GGI Workshop on
‘Randomness, Integrability and Probability’

Program of talks and events[1]

Week 1
April 19-22, 2022

Tuesday April 19

9.00-11.00, Secretariat
Registration

11.15-11.30
Welcome

11.30-12.30
Integrable domain walls in N=4 SYM and ABJM theory
Charlotte Kristjansen *(Niels Bohr Institute, Copenhagen)*

We explain how certain domain walls in respectively N=4 SYM and ABJM theory can be described as integrable boundary states of an underlying super spin chain. Field theoretical one-point functions are expressed as overlaps between Bethe eigenstates and these boundary states which take the form of matrix product or valence bond states, and thanks to integrability closed overlap formulas are found. Furthermore, we discuss how certain ‘microscopic duality relations’, based on the QQ-system of the spin chains, can be used to predict novel overlap formulas.

14.30-15.30
Shuffle algebras and integrable lattice paths
Paul Zinn-Justin *(University of Melbourne)*

We develop a new point of view on shuffle algebras based on solvable lattice models. We establish in this way a nontrivial isomorphism between the center of the Hecke algebra and the shuffle algebra related to toroidal gl(1). As an application we find new formulas for certain partition functions for lattice paths.
This is joint work with A. Garbali.

15.45-16.45
From Gumbel to Tracy–Widom via random (ordinary, plane, and cylindric plane) partitions
Dan Betea *(Université d’Angers)*

We present a few natural measures on partitions, plane partitions, and cylindric plane partitions. We show how extremal statistics of such measures (laws of the largest parts) interpolate—via a ’natural’ finite temperature parameter—between the Gumbel distribution of classical statistics of iid random variables and the Tracy–Widom GUE distribution of correlated systems (eigenvalues) from random matrix theory (RMT). Somewhat strikingly, we also obtain RMT hard-edge behavior (Bessel kernel and distribution) in some cases. Connections to models of directed last passage percolation are discussed throughout, as are connections to the statistical (and quantum) mechanics of (mostly free) fermions at finite temperature. The results are based on joint works with Jérémie Bouttier and Alessandra Occelli.

[1] If not indicated otherwise, events take place in the main Lecture Hall, Aula A.

Wednesday April 20

10.30-11.30

Convergence of the form factor series in the Sinh-Gordon quantum field theory in 1+1 dimensions

Karol Kozlowski *(ENS, Lyon)*

Within the approach of the bootstrap program, the physically pertinent observables in a massive integrable quantum field theory in 1+1 dimensions are expressed by means of the so-called form factor series expansion. This corresponds to a series of multiple integrals in which the n^{th} summand is given by a n-fold integral. While being formally effective for various physical applications, so far, the question of convergence of such form factor series expansions was essentially left open. Still, convergence results are necessary so as to reach the mathematical well-definiteness of such construction and appear as necessary ingredients for the justification of numerous handlings that are carried out on such series.
In this talk, I will describe the technique I recently developed that allows one to prove the convergence of the form factor series that arise in the context of the simplest massive integrable quantum field theory in 1+1 dimensions: the Sinh-Gordon model. The proof amounts to obtaining a sufficiently sharp estimate on the leading large-n behaviour of the n-fold integral arising in this context. This appeared possible by refining some of the techniques that were fruitful in the analysis of the large-n behaviour of integrals over the spectrum of $n \times n$ random Hermitian matrices.

11.45-12.45

Generalized Gibbs ensemble of the Ablowitz-Ladik lattice, and the circular β-ensemble

Guido Mazzuca *(KTE, Stockholm)*

In the theory of generalized hydrodynamics, one of the main object of study is the density of states for the system at hand. In this talk, I consider the Ablowitz-Ladik lattice, which is an integrable system. Specifically, I introduce the Generalized Gibbs ensemble for this lattice, and I relate it with a random matrix model, the Circular β-ensemble. This allows me to compute explicitly the density of states for the Ablowitz-Ladik lattice in terms of the one of this random matrix ensemble. This talk is mainly based on my recent paper with Tamara Grava, 2107.02303, and my other one with Ronan Memin 2201.03429.

14.30-15.30

Free fermions and parafermions

Paul Fendley *(University of Oxford)*

Free fermions are ubiquitous in theoretical physics. Typically such models are found by expressing the Hamiltonian and/or action as a sum or integral over bilinears of local fermionic operators or fields, sometimes requiring a Jordan-Wigner transformation. I describe models that become free fermionic only after under a much subtler transformation that is both non-local and non-linear in the original interacting fermions. This transformation works only for open boundary conditions; for periodic the model remains integrable, but applying traditional techniques there is rather difficult. I will also give explain how free-parafermion chains can be solved in a similar fashion.

15.45-16.45

Limit shape phase transitions

Alexander Abanov *(SUNY at Stony Brook)*

A limit shape phenomenon in statistical mechanics is the appearance of a most probable macroscopic state. This state is usually characterized by a well-defined boundary separating frozen and liquid spatial regions. We will start with a review of a few examples of the appearance of limit shapes. Then we consider a particular class of topological phase transitions in the limit shape problem of statistical mechanics. The problem considered is generally known as the Arctic circle problem. One can visualize the considered transition as merging two melted regions (Arctic circles). We establish the mapping, which identifies the transition as the

Gross-Witten-Wadia transition known in lattice QCD and random matrix problems. It is a continuous phase transition of the third order. We identify universal features of the limiting shape close to the transition using the free fermion and hydrodynamic description. The talk is based on: `https://arxiv.org/abs/2203.05269`

17.30-18.30, Cloister
Wine & Cheese

Thursday April 21

10.30-11.30
On maps with tight boundaries
Jérémie Bouttier *(CEA, Saclay)*

Maps, in the combinatorial sense, are discrete surfaces made of polygons glued together. Over the last 20 years, very precise results on the geometric properties of random maps have been obtained. However, most of the focus has been so far on the spherical (planar) case. Maps of other topologies (higher genus/more boundaries) are well-understood on the enumerative side, thanks to advanced techniques such as topological recursion, but it is unclear how to extend this understanding to geometrical aspects. I will report on an ongoing project with E. Guitter and G. Miermont where we explore this question. Based on arXiv:2104.10084, arXiv:2203.14796 and work in progress.

11.45-12.45
Correlation functions for the doubly periodic Aztec diamond
Maurice Duits *(KTE, Stockholm)*

The purpose of this talk is to report on recent progress on random tilings with doubly periodic weights. With the Aztec diamond as the running example, we will see how the correlation functions can be expressed in terms of matrix-valued orthogonal polynomials or a Wiener-Hopf factorization for the symbols of the transition matrices. These expressions have proved to be promising starting points for asymptotic studies in special cases. For the biased two 2x2 periodic Aztec diamond, finding a Wiener-Hopf factorization amounts to following a linear flow on an elliptic curve, which will be discussed in more detail if time permits.

14.30-15.30
GOE Fluctuations for the maximum of the top path in ASMs
Sunil Chhita *(University of Durham)*

The six-vertex model is an important toy-model in statistical mechanics for two-dimensional ice with a natural parameter Δ. When $\Delta = 0$, the so-called free-fermion point, the model is in natural correspondence with domino tilings of the Aztec diamond. Although this model is integrable for all Δ, there has been very little progress in understanding its statistics in the scaling limit for other values. In this talk, we focus on the six-vertex model with domain wall boundary conditions at $\Delta = 1/2$, where it corresponds to alternating sign matrices (ASMs). We consider the level lines in a height function representation of ASMs. We report that the maximum of the topmost level line for a uniformly random ASMs has the GOE Tracy-Widom distribution after appropriate rescaling. This talk is based on joint work with Arvind Ayyer and Kurt Johansson.

15.45-16.45
Hexagons, QSC and the Bottom bridge
Alessandro Georgoudis *(Nordita, Stockhom)*

In this talk I will present recent developments in the Hexagon formalism for structure constants. I will briefly present integrability in N=4, with all the necessary tools to solve the spectrum problem. I will then introduce the Hexagon formalism in its current picture and finish by presenting our new results to account for wrapping effects.

Friday April 22

10.30-11.30
Determinantal point processes: quasi-symmetries and interpolation
Alexander Bufetov *(CNRS, Marseille)*

For the sine-process, it is proved that almost every realization with one particle removed is a complete and minimal set for the Paley-Wiener space, whereas if two particles are removed, then one obtains a zero set for the Paley-Wiener space. In joint work with Qiu and Shamov, it is proved that almost every realization of a determinantal point process is a uniqueness set for the underlying Hilbert space. Quasi-invariance of the sine-process under compactly supported diffeomorphisms of the line plays a key role.
In joint work with Qiu, the Patterson-Sullivan construction is used to interpolate Bergman functions from a realization of the determinantal point process with the Bergman kernel, in other words, by the Peres-Virág theorem, the zero set of a random series with independent complex Gaussian entries. The invariance of the zero set under the isometries of the Lobachevsky plane plays a key role.

11.45-12.45
Correlation functions of open integrable spin chains with unparallel boundary fields
Véronique Terras *(Université Paris-Saclay, Orsay)*

We consider open integrable quantum chains of spin 1/2. For unparallel boundary magnetic fields, the complete spectrum and eigenstates can be constructed by means of the quantum Separation of Variables, and the scalar products of separate states can be computed as determinants. We more particularly consider the case of a fixed specific value of the boundary field in the last site N of the chain along the z-direction, with any arbitrary boundary magnetic field in the first site of the chain. In this case, the spectrum can be formulated in terms of a homogeneous T-Q equation, and we can compute the action of a basis of local operators on transfer matrix eigenstates as linear combinations of separate states. This enables us to derive the correlation functions of a set of local operators both for the finite and half-infinite chains, with multiple integral formulae in this last case.

14.30-15.30
Q-operators for Open Quantum Integrable Systems
Robert Weston *(Heriot-Watt University, Edinburgh)*

Baxter's Q-operator was introduced in order to obtain Bethe Equations for quantum integrable systems in the absence of a Bethe ansatz. The Q-operator is constructed as the trace of a double-row monodromy matrix over an infinite-dimensional auxiliary space. There are then two alternative routes to the key 'TQ' relations which in turn give the Bethe Equations: short exact sequences and monodromy-matrix factorization. Both routes are well-trodden in the closed case, but far less so in the open case, where the boundary reflection matrices play a key role.
In this talk I will describe, in the most accessible way I can, the construction and properties of the Q-operator for open integrable quantum spin chain. The central ideas are fairly simple and will be explained mostly by using pictures. The presentation will be aimed at a broad mathematics and physics audience.

Week 2
April 26-29, 2022

Tuesday April 26

10.30-11.15
Permutations, moments, measures
Natasha Blitvic *(Lancaster University)*

We will look at a number of interesting examples (some proven, others merely conjectured) of moment sequences in combinatorics. We will consider ways in which this positivity may be expected (or surprising!), the methods of proving it, and the consequences of having it. Topics covered in this talk will range from noncommutative probability to certain hard open problems in combinatrics, but no specialist background will be assumed. Based on recent and ongoing work with Einar Steingrímsson.

11.45-12.30
The rough-smooth boundary in dimer models
Kurt Johansson *(KTH, Stockholm)*

In some two-dimensional dimer or random tiling models we can have three co-existing phases, frozen, rough and smooth. A basic example is the two-periodic Aztec diamond. In this model there is an interface between a rough and a smooth phase which is not completely clear how to define and prove properties about. For example do we get the Airy process as a scaling limit in analogy with a frozen-rough interface? In this talk I will discuss the behavior of the model at this boundary and what we know about it.

14.30-15.15
Algebraic approach to stochastic duality for Markov processes
Chiara Franceschini *(Università di Modena e Reggio Emilia)*

In this talk I will overview the concept of duality for Markov processes and, in particular, some special classes of interacting particle systems. I will explain how such relations arise naturally from an algebraic description of the models and provide some classical examples as well as more recent results. I conclude with some applications in the context of scaling limit for interacting particle systems.

Wednesday April 27

10.30-11.15
Two-dimensional massive integrable models on a torus
Ivan Kostov *(CEA, Saclay)*

The finite-volume thermodynamics of a massive integrable QFT is described in terms of a grand canonical ensemble of loops immersed in a torus and interacting through scattering factors associated with their intersections. The path integral of the loops is evaluated explicitly after decoupling the pairwise interactions by a Hubbard-Stratonovich transformation. The HS fields are holomorphic fields depending on the rapidity and can be expanded in elementary oscillators. The torus partition function is expressed as certain expectation value in the Fock space of these oscillators. In the limit where one of the periods of the torus becomes asymptotically large, the effective field theory becomes mean field type. The mean field describes the infinite-volume thermodynamics which is solved by the Thermodynamical Bethe Ansatz.

11.45-12.30
Exactly solvable anharmonic oscillator, degenerate orthogonal polynomials and Painlevé II
Tamara Grava *(SISSA, Trieste)*

The paper addresses a conjecture of Shapiro and Tater on the similarity between two sets of points in the complex plane; on one side is the values of $t \in \mathbb{C}$ for which the spectrum of the quartic anharmonic oscillator, with potential $V(x,t)$ and certain boundary conditions, has repeated eigenvalues. On the other side is the set of zeroes of the Vorob'ev-Yablonskii polynomials, i.e. the poles of rational solutions of the second Painleve equation. Along the way, we indicate a surprising connection between the anharmonic oscillator problem and certain degenerate orthogonal polynomials. This is a joint work with Marco Bertola and Eduardo Chavez Heredia.

14.30-15.15
Dimers on minimal graphs and maximal Riemann surfaces
Cédric Boutillier *(LPSM, Sorbonne Université, Paris)*

A celebrated result on dimer models is the spectral theorem by Kenyon and Okounkov giving a bijection between dimer models on periodic graph modulo some transformations and the space of Harnack curves with a given Newton polygon with a standard divisor (a point on each oval of the curve). Fock then provided an explicit construction of the inverse spectral map: given an algebraic curve C (not necessarily Harnack) and an appropriate divisor, he defines a periodic minimal graph with a Kasteleyn operator (not necessarily having a combinatorical interpretation) for which C is the spectral curve. In a joint work with David Cimasoni and Béatrice de Tilière, we study the dimer model on arbitrary infinite minimal planar graphs, with Fock's weights, constructed from a fixed compact Riemann surface. The corresponding Kasteleyn operator has a whole family of inverses with an explicit integral representation, with a certain locality property. We define the notion of divisor for a vertex of the graph. We then explain how in the periodic case, we can obtain a parametrization of the spectral curve, compute the phase diagram, the slope and the free energy directly from the Riemann surface. These results can be seen as a generalization of Kenyon's results about isoradial dimers with (genus 0, trigonometric) critical weights to a larger family of graphs, in arbitrary genus.

Thursday April 28

10.30-11.15
Contractions of Integrability Algebras and R-matrices
Niklas Beisert *(ETH, Zurich)*

A large class of R-matrices satisfying the (classical/quantum) YBE is provided by the established tools of classical and quantum algebra (quasi-triangular Lie bialgebras, quantum affine algebras, ...). In particular, R-matrices of difference form based on semi-simple Lie algebras (as well as twists thereof) are well understood. However, there are also prominent examples of R-matrices which do not belong to this class.
In this talk I propose to apply algebraic contraction to semi-simple Lie algebras in order to construct interesting new algebras and R-matrices. We consider the simple example of the contraction of SO(4) to ISO(3). When applied to the quantum algebra structures of Uq(SO(4)) one can obtain (an extension of) the kappa-deformed Poincare algebra including an explicit expression for its R-matrix.
This example can be generalised to explain the algebraic origin of Shastry's R-matrix for the one-dimensional Hubbard model which is also encountered as the worldsheet scattering matrix in the AdS/CFT context. This involves promotion to an affine algebra, adding supersymmetry as well as performing a curious reduction of the non-semi-simple structure of the affine algebra.

11.45-12.10
Conformal bootstrap 2d percolation and logarithmic CFTs
Yifei He *(ENS, Paris)*

A signature example of random geometrical models is the critical percolation, and its fundamental observables involve cluster connectivities which can be related to the correlation functions of the order parameter in the Potts CFT. In this talk I will describe using the conformal bootstrap approach to solve the four-point cluster connectivities. The results further allow analyzing the percolation and polymers logarithmic CFTs with central charge c=0.

12.15-12.40
The open $U_q(\mathfrak{sl}(2))$-invariant staggered six-vertex model
Sascha Gehrmann *(Leibniz Universität Hannover)*

The finite-size spectrum of the critical alternating $\mathbb{Z}_2$-staggered spin-1/2 XXZ model with quantum group invariant boundary conditions is presented. For all values of the staggering parameter the continuum limit has been found to be described in terms of the non-compact SU(2, R)/U(1) Euclidean black hole conformal field theory (CFT) whose scaling dimensions include a continuous component. In addition, we find that levels from the discrete part of the spectrum of this CFT emerge as the anisotropy is varied. The finite size amplitudes of both the continuous and the discrete levels are related to the corresponding eigenvalues of a quasi-momentum operator which commutes with the Hamiltonian and the transfer matrix of the model.

14.30-15.15
Non-compact spin chains and stochastic particle processes
Rouven Frassek *(Università di Modena e Reggio Emilia)*

I will discuss the relation between non-compact spin chains studied in high energy physics and the zero-range processes introduced by Sasamoto-Wadati, Povolotsky and Barraquand-Corwin. The main difference compared to the standard SSEP and ASEP is that in these models several particles can occupy one and the same site. For the models with symmetric hopping rates I will introduce integrable boundary conditions that are obtained from new solution to the boundary Yang-Baxter equation (K-matrix). An explicit mapping of the open SSEP (and the non-compact model cousin) to equilibrium is presented. It allows to obtain closed-form solutions of the probabilities in steady state and of k-point correlations functions.

15.30-15.55
Large deviations for Gibbs ensembles of the classical Toda chain
Ronan Memin *(ENS, Lyon)*

I will explain how we derive the convergence of the empirical measure of the Lax matrix of the Toda chain by a comparison to the beta ensembles of random matrices, using large deviation theory. Based on a joint work with Alice Guionnet.

Friday April 29

10.30-11.15
Time covariance for last passage percolation in half-space
Alessandra Occelli *(ENS, Lyon)*

We consider a last passage percolation model in half space with exponential weights. We show that,when the two are at small macroscopic distance, then the first order correction to the two-time covariance for the point-to-point model is the same as the one of the stationary model. In order to obtain the result, we first derive comparison inequalities of the last passage increments for different models. This is used to prove tightness of the point-to-point process as well as localization of the geodesics. Unlike for the full-space case, for half-space we have to overcome the difficulty that the point-to-point model in half-space with generic start and end points is not known. Based on joint work with Patrik Ferrari.

<u>11.45-12.30</u>
Thermodynamic limits of fishnet graphs with various boundary conditions
Benjamin Basso *(ENS, Paris)*

The fishnet theory is a QFT for matrix scalar fields interacting with a quartic coupling, which is conformally invariant and integrable in the 't Hooft planar limit, despite being non supersymmetric. Besides being a 'baby version' of the maximally supersymmetric Yang-Mills theory in 4 dimensions, to which it relates by deformation, it is also a theory for the so-called fishnet graphs, that are Feynman diagrams with regular square lattice structures, which connect to integrable conformal spin chains. In this talk I will give examples of observables (scaling dimensions and correlation functions) that can be calculated precisely in this theory using integrability and discuss their behaviours in the thermodynamic limits corresponding to large fishnet graphs. These limits reveal interesting connection to 2d sigma models in AdS as well as a strong dependence on the graph boundary conditions, sharing similarities with solvable lattice models subject to the limit shape phenomenon.

<u>14.30-15.15</u>
Limiting current distribution from first principles for a two-species stochastic process
Jan de Gier *(The University of Melbourne)*

I will discuss the rigorous derivation of a Tracy-Widom and a Gaussian distribution in a scaling regime of a two-species integrable stochastic process.

Week 3
May 2-6, 2022

Monday May 2

10.30-11.15
Long range spin chains and freezing
Didina Serban *(CEA, Saclay)*

Long range integrable spin chains are important objects of interest both from a mathematical point of view and for physical applications. The full mathematical structure is understood only for a few particular examples, and the tools used are outside the usual algebraic Bethe Ansatz framework. I will present a method to obtain the wave function for the Haldane-Shastry spin chain by freezing the dynamical degrees of freedom of the Calogero-Sutherland model with spin. This method can be generalised to solve various deformations of the Haldane-Shastry Hamiltonian.

11.45-12.30
Matrix valued orthogonality and random tilings
Arno Kuijlaars *(KU Leuven)*

Matrix valued orthogonal polynomials play a role in random tiling models with periodic weightings. The talk will be focused on lozenge tilings of a hexagon, and it will be shown that the matrix valued orthogonality can be related to orthogonality for meromorphic functions on a Riemann surface. The higher genus cases are of particular interest since these are believed to correspond to random tiling models with three different phases in the large size limit.

14.30-15.15
Symmetry Resolved Entanglement in Integrable Quantum Field Theory
Olalla Castro Alvaredo *(City, University of London)*

In this talk I will review some recent results relating to a measure of entanglement known as symmetry resolved entropy (SRE). This is a measure that can be defined for theories that possess an internal symmetry and which quantifies the amount of entanglement that is contributed by each symmetry sector. In the context of integrable quantum field theory, the SRE can be computed using correlation functions of composite twist fields, extending the standard programme for entanglement measures. In my talk I will give a summary of some results I have contributed to in this direction, which deal with different models and/or states.

Tuesday May 3

10.30-11.15
Introduction to Generalized Hydrodynamics in the Lieb-Liniger gas
Jérôme Dubail *(CNRS Nancy)*

I will give a brief introduction to 'Generalized Hydrodynamics', a hydrodynamic description of one-dimensional integrable systems discovered in 2016 [1,2]. I will describe the theory in the context of the one-dimensional Bose gas, where it is particularly simple. I will briefly review how Generalized Hydrodynamics is successfully used to describe modern cold atoms experiments [3,4]. If time permits, I will also discuss our recent attempt at quantizing Generalized Hydrodynamics, which results in a generalized Luttinger liquid [5].

[1] 1 O. Castro-Alvared, B. Doyon, T. Yoshimura, *Emergent hydrodynamics in integrable quantum systems out of equilibrium*, Phys. Rev. X 6, 041065 (2016).
[2] B. Bertini, M. Collura, J. De Nardis, M. Fagotti, *Transport in Out-of-Equilibrium XXZ Chains: Exact Profiles of Charges and Currents*, Phys. Rev. Lett. 117, 207201 (2016).
[3] M. Schemmer, I. Bouchoule, B. Doyon, J. Dubail, *Generalized Hydrodynamics on an Atom Chip*, Phys.

Rev. Lett. 122, 090601 (2019).
[4] N. Malvania, Y. Zhang, Y. Le, J. Dubail, M. Rigol, D. Weiss, *Quantum Generalized Hydrodynamics*, arXiv:1910.00570.
[5] P. Ruggiero, P. Calabrese, B. Doyon, J. Dubail, *Quantum Generalized Hydrodynamics*, Phys. Rev. Lett. 124, 140603, 2020.

11.45-12.30
How to see the KPZ fluctuations (and Tracy-Widom?) in the Ising model
Semen Shlosman *(CPT Luminy, Marseille)*

I will explain where to look in the Ising model in order to see these nice features.

14.30-15.15
Many new conjectures on Fully-Packed Loop configurations
Andrea Sportiello *(CNRS, LIPN Université Paris 13)*

The Razumov–Stroganov conjecture revolves around Fully-Packed Loop configurations (FPL) and the steady state of the Dense O(1) Loop Model (O(1)DLM). In short, the enumeration of FPL's refined according to the (black) link pattern is proportional to the aforementioned steady state. It exists in two main flavours: 'dihedral' (ASM, HTASM, QTASM,... vs. the DLM on the cylinder), and 'vertical' (VSASM, UASM, UUASM, OSASM, OOASM,... vs. the DLM on the strip). Together with L. Cantini, we gave two proofs (in 2010 and 2012) of the conjecture in the dihedral cases, but, despite the efforts of ourselves and others, the vertical case is still unsolved.
We recently looked back at the FPL configurations pertinent to one of the unsolved cases, namely the UASM (ASM on a 2n x n rectangle with U-turn boundary conditions on one long side), and we had the idea of looking at the refinement according to the black and white link patterns, and the overall number of loops. This doesn't seem to help in understanding the Razumov–Stroganov conjecture, but leads to many more conjectures, suggesting the existence of a remarkable deformation of the Littlewood–Richardson coefficients, somewhat in the same spirit, but apparently by a completely different mechanism, to 'FPL in a triangle' studied by P. Zinn-Justin, and by Ph. Nadeau.

17.30-18.30, Cloister
Wine & Cheese

Wednesday May 4

10.30-11.15
The multispecies totally asymmetric long-range exclusion process and Macdonald polynomials
Arvind Ayyer *(Indian Institute of Science, Bangalore)*

The multispecies totally asymmetric long-range exclusion process (mTALREP) is an interacting particle system with multiple species of particles on a finite ring where the hopping rates are site-dependent. (The homogeneous variant on Z is also known as the Hammersley–Aldous–Diaconis process.) In its simplest variant with a single species, a particle at a given site will hop to the first available site clockwise. We show that the partition function of this process is intimately related to the classical Macdonald polynomial. We also show that well-known families of symmetric polynomials appear as expectations in the stationary distribution of important observables.

11.45-12.30
Thermal form factor expansions for the correlation functions of the XXZ chain
Frank Göhmann *(Bergische Universität Wuppertal)*

The thermodynamic properties and finite temperature correlation functions of 1d quantum chains in the thermodynamic limit can be calculated within the quantum transfer matrix formalism. For the dynamical two-point functions of the Heisenberg-Ising chain we have obtained so-called thermal form factor series that are manifestly different from the form factor expansions with respect to the Hamiltonian basis. These novel series allow us to reconsider a number of longstanding problems. Examples considered so far include the high-T asymptotic analysis of the transverse two-point functions of the XX chain as well as the two-point functions of spin-zero operators of the Heisenberg-Ising chain in the massive antiferromagnetic regime at zero temperature. For the latter we have obtained fully explicit and numerically highly efficient series representations for the longitudinal two-point functions and for the correlation functions of two magnetic current densities that determine the optical conductivity.

14.30-14.55

Transition Probabilities in the Multi-species Asymmetric Exclusion Process

William Mead *(The University of Melbourne)*

The asymmetric simple exclusion process (ASEP) is a well-studied stochastic model of indistinguishable particles which lies within KPZ universality class. However, the multi-species version of the ASEP has yielded very few mathematically rigorous results. In this talk we describe a method for producing transition probabilities for the multi-species version of the ASEP. This is performed via a reduction from a family of integrable stochastic vertex models. From here we obtain crossing probabilities which may be amenable to observing KPZ-like limiting behaviour. Based on joint work with Jan de Gier and Michael Wheeler.

Thursday May 5

10.30-11.15

Correlation functions in many-body systems: Euler hydrodynamics, macroscopic fluctuation theory, and long-range correlations.

Benjamin Doyon *(King's College, London)*

Calculating correlation functions in statistical ensembles of many-body quantum and classical systems is one of the hardest problems in theoretical physics. At large separations in space and / or time, hydrodynamics, which is the theory for the emergent long-wavelength dynamics, gives a lot of information, such as exact asymptotic behaviours. I will present a number of universal results based on, or related to, the "Euler hydrodynamics" of the system. This will include how at equilibrium observables project onto hydrodynamic modes which propagate with the linearised Euler equation (this can even be shown rigorously), and how, in general, large-scale correlations are related to large-deviation theories of fluctuations. I will introduce a ballistic version of the "macroscopic fluctuation theory", which in particular predict that out of equilibrium, long-range correlations in space, apparently not observed until now, generically appear in interacting systems. Without technical details, I will illustrate these concepts in simple examples, such as the classical one-dimensional model of hard rods.
The ballistic macroscopic fluctuation theory will be further explained, and applied to integrable models, in a following talk by Takato Yoshimura.
The present talk is based on works with D. Ampelogiannis, and with G. Perfetto, T. Sasamoto and T. Yoshimura.

11.45-12.30

Asymptotic behavior of transport maps related to the random forced Burgers equation and KPZ problem

Konstantin Khanin *(University of Toronto)*

We'll consider large-scale point fields which naturally appear in the context of the KPZ problem. Such point fields are geometrical objects formed by points of mass concentration, and by shocks separating the sources of these points. We'll also introduce similarly defined point fields for the process of coalescing fractional Brownian motions (cfBM). The main aim is to present theoretical arguments and numerical evidence in support of the conjecture that statistics of these points fields have the same large-time limit. This would indicate that two objects may, in fact, belong to the same universality class.

<u>14.30-15.15</u>
Limit shapes in quantum integrable spin chains
Jean-Marie Stéphan *(Université Claude Bernard, Lyon)*

Limit shapes and arctic curves are known to occur in various models in statistical mechanics, for example dimer models or vertex models. In this talk, I will discuss how those appear in quantum spin chains or quantum fermionic models initialized in a domain wall state. The main example will be the XXZ spin chain, for which exact results may be obtained using Integrability techniques. This is done by taking a non trivial limit in the six vertex model with domain wall boundary conditions, and working out this limit in terms of orthogonal polynomials. If time permits I will also discuss simpler (free) quantum models where positivity is not guaranteed, and the probabilistic meaning is lost.

Friday May 6

<u>10.30-11.15</u>
High-temperature spin transport in the XXZ spin chain: diffusion, KPZ dynamics and sub-diffusion.
Jacopo De Nardis *(CY Cergy Paris Université)*

I will review the recent (surprising) findings for the dynamics of spin transport in the XXZ spin chains. While the Heisenberg chain (and any other chain with non-abelian global symmetry) shows super-diffusive dynamics falling into the celebrated KPZ universality class, the anisotropic case shows a full range of behaviors from ballistic to diffusive and sub-diffusive, the latter emerging whenever the underlying integrability is broken.

<u>11.45-12.30</u>
Nested closed paths in two-dimensional percolation
Bernard Nienhuis *(Leiden University)*

In two-dimensional percolation, three families of exponents have been studied extensively. The so-called *water-melon* exponents govern the probability that selected small patches share a given number of cluster boundaries. The *nested-loop* exponents govern the correlation functions in which the cluster boundaries surrounding one (or more) positions are given a weight. The *monochromatic path* exponents govern the probability that two distant small patches are connected by a given number of non-overlapping paths over open elements only. Here I will discuss a natural extension of these three: the *nested-path* exponents for the correlation function in which the closed non-overlapping paths over open elements surrounding a selected position are given a weight.

<u>14.30-15.15</u>
Quantum Exclusion Processes
Fabian Essler *(University of Oxford)*

I briefly review how to derive quantum master equations for many-particle quantum systems coupled to Markovian quantum noise. I then show how these quantum master equations are related to integrable models. A particular case gives rise to a quantisation of the well-known asymmetric simple exclusion process. I show that the corresponding master equation “fragments” into sectors, and that each sector is described by a different Yang-Baxter integrable model. Finally I show that this operator-space fragmentation into integrable sectors can be used to obtain exact results for dynamical properties in a variant of the quantum ASEP that is amenable to analysis by (non-standard) free-fermion techniques.

Week 4
April 26-29, 2022

Monday May 9

10.30-11.15
Diagonal finite volume matrix elements in the sinh-Gordon model
Fedor Smirnov *(LPTHE, Sorbonne Universite, Paris))*

Using the fermionic basis we conjecture exact expressions for diagonal finite volume matrix elements of exponential operators and their descendants in the sinh-Gordon theory. This conjecture requires verifications. We shall mostly concentrate on the ultra-violet checks against the Liouville three-point functions. Also we shall briefly explain that our expressions sum up the LeClair-Mussardo type infinite series generalized by Pozsgay for excited state expectation values which are appropriate for the long distances.

11.45-12.30
Integrable dynamics on polygons and the dimer integrable system
Sanjay Ramassamy *(CNRS, Saclay)*

On the one hand, several discrete-time dynamical systems on spaces of polygons have been shown in the last twenty years to be integrable. On the other hand, Goncharov and Kenyon introduced ten years ago an integrable system associated with the dimer model on bipartite graphs on the torus. Building upon the notion of triple crossing diagram maps (introduced in recent works of Affolter, Glick, Pylyavskyy and myself), I will describe a framework which encompasses both the geometric dynamics on polygons and the dimer integrable system. This framework makes it possible in particular to identify the conserved quantities of both systems. I will illustrate this paradigm on the example of the pentagram map.
This talk is based on joint work with Niklas Affolter (TU Berlin and École normale supérieure) and Terrence George (University of Michigan).

14.30-15.15
Ballistic macroscopic fluctuation theory for integrable systems
Takato Yoshimura *(All Souls College, Oxford)*

The macroscopic fluctuation theory (MFT) has served as a universal tool for describing the large scale physics pertaining to (rare) fluctuations. While the MFT has been applied only to many-body systems that are purely diffusive so far, the underlying idea can be extended to other transport types too. In this talk I will first explain the basics of the ballistic version of the MFT, which we term the BMFT, and apply it to study the current fluctuations as well as Euler-scale dynamical correlation functions in integrable systems. It turns out that integrability of the system greatly facilitates the application of the BMFT, allowing us to compute the objects of interest exactly. In particular, I will present how the BMFT enables us to evaluate the full Euler-scale dynamical correlation functions in integrable systems explicitly, including their long-range contributions, which had been overlooked so far. Time permitting, I shall also discuss how the structure of the BMFT naturally implies the Gallavotti-Cohen fluctuation theorem. The general idea of the ballistic macroscopic fluctuation theory will also be introduced in the talk by Benjamin Doyon.
The present talk is based on collaboration with B. Doyon, G. Perfetto, and T. Sasamoto.

15.45-16.30
Exact results from the string worldsheet: new lessons from AdS3 superstrings
Alessandro Sfondrini *(Università di Padova)*

It is an outstanding challenge to study superstrings on generic backgrounds: those involving only NSNS fields can be described in terms of a local worldsheet CFT, but RR field strengths make this impossible. This issue arises in all but a handful of backgrounds, preventing the computation of even the simplest observables such as the string spectrum. However, a surprisingly large number of stringy setups can be quantised and solved starting from the Green-Schwarz description of the string and constructing an associated non-relativistic integrable QFT. I will illustrate this in the case of strings in AdS3 backgrounds, which are crucially important in string theory, black-hole physics and holography, and display a unique interplay of RR and NSNS strengths. After a review of the setup, I will describe the related IQFT, which features a mixture of gapped and gapless non-relativistic excitations, illustrate the dynamics of their scattering, their rather unique analytic structure and thermodynamic Bethe ansatz. Based on recent work with Sergey Frolov.

Tuesday May 10

10.30-11.15

Ensembles of Random Matrices with Complex Potentials: Phase Diagrams and Topological Expansion

Pavel Bleher *(Indiana University-Purdue University, Indianapolis)*

We will discuss recent rigorous results on ensembles of random matrices with complex potentials, including topological expansion and phase diagrams in these ensembles in the complex phase space of parameters. This is an ongoing project with Marco Bertola, Alfredo Dea˜no, Maxim Yattselev, Ahmad Barhoumi, Ken McLaughlin, and Roozbeh Gharakhloo.

11.45-12.30

Integrable deterministic dynamics with nonabelian symmetries: From KPZ mean transport of Noether charges to their anomalous fluctuations.

Tomaz Prosen *(University of Ljubljana)*

Recently, accurate numerical and experimental observations suggested super-diffusive high-temperature equilibrium spin transport in Heisenberg XXX spin 1/2 chains (unitary rational six-vertex model) with KPZ 2-point functions and dynamical exponent z=3/2. The phenomenon has been later conjectured to extend to arbitrary classical and quantum integrable systems with non-abelian global symmetries. The dynamical exponent z=3/2 has been explained via a self-consistent argument within the generalised hydrodynamics (GHD) picture of thermodynamic Bethe ansatz. I will discuss a simple integrable model on a discrete space-time lattice allowing for an arbitrary compact Lie group symmetry, exhibiting identical 'mean KPZ physics'. In the second part of my talk, I will discuss finite time fluctuations of the integrated spin current in $SU(2)$ version of the model, the so-called lattice Landau Liftshitz magnet, and show divergence of scaled cumulants of time integrated current and evading the central limit theorem: a phenomenon that has not been observed in widely studied integrable stochastic systems. At the end, I will present a simple minimal interacting model - a reversible charged cellular automaton - with exactly solvable full counting statistics, analytically disclosing the divergence of scaled cumulants and critical behaviour.

14.30-15.15

A new approach to solvable KPZ models via a correspondence to free fermions at positive temperature.

Matteo Mucciconi *(University of Warwick, Coventry)*

In this talk I will present a new way of solving the KPZ equation, producing a direct correspondence between the height function and the edge of a system of free fermions at positive temperature. Such correspondence is naturally stated for solvable discretizations of the KPZ equation and it consists in a bijection that generalizes the celebrated Robinson-Schensted-Knuth algorithm. Considering symmetries of our construction we also find Fredholm Pfaffian formulas for the KPZ equation in half space. In this case asymptotic analysis allow to prove a depinning transition.

17.30-18.30, Cloister
Wine & Cheese

Wednesday May 11

10.30-11.15
Integrable complexity: Hofstadter Butterfly and Bethe Ansatz
Paul Wiegmann *(University of Chicago)*

Hofstadter problem (also known as Harper equation or Almost Mathieu equation) is merely a spectrum of a quantum particle on a 2D lattice in a quantized magnetic field. This simple problem has numerous applications in dynamical systems, localization, quasi-crystals, etc. Despite a seeming simplicity, the problem is notoriously complicated. If the magnetic flux per lattice cell is an irrational number, the spectrum is a singular continuous - a Cantor set of measure zero with no isolated points. This problem became a synonym of unmanageable complexity. At the same time the problem possesses an inspiring and beautiful hierarchical structure revealed by D. Hofstadter in the form of a butterfly. Incidentally or not, the Hofstadter problem can be formulated in terms of the representation theory of the (quantum) deformation of $SL_q(2)$ and for this reason is Bethe Ansatz-integrable. The Bethe-Ansatz equations uncover the hierarchical structure of the topology of the spectrum. In the talk I review these (not-so-recent) developments (made together with with A. Zabrodin and A. Abanov) and formulate the problem of quantitative description of the spectrum in terms of yet to be determined critical exponents.

11.45-12.30
The Heisenberg spin chain and its long-range friends
Jules Lamers *(CEA, Saclay)*

I will discuss the Heisenberg spin chain in the context of its long-range deformations — the inhomogeneous Heisenberg spin chain, the Inozemtsev spin chain and its Haldane–Shastry limit. I will outline some recent progress on these long-range spin chains and highlight a few insights on quantum integrable spin chains offered by the long-range perspective.

14.30-15.15
Exactly Solvable Non-Equilibrium Steady States
Cristian Giardinà *(Università di Modena e Reggio Emilia)*

We discuss two models of boundary-driven systems that can be full be fully solved, i.e. correlation functions in the non-equilibrium steady state can be written in closed-form. The first is an interacting particle system that we called the 'harmonic model' since it involves harmonic numbers. The second is an energy-redistribution model, similar in spirit to the KMP model, with the additional property of being integrable. Both models are of zero-range type; nevertheless, they have a non-trivial stationary state with long-range correlations.
The two models are associated with the open integrable XXX chain with sl(2) pseudo-spin. They emerge from two representations of the sl(2) Lie algebra and they are related by a duality relation arising from the intertwining between these two representations. The solution of both models is obtained by a combination of (i) duality and (ii) quantum inverse scattering method. The 'harmonic model', introduced in J.Stat. Phys. 180, 135-171 (2020) was solved in [arXiv:2107.01720]; the second model is a work in progress with R. Frassek and C. Franceschini.

Thursday May 12

10.30-11.15
Geometrical web models
Jesper Jacobsen *(ENS, Paris)*

We introduce a family of geometrical lattice models generalising the well-known loop model on the hexagonal lattice. These models have a $U_q(sl_n)$ quantum group symmetry, the loop model being the $n = 2$ case. The general models give rise to branching webs and describe, at a special point, the interfaces in Z_n symmetric spin models. We mainly discuss the $n = 3$ case of bipartite cubic webs, which is based on the Kuperberg A_2 spider. We exhibit a local vertex-model reformulation, analogous to the well-known correspondence between the loop model and the nineteen-vertex model. The local formulation allows us in particular to study the model by means of transfer matrices and conformal field theory. We find that it has a rich phase diagram, including a dense and a dilute phase that generalise those known for the loop model.
Based on joint work with Augustin Lafay and Azat Gainutdinov (arXiv:2101.00282 and 2107.10106).

11.45-12.30
Counting statistics of interacting fermions and the Gaussian free field
Pierre Le Doussal *(ENS, Paris)*

14.30-15.15
The dimer model on minimal graphs, the elliptic case and beyond
Béatrice de Tilière *(Université Paris-Dauphine)*

Based on works with C. Boutillier and D. Cimasoni.

Friday May 13

10.30-11.15
Stationary half-space last passage percolation
Patrik Ferrari *(University of Bonn)*

We describe how the limiting process for the half-space stationary last passage percolation is derived. The limiting process is a two-parameter family of distributions: one parameter for the strength of the diagonal bounding the half-space (strength of the source at the origin in the equivalent TASEP language) and the other for the distance of the point of observation from the origin. This is a joint work with Dan Betea and Alessandra Occelli

11.45-12.30
Conformal data of integrable Hamiltonians from non-linear integral equations
Andreas Klümper *(Bergische Universität Wuppertal)*

This talk is about the spectra of Hamiltonians of integrable quantum spin chains and of transfer matrices of related exactly solvable vertex models. I will first review established techniques to set up functional equations for the eigenvalues of transfer matrices, e.g. of the type of T- and Y-systems and their exact truncations. These functional equations are transformed into non-linear integral equations (NLIEs) from which the conformal data can be extracted by use of the dilog-trick.
As a new application of the methods we treat the spin-1/2 Heisenberg chain with off-diagonal boundary fields. It is known that the Heisenberg chain with arbitrary boundary fields is still integrable, but so far defied an explicit solution as even the (minimal) $U(1)$ symmetry is broken. As a consequence the traditional Bethe ansatz fails. In the literature different methods have been reported for the derivation of so-called inhomogeneous $T - Q$ relations. The corresponding Bethe ansatz equations are difficult to solve.
Here we show how the problem can be transformed into a set of NLIEs. Instead of two NLIEs as in the case of the periodically closed chain, we find a set of three NLIEs from which the eigenvalues of the Hamiltonian can be obtained. In contrast to the periodic case, here the integral kernels are long-ranged. We discovered how to treat these integral equations in the thermodynamic limit. These recent results suggest that the

finite size data depend explicitly on the mutual orientation of the boundary fields.

<u>**14.30-15.15**</u>

Effective form factors for free fermionic models at finite temperature

Oleksander Gamayun *(ITF Warsaw University)*

The behavior of dynamical correlation functions in one-dimensional quantum systems at zero temperature is now very well understood in terms of linear and non-linear Luttinger models. The "microscopic" justification of these models consists in exactly accounting for the soft-mode excitations around the vacuum state and at most few high-energy excitations. At finite temperature, or more generically for finite entropy states, this direct approach is not strictly applicable due to the different structure of soft excitations. To address these issues we study the asymptotic behavior of correlation functions in one-dimensional free fermion models. On the one hand, we obtain exact answers in terms of Fredholm determinants. On the other hand, based on "microscopic" resummations, we develop a phenomenological approach that introduces the effective form factors and reduces the problem to the zero temperature case. The information about the initial state is transferred into the scattering phase of the effective fermions. I will demonstrate how this works for correlation functions in the XY model, mobile impurity, and the sine-kernel Fredholm determinants.

Week 5
April 26-29, 2022

Monday May 16

10.30-11.15

Characterizing invariant measures using symmetries of probabilistic integrable models

Guillaume Barraquand *(ENS, Paris)*

I will explain how symmetries with respect to permuting inhomogeneity (or spectral) parameters of stochastic integrable models allows to determine invariant measures of the corresponding probabilistic systems. We will in particular apply this method to the case of the log-gamma polymer and the KPZ equation with a wall, for which invariant measures were characterized recently.

11.45-12.30

Generalised Gibbs ensembles for the Calogero fluid.

Herbert Spohn *(Technical University Munich)*

Over the past years there has been a lot of interest in investigating generalised Gibbs ensembles for integrable many-body systems. Besides some background, in my presentation I will discuss the classical fluid with the interaction potential $1/sinh^2$, whose integrability was established by F. Calogero in the mid seventies. In particular, I will explain the novel method based on the canonical transformation to scattering coordinates.

14.30-15.15

Triangular Ice: Combinatorics and Limit Shapes

Philippe Di Francesco *(University of Illinois, Urbana)*

We consider the two-dimensional 20-Vertex 'ice' model on the triangular lattice with special 'domain wall' type boundary conditions on various domains. Like the square lattice (6-Vertex) version, this model has a fascinating combinatorial content. We review the known exact results and conjectures relating its configurations to domino tilings of certain plane domains.
Finally, we study the limit shape of large configurations, delimited by a piecewise analytic arctic curve.
(Based on works with E. Guitter, Institut de Physique Théorique, Université Paris Saclay, France and with B. Debin, Université catholique de Louvain, Belgium).

Tuesday May 17

10.30-11.15

Skew Howe duality and limit shapes of Young diagrams

Olga Postnova *()*

Consider the exterior algebra of the tensor product of two complex vector spaces of dimension n and k. This space could be regarded as a bimodule for the action of dual pairs of Lie groups. For example, for GL(n) x GL(k) - case this exterior algebra decomposes into direct sum of bimodules parametrised by conjugate partitions inside the n x k rectangle. This is the skew Howe duality. On the level of characters the skew Howe duality yields the dual Cauchy identity for the Schur functions which could be viewed via the free fermions.
We interpret the skew Howe duality as a natural consequence of lattice paths on lozenge tilings of certain partial hexagonal domains. This combinatorial approach also allows to obtain product formulas for the q-deformations of multiplicities or different dual pairs of Lie groups . We consider the corresponding probability measures on Young diagrams and prove the uniform convergence to the limit shape of Young diagrams in the limit when n and k go to infinity.

11.45-12.30

Dynamics of symmetry-resolved entanglement measures in free fermionic systems

Riccarda Bonsignori *((Rudjer Boskovic Institute, Zagreb)*

The presence of a global internal symmetry in a quantum many-body system is reflected in the fact that the entanglement between its subparts is endowed with an internal structure, namely it can be decomposed as sum of contributions associated to each symmetry sector. The study of the symmetry resolution of entanglement measures provides a formidable tool to probe the out-of-equilibrium dynamics of quantum systems.
In this talk I will present the results of a series of works, done in collaboration with Gilles Parez and under the supervision of Pasquale Calabrese, devoted to the study of the dynamics of three different symmetry-resolved entanglement measures after a global quantum quench, namely the symmetry-resolved entanglement entropy, mutual information and negativity. In the context of free fermions, we are able to provide analytical results for the relevant time-dependent quantities. Moreover, we argue that our results can be understood in the framework of the quasiparticle picture for the entanglement dynamics, and provide a conjecture that we expect to be valid for generic integrable models.

14.30-15.15

Hydrodynamics of a box-ball system

Vincent Pasquier *(CEA-CNRS, Gif sur Yvette)*

I present a model of cellular automaton which describes the motion of balls arranged on a line. It results from the discretization of soliton equa- tions on the one hand and the crystal limit of the six vertex model on the other hand. With the help of a transformation due to Kerov, Kirillov and Reshetikhin, we bring the movement of the balls back to that of the solitons. I will describe the thermodynamics of randomly placed balls and I will study the evolution of a partition of the system into two subsystems of different ball densities. The agreement between the theory based on the prediction of the speed of solitons (GHD) and numerical simulations is astounding. Work in collaboratin with Grégoire Misguich and Atsuo Kuniba.

17.30-18.30, Cloister

Wine & Cheese

Wednesday May 18

10.30-11.15

Integrable quantum cluster algebras associated with affine root systems

Rinat Kedem *(University of Illinois, Urbana)*

I will describe how the collection of discrete evolutions known as Q-systems, associated with each affine root system, can be shown to be integrable, after quantization. The quantization is canonical in almost all cases, by the identification of the evolution equations with cluster algebra mutations. Once quantized, their solutions are seen to be limits of Macdonald-Koornwinder q-difference operators, and the conserved quantities of the discrete time evolutions are generalized q-Toda Hamiltonians.
Joint work with P. Di Francesco.

11.45-12.30

Exact solution for the macroscopic fluctuation theory for the symmetric simple exclusion process

Tomohiro Sasamoto *(Tokyo Institute of Technology, Tokyo)*

It has been known for a long time that large deviations of symmetric simple exclusion process(SEP) are described by MFT (macroscopic fluctuation theory) equations. They are coupled nonlinear partial differential equations and have resisted exact analysis except for stationary situation. In this talk we present the first

exact solution to the MFT equations for SEP in time dependent regime. The key in our arguments is the introduction of a novel generalization of the Cole-Hopf transformation, which maps the MFT equations to the AKNS equations. They are a well-known classical integrable system and can be solved exactly using standard ideas of inverse scattering method. For the step initial condition with two densities, we obtain exact and compact formulas for the optimal density profile and the response field which produce a required fluctuation, both at initial and final times. The talk is based on a joint work with Kirone Mallick and Hiroki Moriya [1].
Reference [1] Kirone Mallick, Hiroki Moriya, Tomohiro Sasamoto, Exact solution of the macroscopic fluctuation theory for the symmetric exclusion process, arXiv: 2202.05213

14.30-15.15
A case study: the tangent method applied to two-periodic Aztec diamonds
Philippe Ruelle *(Université catholique de Louvain, Louvain-la-Neuve)*

We use the octahedron recurrence in order to compute boundary one-refined and two-refined partition functions for two-periodic Aztec diamonds. The simple and two-refined tangent methods allow to derive the arctic curve, separating the solid and liquid phases. The curve satisfies the known algebraic equation of degree 8, of which either tangent method gives an explicit parametrization.

15.45-16.10
Hermitian Schur measures: from quantum mechanics to new asymptotic statistics for random partitions
Harriet Walsh *(ENS, Lyon)*

We consider a class of probability measures on integer partitions which arise not from combinatorics but from certain natural quantum mechanical models. We show that these measures can be tuned to have "multicritical" asymptotic edge fluctuations outside of the GUE universality class, governed by natural higher order generalisations of the Tracy—Widom distribution considered by Le Doussel, Majumdar and Schehr, and discuss how this relates to integrable differential equations and to a family of unitary matrix models, which are multicritical in their own way. When tuned in a different way, these measures exhibit decorrelation, and new asymptotic behaviour in the bulk. There is a possible combinatorial interpretation for these models involving discretised surfaces, or maps. Based on joint work with Dan Betea and Jérémie Bouttier, and on work in progress.

Thursday May 19

10.30-11.15
Dynamical universality classes: Recent results and open questions
Gunter Schuetz *(Forschungszentrum Juelich, Juelich)*

Universality asserts that, especially near phase transitions, the macroscopic properties of a physical system do not depend on its details such as the precise form of microscopic interactions. We show that the two best-known examples of dynamical universality classes, the diffusive and KPZ-classes, are only part of an infinite discrete family. The members of this family can be identified by their dynamical exponent which surprisingly can be expressed by a Kepler ratio of Fibonacci numbers. This strongly indicates the existence of a simpler but still unknown underlying mechanism that determines the different classes.

11.45-12.30
Operator spreading in quantum hard-core gases
Marko Medenjak *(University of Geneva)*

In the talk we will first discuss different features of the operator dynamics that are thought to distinguish between integrable and non-integrable systems, which will motivate our subsequent analysis. Following the introduction we will present a matrix product ansatz based approach for solving the dynamics of local quantum circuits, which will allow us to discern aforementioned salient features in integrable hardcore gases analytically.

14.30-15.15
Arctic Boundaries in Ice Models
Amol Aggarwal *(Columbia University, New York / IAS, Princeton)*

Certain two-dimensional models in statistical mechanics are widely known or believed to exhibit arctic boundaries, which are sharp transitions from ordered (frozen) to disordered (temperate) phases. In this talk we will explain a general heuristic devised by Colomo-Sportiello in 2016, known as the geometric tangent method, for locating these arctic boundaries in such models. We will also outline a way of making this tangent method rigorous (for concreteness, in the example of the domain-wall six-vertex model at ice point), which is based on a probabilistic analysis of non-crossing directed path ensembles.

Friday May 20

10.30-11.15
Non-intersecting Brownian bridges in the flat-to-flat geometry
Grégory Schehr *(CNRS/Sorbonne Université, Paris)*

In this talk, I will discuss N non-intersecting Brownian bridges propagating from an initial configuration $\{a_1 < a_2 < \ldots < a_N\}$ at time $t = 0$ to a final configuration $\{b_1 < b_2 < \ldots < b_N\}$. I will first show that this problem can be mapped to a non-intersecting Dyson's Brownian bridges with Dyson index $\beta = 2$. For the latter I will derive an exact effective Langevin equation that allows to generate very efficiently the non-intersecting bridge configurations. In particular, for the flat-to-flat configuration in the large N limit, where $a_i = b_i = (i-1)/N$, for $i = 1, \cdots, N$, I will use this effective Langevin equation to derive an exact Burgers' equation (in the inviscid limit) for the Green's function and solve this Burgers' equation for arbitrary time $0 \leq t \leq t_f$. Finally, I will discuss connections to some well known problems, such as the Chern-Simons model, the related Stieltjes-Wigert orthogonal polynomials and the Borodin-Muttalib ensemble of determinantal point processes.

11.45-12.30
Lozenge tilings via the dynamical loop equation
Vadim Gorin *(University of Wisconsin, Madison)*

I will present a general framework for studying fluctuations of high-dimensional discrete Markov chains with random matrix type interactions between particles. The approach is based on a novel holomorphic observable for the transition probabilities. As an application we will discuss inhomogeneous (q,kappa)-distributions on lozenge tilings and uncover their rich asymptotic behavior.

Week 6
May 23-27, 2022

Monday May 23

10.30-11.15
Random Young tableaux and the tangent plane method
Istvan Prause *(University of Eastern Finland)*

In the first part of the talk I'll describe joint work with Rick Kenyon which can be dubbed the "tangent plane method". It is based on the variational approach for limit shapes and applies across a variety of models also beyond free fermions such as the 5-vertex model. In the second part of the talk, I'll consider random Young tableaux and use the tangent plane method to solve the limit shape problem for tableaux with arbitrary boundary profile. That is, we are interested in the typical shape of a large random tableaux on a given Young diagram shape.

11.45-12.30
Random eigenvalues at high temperature
Cesar Cuenca *(Harvard University)*

The eigenvalue distributions of random matrices often admit a generalization involving the 'inverse temperature' parameter $\beta > 0$. The talk focuses on two examples: the Hermite ensemble, and the spectra of sums of Hermitian random matrices. For both examples, we discuss a Law of Large Numbers in the high temperature regime: the number of eigenvalues tends to infinity, while the inverse temperature β tends to zero. This talk is based on joint work with Florent Benaych-Georges and Vadim Gorin.

14.30-15.15
Crosscap States in Integrable Theories
Joao Caetano *(CERN, Geneva)*

In this talk, I will describe crosscap states in integrable field theories and spin chains in 1+1 dimensions. I will derive an exact formula for overlaps between the crosscap state and any excited state in integrable field theories with diagonal scattering. I will then compute the crosscap entropy, i.e. the overlap for the ground state, in some examples. In the examples analyzed, the result turns out to decrease monotonically along the renormalization group flow except in cases where the discrete symmetry is spontaneously broken in the infrared. I will discuss crosscap states in integrable spin chains, and obtain determinant expressions for the overlaps with energy eigenstates. I will comment on the realization of crosscap states in holography.

Tuesday May 24

10.30-11.15
Introduction to instanton calculus
Nikita Nekrasov *(Simons Center for Geometry and Physics, Stony Brook)*

I will review the problems in quantum gauge theory in four dimensions which, on the other hand, produce Donaldson invariants of smooth four-manifolds, and, on the other hand, produce the probability measures (and their complex extensions) on the Young graph and its generalizations.

11.45-12.30
Shuffle algebras and integrability
Alexandr Garbali *(University of Melbourne)*

I will discuss shuffle algebras and their connections with integrable models. The main example will be the trigonometric shuffle algebra. This algebra is related to the Macdonald operators and functions and to the quantum toroidal algebra.

14.30-15.15
Emptiness formation in polytropic quantum liquids
Dimitri Gangardt *(University of Birmingham)*

Emptiness formation probability (EFP) is probably the most iconic and widely studied example of large deviations in statistical mechanics of many-body systems. It allows to characterise effects of interactions and serves as the litmus test for validity of approximate non-perturbative tecniques such as instanton calculus. The leading asymptotics of EFP are reliably captured by imaginary time hydrodynamics in which interactions enter via the thermodynamic equation of state.
In my talk I will present recent results of calculating emptiness formation probability in interacting quantum liquids with the polytropic equation of state. i.e. power-law pressure-density dependence characterised by polytropic index. The large deviation function is given by an optimal fluctuation (instanton) of hydrodynamic action which is obtained by solving hydrodynamic equations of motion in imaginary time. While solution can be obtained only for special values of the polytropic index, the final result can be analytically continued to arbitrary power-law equation of state.
The optimal instanton configuration has interesting universal spacetime features and I will discuss them in the context of limit shape phenomena.
[1] H-C Yeh, DMG and A Kamenev, JPhys A 55 064002, 2022

17.30-18.30, Cloister
Wine & Cheese

Wednesday May 25

10.30-11.15
XXX chain: spinons, bound states and form factors
Nikolai Kitanine *(Université de Dijon)*

The goal of this talk is to give an overview of the role of the holes (spinons) and complex Bethe roots (corresponding to the bound states) for the computation of form factors for the isotropic XXX spin chain. The Heisenberg spin chain is one of the best studied examples of quantum integrable systems, however the crucial problem of analytic description of equilibrium and out-of equilibrium dynamics remains mostly unsolved for this fundamental model. In particular, there exist very few analytic results for time-dependent correlation functions even for the equilibrium case. The main obstacle is the lack of manageable analytic representations for the relevant form factors and overlaps. I'll discuss the new method we developed to compute explicitly the form factors for low-energy excited states of the XXX chain. I will show how to use this technique for simplest excitations and how to take into account the complex roots of the Bethe equations.

11.45-12.30
$N = 4$ **Super Yang-Mills as a 'generating theory' for integrable systems**
Shota Komatsu *(CERN, Geneva)*

I will give a (hopefully-)pedagogical overview of the application of integrability to N=4 supersymmetric Yang-Mills theory emphasizing its role as a "generating theory" for various integrable structures. I will showcase connections to various concepts in integrability without delving into technical details. The topics to be covered are 1. the connection between large N gauge theory and spin chain, 2. the branch-point twist operator and correlation functions, 3. integrable boundary states and integrable quenches and D-branes.

14.30-15.15
The Floquet Baxterisation
Yuan Miao *(GGI, Florence)*

In this talk I will focus on the Floquet dynamics of quantum many-body systems and demonstrate a universal way of constructing quantum Floquet circuits that are integrable, i.e. commuting with the transfer matrices of a class of inhomogeneous integrable vertex models. I will explain the details using the example of the renowned six-vertex model (quantum spin-1/2 XXZ chain), where a spontaneous breaking of an anti-unitary symmetry is present in the easy-plane regime.

Thursday May 26

10.30-11.15
Interlacing particles, vertex models and tilings
Sylvie Cortéel *(CNRS, Université de Paris, & UC Berkeley)*

This talk will be an introductory talk on the combinatorics of interlacing partitions and five vertex models. I will explain how you can use these tools to enumerate lozenge and domino tilings of certain regions, including the Aztec diamond.

11.45-12.30
Semilocal Gibbs ensembles and nonequilibrium symmetry-protected topological order
Lenart Zadnik *(Université Paris Saclay)*

I will revisit the problem of nonequilibrium time evolution of 1D quantum systems. In the last decade we have come to understand that the extended quantum system acts as an effective bath for its local constituents, thus causing their relaxation to standard ensembles of statistical mechanics: stationary expectation values of local observables can then be described by the (generalised) Gibbs ensemble. I will show that global symmetries (e.g. a spin flip symmetry in spin-1/2 chains) can invalidate this picture. This occurs in models whose Hamiltonians possess conservation laws that are not local but act as such in the symmetry-restricted space where time evolution occurs. Such conservation laws constitute 'semilocal (generalised) Gibbs ensembles' that can be interpreted as a hallmark of a nonequilibrium symmetry-protected topological order. Among their exceptional features is the logarithmic scaling of the excess entropy of a spin block, triggered by a local perturbation in the initial state, as well as slow melting of the order induced either by a (symmetry-breaking) rotation of the initial state, or by an increase of the temperature.

14.30-15.15
Random partition models from gauge theory
Nikita Nekrasov *(Simons Center for Geometry and Physics, Stony Brook)*

I will review the techniques of deriving differential equations on expectation values of certain observables in statistical models, where random variables are collections of Young diagrams. Two celebrated equations of mathematical physics, Knizhnik-Zamolodchikov-Bernard, and Belavin-Polyakov-Zamolodchikov equations, as well as their numerous reductions and limits, e.g. Painleve VI, show up.

Friday May 27

10.30-11.15
A Pfaffian - determinantal duality in random matrices and last passage percolation
Nikos Zygouras *(University of Warwick)*

It is known that random matrix distributions such as those that describe the largest eignevalue of the Gaussian Orthogonal and Symplectic ensembles (GOE, GSE) admit two types of representations: one in terms of a Fredholm Pfaffian and one in terms of a Fredholm determinant. The equality of the two sets of expressions has so far been established via involved computations of linear algebraic nature. We provide a structural explanation of this duality via links (old and new) between the model of last passage percolation and the irreducible characters of classical groups, in particular the general linear, symplectic and orthogonal groups, and by studying, combinatorially, how their representations decompose when restricted to certain subgroups. Based on joint work with Elia Bisi.

11.45-12.10

Measurement catastrophes in quantum jammed states

Saverio Bocini *(Université Paris-Saclay)*

Local measurements can sometimes lead to unexpected macroscopic behaviours. Such "measure- ment catastrophes" in integrable models go beyond generalized hydrodynamics, that is arguably the most effective large-scale description of dynamics in integrable models in the presence of inho- mogeneities. A noteworthy occurrence of this phenomenon is found in systems exhibiting quan- tum jamming. I will provide a simple and solvable example by considering a particular class of the jammed states of the large-anisotropy limit of the Heisenberg magnet. That will allow me to present the microscopic dynamics behind the emergence of ballistic profiles of local observables following a local measurement in that particular model.

14.30-15.15

First-passage resetting, and some other first-passage problems

Julien Randon-Furling *(Université Pantheon Sorbonne, Paris)*

This talk will present results obtained recently with B. de Bruyne and S. Redner on first-passage resetting, where the resetting of a random walk to a fixed position is triggered by a first-passage event of the walk itself. In an infinite domain, we calculate the resulting spatial probability distribution of the particle analytically, and also obtain this distribution by a path decomposition. In a finite interval, we define an optimization problem that is controlled by first-passage resetting: a cost is incurred whenever the particle is reset and a reward is obtained while the particle stays near the reset-trigger point. This scenario is motivated by reliability theory. We derive the condition to optimize the net gain in this system, namely, the reward minus the cost. Time permitting, I will also talk about two other first-passage problems: a) an extension of first-passage resetting into a minimalist dynamical model of wealth evolution and wealth sharing among N agents as a platform to compare the relative merits of altruism and individualism; b) the distribution of the first-passage time of the Brownian maximum below the diagonal, that is: considering the running maximum S_t of a standard Brownian motion B_t, we compute the distribution of the first time T when it becomes less than the running time t. The motivation for our work comes from a mathematical model for animal foraging. We also compute the joint distribution of T and B_T.

- B de Bruyne, J R-F, S Redner, Phys. Rev. Letters 125 (2020) 050602;
- B de Bruyne, J R-F, S Redner, J. Stat. Mech. 2021(10) 103405;
- J R-F, P Salminen, P Vallois, Stoch. Proc. Their Appl. (2022) https://doi.org/10.1016/j.spa.2021.12.015.

Week 7
May 30-June 3, 2022

Monday May 30

11.45-12.30
k-tilings of the Aztec diamond
David Keating *(University of Wisconsin, Madison)*

We will study k-tilings (k-tuples of domino tilings) of the Aztec diamond. We assign a weight to each k-tiling, depending on the number of 'interactions' between the dominos of the different tilings. We will compute the generating polynomials of the k-tilings by relating them to the LLT polynomials of Lascoux, Leclerc, and Thibon. The required properties of the LLT polynomials will be proved by showing that they can be seen as the partition function of an integrable colored vertex model. We will then prove some combinatorial results about k-tilings in certain limits of the interaction strength.

14.30-15.15
Rigorous results on a frustration-free quantum fully packed loop model
Zhao Zhang *(SISSA, Trieste)*

Exact analytical results of classical statistical mechanics models can be converted to that of the ground state of a quantum model by designing a frustration-free Hamiltonian in terms of projection operators, the ground state of which is given by the uniform superposition of classical configurations, as has been done with quantum dimer model. Such construction relies on the ergodicity of the Hamiltonian, which hasn't been well understood in the past at least for quantum dimer model. In light of relevance to recent developments of Hilbert space fragmentation and quantum many-body scars, we systematically study the ergodicity within Krylov subspaces and exact eigenstates of a fully packed loop model on square lattice with domain wall boundary condition. Using exact enumeration of six-vertex configurations with a free boundary, we compute numerically the entanglement entropy. An upper bound on the spectral gap in the constrained Hilbert space is proven by constructing a trial wave function with a twist near the ideal state under domain wall boundary condition.

17.30-18.30, Cloister
Wine & Cheese

Tuesday May 31

11.45-12.30
The $\hbar \to 0$ limit of the Entanglement Entropy
Jacopo Viti *(UFRN, Natal)*

It may be naively assumed that certain entanglement measures, like the entanglement entropy of a pure state, approach zero in the classical limit. In this talk, we show that the limit (when properly defined) is both well-behaved and nonzero: for a bipartite system of N particles, it coincides with the Shannon entropy of N bits. These results have been demonstrated previously in the context of quantum field theory, but here we derive them in an elementary way and also provide a physical interpretation. Work in collaboration with G. Mussardo, to be published in PRA as Editor suggestion, see https://arxiv.org/abs/2112.06840.

14.30-15.15
Origin and applications of the correspondence between classical and quantum integrable theories
Davide Fioravanti *(INFN, Bologna)*

We show how functional relations, which can be considered as a definition of a quantum integrable theory, entail an integral equation that can be extended upon introducing dynamical variables to a Marchenko-like equation. Then, we naturally derive from the latter a classical Lax pair problem. We exemplify our method by focusing on the massive version of the ODE/IM (Ordinary Differential Equations/Integrable Models) correspondence involving the sinh-Gordon model, first emerged in the gauge theories and scattering amplitudes/Wilson loops AdS3 context with many moduli/masses, but in a way which reveals its generality. In fact, we give some hints, in the end, to its application to spin chains.

15.45-16.10
Quantum Spectral Curve: from anomalous dimensions to structure constants
Arthur Klemenchuk Sueiro *(ENS, Paris)*

In this seminar I will introduce the Quantum Spectral Curve (QSC), an integrability based construction that exactly solves the spectrum of anomalous dimensions in planar N=4 Super Yang-Mills theory. Then I will present a new conjecture in which QSC is used to compute structure constants by taking into account finite size corrections to the so-called Hexagon formalism. Lastly, the asymptotic and strong coupling limits of QSC will be discussed and used to check our conjecture. Based on work in progress with Benjamin Basso and Alessandro Georgoudis.

Wednesday June 1

11.45-12.30
Entropy geometry in higher vertex models
Kari Eloranta *(University of Helsinki)*

We report on two higher order relatives of the Ice, the 15- and the 19-vertex models, defined on a bounded domain. In the 15-vertex case it is shown that the Domain Wall Boundary Condition can be relaxed yet still the limit shape phenomenon takes place. With increasingly more general boundary conditions one can test what the equilibrium consequences qualitatively are. In the dynamic 19-vertex case limit shapes appear already under DWBC in great variety since this is much more complex rule. By further parametrization interesting weighted cases can be unveiled. One can e.g. erase the six-vertex dynamics yet still the limit shape phenomenon prevails. At the core of the study for the models is their dynamization through the decomposition of the action dynamics to a minimal set of prime components. Dynamic parametrizations can be introduced to emulate vertex weightings. At the equilibrium the random actions have their own intensities and supports which nest within the limit shape.

14.30-15.15
Exact results for quenched randomness at criticality
Gesualdo Delfino *(SISSA, Trieste)*

The study of systems with short range quenched disorder has been a notoriously challenging problem for the theory of critical phenomena and universality, and analytical insight on random critical points has been limited to rare perturbative limits. We will explain how exact access to random criticality has recently been gained in two dimensions through a new way of exploiting conformal invariance.