Fake vs Genuine Sugra

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OR

Fake (pseudo) susy of domain walls (cosmologies) as Hamilton-Jacobi theory & lessons from some simple models

OR

Part 1 Background

inc. work with K. Skenderis, J. Sonner

DW/C correspondence

HJ & fake sugra

adS

Part 2 Lessons (Julian Sonner & PKT, hep-th/0703276)

Dilaton walls (& branched superpotentials)

Axion-Dilaton walls (& inconsistency of adapted truncation)

OR

Talk

Domain Walls & Cosmologies

$$\mathcal{L} = \sqrt{-\det g}\left[R - \frac{1}{2}|\partial\Phi|^2 - V(\Phi) \right]$$

d-dim. metric target space norm multi-cpt. scalar field

Set $\eta = \begin{cases} 1 & \text{domain wall} \\ -1 & \text{FLRW cosmology} \end{cases}$

$$\alpha = (d-1)\beta \quad , \quad \beta = \frac{1}{\sqrt{2(d-1)(d-2)}}$$

$$\boxed{ds_d^2 = \eta\left(f e^{\alpha\varphi} dz\right)^2 + e^{2\beta\varphi}\underbrace{\left[\frac{-\eta\, dr^2}{1+\eta k r^2} + r^2 d\Omega_\eta^2\right]}_{\text{max. sym space(time) of radius } k}}$$

arbitrary fn. of z

scale factor variable $\varphi(z)$

$$\Phi = \Phi(z)$$

curve in target space

$$\boxed{L_{eff} = \frac{1}{2} f^{-1}\left(\dot\varphi^2 - |\dot\Phi|^2\right) - f e^{2\alpha\varphi}\left(\eta V - \frac{\eta k}{2\beta^2} e^{-2\beta\varphi}\right)}$$

$\dot\varphi = \frac{d}{dz}\varphi$ etc einbein

(cf. relativistic particle)

The DW/C correspondence (Skenderis & PKT)

$$\left\{\begin{array}{l} \eta \to -\eta \\ V \to -V \\ k \to -k \end{array}\right\} \text{ is \underline{symmetry} of } L_{eff}$$

$$\Rightarrow \quad \boxed{\text{Domain Wall}} \xleftrightarrow[k \to -k]{V \to -V} \boxed{\text{Cosmology}}$$

related by analytic continuation

e.g. $V = \Lambda$, const. Some special (well-known) cases:

	DW	C
$\Lambda < 0$	adS $\left\{\begin{array}{l} k=0 \\ k=-1 \\ k=1 \end{array}\right.$ dS-slicing	$\left.\begin{array}{l} k=0 \\ k=1 \\ k=-1 \end{array}\right\}$ dS
$\Lambda > 0$	dS $k=1$	$k=-1$ adS
$\Lambda = 0$	$\left.\begin{array}{c} k=1 \\ \text{d-dim. Rindler} \end{array}\right\}$	$\left\{\begin{array}{l} k=-1 \\ \text{Milne} \end{array}\right.$

2 regions of Mink. separated by horizon

Hamilton - Jacobi

(Salopek & Bond
Skenderis & PKT)

In Hamiltonian form,

einbein

$$L_{eH} = \dot{\varphi}\pi + \dot{\Phi}\cdot P - f\,\mathcal{H}(\varphi, \Phi; \pi, P)$$

$$\mathcal{H} = \frac{1}{2}\left(-\pi^2 + |P|^2\right) - e^{2\alpha\varphi}\left(\eta V(\Phi) - \frac{\eta k}{2\beta^2} e^{-2\beta\varphi}\right)$$

Hamilton Jacobi eq. for Hamilton's "principal" fn. S is

$$\boxed{-\left(\frac{\partial S}{\partial \varphi}\right)^2 + \left|\frac{\partial S}{\partial \Phi}\right|^2 = 2e^{2\alpha\varphi}\left(\eta V(\Phi) - \frac{\eta k}{2\beta^2} e^{-2\beta\varphi}\right)}$$

For $k = 0$, solve by writing

$$S(\varphi, \Phi) = \pm 2 e^{\alpha\varphi} W(\Phi)$$

where $W(\Phi)$ must solve "reduced HJ" eq.

$$\boxed{2\left[\left|\frac{\partial W}{\partial \Phi}\right|^2 - \alpha^2 W^2\right] = \eta V}$$

i.e. $\boxed{\eta V = 2\left[(W')^2 - \alpha^2 W^2\right]}$ for single scalar σ, where $W'(\sigma) = \frac{\partial W(\sigma)}{\partial \sigma}$

'Genuine' sugra : $d = 3, 4$

(Sonner & PKT)

For $\left\{\begin{matrix} \eta = 1 \\ d = 3 \end{matrix}\right\}$ W is $d=3$ sugra superpotential for real scalar superfield σ

For $d = 4$ sugra coupled to chiral superfield τ :

$$\tau = \underset{\text{'axion'}}{\chi} + i \, \underset{\text{'dilaton'}}{\Sigma(\sigma)}$$

holomorphic superpotential

$$V = \frac{1}{2} e^{K} \left[\left| \partial_\tau P + \partial_\tau K \, P \right|^2 G^{-1} - 4\alpha^2 |P|^2 \right]$$

inverse of target space metric

Kähler pot.
$(G = \partial_\tau \partial_{\bar\tau} K)$

- Choose $P = 1$
- " $K = 2 \log W(\sigma)$ (i.e. special Kähler)
- " Σ such that $\left(\frac{W'}{W}\right)' - \left(\frac{\Sigma''}{\Sigma'}\right)\left(\frac{W'}{W}\right) = \frac{1}{2}$

Then $\boxed{V = 2\left[(W')^2 - \alpha^2 W^2 \right]}$

(& similar agreement with Killing spinor eq. of $d=3$ sugra generalized to $d=4$)

Axion - Dilaton sugra / non-sugra

Choose $\Sigma(\sigma) = \pm \frac{2}{\mu} e^{-\frac{\mu}{2}\sigma}$

$$\Longrightarrow \quad \left(\frac{W'}{W}\right)' + \frac{\mu}{2}\left(\frac{W'}{W}\right) = \frac{1}{2}$$

One solution is $W = W_0 \, e^{\mu^{-1}\sigma}$

Then

$$\underbrace{4G|dz|^2}_{\text{target space metric}} = \underbrace{d\sigma^2 + e^{\mu\sigma} d\chi^2}_{\text{hyperbolic space, radius} \sim \frac{1}{|\mu|}}$$

$$V = \Lambda e^{-\lambda\sigma}$$

$$\boxed{\begin{aligned} \lambda &= -\frac{2}{\mu} \\ \Lambda &= \frac{1}{2} W_0^2 \left(\lambda^2 - \lambda_u^2\right) \end{aligned}}$$

($\lambda_u = 2\alpha$ – see later for significance)

This is special case of general axion-dilaton model

$$\boxed{\begin{aligned} |d\Phi|^2 &= d\sigma^2 + e^{\mu\sigma} d\chi^2 \\ V &= \Lambda e^{-\lambda\sigma} \end{aligned}}$$

More later on this model

axion-dilaton coupling

'cosmological constant'

dilaton coupling const.

Fake Susy (k=0)

For d=3 sugra (& d=4,5 suitably understood)

$$\delta\psi_\mu\Big| = 0 \;\Rightarrow\; \boxed{\left(D_\mu - \frac{1}{2(d-2)} W \Gamma_\mu\right)\epsilon = 0}$$

$\left.\begin{array}{l}\text{fermions}\\ \text{gauge fields}\end{array}\right\} = 0$

Killing spinor eq.

Killing spinor

For domain wall solution of eq. of motion (in gauge f) the integrability conditions are

$$\boxed{\begin{aligned} f^{-1}\dot\varphi &= \mp 2\alpha e^{\alpha\varphi} W \\ f^{-1}\dot\sigma &= \pm 2 e^{\alpha\varphi} W' \end{aligned}}$$

'BPS' eqs.

But these follow from HJ theory!

$$S = \pm 2 e^{\alpha\varphi} W \;\Rightarrow\; \begin{cases} \Pi = \dfrac{\partial S}{\partial\varphi} = \pm 2\alpha e^{\alpha\varphi} W \\[2ex] P = \dfrac{\partial S}{\partial\sigma} = \pm 2 e^{\alpha\varphi} W \end{cases}$$

eq. of motion $\left\{\begin{array}{l}\Pi = -\dot\varphi \\ P = \dot\sigma\end{array}\right\} \longrightarrow$ BPS eq.

> N.B. (i) W complex for $k \neq 0$
>
> (ii) DW $\to$ Cos. : Susy $\to$ Pseudosusy

Hamilton vs Jacobi: Historical aside

Hamilton (1834): From soln. to mechanical problem can construct S satisfying HJ eq.

Jacobi (1936): From soln. S of HJ eq. can construct soln. to mechanical problem.

Fake susy history reverses this order!

1999 Skenderis & PKT
De Wolfe, Freedman, Gubser & Karch

From soln W of 'reduced' HJ eq. get susy domain wall by solving 'BPS' eqs.

2003 Freedman, Nuñez, Schnabl & Skenderis
2005 Sonner & PKT
2006 Skenderis & PKT

From domain wall solution can construct W satisfying reduced HJ eq. such that 'BPS' eqs are satisfied.

Construction of superpotential ($k=0$)

- Start from solution $(\phi(z), \sigma(z))$ of eqs. of motion & constraint in given gauge
  e.g. $f = e^{\alpha\phi}$ ($\Rightarrow z$ is affine distance parameter)

- Provided that $\dot{\sigma} \neq 0$, we have inverse fn $z(\sigma)$

- Define $W(\sigma)$ by

$$\boxed{W(\sigma) = \mp \frac{1}{2\alpha} \dot{\phi}(z(\sigma))}$$

It then follows from eqs. of motion & constraint that

$$\text{(i)} \quad W' = \pm \frac{1}{2} \dot{\sigma}$$

$$\text{(ii)} \quad 2\left[(W')^2 - \alpha^2 W^2\right] = V(\sigma)$$

[N.B. Similar construction for $k \neq 0$ but for <u>complex</u> W]

Unstable asymptotically adS walls (Skenderis & PKT)

$$V(\sigma) = -\frac{1}{2\beta^2 \ell^2} + \frac{1}{2} m^2 \sigma^2 + \cdots \quad (\text{near } \sigma = 0)$$

adS radius

mass of σ-particle in adS vac.

For domain wall asymptotic to adS vacuum

$$\boxed{\ddot{\sigma} + \frac{(d-1)}{\ell}\dot{\sigma} - m^2\sigma \to 0 \quad \text{as } z \to \infty}$$

$$\therefore \quad \sigma \sim e^{-\nu z/\ell}, \quad \nu^2 - (d-1)\nu - m^2\ell^2 = 0$$

$$\therefore \quad \nu = \nu_\pm = \frac{d-1}{2} \pm \sqrt{\frac{(d-1)^2}{4} + m^2\ell^2}$$

- Breitenlohner-Freedman stability bound is

$$\boxed{m^2 > -\frac{(d-1)^2}{4\ell^2}}$$

- Instability of adS vacuum $\Rightarrow$ $\sigma(z)$ oscillates s.t $z = \infty$ is accumulation point of zeros of $\dot{\sigma}$

$\therefore \nexists\, W$ for wall asymptotic to unstable adS

Near-adS superpotentials

(Sonner & PKT)

Assume BF bound <u>satisfied</u>

Then $\sigma(z) \to 0$ <u>monotonically</u> for asy.-adS wall

$\therefore$ Construction applies $\to$

$$\boxed{W = W_\pm = \frac{1}{2\alpha\beta\ell} + \frac{\nu_\pm}{4\ell}\sigma^2 + \dots}$$

✓ satisfies $2\left[(W')^2 - \alpha^2 W^2\right] = V$

N.B. $W'(0) = 0$, so adS vacuum is susy with respect to both W_+ and W_-

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If $W'(0) \neq 0$, then $2\left[(W')^2 - \alpha^2 W^2\right] = V$ has soln

$$\boxed{W = W_M \equiv \sqrt{M^2 + \frac{(d-2)^2}{\ell^2}} + \alpha M \sigma + \frac{1}{2}\alpha^2 \sqrt{M^2 + \frac{(d-2)^2}{\ell^2}}\,\sigma^2 + \frac{1}{6}\left(\alpha^2 M + \frac{m^2}{4\alpha M}\right)\sigma^3 + \dots}$$

1-parameter family

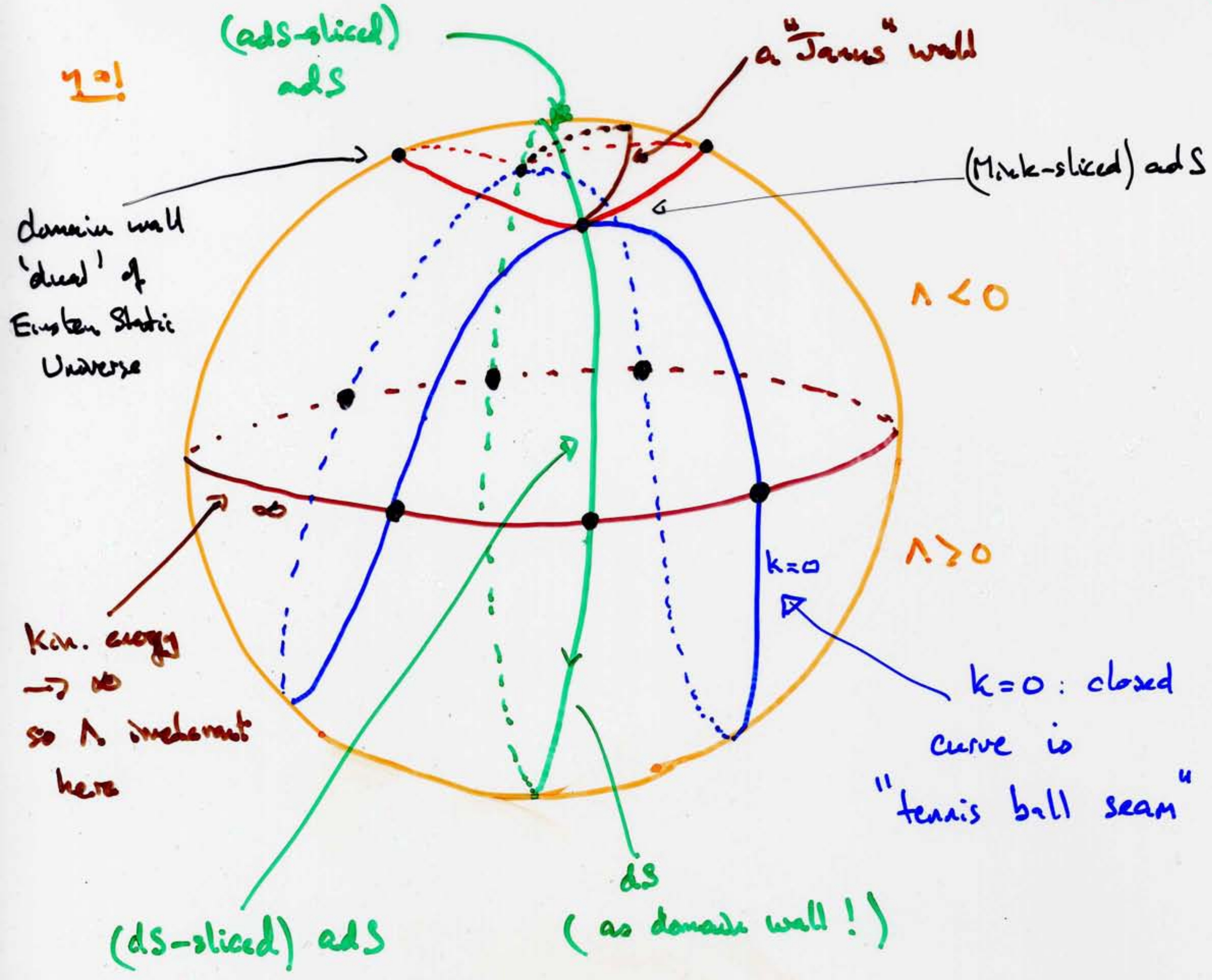
linear term

diverges at $M = 0$ $\therefore$ $M \neq 0$

Domain wall that is susy w.r.t. <u>this</u> W is <u>not</u> asymptotic to adS vacuum

$V = \Lambda$

- For $f = e^{-\alpha\phi}$, eqs of motion define 2-dim. autonomous dyn. system for $(\dot{\sigma}, \dot{\phi})$ (Halliwell)
- Global phase space is sphere with 10 fixed pts (Sonner & PKT)



N.B. Similar diagram for cosmology

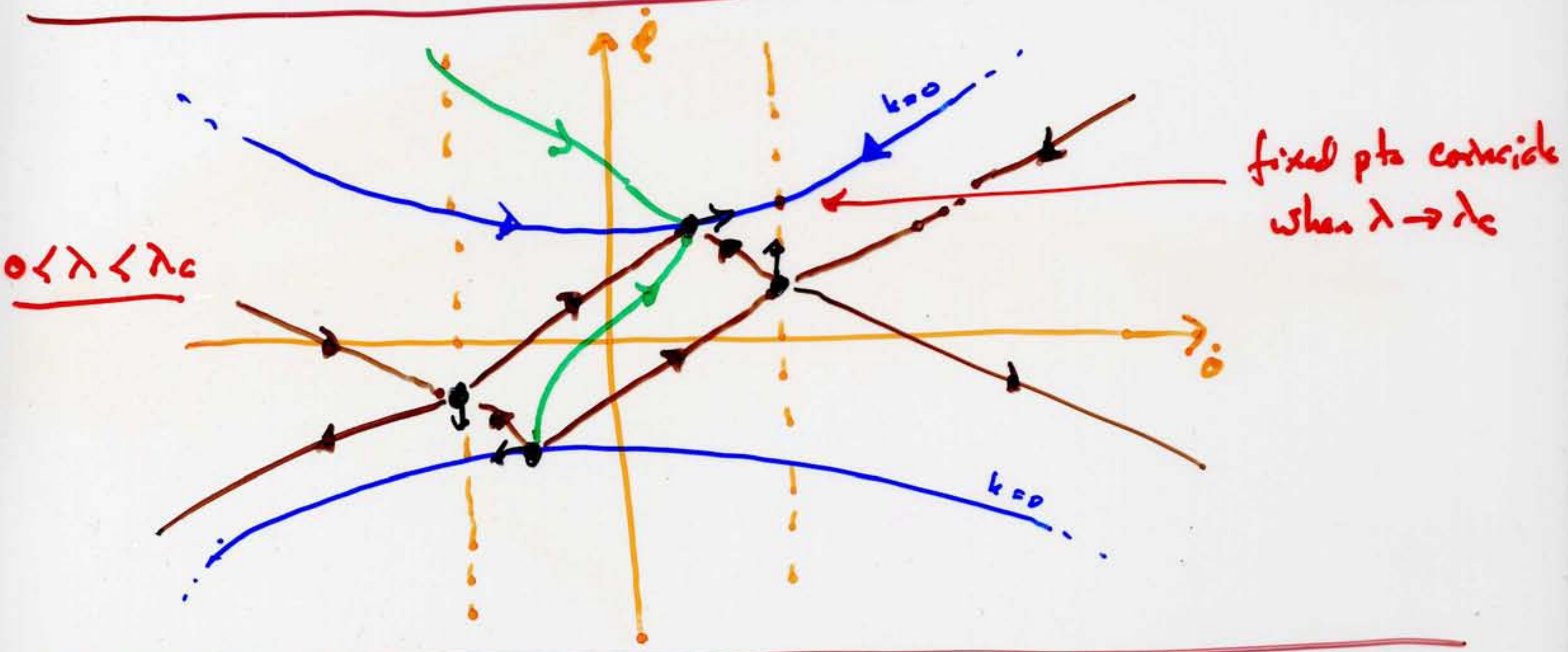
$\Lambda \to -\Lambda$ & adS $\to$ dS

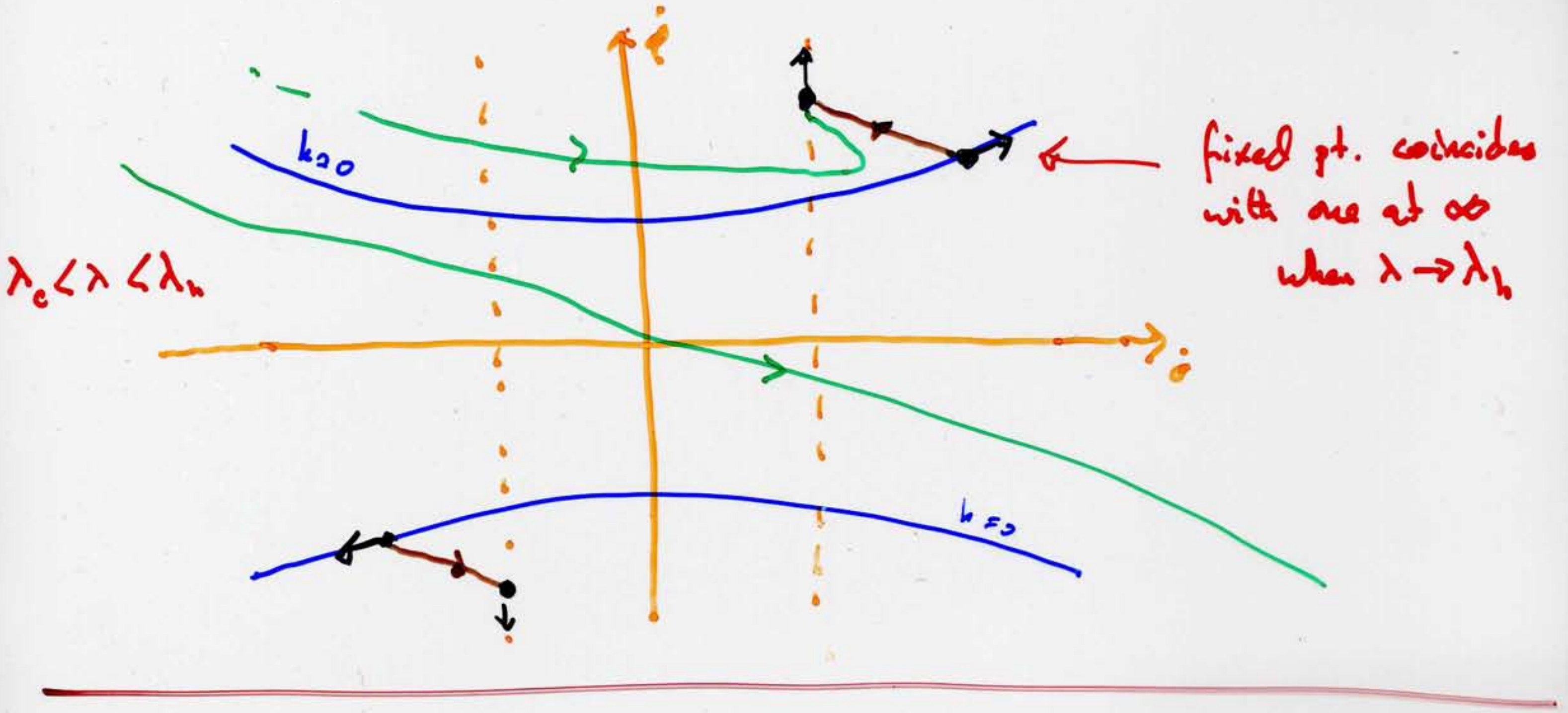
$k \to -k$ dS $\to$ adS

$$\underline{V = \Lambda e^{-\lambda\sigma}}$$

- Same global phase space but some fixed points have λ-dependent positions (& eigenvalues)
- Get '<u>transcritical</u>' bifurcations at $\lambda = \begin{cases} \lambda_c = 2\sqrt{\alpha\beta} \\ \lambda_h = 2\alpha > \lambda_c \end{cases}$

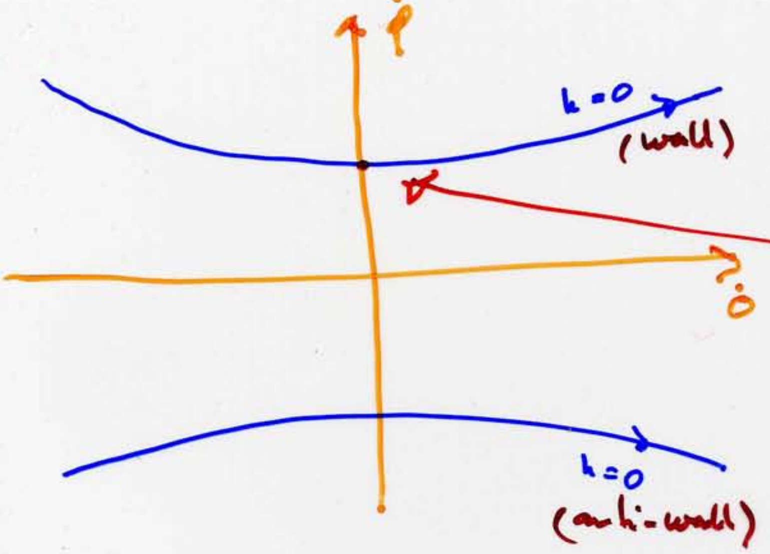
<u>View from above 'north pole' ($\Lambda < 0$)</u>





$\lambda = \lambda_h$

No $k=0$ fixed points
$\therefore$ unique flat wall
& anti-wall



wall has one isolated zero of $\dot\sigma$

Simple explicit soln. for $\lambda = \lambda_h$

$$\Lambda = -2/\lambda_h^2$$

$$ds^2 = \underbrace{e^{-\lambda\sigma} dz^2}_{d\tilde{z}^2} + e^{2\beta\varphi} ds^2(\text{Mink})$$

$$e^{\lambda_h \varphi} = z\, e^{\frac{1}{2} z^2} \qquad \boxed{e^{\lambda_h \sigma} = \frac{1}{z} e^{\frac{1}{2} z^2}}$$

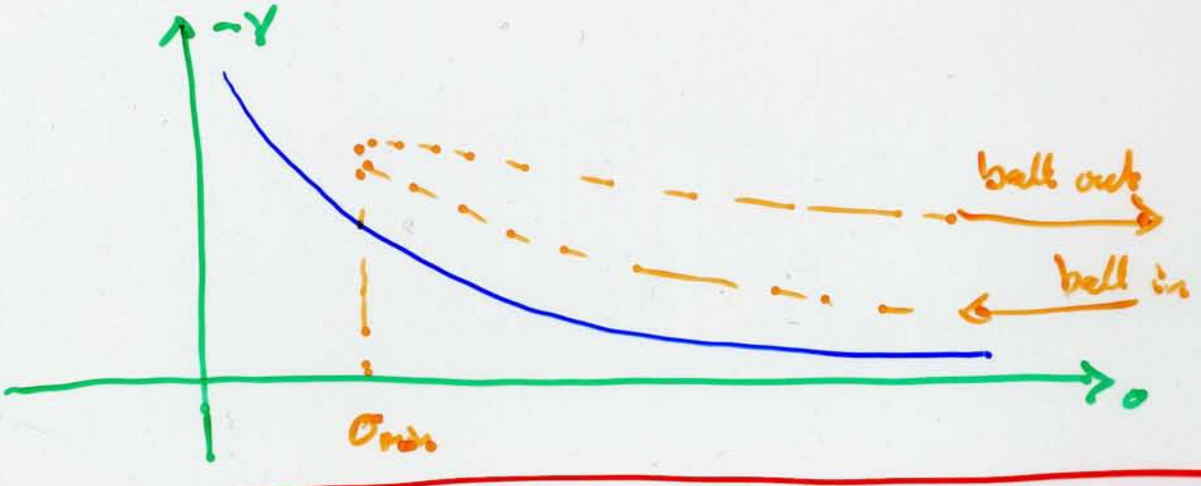
$\dot\sigma = 0$ at $z = 1$

$\therefore$ $\boxed{\sigma \geq \sigma_{min}}$

$\tilde{z} \sim \sqrt{z}$ as $z \to 0$

$\therefore$ sing. at $z = 0$ is at finite distance

Cosmological 'dual', $V \to -V$, has simple mechanical interpretation



Branched Superpotentials (Sonner & PKT)

Use $\lambda = \lambda_n$ flat wall to construct $W(\sigma)$:

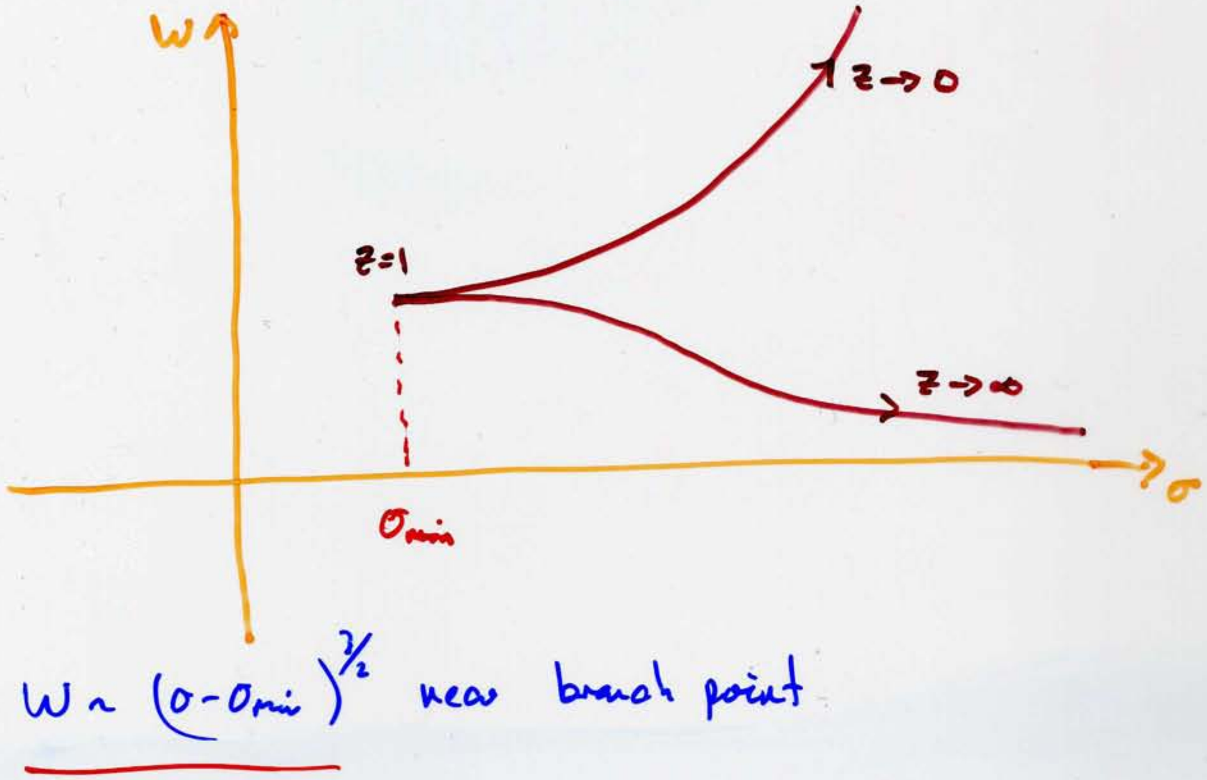
$$\boxed{W = \frac{(1+z^2)}{\lambda_n^2 z} e^{-\frac{1}{2}\lambda_n \sigma}}$$

with $z(\sigma)$ defined implicitly by $\boxed{e^{\lambda_n \sigma} = \frac{1}{z} e^{\frac{1}{2} z^2}}$

$$W' = -\frac{(1-z^2)}{2\lambda_n z} e^{-\frac{1}{2}\lambda_n \sigma} = 0 \quad \text{at } z = 1$$

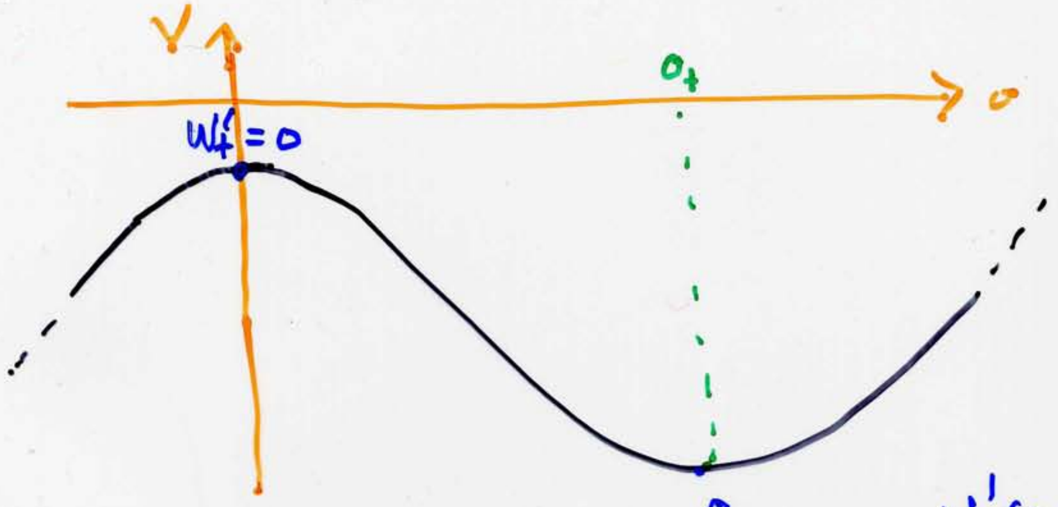
<u>but</u> $W'' - \alpha^2 W = -\frac{z}{1-z^2} e^{-\frac{1}{2}\lambda_n \sigma} \to \infty$ at $z = 1$

such that $4W'(W'' - \alpha^2 W) = \frac{2}{\lambda_n} e^{-\lambda_n \sigma} = V'$, <u>as req'd</u>



$W \sim (\sigma - \sigma_{min})^{3/2}$ near branch point

Lessons for adS superpotentials



$W_+ > W_-$ near $\sigma = 0$

assume $W_+'(\sigma_+) = 0$

(no known sugra. example!)

$$W_+' = \frac{1}{\sqrt{2}}\sqrt{V + 2\alpha W_+^2} > \frac{1}{\sqrt{2}}\sqrt{V + 2\alpha W_-^2} = W_-'$$

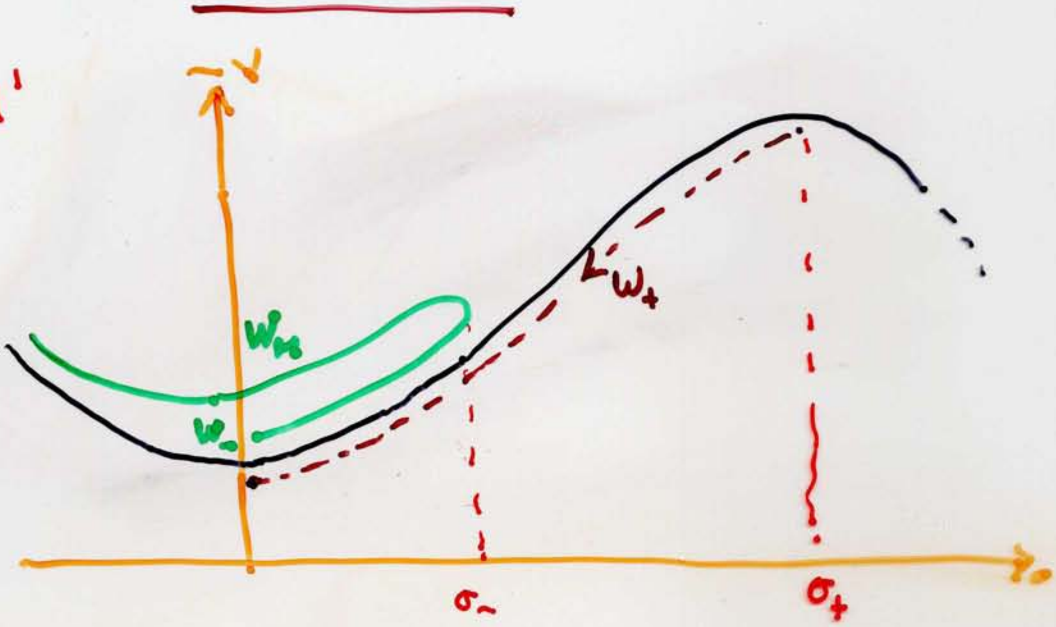
$\therefore$ $W_+ > W_-$ as long as neither has stationary pt.

(Amsel, Hertog, Hollands & Marolf)

$\therefore$ $\exists$ $\sigma_- < \sigma_+$ s.t. $W'(\sigma_-) = 0$

But $V'(\sigma_-) \neq 0$ $\Rightarrow$ W_- is branched

For cosmol. 'dual' there is simple mechanical explanation.

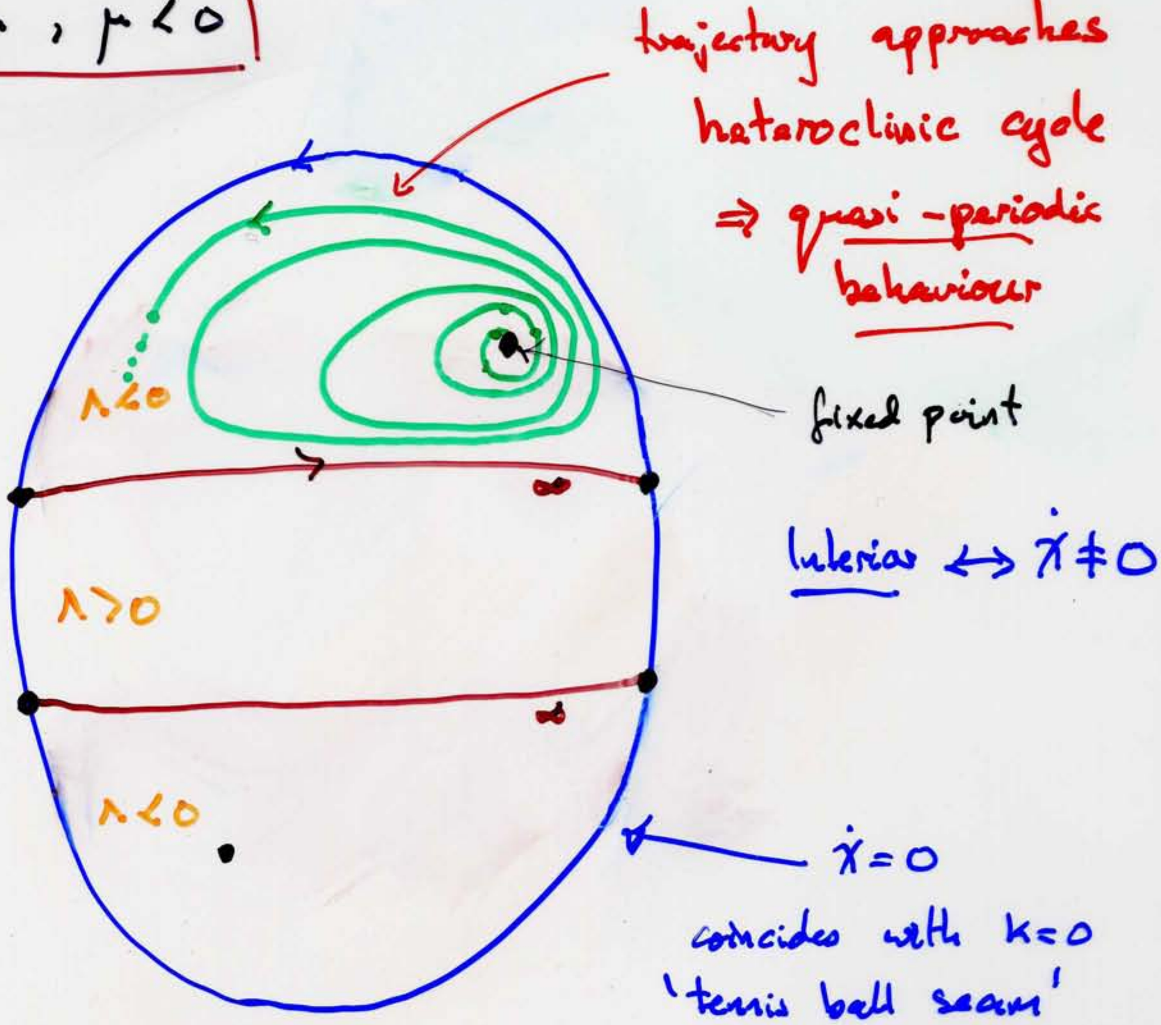


Axion-Dilaton Redux

(Sonner & PKT)

For <u>flat</u> walls can again reduce to 2-dim autonomous dyn. system. Global phase space is <u>topologically a disc</u>

e.g $\boxed{\lambda > \lambda_h \,,\ \mu < 0}$

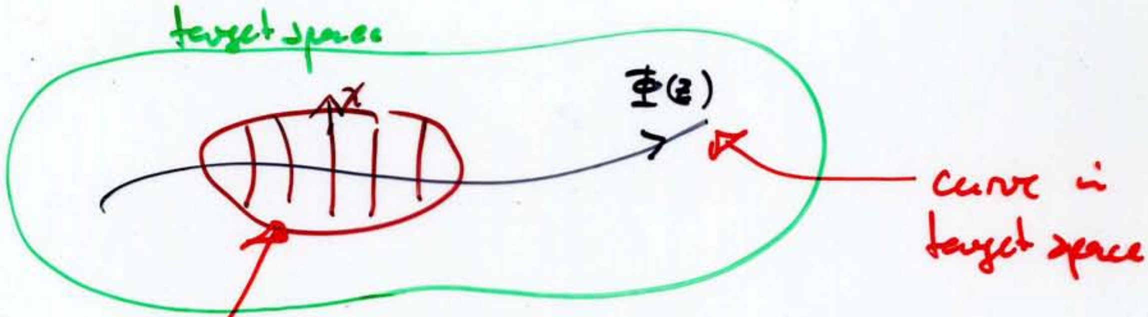


N.B (i) recurrent cosmic acceleration in cosmology 'dual' (Sonner & PKT)

(ii) Fixed pt. solution $\to$ <u>non-geodesic</u> motion on hyperbolic target space (Rosseel, Van Riet & Westra)

E pluribus unum? (Sonner & PKT)

Let $(\phi(z), \Phi(z))$ be DW/C for <u>multi-scalar</u> model



patch with "adapted" coords. (σ, χ)
s.t. $\sigma = \sigma(z)$, $\chi \equiv 0$

(Celi, Ceresole, Dall'Agata, Van Proeyen & Zagermann)

Can now apply constructions of 1-scalar model

<u>But</u> 1. Adapted coords. only local.

2. Truncation may not be consistent

Can check for 'non-geodesic' fixed pt. soln. of axion-dilaton model as soln. known <u>exactly</u>

(i) adapted truncation is <u>inconsistent</u>

(ii) soln. is <u>not</u> susy in context of fake axion-dilaton sugra ($\mu\lambda = -2$)