“Random Interfaces and Integrable Probability” GGI conference, Florence, June 22-26, 2015

Abstracts of talks

Asymptotics of representations of classical Lie groups

Alexei Bufetov *(Higher School of Economics, Moscow, and Institute for Information Transmission Problems, Moscow)*

We study the decompositions into irreducible components of tensor products and restrictions of irreducible representations of classical Lie groups as the rank of the group goes to infinity. We prove the Law of Large Numbers and the Central Limit Theorem for the random counting measures describing the decomposition. It turns out that this problem is intrinsically connected with random lozenge and domino tilings, and also with free probability.
The talk is based on a joint work with V. Gorin.

Stochastic quantum integrable systems

Ivan Corwin *(Columbia University, New York)*

We describe recent work involving interacting particle systems related to $U_q(sl_2)$ quantum integrable systems. This theory serves as an umbrella for exactly solvable models in the Kardar Parisi-Zhang universality class, as well as provides new examples of such systems, and new tools in their analysis.

Gradient Gibbs measures with disorder

Codina Cotar *(University College of London)*

We consider - in uniformly strictly convex potentials case - two versions of random gradient models. In model A) the interface feels a bulk term of random fields while in model B) the disorder enters though the potential acting on the gradients itself. It is well known that without disorder there are no Gibbs measures in infinite volume in dimension d=2, while there are gradient Gibbs measures describing an infinite-volume distribution for the increments of the field, as was shown by Funaki and Spohn. Van Enter and Kuelske proved that adding a disorder term as in model A) prohibits the existence of such gradient Gibbs measures for general interaction potentials in d=2. Cotar and Kuelske proved the existence of shift-covariant gradient Gibbs measures for model A) when d>=3 and the expectation with respect to the disorder is zero, and for model B) when d>=2.
In recent work with Kuelske, we proved uniqueness of shift-covariance gradient Gibbs measures with expected given tilt under the above assumptions. We also proved decay of covariances for both models. We will also discuss in our talk new work on non-convex potentials with disorder.

Planar maps, circle patterns, conformal point processes and 2D gravity

Francois David (*CEA-Saclay*)

I present a model of random planar triangulations (planar maps) based on circle patterns and discuss its properties. It exemplifies the relations between discrete random geometries in the plane, conformally invariant point processes and two dimensional quantum gravity (Liouville theory and topological gravity). This is joint work with Bertrand Eynard.

The Z-invariant massive Laplacian on isoradial graphs.
Béatrice de Tilière *(UPMC-Jussieu, Paris)*

After having explained the notion of Z-invariance for models of statistical mechanics, we introduce a one-parameter family (depending on the elliptic modulus *k*) of Z-invariant weights for the discrete massive Laplacian defined on isoradial graphs. We prove an explicit formula for its inverse, the massive Green function, which has the remarkable property of only depending on the local geometry of the graph. We mention consequences of this result for the model of spanning forests, in particular the proof of an order two phase transition with the critical spanning tree model on isoradial graphs introduced by Kenyon. Finally, we consider the spectral curve of this massive Laplacian, and prove that it is a Harnack curve of genus 1. This is joint work with Cédric Boutillier and Kilian Raschel.

Global fluctuations of non-colliding processes and non-intersecting paths
Maurice Duits (*Royal Institute of Technology, Stockholm)*

Ensembles of non-intersecting paths and non-colliding processes give rise to random surfaces in a natural way, by viewing the paths or trajectories as the level lines. The fluctuations of such random surfaces are expected to be universally governed by the Gaussian Free Field, which has been verified for number of models in recent years. In this talk I will discuss a new approach to establishing this universality. In particular, a general Central Limit Theorem for smooth multi-time linear statistics for determinantal point process with extended kernels will be presented.

Integrability, Topology and Discrete "Holomorphicity"
Paul Fendley *(University of Oxford)*

Integrable systems have applications ranging from experimental physics to profound mathematics. An example of the latter is the fundamental role of the Temperley-Lieb algebra of statistical mechanics in evaluating the Jones polynomial of knot and link invariants. A seemingly distinct example is discrete "holomorphicity", which gives a powerful tool both for proving conformal invariance and finding integrable Boltzmann weights. In this talk I will explain the deep relations between these two examples. In particular, I will show how utilizing topological invariants enables discretely "holomorphic" quantities to be found easily. This allows both a deeper understanding of why they occur, and a great generalization of where they occur. Applying these results to quantum spin chains yields exact zero modes, such as a Fibonacci zero mode in the hard-square/golden chain.

Height fluctuations for the stationary KPZ equation
Patrick Ferrari *(University of Bonn)*

In this talk I will discuss the height fluctuations of models in the KPZ (Kardar-Parisi-Zhang) universality class with stationary initial conditions. In particular, I will present the solution for the KPZ equation, where the stationary initial height profile is a two-sided Brownian motion. This is a joint work with Alexei Borodin, Ivan Corwin, and Bálint Vetö (arXiv: 1407.6977).

Ideal bipartite graphs and moduli spaces of flat connections on surfaces
Alexander B. Goncharov *(Yale University, New Haven)*

We consider bipartite graphs of special kind on a topological surface with boundary S which we call (rank m) ideal bipartite graphs. We show that each such graph gives rise to a cluster coordinate system on the space of flat PGL(m)-connections on S. The coordinate systems related to different graphs are related by cluster transformations.

Central Limit Theorem for discrete log-gases
Vadim Gorin *(Massachusetts Institute of Technology)*

A log-gas is an ensemble of *N* particles on the real line, for which the probability of a configuration is the power of the Vandermonde determinant times the product of a weight w(x) over the positions of particles. Such ensembles are widespread in the random matrix theory, while their discrete counterparts appear in numerous statistical mechanics models such as random tilings and last passage percolation.
I will explain a new approach which gives Central Limit Theorems for global fluctuations of discrete log-gases for a wide class of the weights w(x). The approach is based on novel discrete equations, which are analogues of the loop equations known in the continuous settings.

Logarithmic correlations in percolation and other geometrical critical phenomena
Jesper Jacobsen *(Ecole Normale Supérieure, Paris)*

The purpose of renormalisation group and quantum field theory approaches to critical phenomena is to diagonalise the dilatation operator. Its eigenvalues are the critical exponents that determine the power law decay of correlation functions. However, in many realistic situations the dilatation operator is, in fact, not diagonalisable. Examples include geometrical critical phenomena, such as percolation, in which the correlation functions describe fluctuating random interfaces. These situations are described instead by logarithmic (conformal) field theories, in which the power-law behavior of correlation functions is modified by logarithms. Such theories can be obtained as limits of ordinary quantum field theories, and the logarithms originate from a resonance phenomenon between two or more operators whose critical exponents collide in the limit. We illustrate this phenomenon on the geometrical Q-state Potts model (Fortuin-Kasteleyn random cluster model), where logarithmic correlation functions arise in any dimension. The amplitudes of the logarithmic terms are universal and can be computed exactly in two dimensions, in fine agreement with numerical checks. In passing we provide a combinatorial classification of bulk operators in the Potts model in any dimension.

Limiting shapes of Ising droplets, fingers, and corners
Pavel L. Krapivsky *(Boston University)*

In this talk I will discuss limiting shapes arising in the context of Ising model endowed with zero-temperature spin-flip dynamics. Some limiting shapes are known in two dimensions. I'll show how to determine limiting shapes of the shrinking droplet, translating finger, and `melting' corner. In the latter example, I'll also discuss fluctuations of the melted area and show how to compute the first four cumulants of the area. Finally, I'll outline attempts to guess evolution equations for limiting shapes in three and higher dimensions.

A factorisation theorem for the number of rhombus tilings of a hexagon with trianglar holes
Christian Krattenthaler *(Wien University)*

I shall present a curious factorisation theorem for the number of rhombus tilings of a hexagon with vertical and horizontal symmetry axes, with triangular holes along one axis. I shall set this theorem in relation with other factorisation theorems, and discuss some consequences and open questions. This is joint work with Mihai Ciucu.

Exact results and conjectures from the replica Bethe ansatz for KPZ growth and random directed polymers

Pierre Le Doussal *(Ecole Normale Supérieure, Paris)*

We describe the replica Bethe Ansatz (RBA) method for the continuum KPZ equation and directed polymer (DP) and recall some results for the height distributions of the main classes of initial conditions and relations to random matrices statistics. We mention more recent applications of RBA for transition classes, non-crossing paths and lattice DP models.

Stochastic quantum integrable systems in infinite volume

Leonid Petrov *(University of Virginia, Charlottesville)*

I will discuss the higher spin vertex model, which is a (discrete-time) stochastic quantum integrable system on the (half-)line. In various regimes, this model degenerates to both ASEP and q-TASEP, two well-known integrable discretizations of the Kardar-Parisi-Zhang equation. Bethe ansatz integrability provides eigenfunctions of the model, which are nice symmetric rational functions generalizing the Hall-Littlewood symmetric polynomials. A certain Markov (self-)duality plus Fourier-like transforms associated with the eigenfunctions allow to write down exact formulas for observables of the system started from an arbitrary initial data.

Are limit shape equations integrable?

Nicolai Reshetikhin *(University of California, Berkeley)*

We will argue that limit shape equations are integrable for models in statistical mechanics where tranfer-matrices form commuting families.

Simple approaches to arctic curves for Alternating Sign Matrices

Andrea Sportiello *(Université Paris 13, Villetaneuse)*

We consider Alternating Sign Matrices (ASM) with a weight w per -1 in the matrix. Many things are known from the mapping on the 6-Vertex Model with domain-wall boundary conditions (DWBC). Differently from models on periodic tori, the DWBC force extensive regions to be almost-surely "frozen", and an asymptotic boundary between the frozen and unfrozen regions, called Arctic Curve, emerges.
At w=2, we are at the free-ermion point. We have quite detailed informations on the system. In particular, the arctic curve is found to be a circle. At generic w, (and notably at w=1, the uniform case), the arctic curve has been established in a series of papers of Colomo and Pronko. For a certain F=F(z;x,y), involving the 1-boundary refined enumeration, the curve (in x-y plane) is determined by F = d/dz F = 0 (call this the "Colomo-Pronko formula"). The derivation involves both exact results from Integrable Systems, and a (non-rigorous) asymptotic analysis through a mapping to a quite complicated random-matrix model.
Here we describe a procedure, that we call "the tangent method", which provides a rederivation of the Colomo-Pronko formula, through a completely different method avoiding the détour on random matrices. Joint work with Filippo Colomo.

A class of (2+1)-dimensional growth process with explicit stationary measure

Fabio Toninelli *(Université de Lyon)*

We introduce a class of (2+1)-dimensional random growth processes, that can be seen as asymmetric random dynamics of discrete interfaces. Interface configurations correspond to height functions of dimer coverings of the infinite hexagonal or square lattice. “Asymmetric” means that the interface has an average non-zero drift. When the asymmetry parameter p − 1/2 equals zero, the infinite-volume Gibbs measures π_ρ (with given slope ρ) are stationary and reversible. When p\neq 1/2, \pi_\rho is not reversible any more but, remarkably, it is still stationary. In such stationary states, one finds that the height function at a given point x grows linearly with time t with a non-zero speed := <(hx(t) − hx(0))> = v t and that the typical fluctuations of Q_x(t) are smaller than any power of t. For the specific case p = 1 and in the case of the hexagonal lattice, the dynamics coincides with the “anisotropic KPZ growth model” studied by A. Borodin and P. L. Ferrari. For a suitably chosen initial condition (that is very different from the stationary state), they were able to determine the hydrodynamic limit and the interface fluctuations, exploiting the fact that some space-time correlations can be computed exactly, and predicted stationarity of Gibbs measures.

A Bethe Ansatz Approach to ASEP

Craig Tracy *(University of California, Davis)*

In the one-dimensional, asymmetric simple exclusion process (ASEP), particles are at integer sites on the line. Each particle waits exponential time, and then

1. with probability *p* is moves one step to the right if the site is occupied, otherwise it does not move;
2. with probability *q*=1-*p* it moves one step to the left if the site in unoccupied, otherwise it does not move.

For the special case of step initial conditions, we explain how ideas from Bethe Ansatz, coupled with some combinatorial identities and functional analysis, lead to an exact formula for $\mathbb{P}(x_m(t) \leq x)$, the probability that the *m*th particle's location is less than or equal to *x* at time *t*. This work, done in the period 2007-08, is joint work with Harold Widom. An overview of this work can be found in arXiv:1101.2682.

Asymptotic behaviors in Schur processes

Mirjana Vuletic *(University of Massachusetts Boston)*

We present some asymptotic results, like the limit shape and fluctuation behavior at the edge, of models falling in the class of Schur processes. These include pyramid partitions introduced by Kenyon, Szendroi and Young, Aztec diamonds with non-uniform measures and steep tilings recently introduced by Bouttier, Chapuy and Corteel. This is a joint work with D. Betea, and C. Boutillier. We will also discuss results for models falling in the class of symmetric Schur processes. Various limit shape results, confirming our analytic results, were obtained experimentally using a perfect sampling algorithm for Schur processes. The algorithm which is a joint work D. Betea, C. Boutillier, J. Bouttier, G. Chapuy and S. Corteel will also be discussed.

Random Matrices, Growth and Hydrodynamics Singularities

Paul Wiegmann *(University of Chicago)*

I review the approach to Hele-Shaw problem, stochastic aggregation and hydrodynamics singularities through models of random matrices.

Posters

Spontaneous Breaking of U(N) Symmetry in Invariant Matrix Models
Fabio Franchini *(INFN, Sezione di Firenze)*

We introduce the study of the eigenvectors of a random matrix. Traditionally, the requirement of base invariance has lead to the conclusion that invariant models describe extended systems. We show that deviations of the eigenvalue statistics from the Wigner-Dyson universality reflects itself on the eigenvector distribution.
In particular, gaps in the eigenvalue density spontaneously break the U(N) symmetry to a smaller one. Models with log-normal weight, such as those emerging in Chern-Simons and ABJM theories, break the U(N) in a critical way, resulting into a multi-fractal eigenvector statistics.
These results pave the way to the exploration of localization problems using random matrices via the study of new classes of observables and potentially to novel, interdisciplinary, applications of matrix models.
- F. Franchini; “On the Spontaneous Breaking of U(N) symmetry in invariant Matrix Models”; arXiv:1412.6523.
- F. Franchini; “Toward an invariant matrix model for the Anderson Transition”; arXiv:1503.03341.

On the Integrability of B-type KdV Equations
Jianqin Mei *(Dalian University of Technology)*

The Lax integrability, bilinear integrability for B-type KdV equation have been explored. The tau function, Backlund transformation, N-soliton solutions and Riemann-theta function solutions have been constructed.

Asymmetric six-vertex model and stochastic processes
Daria Rudneva *(Higher School of Economics, Moscow)*

A six-vertex model is a widely used model in solid state physics. It is also famous for its integrability properties. Starting from symmetric 6-vertex model, we will show how to get asymmetric six-vertex model in external electrical fields and prove its integrability. Connections between other models such as XXZ spin chain and ASEP will be shown in pictures and formulae, parameters required for integrability will be found.

Off-critical interfaces and phase separation in 2D. Exact results from field theory
Alessio Squarcini *(SISSA, Trieste)*

We consider phase separation and interfaces of systems of classical statistical mechanics in two dimensions below criticality. While interfaces in 2D at criticality are described by SLE, exact results for the scaling limit of the order parameter profile in the off-critical regime have been obtained only by means of exact lattice computations for the 2D Ising model; a circumstance that raises the question about the role of Ising solvability. We will show how low-energy properties of two-dimensional field theory yield exact results for order parameter profiles, passage probabilities and interface structure in presence of single and double interfaces for different universality classes in 2D. Results available from the lattice solution of the Ising model in the plane and in the half-plane are recovered as a particular case. This is joint work with G. Delfino.

Fused RSOS as Higher Level Non-Unitary Minimal Cosets
Elena Tartaglia *(The University of Melbourne)*

It is well known that the continuum scaling limit of the critical Forrester-Baxter Restricted-Solid-on-Solid (RSOS) model, with crossing parameter $\lambda = (m' - m)\pi/m'$, is described by the non-unitary minimal model $M(m,m')$. More generally, we argue that at criticality the $n \times n$ fused RSOS models with n¿1 relate to the higher-level non-unitary coset minimal models. Specifically, starting with the off-critical elliptic models for n=1,2,3, we calculate the Corner Transfer Matrix 1-dimensional configurational sums to obtain finitised fermionic branching functions and infer a general formula for the central charges $c{=}c^{m,m';n}$. In this way we identify the continuum scaling limit of $n \times n$ fused RSOS models with known level-n non-unitary coset theories. (This is joint work with Paul A. Pearce)

Bayesian shrinkage approach in variable selection for mixed effects models.
Mingan Yang *(Central Michigan University, Mount Pleasant)*

Recently, many shrinkage priors have been proposed and studied in linear models to address massive regression problems. However, shrinkage priors are rarely used in mixed effects models. In this article, we address the problem of joint selection of both fixed and random effects with the use of several shrinkage priors in linear mixed models. The idea is to shrink small coefficients to zero while {keeping} larger coefficients through the use of heavy-tailed shrinkage priors. The shrinkage priors can be obtained via a scale mixture of normal distributions to facilitate computation. We use a stochastic search Gibbs sampler to implement a fully Bayesian approach for variable selection. The approach is illustrated using simulated data and a real example.