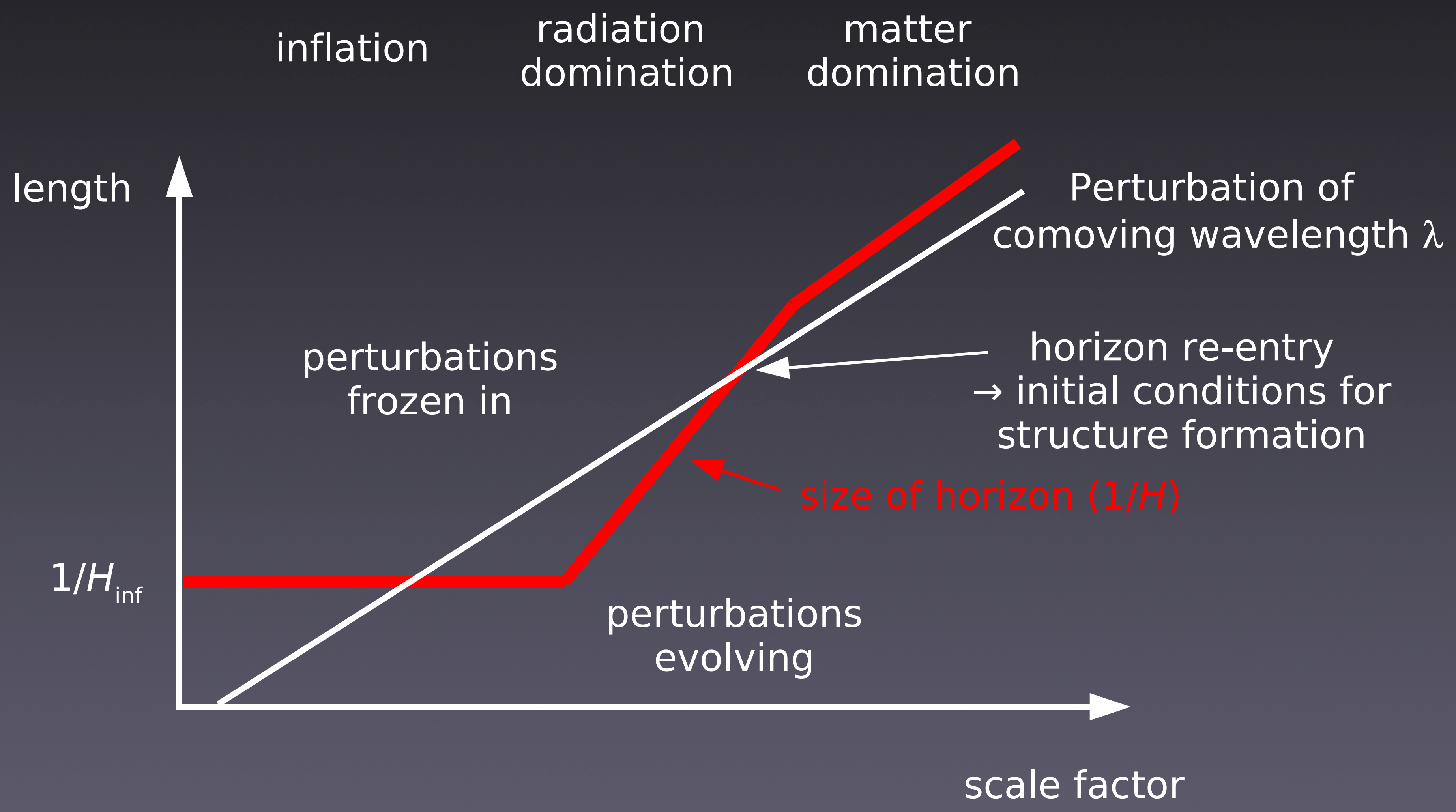
Inflationary perturbations

- Fourier modes evolve independently
- Mode equation $u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0$
- z depends on background (i.e., $V(\phi)$)

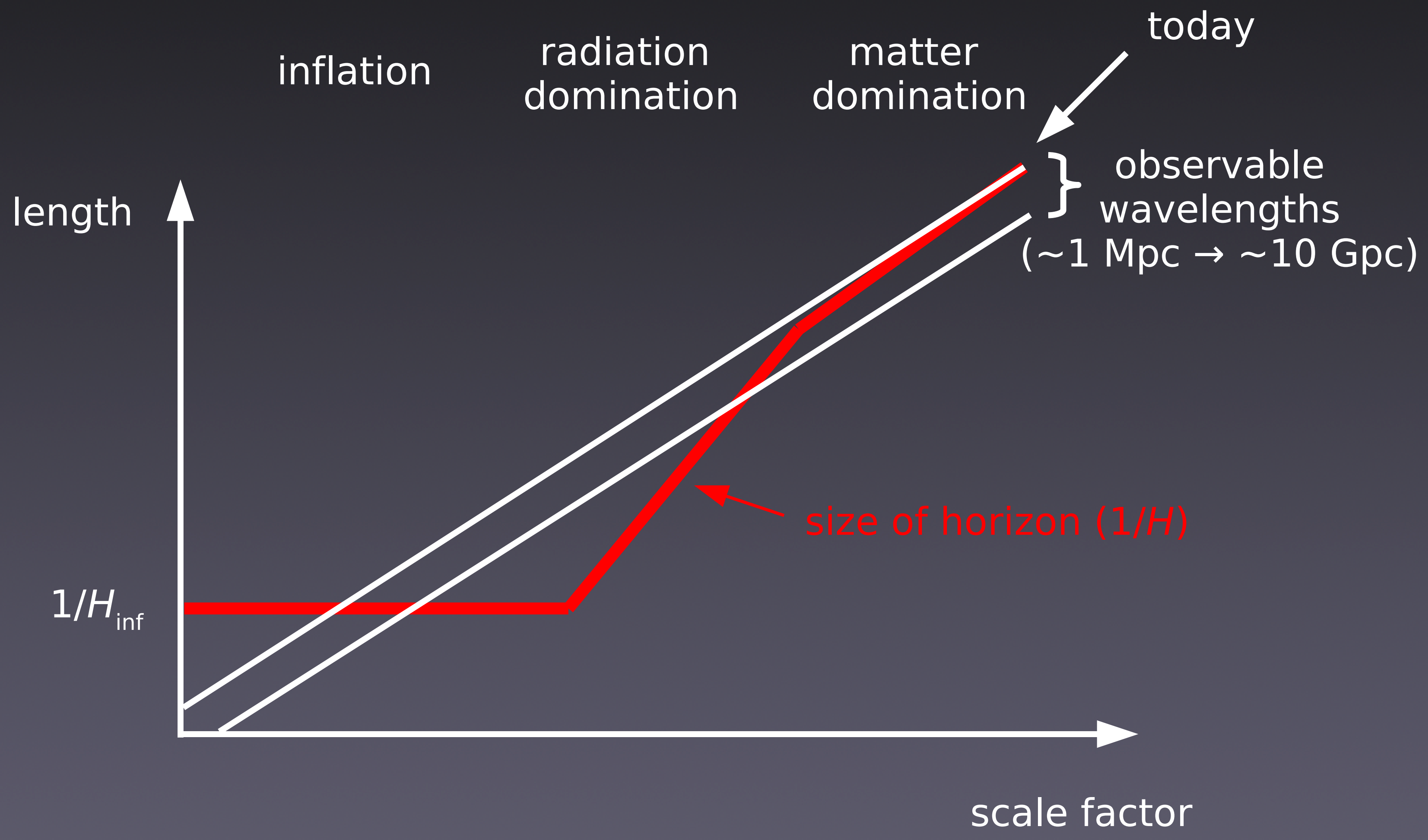
  scalars: $z = a\dot{\phi}/H$ tensors: $z = a$
- Power spectrum: $P(k) = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|^2$
- For $k^2 \ll z''/z \approx 2a^2H^2$, $u_k \to z$

  $\Rightarrow$ $P(k) \to$ const. ("freeze in")

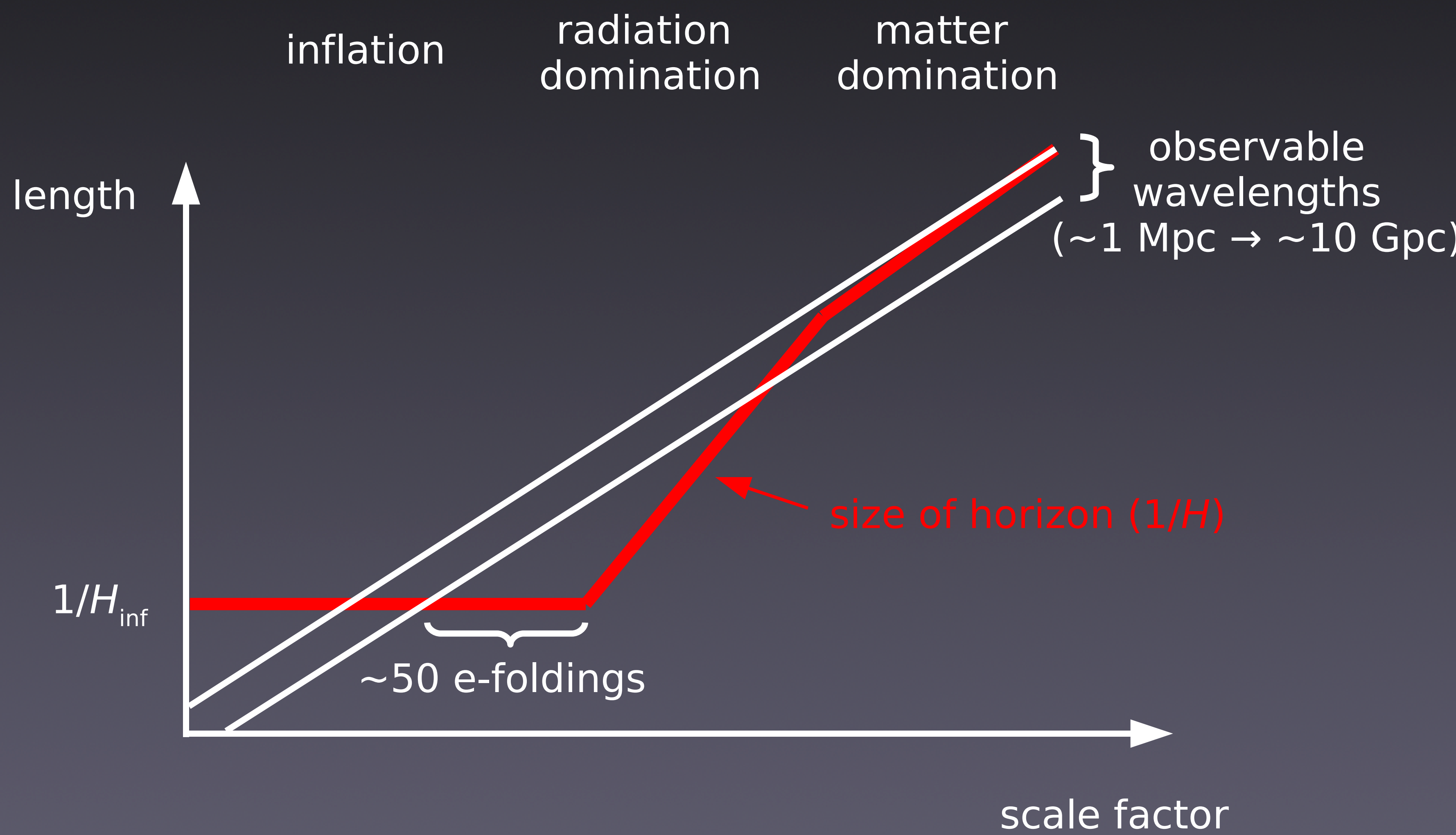
Inflationary perturbations



Inflationary perturbations

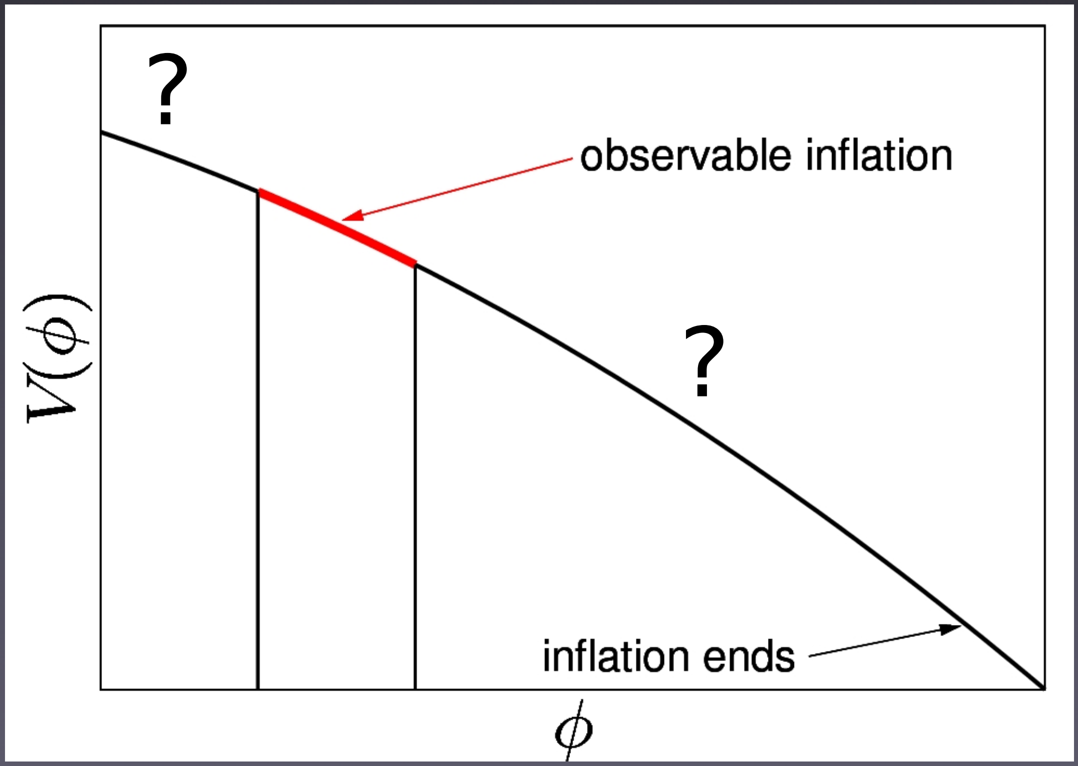


Inflationary perturbations



Observable inflation?

- We can only "see" a small part of the inflaton potential



Slow-roll inflation

- Sufficient (but not necessary) condition for inflation to last long enough:

Slow roll conditions

$$\epsilon = \frac{١}{٢}\left(\frac{H'}{H}\right)^{٢} \ll ١ \qquad |\eta| = \left|\frac{H''}{H}\right| \ll ١ \qquad |\xi| = \left|\frac{H'''H}{H^{٢}}\right| \ll ١$$

$$\vdots$$

- Equivalently: potential needs to be flat enough

Shape of primordial spectra

- Taylor-expansion

$$\ln P_s(\ln k) \simeq \overbrace{\ln A_s}^{\text{scalar amplitude}} + (\underbrace{n_s}_{\text{scalar spectral index}} - 1)(\ln k - \ln k_0) + \frac{1}{2}\overbrace{\alpha_s}^{\text{running of spectral index}}(\ln k - \ln k_0)^2 + \ldots$$

$$\ln P_T(\ln k) \simeq \overbrace{\ln A_T}^{\text{tensor amplitude}} + \underbrace{n_T}_{\text{tensor spectral index}}(\ln k - \ln k_0) + \ldots$$

- Tensor-to-scalar ratio $r = A_T/A_S$

Shape of primordial spectra

- Taylor-expansion

Good approximation for slow-roll

$$\left.\begin{array}{l} A_S \approx H^2/\epsilon\pi \\ \\ A_T \approx 16 H^2/\pi \end{array}\right\} \quad r \approx 16\epsilon$$

$$n_S \approx ١ + ٢\epsilon - 4\eta$$

$$\alpha_S \approx -2\xi$$

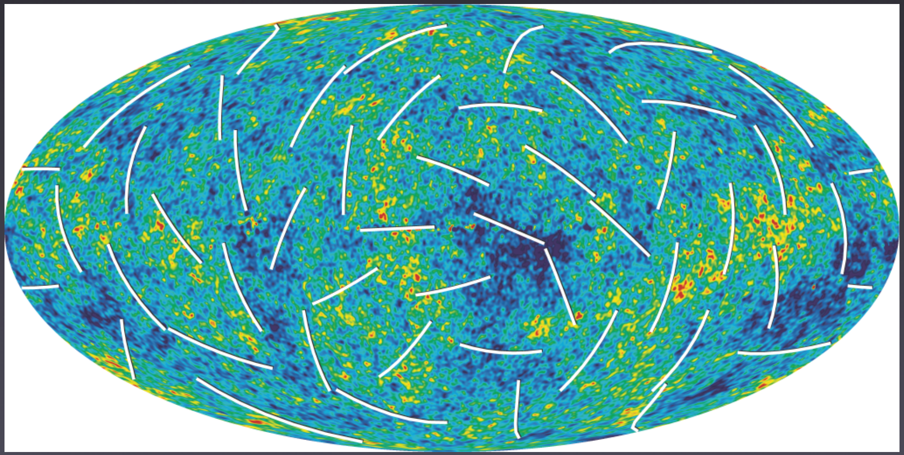
$$n_T \approx -2\epsilon$$

Good enough...?

II. Slow-roll inflation and the real world

Probing perturbations with the CMB

- Can measure temperature and polarisation anisotropies of CMB

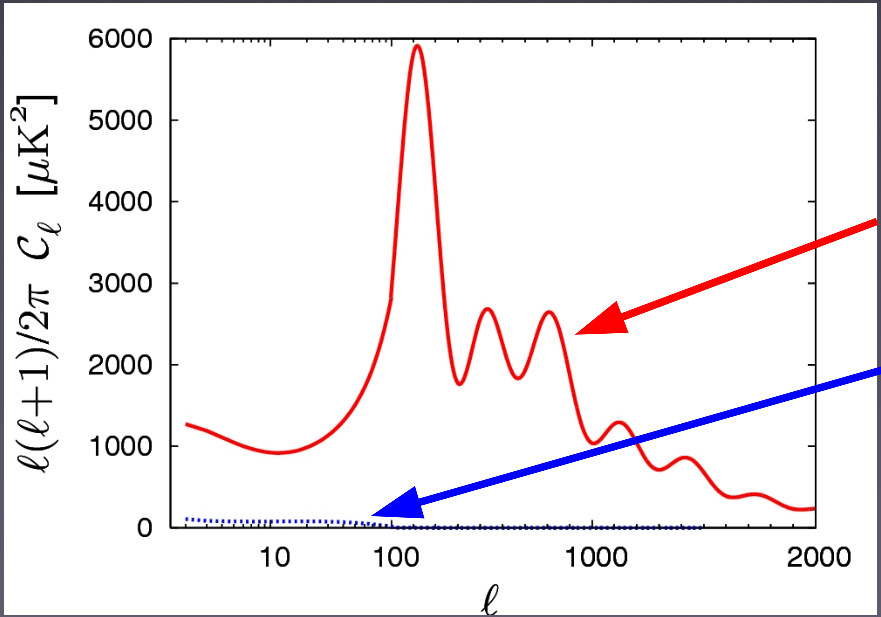


[WMAP 2008]

- Polarisation: *E*- and *B*-modes
  - $\nabla \times E = 0$ (curl-free)
  - $\nabla . B = 0$ (divergence-free)

Probing perturbations with the CMB

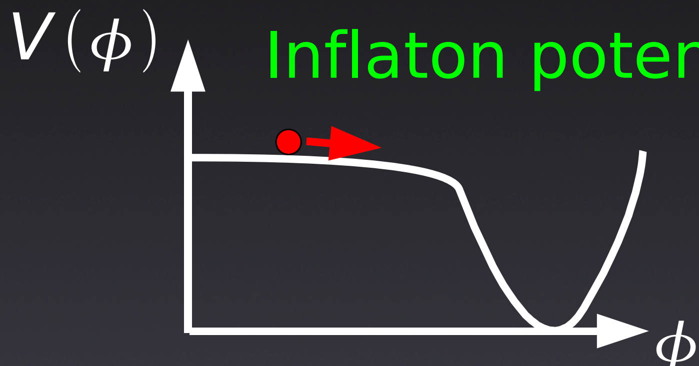
- At linear order in perturbation theory:
  - T → scalar, (vector), tensor
  - E → scalar, (vector), tensor
  - B → (vector), tensor



Scalar TT power spectrum

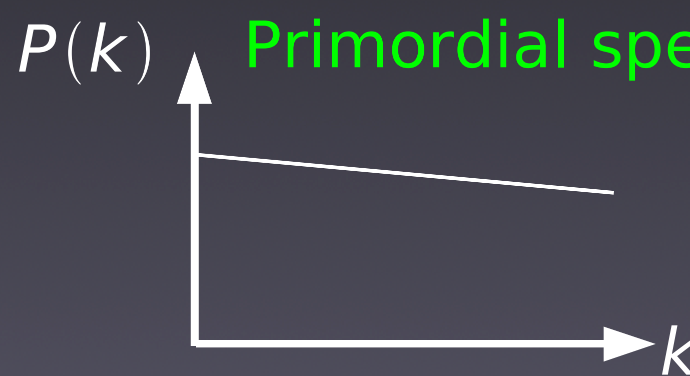
Tensor TT power spectrum, r = 0.2

Probing perturbations with the CMB



Inflaton potential

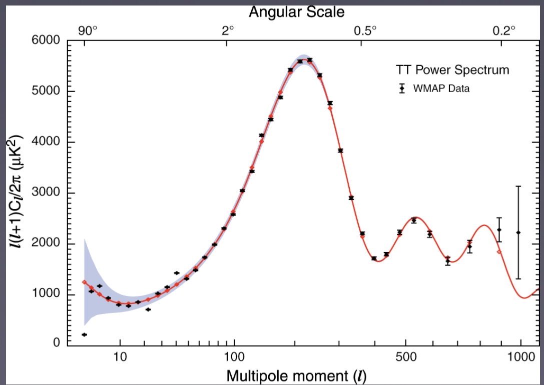
Exact solution
or
analytic approximation



Primordial spectrum

CMB spectrum

Transfer function
$(\Omega_b, \Omega_m, \Omega_\Lambda, \ldots)$



Inference (the Bayesian way)

- Given data D and model M with free parameters x, we want to infer the probability density

$$P(x|D,M)\, dx$$

- We can calculate

$$P(D|x,M)$$

(through combinatorics, Monte Carlo...)

- Apply *Bayes*' *Theorem*:

$$P(A|B)\, P(B) = P(B|A)\, P(A)$$

Inference (the Bayesian way)

$$P(x|D,M)\,dx \propto L(D|x,M) \cdot \pi(x|M)\,dx$$

$$\text{Posterior} \propto \text{Likelihood} \cdot \text{Prior}$$

- Prior: What we know about x before we measure D
- Likelihood: What the data tell us about x
- Posterior: What we know about x after measuring D

  → *used to construct credible intervals, etc.*
- What to choose for $\pi(x)\,dx$ if we do not have any prior knowledge?
- → usually: *flat prior*, i.e., $\pi(x)\,dx = \text{const.}\,dx$

Inference (the Bayesian way)

$$P(x|D,M)\,dx \propto L(D|x,M)\cdot\pi(x|M)\,dx$$

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Warning!

Results should be taken *cum grano salis*

→ examples later...

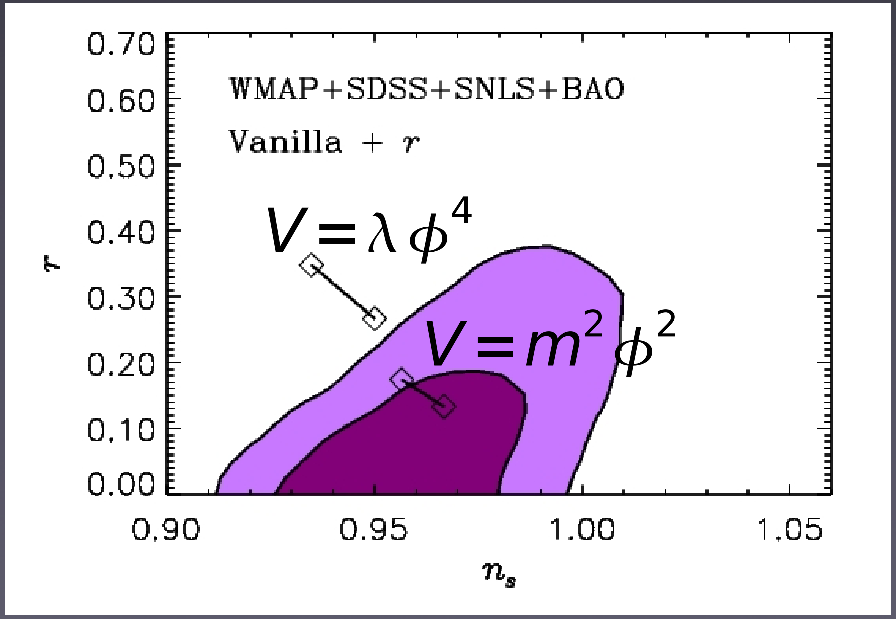
What to constrain?

- Conventional way: reconstruct power spectra in some parameterisation
- Compare with prediction of your favourite models

- Presently:

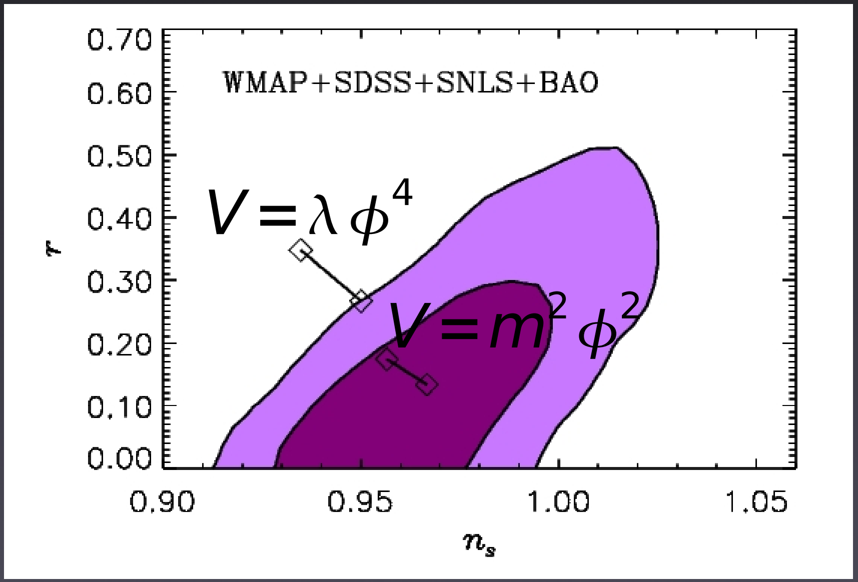
  A few models ruled out (or under pressure)

  Still many compatible, no clear verdict possible

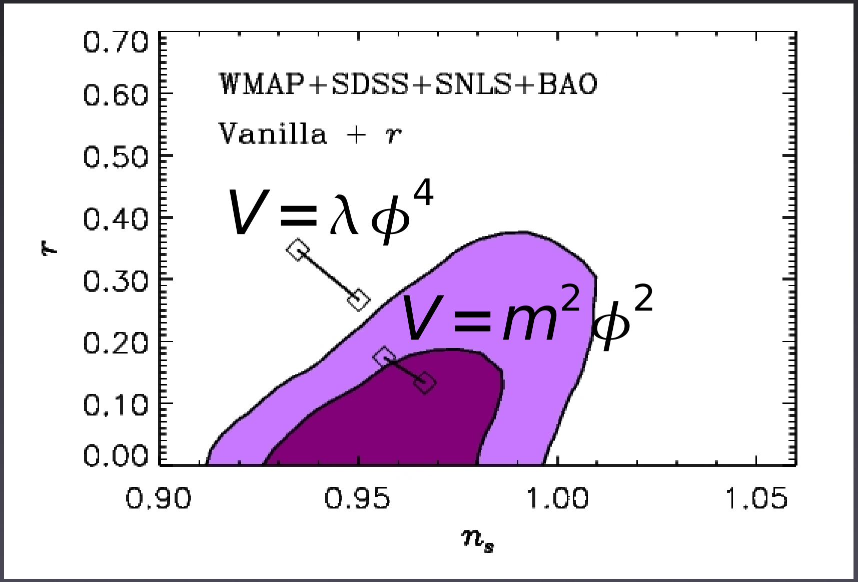


Side comment
Caveat #1: model dependence

ΛCDM-6 + r + m_ν + w



ΛCDM-6 + r



[JH, Hannestad, Sloth, Wong 2006]

- Example for model-dependence of inference: Even adding physically unrelated parameters (*here*: neutrino mass and DE equation of state parameter) can affect results

What to constrain?

- Alternative: skip the power spectrum
- Constrain directly inflationary dynamics

Examples:

- Inflaton potential $V(\phi) = V_0 + V'\phi + \frac{1}{2}V''\phi^2 + \ldots$

[Grivell & Liddle 1999]

- Hubble parameter $H(\phi) = H_0 + H'\phi + \frac{1}{2}H''\phi^2 + \ldots$

- Slow-roll parameters $\{H^2/\epsilon, \epsilon, \eta, \xi, \ldots\}$

[Easther & Peiris 2006]

→ solve mode equations

[Lesgourgues & Valkenburg 2007; JH, Lesgourgues, Valkenburg 2008]

What are the advantages?

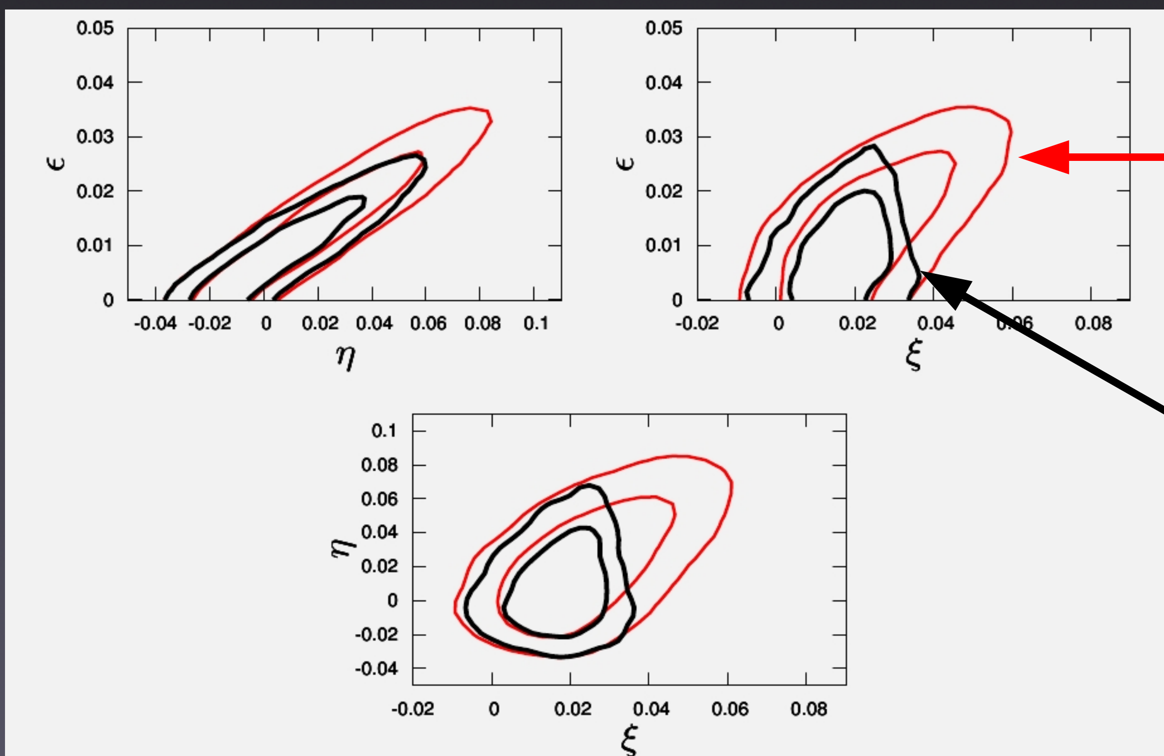
- More natural basis of inflationary parameter space

  → more realistic priors (?)

- Avoids the use of approximations:
  - Don't need to assume slow roll
  - Parameterisation of spectrum may be insufficient to describe all models
- Can weed out inconsistent parameter combinations and impossible spectra

Comparison with usual approach

68%- and 95%-credible contours



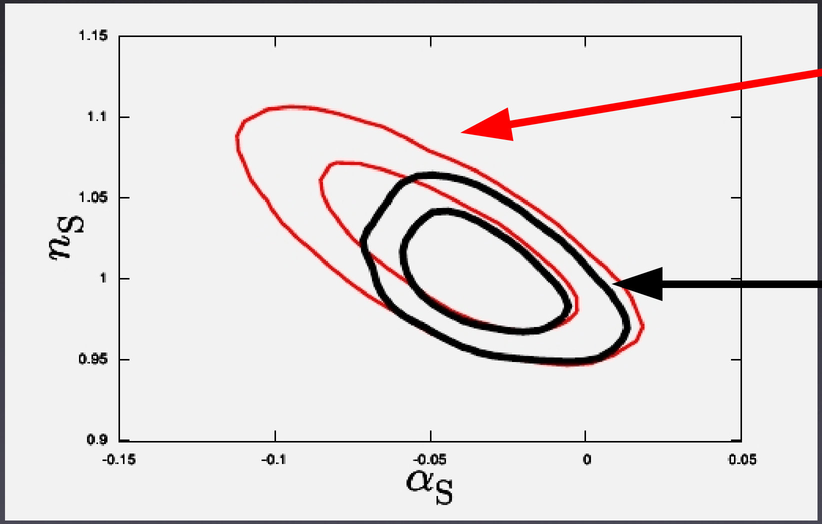
Slow-roll approximation
+
Taylor expanded spectra

exact solution

Difference due to inaccuracy of slow-roll approximation?

Comparison with usual approach

Running vs. spectral index



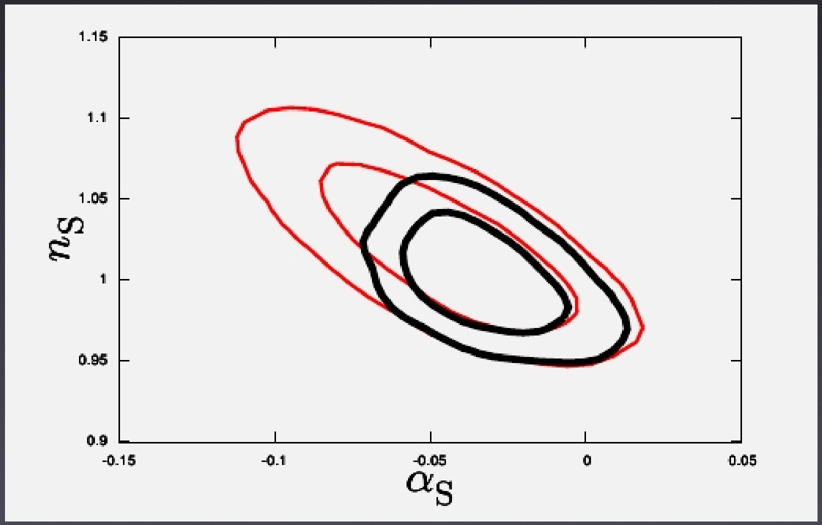
tilt and running from slow-roll approximation

tilt and running from exact solution

- Large ξ, large negative running
  → inflation stops too early
  → slow-roll assumption inconsistent!

Comparison with usual approach

Running vs. spectral index



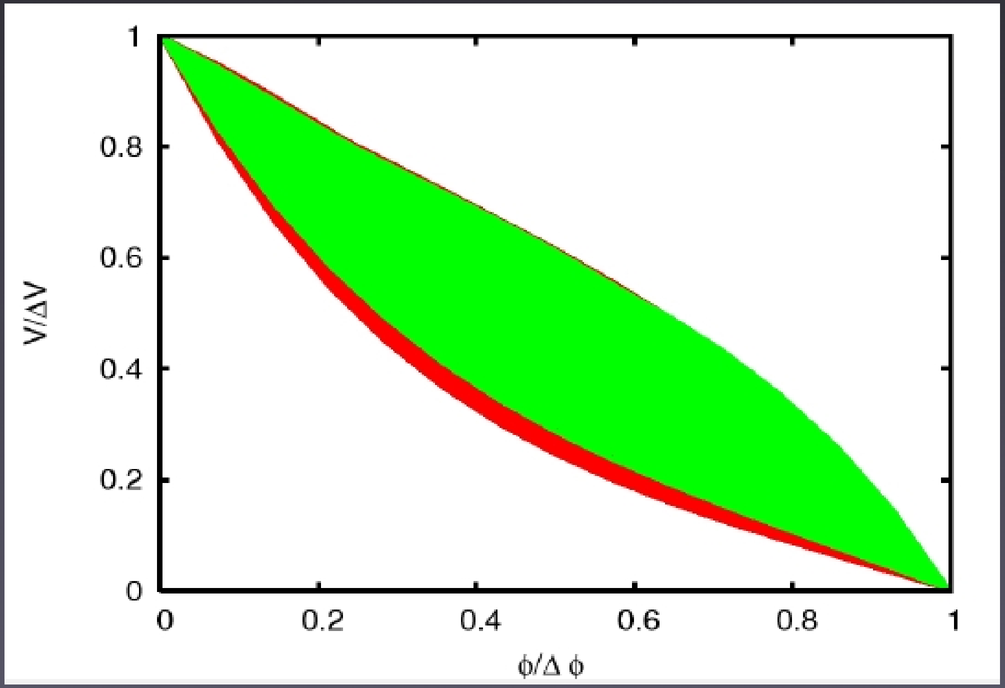
- Difference not due to approximation, rather due to implicit prior assumption
- At the moment, prior more constraining than data
- Our approach self-consistent

- Large ξ, large negative running
  → inflation stops too early
  → slow-roll assumption inconsistent!

Reconstructing the inflaton potential

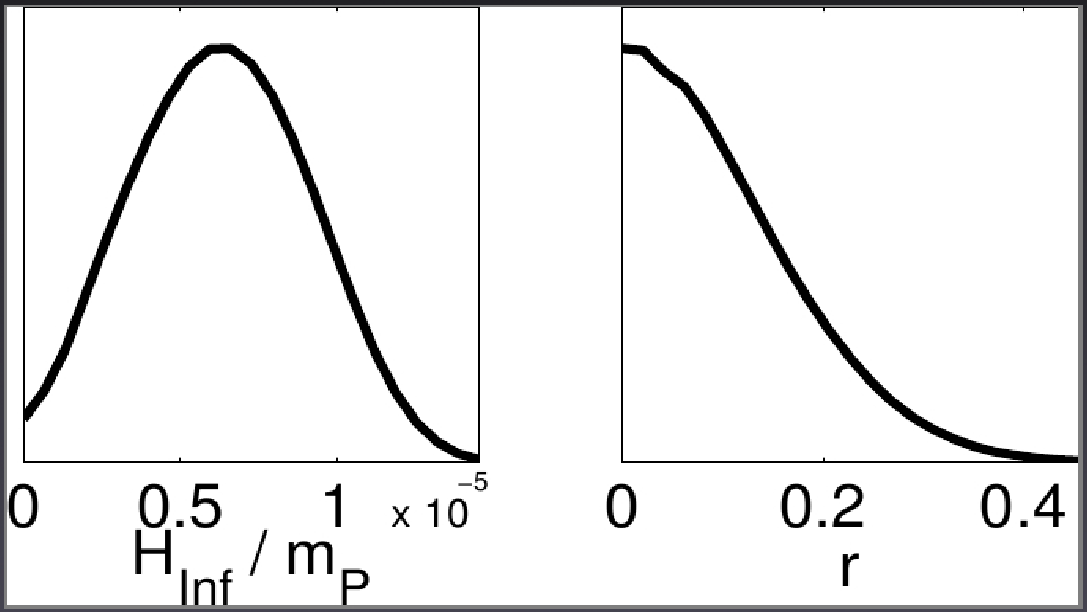
Reconstructed potentials from

$\{H^2/\epsilon, \epsilon, \eta, \xi\}$ - parameterisation



[Lesgourgues, Starobinsky, Valkenburg 2007]

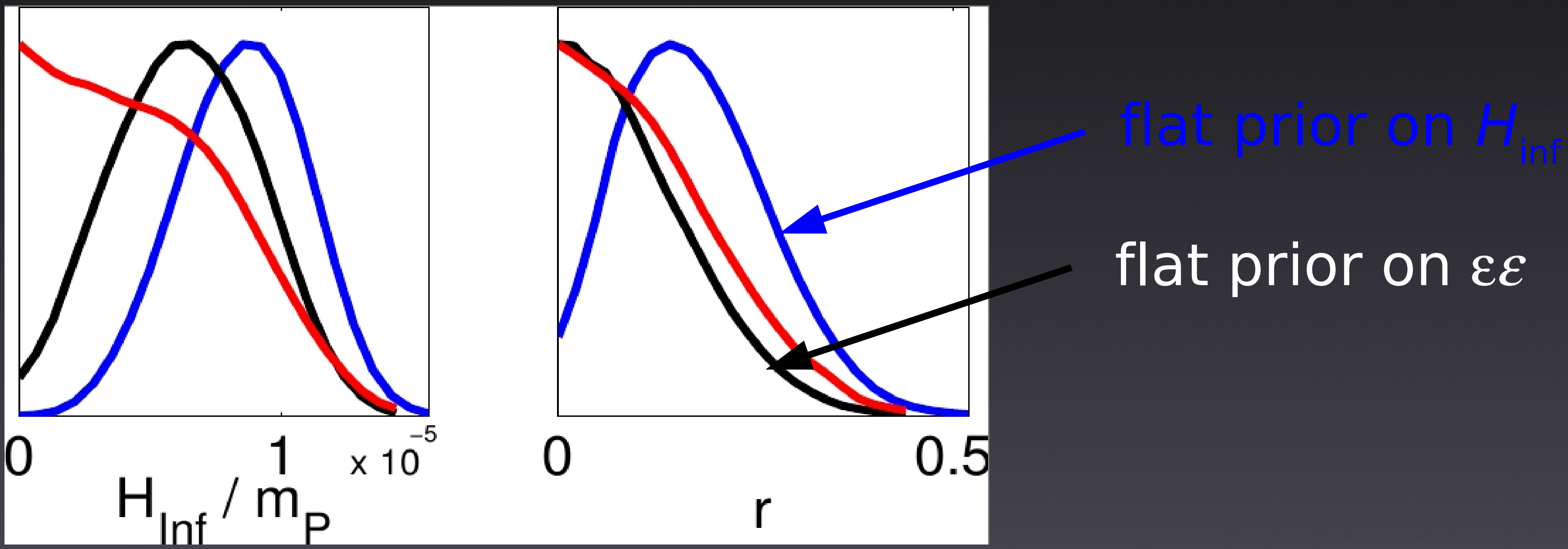
Direct constraints on scale of inflation



$A_T \propto H_{inf}^2$

- Values of $H_{inf} < 10^{-6}\ m_P$ disfavoured?
- This is for flat prior on ε, not on H_{inf}!
- What would have happened if we had chosen a prior that's flat on H_{inf}?

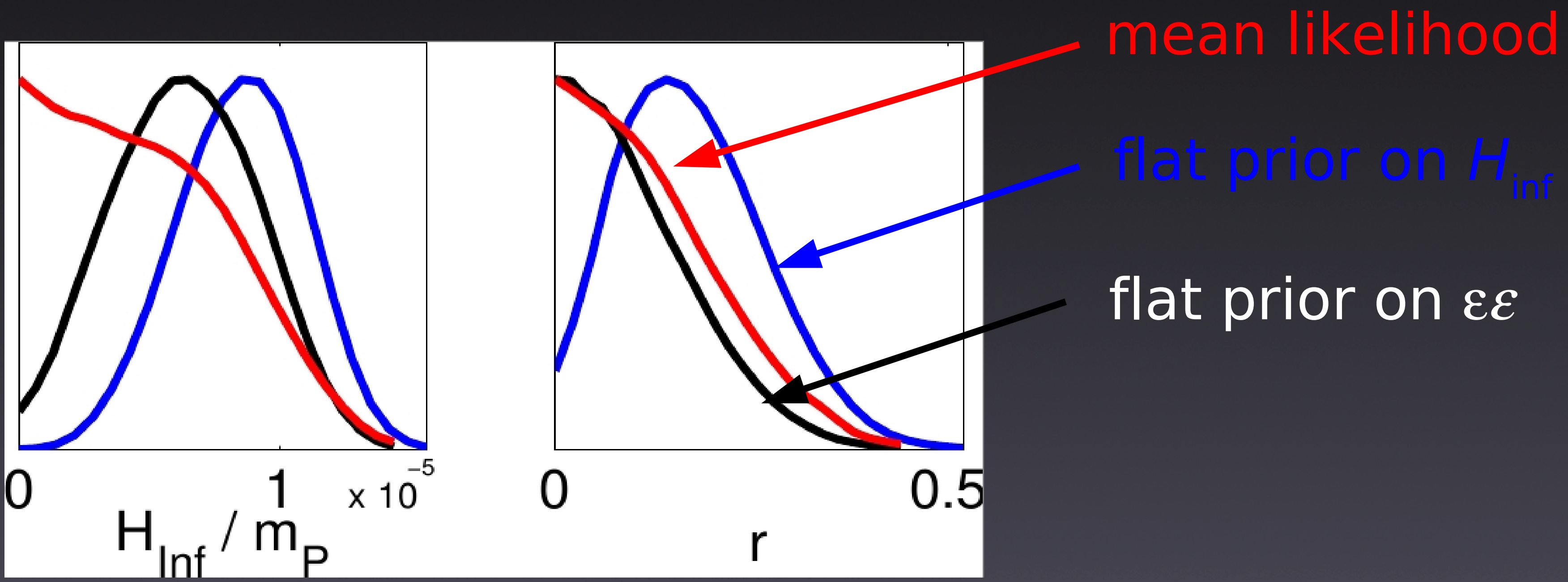
Direct constraints on scale of inflation



[Valkenburg, Krauss, JH 2008]

- Low H_{inf} even more unlikely?!

Direct constraints on scale of inflation



[Valkenburg, Krauss, JH 2008]

- Low H_{inf} even more unlikely?!
- Sanity check: Look at *mean likelihood*
  - prior independent, but no probabilistic interpretation
  - data show no preference for high H_{inf} or non-zero r

Caveat #2: prior dependence

- Flat prior in one parameterisation is usually not flat in a different one

$$\boxed{\begin{array}{c}\text{Choice of parameterisation}\\ = \\ \text{Choice of prior}\end{array}}$$

- Particularly problematic for badly constrained parameters and cases with no obvious canonical parameterisation

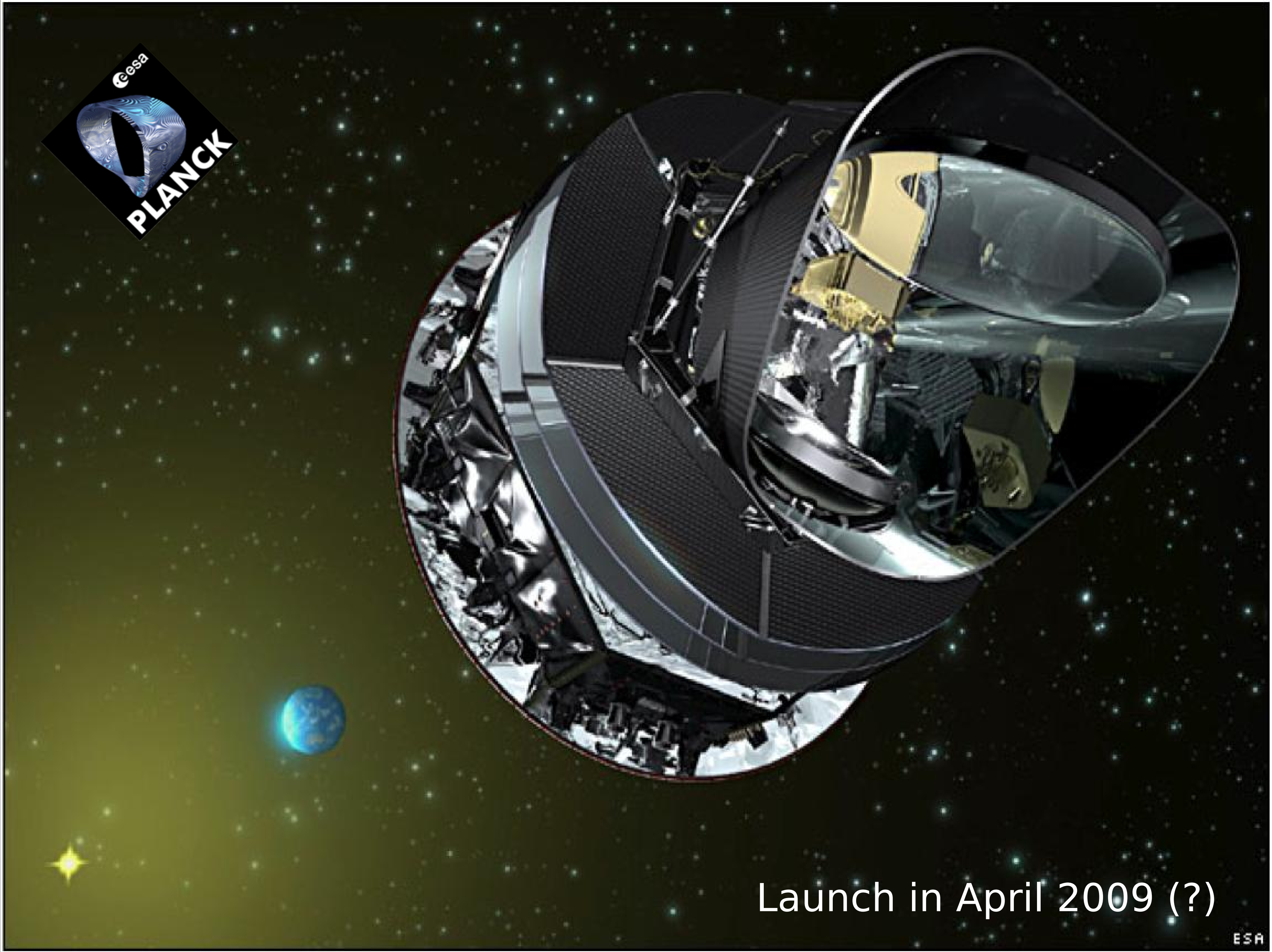
But...

- More constraining data will alleviate problems associated with inference
  - Model dependence:

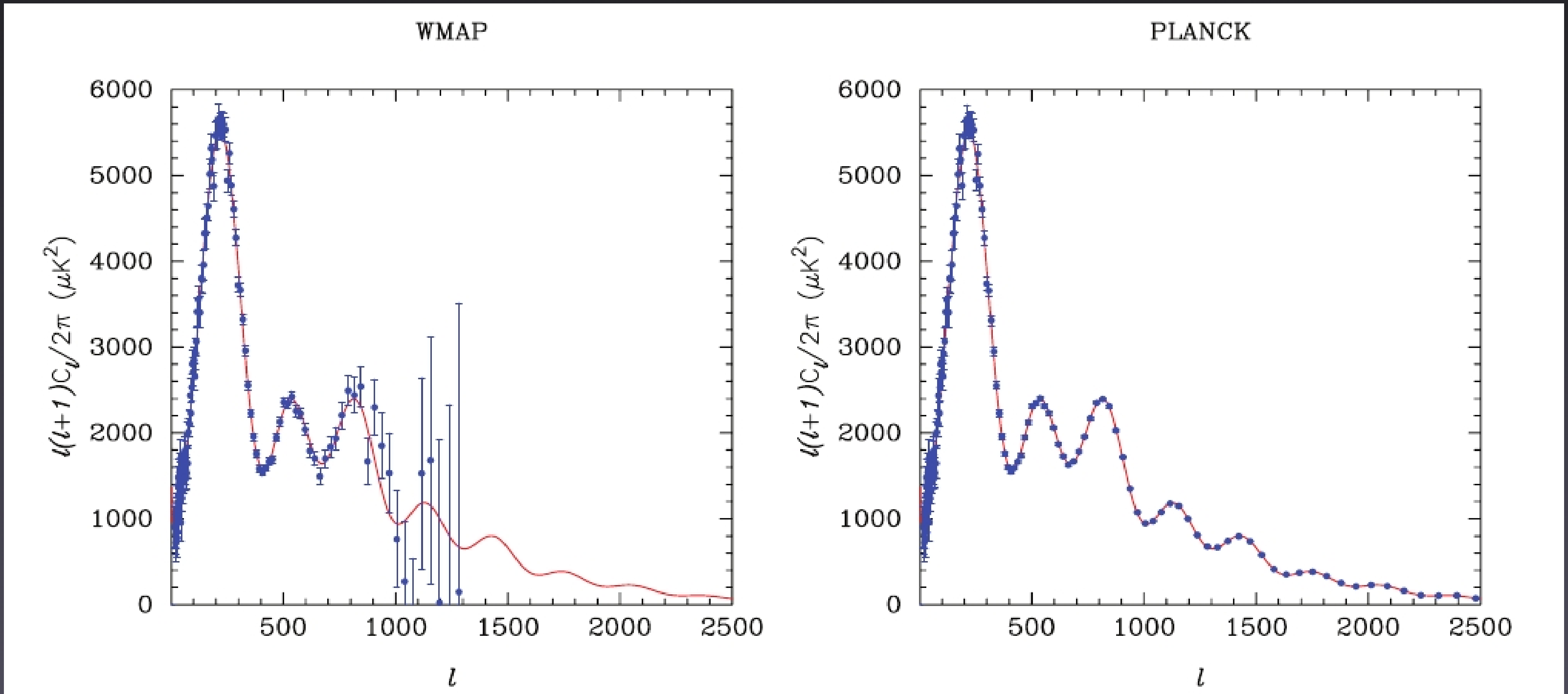
    Parameter degeneracies will be broken
  - Prior dependence:

    Decreases for reasonably smooth priors

III. What does the future hold for slow-roll inflation?

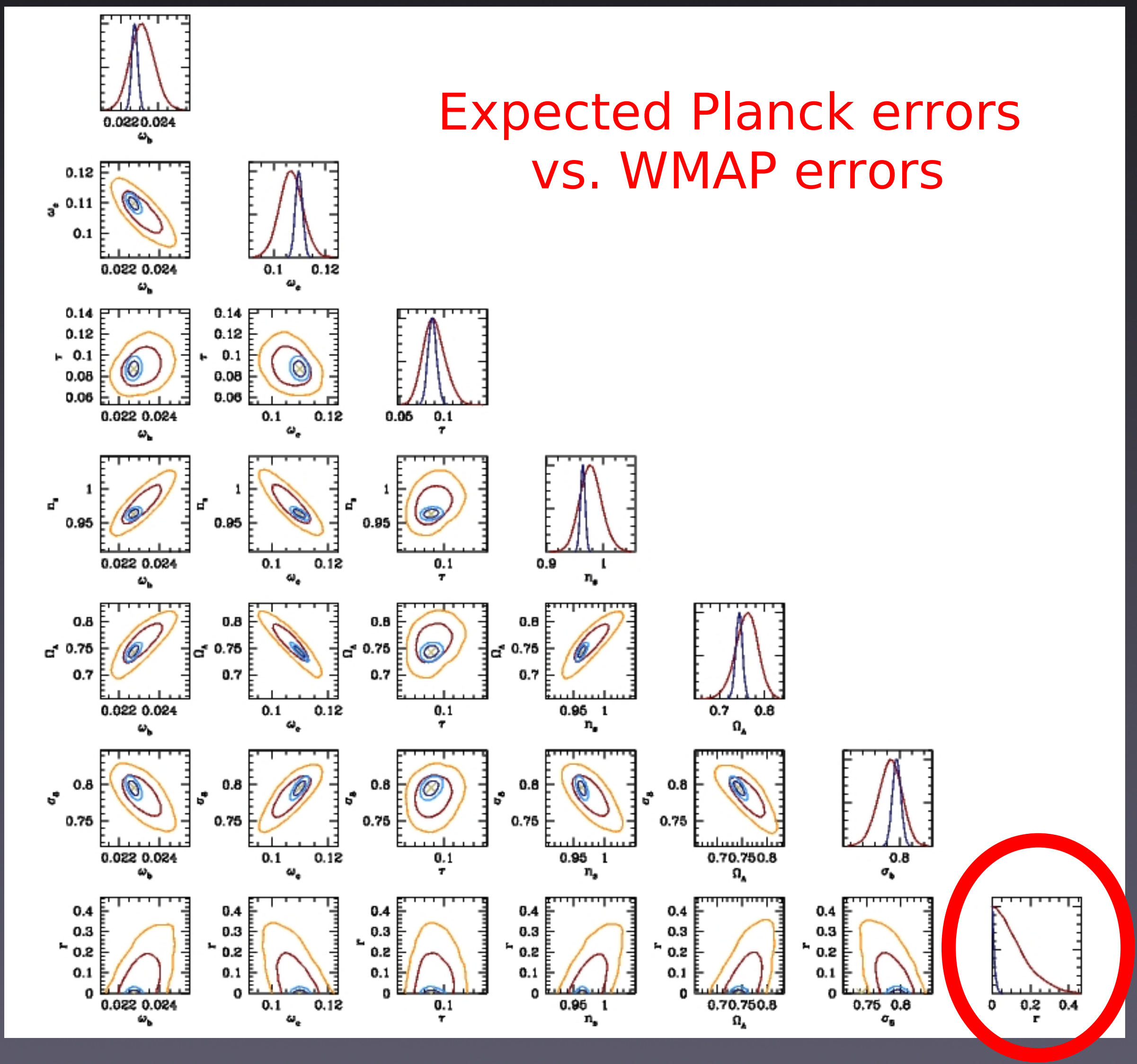

esa
PLANCK
Launch in April 2009 (?)
ESA

TT spectrum with Planck



- Essentially limited by cosmic variance

Parameter constraints with Planck



[Colombo, Pierpaoli, Pritchard 2008]

- Most parameter constraints will improve by factor 2-3 wrt WMAP
- Tensor to scalar ratio: Factor 9 possible... ... if *B*-mode information can be retrieved otherwise ''only'' factor 3
- After Planck: CMBPol (?) ultimate *E*-polarisation

Beyond the CMB?

- Need larger *k*-range and/or smaller errors...
- At multipoles >2000: primordial signal swamped by secondary perturbations (thermal and kinetic SZ-effect)
- CMB eventually limited by cosmic variance: last scattering surface is only 2d
  → need to trace perturbations in 3d for further improvements

21cm tomography

- Trace neutral hydrogen at redshifts < 15 by observing 21cm emission (spin flip)
- Technically and theoretically challenging
    - Foregrounds, weak signal
    - Need to understand reionisation...
- Potential rewards: order of magnitude improvement on sensitivity to parameters

[Tegmark & Zaldarriaga 2008; Barger et al. 2008]

- Technology being developed...

LOFAR → MWA → SKA → FFTT

IV. Beyond the simplest model

Beyond the simplest model

List of ingredients

- 1 inflaton field
- Lagrangian, consisting of
  - Canonical kinetic term
  - Smooth and flat potential
- ''Standard'' initial conditions

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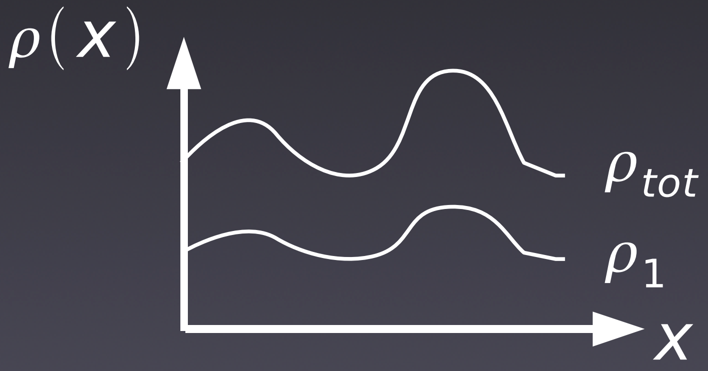
More than one field?

Multi-field models

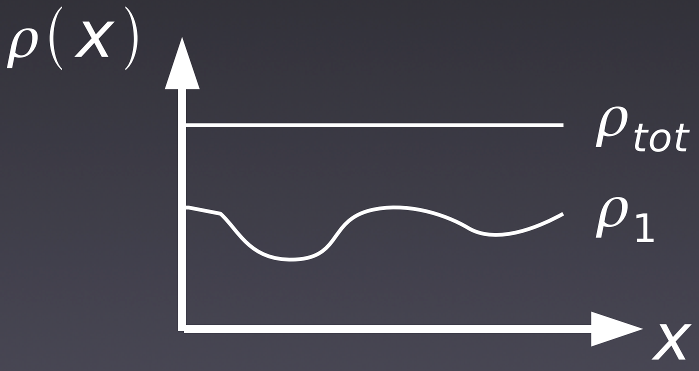
Smoking gun...

Adiabatic perturbation



$$\delta\rho_1/\rho_1 = \delta\rho_2/\rho_2 = const.$$
$$\delta\rho_{tot} \neq 0$$

Isocurvature perturbation

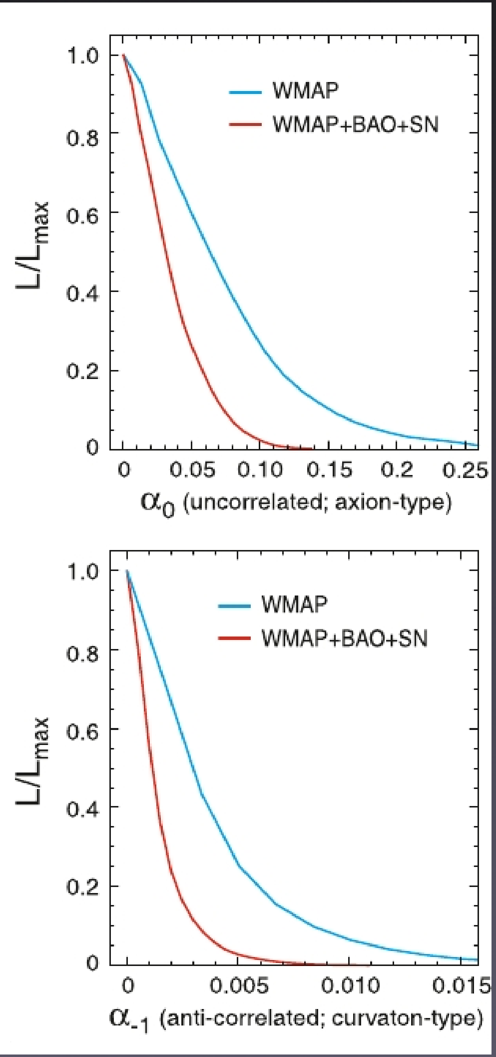


$$\delta\rho_1/\rho_1 \neq \delta\rho_2/\rho_2$$
$$\delta\rho_{tot} = 0$$

Single field: purely adiabatic perturbations

Multi-field models

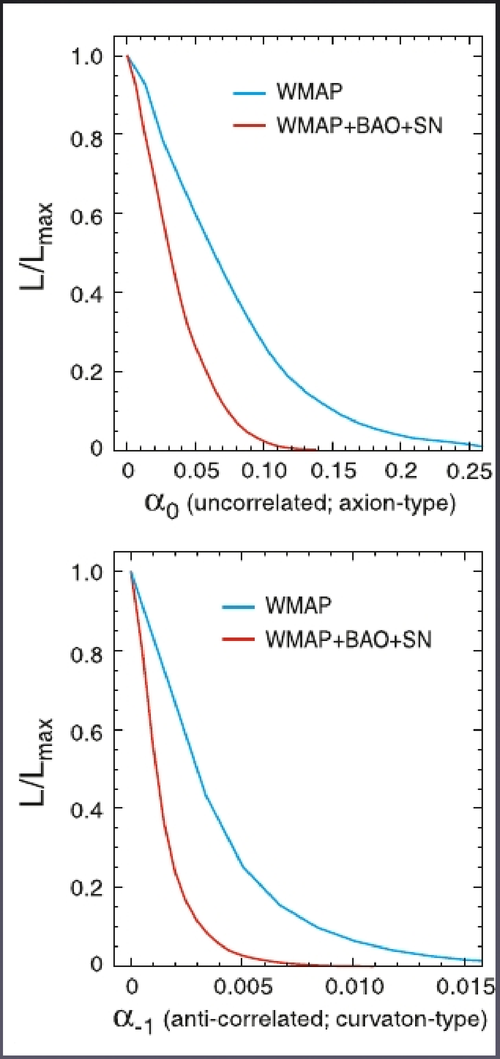
- Presently: no evidence
- Tight constraints on isocurvature fraction α
- Does not mean inflation cannot have been multi-field



[Komatsu et al. 2008]

Multi-field models

- Presently: no evidence
- Tight constraints on isocurvature fraction α
- Does not mean inflation cannot have been multi-field
- Ockham's razor might prefer single field inflation



[Komatsu et al. 2008]

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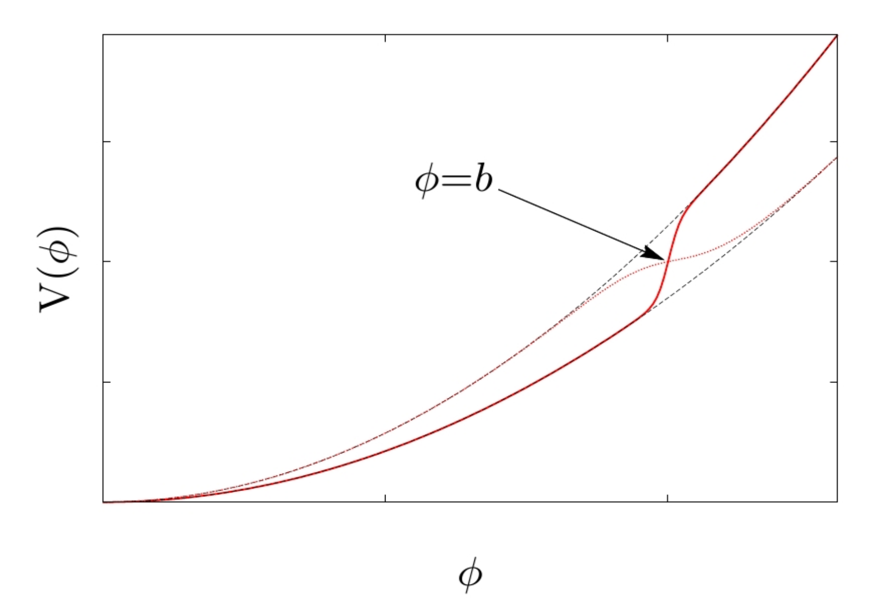
- "Standard" initial conditions

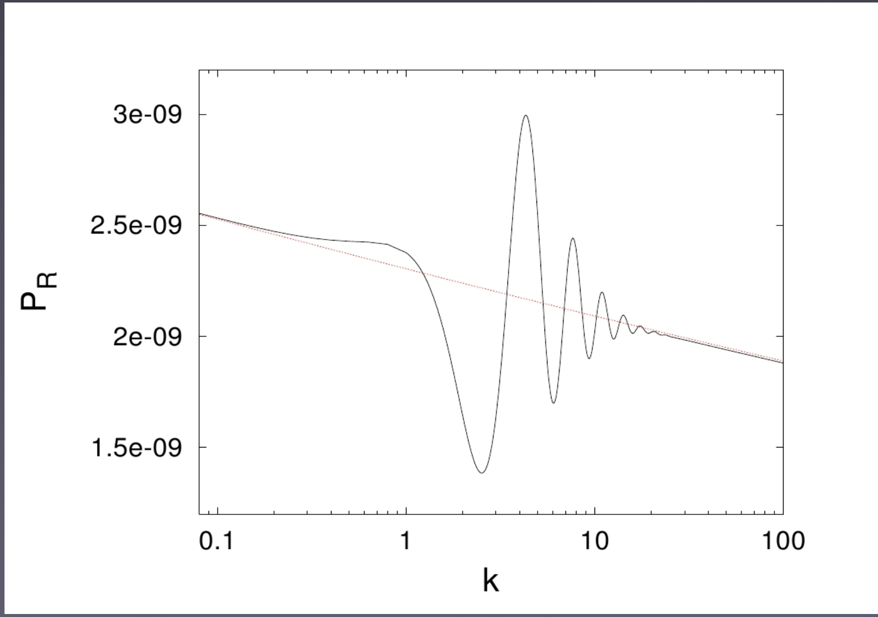
Slow roll violated?

Violation of slow-roll

- No need for slow-roll conditions to always hold
- Can be broken by, e.g, phase transitions

→ features in spectrum





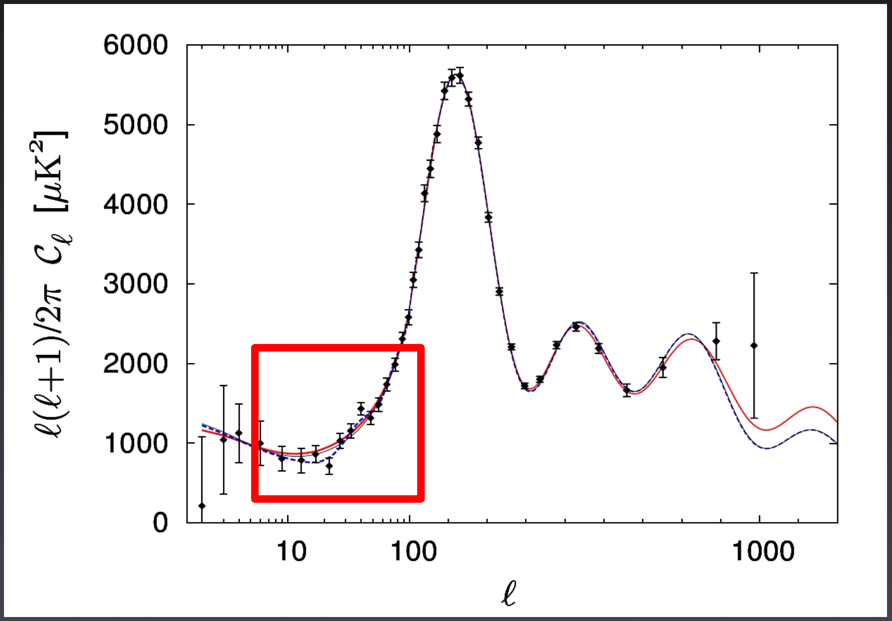
Example:

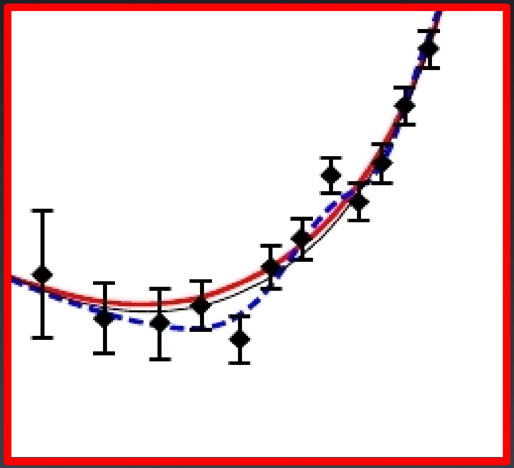
step inflation toy model

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \left(1 + c \tanh\left(\frac{\phi - b}{d}\right)\right)$$

[Adams, Cresswell, Easther 2001]

Features in the spectrum?





- Step model yields modest improvement in fit ($\Delta\chi^2 \sim 7$) compared to smooth spectrum
- May explain glitches in data?

[JH, Covi, Melchiorri, Slosar 2007]

Beyond the simplest model

List of ingredients

- 1 inflaton field
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    - Slow-roll potential
- ''Standard'' initial conditions

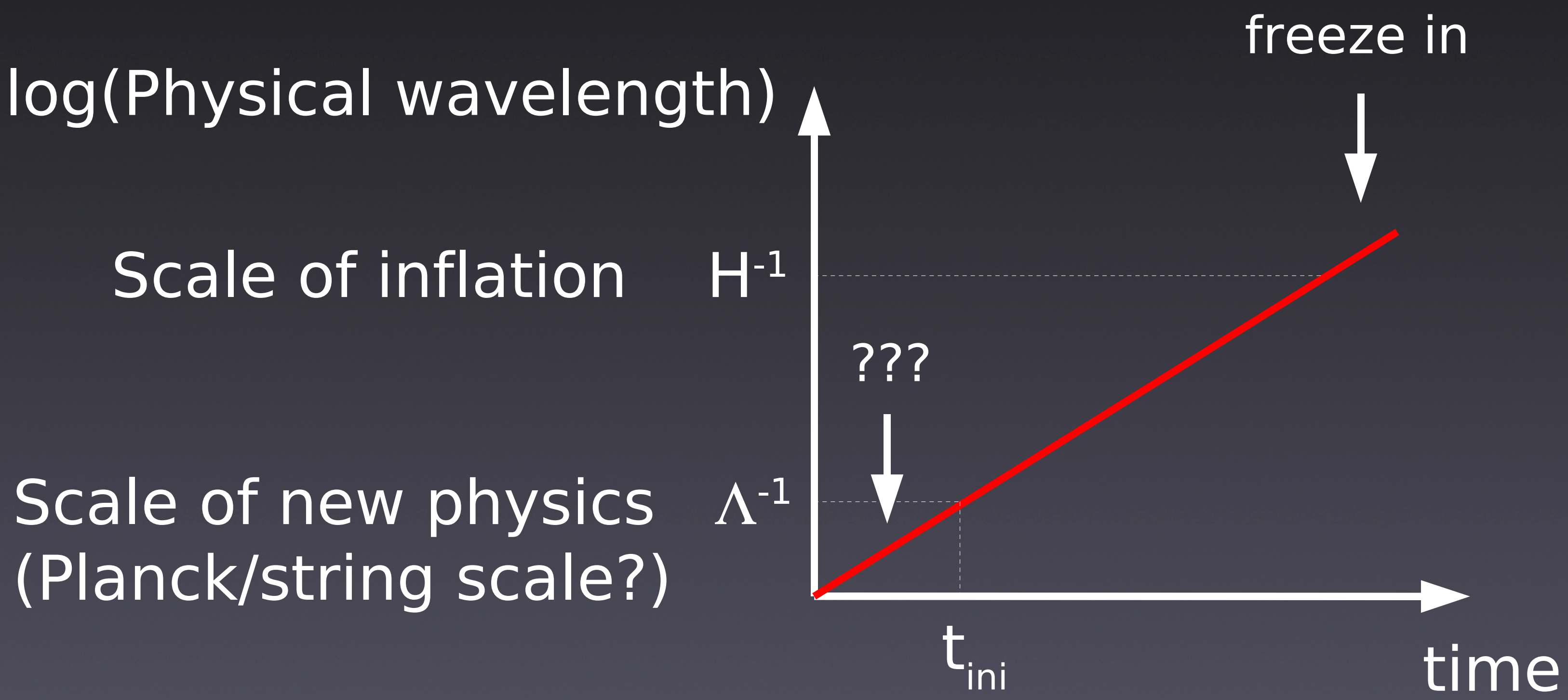
Non-standard
Initial conditions?

Inflation and initial conditions

- Inflation sets initial conditions for structure formation
- What about the initial conditions of inflation itself?
  - Classical level: attractor solution exists
  - Quantum level: no unique choice

  Typically impose Bunch-Davies vacuum of de Sitter space at sub-Hubble scales

Transplanckian origin of fluctuations



- At early times, wavelength is shorter than Planck scale (or other new physics scale)
- Impose initial conditions at scale Λ^{-1} (not necessarily Bunch-Davies)

Signatures of non-BD initial conditions

- Depends on new physics...
- Many suggestions:

[Danielsson; Easther, Greene, Kinney, Shiu; Martin, Brandenberger; Bozza, Giovannini, Veneziano; Kaloper, Kleban, Lawrence, Shenker; ...]

- Generic prediction:
  - Oscillatory modulation of perturbation spectra
  - Amplitude suppressed by some power of

$$\boxed{\xi = H/\Lambda}$$

Transplanckian ripples

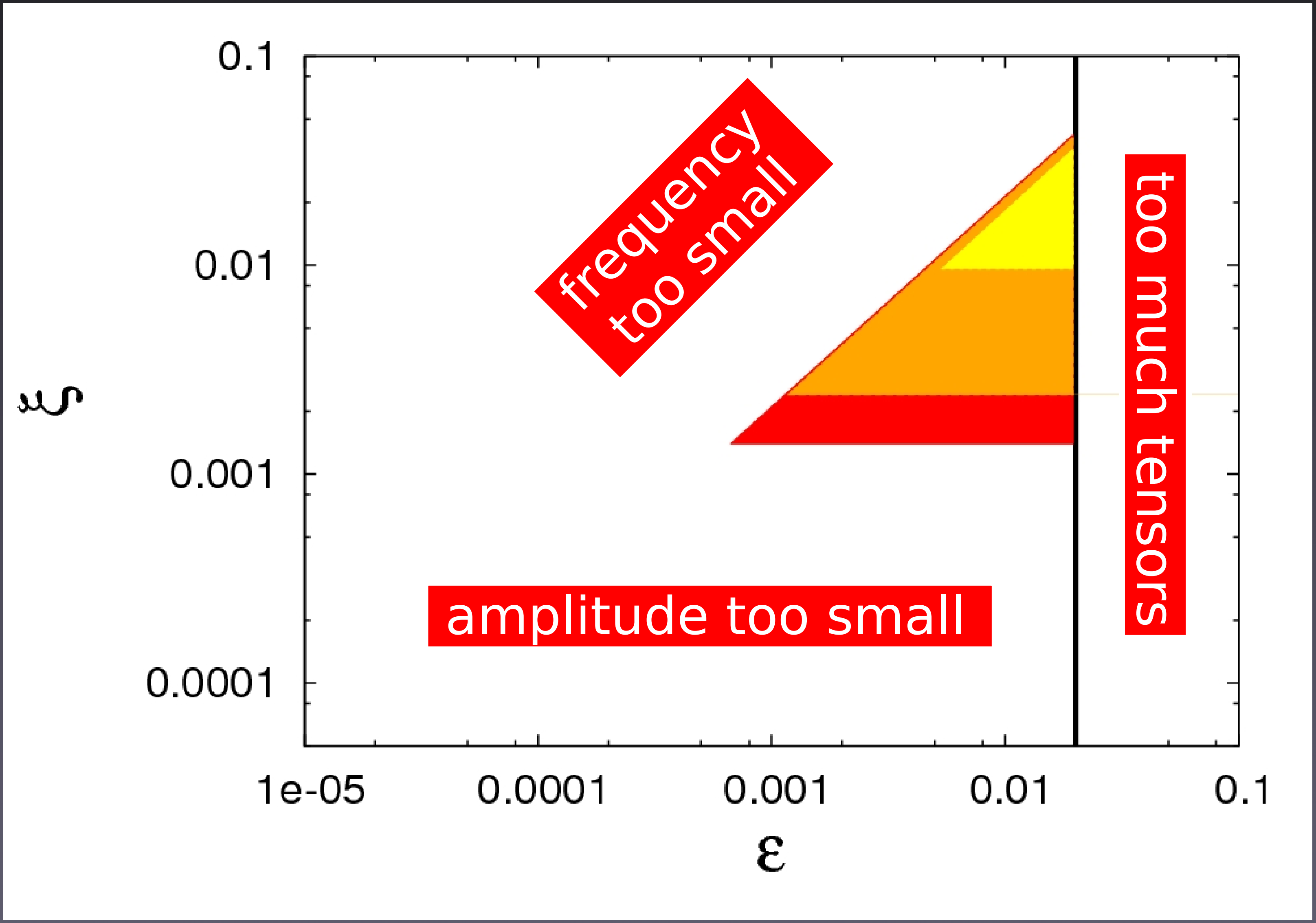
(Danielsson model + slow roll inflation)

$$\mathcal{P}(k) \simeq \mathcal{P}^{\mathrm{BD}}(k)\left\{1 + \xi\left(\frac{k}{k_0}\right)^{-\epsilon}\sin\left[\frac{2\epsilon}{\xi}\ln\left(\frac{k}{k_0}\right) + \varphi\right]\right\}$$

- ξ = H/Λ: amplitude, frequency
- ϵ (first slow-roll parameter): frequency

  (NB: tensor-to scalar ratio r = 16 ϵ)
- φ: phase

Detectability of trans-Planckian effects (optimistic estimate)



"WMAP"

"Planck"

CVL_{2000}

Non-standard initial conditions

- Have unique signature
- Possibly detectable
- Discovery not very likely, unless scale of inflation is large (i.e. tensors can be found), and scale of new physics is a few orders of magnitude below Planck scale

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Non-canonical kinetic term?

Arbitrary kinetic term

$$L(\phi) = F\left(\frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi, \phi\right)$$

- k-essence, DBI-inflation...
- Not distiguishable via power spectrum
- However: possibly detectable higher order correlations (bi-, trispectrum) → non-Gaussianity

Non-Gaussianity

$$\Phi = \Phi_L + f_{nl}\,\Phi_L^2$$

curvature perturbation
during matter domination

Gaussian part

- Condense bispectrum information into one number
- Easier to estimate from data
- Slow-roll inflation predicts $f_{nl} \sim O(1)$

Non-Gaussianity

$$-9 < f_{nl} < 111 \qquad (@\ 95\%\ \text{c.l.})$$

[Komatsu et al. 2008]

- No strong evidence at the moment
- Also other possible sources of f_{nl}:
    - multi-field inflation
    - non-slow-roll potentials
    - initial conditions
- f_{nl} alone might not be enough to distinguish between scenarios

Conclusions

- Cosmological data provide direct window to the physics of inflation, at energies way beyond the capabilities of laboratory experiments
- Data are starting to allow us to distinguish between classes of models
- We can expect interesting results in the near future